Nonfactorizable charm – loop effects in exclusive FCNC $B$ – decays

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We revisit the calculation of nonfactorizable contributions induced by charm-quark loops to the amplitudes of exclusive FCNC $B$-decays. Nonfactorizable contributions may be expressed via the three-particle quark-antiquark-gluon distribution amplitude (3DA) of the $B$-meson $\langle 0|\bar{q}(x)G_{\mu\nu}(y)b(0)|B(p)\rangle$. We show that the knowledge of the full generic 3DA with non-aligned arguments, is necessary for a proper summation of large $\left(\frac{\Lambda_{QCD}m_b}{m_c^2}\right)^n$ corrections. In particular, the dependence of 3DA on variable $(x - y)^2$ is crucial.

FCNC $b \rightarrow s$ and $b \rightarrow d$ transitions do not occur at the tree level in SM and proceed via loops, where $t$, $c$ and $u$-quarks contribute. BRs of FCNC decays are small; on the other hand, new particles may show up virtually in the loops. Therefore, FCNC decays are most popular candidates for indirect search of physics BSM.

**Illustration:** $B \rightarrow Kl^+l^-$ decay $0 < \sqrt{s} < (M_B - M_K)$, $s$ - momentum squared of $l^+l^-$ pair.

- Charmonia appear in the kinematical decay region. In the charmonia region, charm contribution dynamically enhanced and dominates.

- Far from the charmonia region, top dominates (black dashed). Still, to study possible NP effects, *Need to gain theoretical control over charm contributions*
Charm generates two different topologies: (a) penguin topology (b) weak annihilation topology

![Diagrams for penguin and weak annihilation topologies]

- Account of hard gluon exchanges lead to the four-quark operators

\[ H_{\text{eff}}^{b \to s\bar{c}c} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \{ C_1(\mu)O_1 + C_2(\mu)O_2 \} \]

with

\[ O_1 = \bar{s}^i \gamma_\mu (1 - \gamma_5) c^i \bar{c}^j \gamma^\mu (1 - \gamma_5) b^j, \quad O_2 = \bar{s}^i \gamma_\mu (1 - \gamma_5) c^i \bar{c}^j \gamma^\mu (1 - \gamma_5) b^j, \]

and the similar terms with \( c \to u \) (\( i, j \) color indices). The SM Wilson coefficients at the scale \( \mu_0 = 5 \) GeV [corresponding to \( C_2(M_W) = -1 \)]: \( C_1(\mu_0) = 0.241, C_2(\mu_0) = -1.1 \).

These operators lead to factorizable contributions to the amplitudes of exclusive FCNC \( B \)-decays.

- Soft gluon exchanges between the charm-quark loop and the \( B \)-meson loop lead to nonfactorizable contributions to the amplitudes.
Nonfactorizable charm contributions are comparable with factorizable contributions

How do we know that? Compare charmonia in $l^+l^-$-annihilation and in FCNC $B$-decays:

The patterns of charmonia in charm contribution to vacuum polarization (left) and in $B \rightarrow KL^+l^-$ (right) are different. The difference is due to nonfactorizable contributions.

In some cases, factorizable charm contribution vanishes and thus only nonfactorizable charm contributes (e.g in $B \rightarrow K^*\gamma$)

We need formalism to calculate nonfactorizable charm effects in QCD.
At $q^2 \ll 4m_c^2$, the charm loop may be calculated in pQCD.

**Factorizable part:** product of $B \rightarrow M_f$ form factor and the charm polarization function.

**Nonfactorizable part** (illustration for scalar “quarks” and scalar “gluon”):

$$A(q, p) = \frac{1}{(2\pi)^8} \int \frac{dk}{m_s^2 - k^2} \int dy e^{-i(k-p')y} \int dx e^{-ikx} dk \Gamma_{cc}(\kappa, q) \langle 0|\bar{s}(y)G(x)b(0)|B_s(p)\rangle.$$

The 3DA depends on 5 variables $xp, yp, x^2, y^2, xy \ (p^2 = M_B^2)$ and may be parametrized as follows:

$$\langle 0|s^+(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda yp} \int d\omega e^{-i\omega xp} \Phi(\omega, \lambda) \left[ 1 + O(\Lambda_{QCD}^2 x^2, \Lambda_{QCD}^2 y^2, \Lambda_{QCD}^2 (x - y)^2) \right].$$

$\Phi(\omega, \lambda)$ is peaked at $\lambda, \omega \sim \Lambda_{QCD}/m_b$.

The contribution of $\Phi(\omega, \lambda)$ to $A(q, p)$ is easy to calculate (see the right plot).

Notice: the gluon is soft as the major part of the $B$-meson momentum is carried by $b$-quark.
Contributions of other terms to the amplitude $A(q, p)$ relative to the $\Phi(\omega, \lambda)$ term:

$$
\Lambda_{QCD}^2 y^2 \rightarrow \frac{\Lambda_{QCD}}{m_b}, \quad \Lambda_{QCD}^2 x^2 \rightarrow \frac{\Lambda_{QCD}^3 m_b}{m_c^4}, \quad \Lambda_{QCD}^2 xy \rightarrow \frac{\Lambda_{QCD} m_b}{m_c^2}.
$$

Nonfactorizable corrections are expressed via

$$
\langle 0| s^\dagger (y) G(x) b(0) | B_s(p) \rangle = \int d\lambda e^{-i\lambda y} \int d\omega e^{-i\omega x} \Phi(\omega, \lambda) \left[ 1 + O(\Lambda_{QCD}^2 x^2, \Lambda_{QCD}^2 y^2, \Lambda_{QCD}^2 (x - y)^2) \right],
$$

The new result is that the knowledge of its functional dependence on $(x - y)^2$ is essential for a proper resummation of large $\Lambda_{QCD} m_b/m_c^2$ corrections.

Previously, it was believed that the 3DA with aligned arguments $x_\mu = uy_\mu$, on the LC $x^2 = 0, y^2 = 0$ and $(x - y)^2 = 0$

is sufficient to calculate nonfactorizable contributions.

One needs the off-LC contributions. A challenge for future calculations
Conclusions and outlook

- A serious open theoretical problem in FCNC B-decays is the contribution of charming loops which “pollute” the differential distributions.

- At $q^2 \ll 4m_c^2$, a consistent description of charming loops requires the knowledge of off-LC 3DAs.

- In QCD, B-meson 3DAs with non-aligned arguments involve new Lorentz structures compared to LC 3DAs. Respectively, new invariant amplitudes arise.

\[
\langle 0|\bar{s}(y)G_{\alpha\beta}(uy)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda y} \int d\omega e^{-i\omega yv} \left[ \frac{y_\alpha y_\beta}{y^2} - \frac{y_\beta y_\alpha}{y^2} \right] \Phi(\lambda, \omega). \tag{1}
\]

For non-aligned case, new structures and new amplitudes arise:

\[
\langle 0|\bar{s}(y)G_{\alpha\beta}(x)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xv} \int d\omega e^{-i\omega yv} \left[ \frac{1}{2} \left( \frac{x_\alpha y_\beta}{x^2} - \frac{x_\beta y_\alpha}{x^2} + \frac{y_\alpha y_\beta}{y^2} - \frac{y_\beta y_\alpha}{y^2} \right) \Phi_S(\lambda, \omega) + \left( \frac{x_\alpha y_\beta}{x^2} - \frac{x_\beta y_\alpha}{x^2} - \frac{y_\alpha y_\beta}{y^2} + \frac{y_\beta y_\alpha}{y^2} \right) \Phi_A(\lambda, \omega) \right]. \tag{2}
\]

$\Phi_S = \Phi$ from (1), whereas $\Phi_A$ is new. If the contributions induced by $\Phi_A$ are not suppressed, a consistent calculation of the decay amplitude $A$ needs further inputs.

Detailed investigations are necessary.