Charm-loop effect in $B \to K^{(*)}ll$ decays beyond leading twist

Nico Gubernari

Technische Universität München
In collaboration
with D. van Dyk and J. Virto
EOS-2019-01

European Physical Society Conference on High Energy Physics
Ghent, 13-Jul-2019
What’s new?

• revisit **soft gluon contributions to the “charm-loop”** effect in rare $B \rightarrow K^{(*)}ll$ decays

  [Khodjamirian et al. ’10]

• method of **$B$-meson Light-Cone Sum Rules**
What’s new?

- revisit soft gluon contributions to the "charm-loop" effect in rare $B \rightarrow K^{(*)}ll$ decays

  [Khodjamirian et al. ‘10]

- method of $B$-meson Light-Cone Sum Rules

- inclusion of higher-twist corrections and complete set of $B$-meson light-cone distribution amplitudes ($B$-LCDAs)

- preliminary results show a smaller soft gluon contributions to the charm-loop comparing with the previous calculation
Introduction
Anomalies in $b \rightarrow s l l$

FCNC are loop, GIM and CKM suppressed in the SM.

Sensitive to new physics contributions.

Tension between experiment and SM in several observables:

- lepton flavour universality ratios $R_{K^{(*)}}^{\mu e}$
- angular observables $B \rightarrow K^{*} \mu \mu$ ($P_{5}^{I}, \ldots$)
- ...

[Bobeth/Chrzaszcz/van Dyk/Virto '17]
transitions described by the effective Hamiltonian

\[ H(b \rightarrow s l^+ l^-) = -\frac{4G_F}{\sqrt{2}} v_{tb} v_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \quad \mu = m_b \]

main contribution to \( B \rightarrow K(\ast) ll \) in the SM given by local operators \( O_7, O_9, O_{10} \)
$b \rightarrow sll$ effective Hamiltonian

Transitions described by the effective Hamiltonian

$$H(b \rightarrow s l^+ l^-) = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

$\mu = m_b$

Main contribution to $B \rightarrow K^{(*)} ll$ in the SM given by local operators $O_7, O_9, O_{10}$

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$
$$O_9 = \frac{e}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum l (\bar{l} \gamma_\mu l)$$
$$O_{10} = \frac{e}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum l (\bar{l} \gamma_\mu \gamma_5 l)$$

Hadronic matrix elements computed in a previous work ($B \rightarrow K^{(*)}$ form factors) [NG/Kokulu/van Dyk ’18]
Charm-loop in $B \to K(\ast)ll$

additional non-local contributions come from $O_1$ and $O_2$ combined with the e.m. current (charm-loop contribution)

\[ O_1 = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L) \quad O_2 = (\bar{s}_L^i \gamma^\mu c_L^i)(\bar{c}_L^i \gamma_\mu b_L^i) \]
additional non-local contributions come from $O_1$ and $O_2$ combined with the e.m. current (charm-loop contribution)
Operator Product Expansion (OPE)

expand correlator near the light-cone ($x^2 \approx 0$)

\[
\int d^4y \ e^{iky} \langle K^{(*)}(k) | T\{O_{1,2}(0), \bar{c}\gamma_\mu c(x)\} | B(q + k) \rangle
\]

expansion yields new, non-local matrix elements, to be computed
Operator Product Expansion (OPE)

expand correlator near the light-cone \((x^2 \approx 0)\)

\[
\int d^4 y \, e^{i k y} \langle K^{(*)}(k) | T\{O_{1,2}(0), \bar{c} \gamma_{\mu} c(x)\} | \bar{B}(q + k) \rangle
\]

expansion yields new, non-local matrix elements, to be computed
Charm-loop soft gluon emission

two diagrams with a soft-gluon

effects not included in QCD factorization (soft effects that are not included in the FFs)

non-local hadronic matrix element $q^2$ dependent ($q^2$ is the dilepton mass squared)
Our approach to the calculation
QCD perturbation theory breaks down at low energies. Non-perturbative techniques are needed to compute hadronic matrix elements.
QCD perturbation theory breaks down at low energies, so non-perturbative techniques are needed to compute hadronic matrix elements.

**Light-cone sum rules (LCSRs)**
- quark-hadron duality approximation
- universal $B$-meson matrix elements
- applicable for both local and non-local matrix elements
QCD perturbation theory breaks down at low energies, non-perturbative techniques are needed to compute hadronic matrix elements.

**Light-cone sum rules (LCSRs)**
- quark-hadron duality approximation
- universal $B$-meson matrix elements
- applicable for both local and non-local matrix elements

**Lattice QCD**
- numerical evaluation of correlators in a finite and discrete space-time
- non-local matrix elements still work in progress ($K \to \pi\pi$; not yet possible for heavy mesons)
LCSRs are used to determine hadronic matrix elements from a correlation function $\Pi(k, q)$

$$\Pi(k, q) = i \int d^4 y \ e^{iky} \langle 0 | T \{ J_{\text{int}}(y), \bar{\partial}_\mu(0, x) \} | B(q + k) \rangle$$

where

$$J_{\text{int}} = \bar{d} \gamma_\nu (\gamma_5) s$$

$$\bar{\partial}_\mu(0, x) = I_{\mu\rho\alpha\beta} (\bar{q}) \bar{s} \gamma_\rho b_L(0) G(ux)$$
LCSRs are used to determine hadronic matrix elements from a correlation function $\Pi(k, q)$

$$\Pi(k, q) = i \int d^4y \ e^{iky} \langle 0 | T \{ J_{\text{int}}(y), \tilde{O}_\mu(0, x) \} | B(q + k) \rangle$$

where

$$J_{\text{int}} = d \gamma_\nu (\gamma_5) s$$

$$\tilde{O}_\mu(0, x) = I_{\mu\rho\alpha\beta} (\tilde{q}) \bar{s} L \gamma_\rho b_L(0) G(ux)$$

we want to compute the matrix element

$$\langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q + k) \rangle$$

soft gluon emitted by the $B$-meson (no need to consider a soft gluon emitted by the Kaon)
Light-cone Sum Rules in a nutshell 2

two ways to compute the correlator

$$\Pi(k, q) = i \int d^4x e^{ikx} \langle 0| T\{J_{\text{int}}(y), \bar{O}_\mu(0, x)\}|B(q + k)\rangle$$

1. Hadronic representation for positive $k^2$

2. OPE for large negative $k^2$ and $q^2 \ll 4m_c^2$
two ways to compute the correlator

\[
\Pi(k, q) = i \int d^4x \, e^{ikx} \langle 0 \mid T \{ J_{\text{int}}(y), \bar{\partial}_\mu(0, x) \} \mid B(q + k) \rangle
\]

1. Hadronic representation for positive \( k^2 \)

2. OPE for large negative \( k^2 \) and \( q^2 \ll 4m_c^2 \)

the sum rule is obtained matching the result the two different calculations of \( \Pi(k, q) \) and using semi-global quark-hadron duality
Hadronic calculation

for positive $k^2$

\[
\Pi(k, q) = i \int d^4 y e^{iky} \langle 0 | T \{ U_{int}(y), \bar{O}_\mu(0, x) \} | B(q + k) \rangle
\]

insert a complete set
of hadronic states
Hadronic calculation

\[ \Pi(k, q) = i \int d^4y \, e^{iky} \langle 0 | T \{ J_{\text{int}}(y), \tilde{O}_\mu(0, x) \} | B(q + k) \rangle \]

for positive \( k^2 \)

insert a complete set of hadronic states

\[ \frac{f_{K^(*)}}{\langle 0 | J_{\text{int}} | K^(*) \rangle} \langle K^(*) (k) | \tilde{O}_\mu(0, x) | B(q + k) \rangle \]

\[ \frac{k^2 - m_{K^(*)}}{k^2 - m_{K^(*)}} + \text{continuum} \]

hadronic dispersion relation
OPE calculation

For large negative $k^2$ and $q^2 \ll 4m_c^2$, the $B$-meson treated in HQET

$$\Pi(k, q) = i \int d^4y \ e^{iky} \langle 0 | T \{ j_{int}(y), \bar{\partial}_\mu (0, x) \} | B(v) \rangle$$

Factorize hard and soft contributions

$$\Pi(k^2, q^2) = \int_0^\infty ds \ \sum_n \frac{l_n^{\alpha \beta} (s, q^2)}{(s - k^2)^n} \langle 0 | \bar{d}(y) G_{\alpha \beta} (ux) h_v (0) | B(v) \rangle$$
for large negative $k^2$ and $q^2 \ll 4m_c^2$, the $B$-meson treated in HQET

$$\Pi(k, q) = i \int d^4y \ e^{iky} \langle 0 | T \{ J_{int}(y), \bar{O}_\mu(0, x) \} | B(v) \rangle$$

factorize hard and soft contributions

$$\Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{l_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$$

- compute $I_n$ from a perturbative hard scattering kernel
- $B$-to-vacuum non-local matrix element is a necessary non-perturbative input
Three-particle distribution amplitudes

traditional set three-particle \textit{B}-meson light-cone distribution amplitudes (B-LCDA) [Kawamura et al. '01]

\begin{equation}
\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle
= \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[ (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha)(\Psi_A - \Psi_V) - i\sigma_{\alpha\beta} \Psi_V - (v_\alpha v_\beta - v_\beta v_\alpha) \frac{X_A}{v \cdot y} + (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) \frac{Y_A}{v \cdot y} \right] \right\}(y, ux)
\end{equation}

used in the previous calculation Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMWP2010)
Three-particle distribution amplitudes

basis with all independent Lorentz structures, 8 independent $B$-LCDAs [Braun/Ji/Manashov '17]

$$\langle 0 | \bar{d}(y)G_{\alpha \beta}(ux)h_v(0)|B(v) \rangle = \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[ (v_\alpha y_\beta - v_\beta y_\alpha) (\Psi_A - \Psi_V) - i \sigma_{\alpha \beta} \Psi_V - (y_\alpha v_\beta - y_\beta v_\alpha) \frac{X_A}{v \cdot y} + (y_\alpha y_\beta - y_\beta y_\alpha) \frac{W + Y_A}{v \cdot y} \right. \right.$$

$$\left. - i \epsilon_{\alpha \beta \rho \sigma} y^\rho v^\sigma y_5 \frac{X_A}{v \cdot y} + i \epsilon_{\alpha \beta \rho \sigma} y^\rho y_5 \frac{Y_A}{v \cdot y} - (y_\alpha v_\beta - y_\beta v_\alpha) y_\sigma y^\sigma \frac{W}{(v \cdot y)^2} + (y_\alpha y_\beta - y_\beta y_\alpha) y_\sigma y^\sigma \frac{Z}{(v \cdot y)^2} \right\} \right( y, ux)$$

the LCDAs $\Psi_A, \Psi_V, X_A, Y_A, \ldots$ have no definite twist (twist = dimension – spin)

higher twists are suppressed by power of $\Lambda_{\text{had}}/m_B$

express the LCDAs in the traditional form in LCDAs with definite twist
three-particle LCDAs twist basis

models given for LCDAs up to twist 4, twist 5 or higher give corrections of the order $1/m_b^2$

[Braun/Ji/Manashov ’17]

\[
\psi_A = \frac{1}{2}(\phi_3 + \phi_4) \\
\psi_V = \frac{1}{2}(-\phi_3 + \phi_4) \\
x_A = \frac{1}{2}(-\phi_3 - \phi_4 + 2\psi_4) \\
y_A = \frac{1}{2}(-\phi_3 - \phi_4 + \psi_4 - \psi_5) \\
\tilde{x}_A = \frac{1}{2}(-\phi_3 + \phi_4 - 2\tilde{\psi}_4) \\
\tilde{y}_A = \frac{1}{2}(-\phi_3 + \phi_4 - \tilde{\psi}_4 + \tilde{\psi}_5) \\
w = \frac{1}{2}(\phi_4 - \psi_4 - \tilde{\psi}_4 + \psi_5 + \tilde{\psi}_5 + \phi_6) \\
z = \frac{1}{4}(-\phi_3 + \phi_4 - 2\tilde{\psi}_4 + 2\tilde{\psi}_5 + \tilde{\psi}_5 + \phi_6)
\]
three-particle LCDAs twist basis

models given for LCDAs up to twist 4, twist 5 or higher give corrections of the order $1/m_b^2$

[Braun/Ji/Manashov ‘17]

$$
\begin{align*}
\Psi_A &= \frac{1}{2} (\Phi_3 + \Phi_4) \\
\Psi_V &= \frac{1}{2} (-\Phi_3 + \Phi_4) \\
X_A &= \frac{1}{2} (-\Phi_3 - \Phi_4 + 2\Psi_4) \\
Y_A &= \frac{1}{2} (-\Phi_3 - \Phi_4 + \Psi_4 - \Psi_5)
\end{align*}
$$

$$
\begin{align*}
\tilde{X}_A &= \frac{1}{2} (-\Phi_3 + \Phi_4 - 2\Psi_4) \\
\tilde{Y}_A &= \frac{1}{2} (-\Phi_3 + \Phi_4 - \Psi_4 + \Psi_5) \\
W &= \frac{1}{2} (\Phi_4 - \Psi_4 - \Psi_5 + \Psi_5 + \Phi_5) \\
Z &= \frac{1}{4} (-\Phi_3 + \Phi_4 - 2\Psi_4 + 2\Psi_5 + \Phi_5 + \Phi_6)
\end{align*}
$$

use to compute the sum rule

- all 8 independent Lorentz structures (four of them considered for the first time)
- results using LCDAs up to twist 4
- new models for the LCDAs
OPE result

obtaining the OPE result

$$\Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0|\bar{d}(y)G_{\alpha\beta}(u_x)h_v(0)|B(v)\rangle$$

insert the LCDAs in the OPE
obtaining the OPE result

\[
\Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0 | \tilde{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle
\]

insert the LCDAs in the OPE

\[
\Pi(k^2, q^2) = f_B \int_0^\infty ds \sum_{n,t=3,4} \frac{I_{n,t}(s, q^2)}{(s - k^2)^n} \psi_t(y, ux)
\]

integrate by parts
to obtain a dispersive integral
obtaining the OPE result

\[ \Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \left\langle 0|\tilde{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\right\rangle \]

insert the LCDAs in the OPE

\[ \Pi(k^2, q^2) = f_B \int_0^\infty ds \sum_{n,t=3,4} \frac{I_{n,t}(s, q^2)}{(s - k^2)^n} \Psi_t(y, ux) \]

integrate by parts to obtain a dispersive integral

\[ \Pi(k^2, q^2) = f_B \int_0^\infty ds \sum_{t=3,4} \frac{I_t(s, q^2)}{s - k^2} \Psi_t(y, ux) \]
The Sum Rule

matching of the hadronic representation onto the OPE result

\[
\frac{f_{K^(*)}}{\langle 0|J_{\text{int}}|K^(*)\rangle} \frac{\langle K^(*)|\bar{\phi}_\mu(0, x)|B(q + k)\rangle}{k^2 - m_{K^(*)}} + \text{continuum} + f_B \int_0^\infty ds \sum_{t=3,4} \frac{I_t(s, q^2)}{s - k^2} \psi_t(y, ux)
\]
The Sum Rule

\[
\frac{f_{K^+}}{0|J_{int}|K^+(s_0)} \langle K^+(k)|\tilde{O}_\mu(0, x)|B(q + k)\rangle \Bigg| \frac{1}{k^2 - m_{K^+}} \Bigg] + \text{continuum}
\]

\[
f_B \int_0^\infty ds \sum_{t=3,4} \frac{I_t(s, q^2)}{s - k^2} \Psi_t(y, ux)
\]

use semi-global quark-hadron duality

\[
s_0 = \text{effective threshold}
\]

\[
\frac{\langle K^+(k)|\tilde{O}_\mu(0, x)|B(q + k)\rangle}{k^2 - m_{K^+}} = f_B \frac{1}{f_{K^+}} \int_0^{s_0} ds \sum_{t=3,4} \frac{I_t(s, q^2)}{s - k^2} \Psi_t(y, ux)
\]
Matching of the hadronic representation onto the OPE result

\[
\frac{f_{K^(*)}}{\langle 0 | J_{\text{int}} | K^(*) \rangle} \left\langle K^(*)| (k) | \bar{\Omega}_\mu (0, x) | B (q + k) \right\rangle \cdot \frac{k^2 - m_{K^(*)}}{\text{continuum}}
\]

\[
f_B \int_0^\infty ds \sum_{t=3,4} \frac{I_t (s, q^2)}{s - k^2} \Psi_t (y, ux)
\]

Use semi-global quark-hadron duality

\[
s_0 = \text{effective threshold}
\]

Apply Borel transformation

\[
\left\langle K^(*)| (k) | \bar{\Omega}_\mu (0, x) | B (q + k) \right\rangle = \frac{f_B}{f_{K^(*)}} \int_0^{s_0} ds e^{\frac{m_{K^(*)} - s}{M^2} - s} \sum_{t=3,4} I_t (s, q^2) \Psi_t (y, ux)
\]
Numerical Results
Preliminary results and comparison

\[
\begin{array}{ccc}
\Delta C^9(q^2) & \text{KMPW2010} & \text{GvDV2019} \\
\hline
\text{factorizable contr.} & 0.27 & 0.27 \\
B \rightarrow Kll & \tilde{\mathcal{A}}(q^2 = 1) & -0.09^{+0.06}_{-0.07} & (1.9^{+0.6}_{-0.6}) \cdot 10^{-4} \\
B \rightarrow K^*ll & \tilde{\nu}_1(q^2 = 1) & 0.6^{+0.7}_{-0.5} & (1.2^{+0.4}_{-0.4}) \cdot 10^{-3} \\
 & \tilde{\nu}_2(q^2 = 1) & 0.6^{+0.7}_{-0.5} & (2.1^{+0.7}_{-0.7}) \cdot 10^{-3} \\
 & \tilde{\nu}_3(q^2 = 1) & 1.0^{+1.6}_{-0.8} & (3.0^{+1.0}_{-1.0}) \cdot 10^{-3} \\
B_s \rightarrow \phi ll & \text{...} & - & \text{???} \\
\end{array}
\]

results represented as a $q^2$ dependent correction to $C^9$

we fully reproduce the results given in KMWP2010

[q^2 is the dilepton mass square]
Preliminary results and comparison

<table>
<thead>
<tr>
<th>$\Delta C^9(q^2)$</th>
<th>KMPW2010</th>
<th>GvDV2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>factorizable contr.</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>$B \to Kll$ $\bar{A}(q^2 = 1)$</td>
<td>$-0.09^{+0.06}_{-0.07}$</td>
<td>$(1.9^{+0.6}_{-0.6}) \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$B \to K^*ll$ $\bar{V}_1(q^2 = 1)$</td>
<td>$0.6^{+0.7}_{-0.5}$</td>
<td>$(1.2^{+0.4}_{-0.4}) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$B \to K^*ll$ $\bar{V}_2(q^2 = 1)$</td>
<td>$0.6^{+0.7}_{-0.5}$</td>
<td>$(2.1^{+0.7}_{-0.7}) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$B \to K^*ll$ $\bar{V}_3(q^2 = 1)$</td>
<td>$1.0^{+1.6}_{-0.8}$</td>
<td>$(3.0^{+1.0}_{-1.0}) \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

$B_s \to \phi ll$ ...

results represented as a $q^2$ dependent correction to $C^9$
we fully reproduce the results given in KMWP2010

matrix elements parametrized analogously to the form factors:

$$\langle K (k)|\bar{\Theta}_\mu(0,x)|B(q + k)\rangle = \left((k \cdot q)q_\mu - q^2 k_\nu\right)\bar{A}(q^2) + \ldots$$

$$\langle K^*(k,\eta)|\bar{\Theta}_\mu(0,x)|B(q + k)\rangle = \epsilon_{\mu\alpha\beta\gamma}\eta^*\alpha q^\beta k^\gamma \bar{V}_1(q^2) + i \left((m_B^2 - m_{K^*}^2)\eta^*_\mu - (\eta^* \cdot k)(2k + q)_\mu\right)\bar{V}_2(q^2)$$

$$+ i(\eta^* \cdot q)\left(q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(2k + q)_\mu\right)\bar{V}_3(q^2) + \ldots$$

$q^2$ is the dilepton mass square
Why such different results?

different inputs: \( B \)-LCDAs models depend on \( \lambda_H^2, \lambda_E^2 \) →

KMPW10:

\[
\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15 \text{ GeV}^2
\]

\( \Rightarrow \) twist 3 does not contribute

we use

\[
\lambda_E^2 = 0.03 \pm 0.02 \text{ GeV}^2
\]
\[
\lambda_H^2 = 0.06 \pm 0.03 \text{ GeV}^2
\]

\( \Rightarrow \sim 10 \) times smaller [Nishikawa/Tanaka '14]
Why such different results?

different inputs: $B$-LCDAs models depend on $\lambda_H^2$, $\lambda_E^2$ →

KMPW10:

$\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15 \text{ GeV}^2$

⇒ twist 3 does not contribute

we use $\lambda_E^2 = 0.03 \pm 0.02 \text{ GeV}^2$

$\lambda_H^2 = 0.06 \pm 0.03 \text{ GeV}^2$

⇒ ~$10$ times smaller [Nishikawa/Tanaka '14]

3-particle $B$-LCDAs heavy quark (twist) expansion →

KMPW10: the 3-pt $B$-LCDAs twist expansion was not known

we use Braun/Ji/Manashov '17
## Why such different results?

| Different inputs: $B$-LCDAs models depend on $\lambda_H^2, \lambda_E^2$ | KMPW10: $\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15 \text{ GeV}^2$
| | $\Rightarrow$ twist 3 does not contribute |
| | we use $\lambda_E^2 = 0.03 \pm 0.02 \text{ GeV}^2$
| | $\lambda_H^2 = 0.06 \pm 0.03 \text{ GeV}^2$
| | $\Rightarrow \sim 10$ times smaller [Nishikawa/Tanaka ’14] |

| 3-particle $B$-LCDAs heavy quark (twist) expansion | KMPW10: the 3-pt $B$-LCDAs twist expansion was not known |
| | we use Braun/Ji/Manashov ’17 |

| Lorentz structures considered in $\langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$ | KMPW10: 4 Lorentz structures |
| | $\Rightarrow$ partial cancelation (new structures come with an opposite sign) |

| | all 8 independent Lorentz structures |
Summary

- update the previous calculation to the soft gluon contributions to the charm-loop effect in $B \to K^{(*)}\ell\ell$ decays
- add results for $B_s \to \phi\ell\ell$
- numerical $\sim$100 smaller comparing with KMPW2010 (inputs, Lorentz structures, twist expansion, new $B$-LCDAs models)
- results will be easily accessible in the open source software EOS (https://github.com/eos/eos)
Thank you!
Three-particle LCDAs models and $\lambda_{H,E}^2$

\[ \Psi_A(\omega_1, \omega_2) = \Psi_V(\omega_1, \omega_2) = \frac{\lambda_E^2}{6\lambda_B^4} \omega_2^2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}} \]

\[ X_A = \frac{\lambda_E^2}{6\lambda_B^4} \omega_2 (2\omega_1 - \omega_2) e^{-\frac{\omega_1 + \omega_2}{\lambda_B}} \]

\[ Y_A = -\frac{\lambda_E^2}{24\lambda_B^4} \omega_2 (7\lambda_B - 13\omega_1 + 3\omega_2) e^{-\frac{\omega_1 + \omega_2}{\lambda_B}} \]

\[ \lambda_{H,E}^2 \text{ definition} \]

\[ \langle 0| \bar{a}(0) G_{\alpha\beta}(0) h_v(0) |B(v)\rangle = -\frac{i}{6} f_B \lambda_H^2 \text{Tr} \left[ \gamma_5 \Gamma P_+ \sigma_{\alpha\beta} \right] - \frac{i}{6} f_B (\lambda_H^2 - \lambda_E^2) \text{Tr} \left[ \gamma_5 \Gamma P_+ (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) \right] \]
Threshold $s_0$ determination

\[
 f_{K^{(*)}} e^{-\frac{m_{K^{(*)}}}{M^2}} \langle K^{(*)}(k) | \bar{\phi}_\mu(0,x) | B(q+k) \rangle = f_B \int_0^{s_0} ds \ e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2) \Psi_t(y,ux) \tag{1}
\]

derive with respect to $1/M^2$
and divide by (1)

\[
 m_{K^{(*)}} = \frac{\int_0^{s_0} ds \ s \ e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2) \Psi_t(y,ux)}{\int_0^{s_0} ds \ e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2) \Psi_t(y,ux)}
\]

daughter sum rule to extract $s_0$
Alignment of the gluon with the $K^{(*)}$ meson

We are interested in the dominant effect of the nonvanishing gluon momenta generated by the exponent in (3.9). Decomposing the covariant derivative in the light-cone vectors

$$\mathcal{D} = \frac{n_-}{2} + \frac{n_+}{2} + \mathcal{D}_\perp,$$

(3.10)

we retain only the $n_-$ component, which corresponds to the gluons emitted antiparallel to $q$, that is, in the same direction as the s-quark in the $B$-meson rest frame. We then have

$$G^\alpha_\beta(u x) \simeq \exp[-i u(n_- x) \frac{(n_+ \mathcal{D})}{2}] G^\alpha_\beta$$

$$= \int d\omega \exp[-i u(n_- x)\omega] \delta[\omega - \frac{(n_+ \mathcal{D})}{2}] G^\alpha_\beta.$$

(3.11)

[...]

is represented in a compact unintegrated form, and we use the notation $\bar{q} = q - u\omega n_-$, so that $\bar{q}^2 \simeq q^2 - 2u\omega m_b$. Here we take into account that $\omega \ll m_b$, after the hadronic matrix element is taken. Note that the neglected components of $\mathcal{D}$ in (3.10) produce small, $O(\omega/m_b)$ corrections to $\bar{q}^2$, hence our approximation is well justified.

[Khodjamirian et al. ‘10]
Where is the mistake?

\[
\langle 0 | \bar{d}(y) G_{\alpha \beta}(ux) n^\beta h_v(0) | B(v) \rangle
\]

\[
= \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[ (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) (\Psi_A - \Psi_V) - i \sigma_{\alpha \beta} \Psi_V - (v_\alpha v_\beta - v_\beta v_\alpha) \frac{X_A}{v \cdot y} + (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) \frac{Y_A}{v \cdot y} \right] \right\} (y, ux)
\]

\[
= \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[ (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) (\Psi_A - \Psi_V) - i \sigma_{\alpha \beta} \Psi_V - (v_\alpha v_\beta - v_\beta v_\alpha) \frac{X_A}{v \cdot y} + (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) \frac{Y_A}{v \cdot y} \right] \right\} (y, ux)
\]

[Kawamura et al. ’01]

[Khodjamirian et al. ’06]