Charm-loop effect in $B \to K^{(*)}ll$ decays beyond leading twist

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In collaboration
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What's new?

- revisit soft gluon contributions to the "charm-loop" effect in rare $B \to K^{(*)}ll$ decays

  [Khodjamirian et al. '10]

- method of *B*-meson Light-Cone Sum Rules

What's new?

- revisit soft gluon contributions to the "charm-loop" effect in rare $B \to K^{(*)}ll$ decays

  [Khodjamirian et al. '10]

- method of *B*-meson Light-Cone Sum Rules

- inclusion of higher-twist corrections and complete set of *B*-meson light-cone distribution amplitudes (*B*-LCDAs)

- preliminary results show a smaller soft gluon contributions to the charm-loop comparing with the previous calculation

Introduction

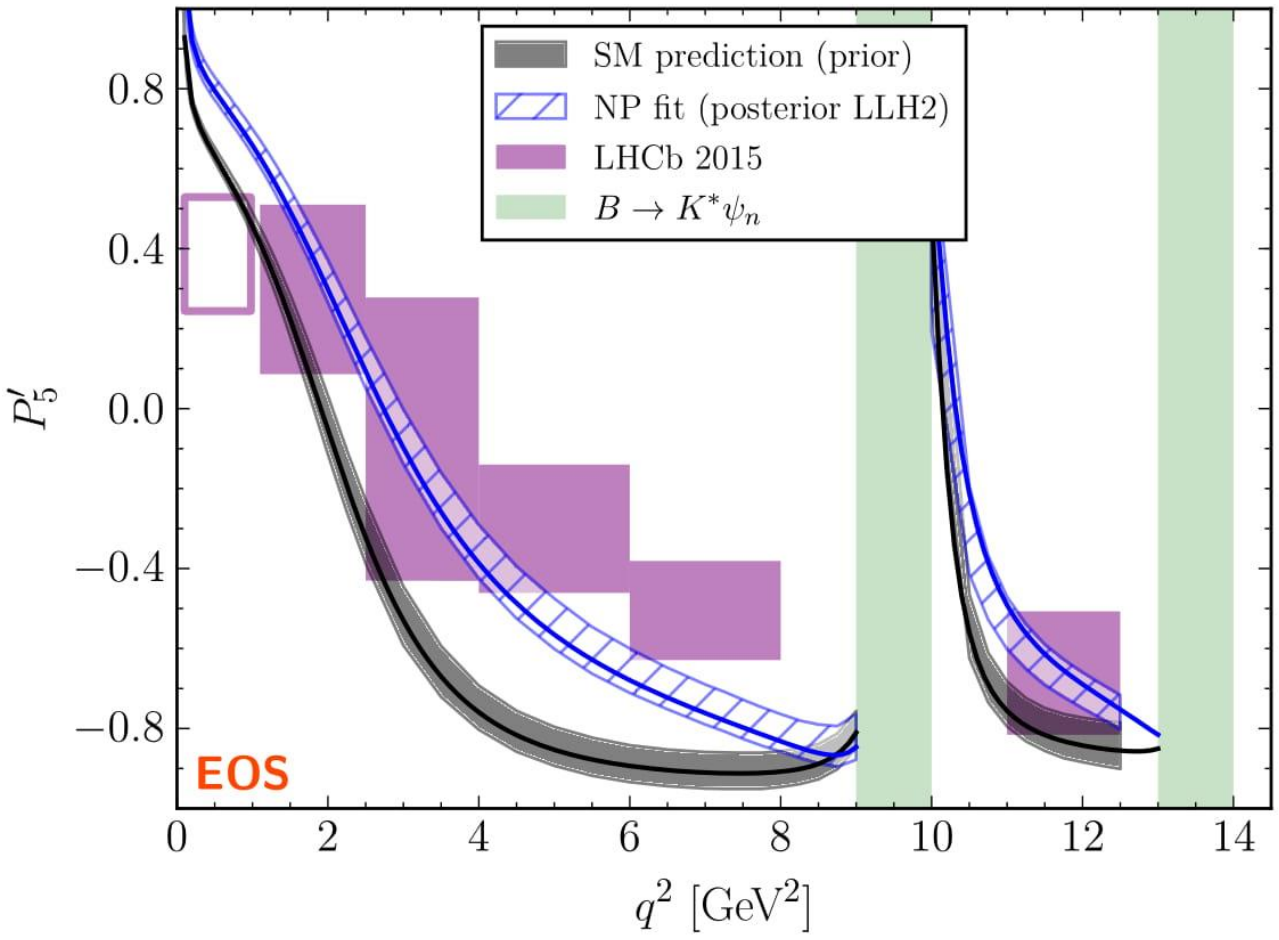
Anomalies in $b \to sll$

FCNC are loop, GIM and CKM **suppressed in the SM**.

Sensitive to new physics contributions.

Tension between experiment and SM in several observables:

- lepton flavour universality ratios $R^{\mu e}_{K^{(*)}}$
- angular observables $B \to K^* \mu\mu$ (P_5', ...)
- ...



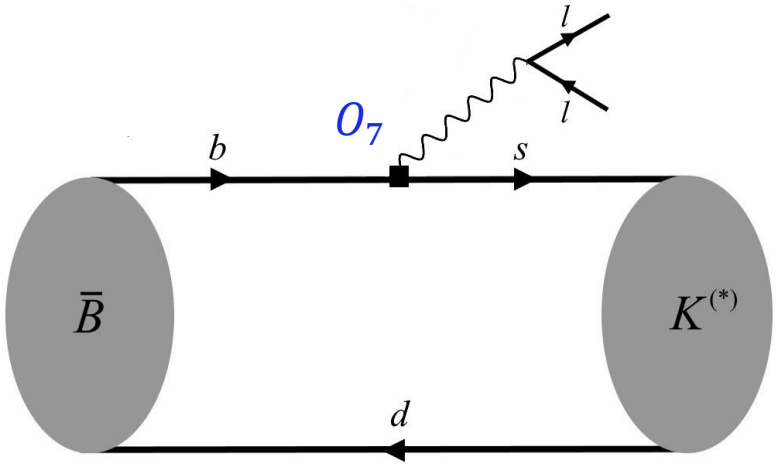
[Bobeth/Chrzaszcz/van Dyk/Virto '17]

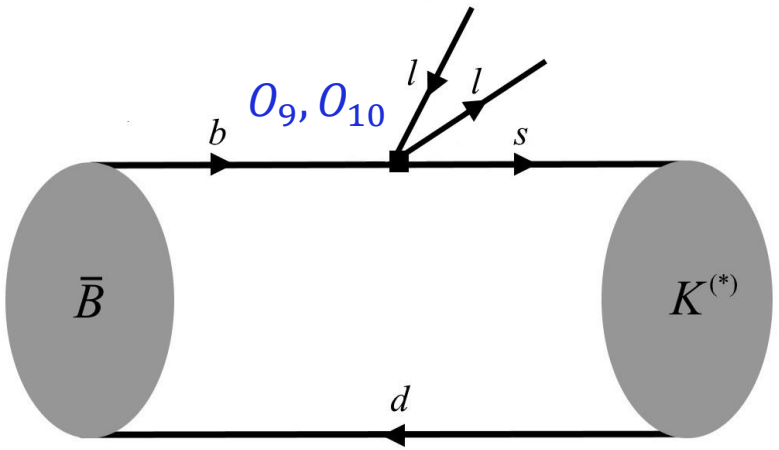
$b \to sll$ effective Hamiltonian

transitions described by the **effective Hamiltonian**

$$H(b \to sl^+l^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)O_i(\mu) \qquad \mu = m_b$$

main contribution to $B \to K^{(*)}ll$ in the SM given by local operators O_7, O_9, O_{10}





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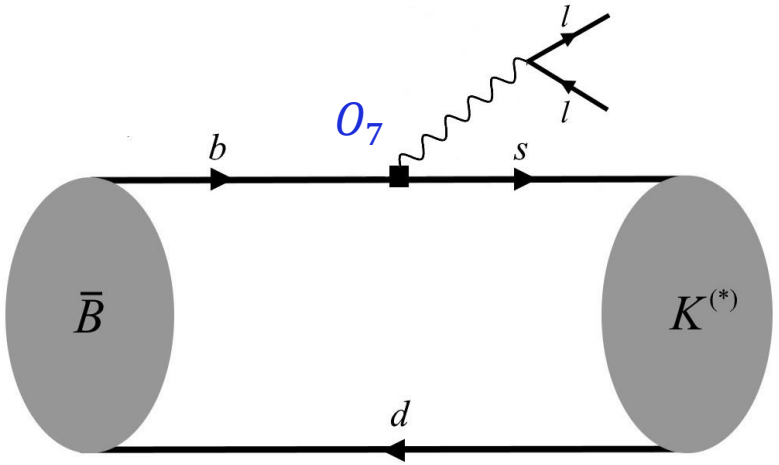
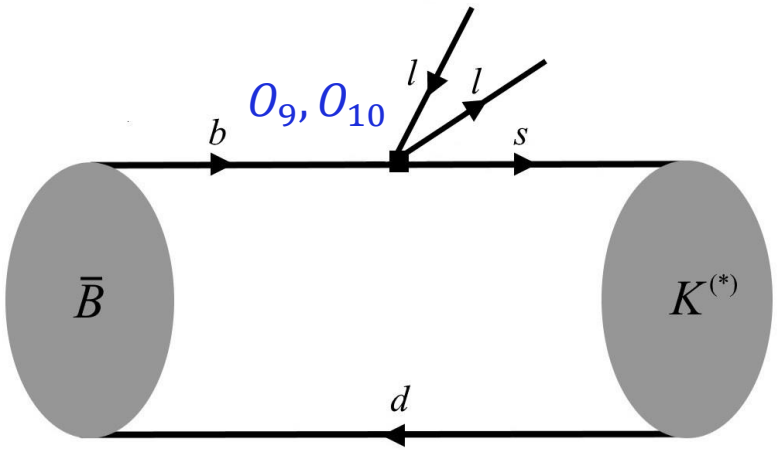
$$O_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu} \qquad O_9 = \frac{e}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\sum_l(\bar{l}\gamma_\mu l) \qquad O_{10} = \frac{e}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\sum_l(\bar{l}\gamma_\mu\gamma_5 l)$$

hadronic matrix elements computed in a previous work ($B \to K^{(*)}$ form factors) [NG/Kokulu/van Dyk '18]

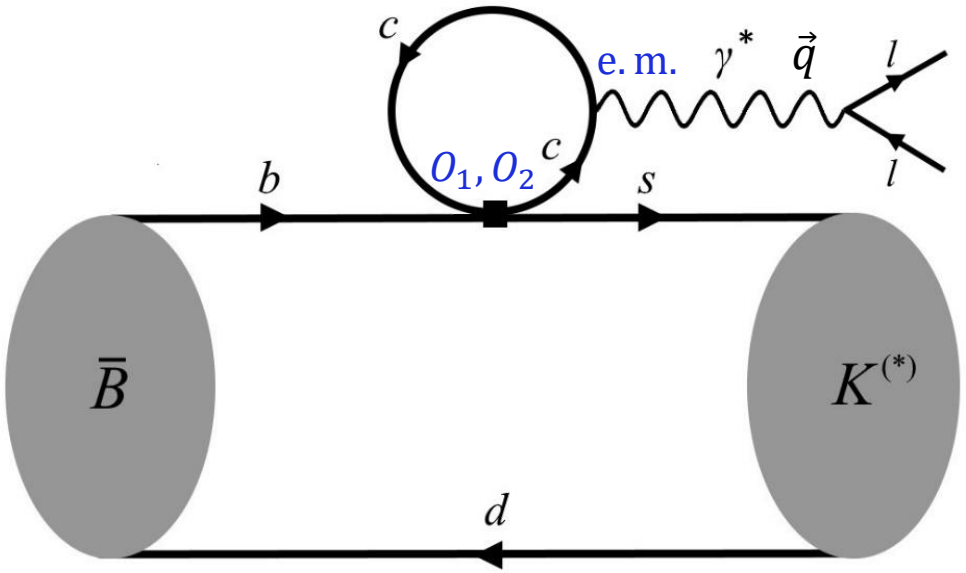




Charm-loop in $B \to K^{(*)}ll$

additional non-local contributions come from O_1 and O_2 combined with the e.m. current (charm-loop contribution)

$$O_1 = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L) \qquad O_2 = \left(\bar{s}_L^j \gamma^\mu c_L^i\right)\left(\bar{c}_L^i \gamma_\mu b_L^j\right)$$



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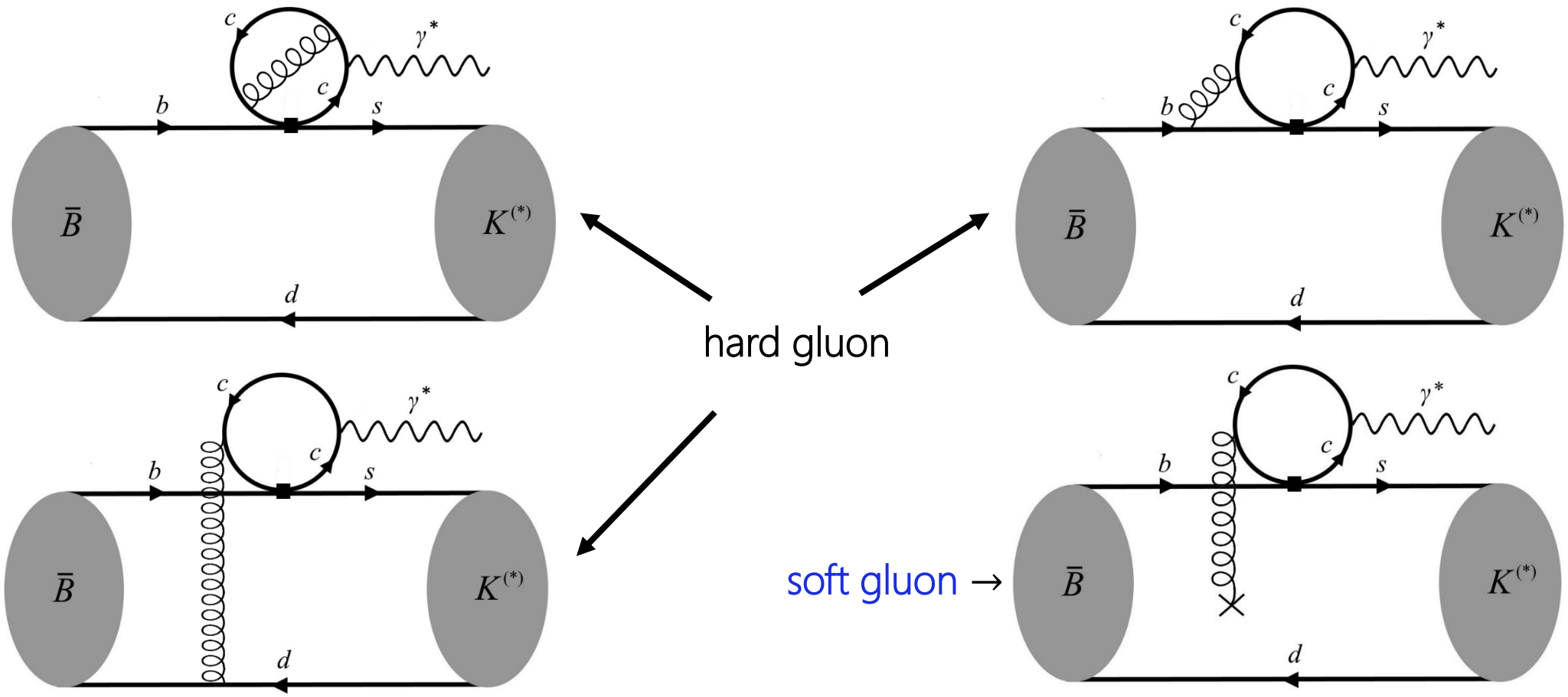
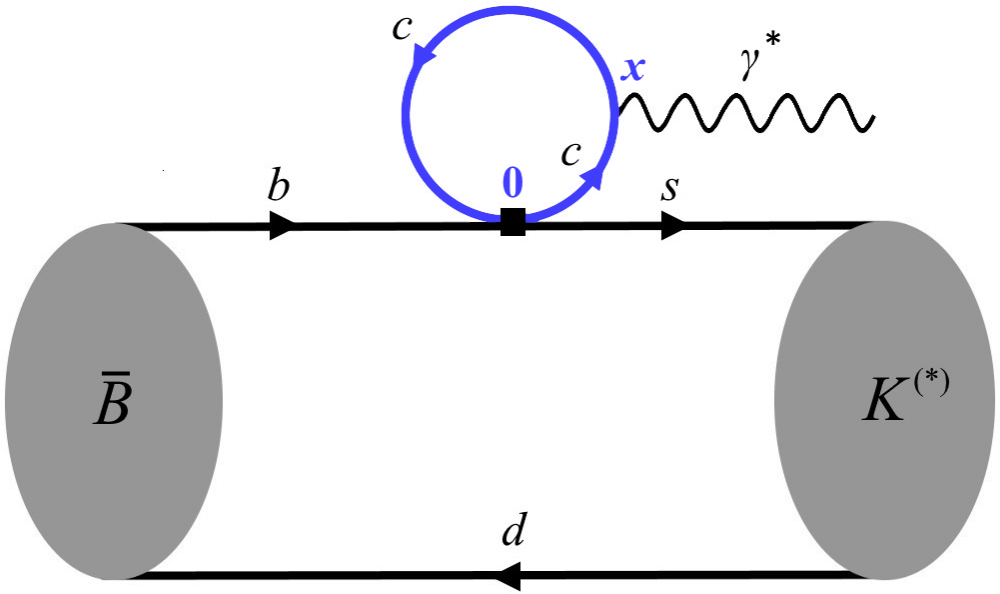


Operator Product Expansion (OPE)

expand correlator near the light-cone ($\boldsymbol{x^2 \simeq 0}$)

$$\int \mathrm{d}^4 y\, e^{iky} \langle K^{(*)}(k) | T\{O_{1,2}(0), \bar{c}\gamma_\mu c(x)\} | B(q+k) \rangle$$

expansion yields new, non-local matrix elements, to be computed



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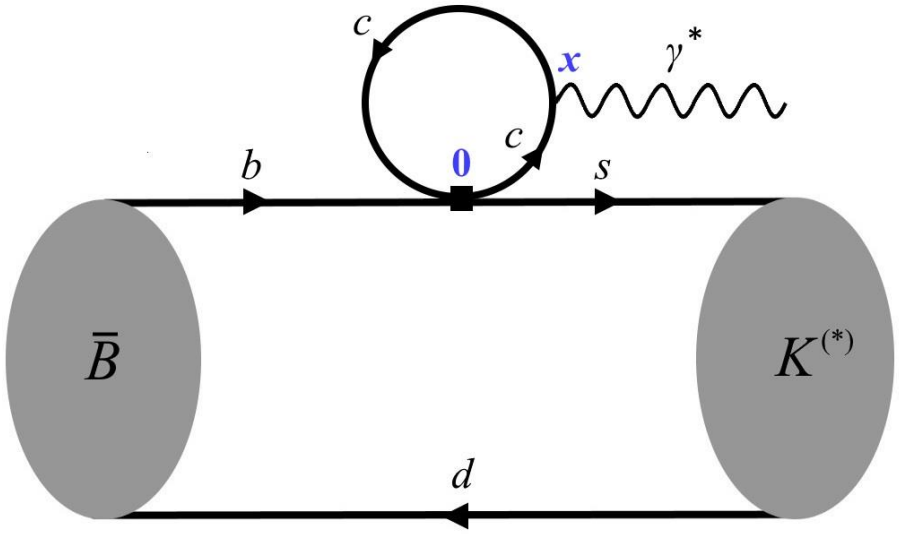
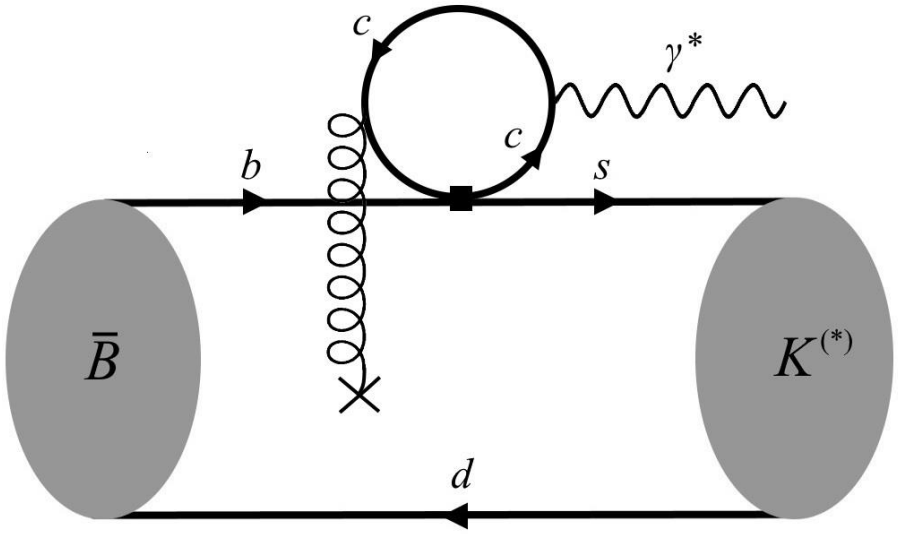


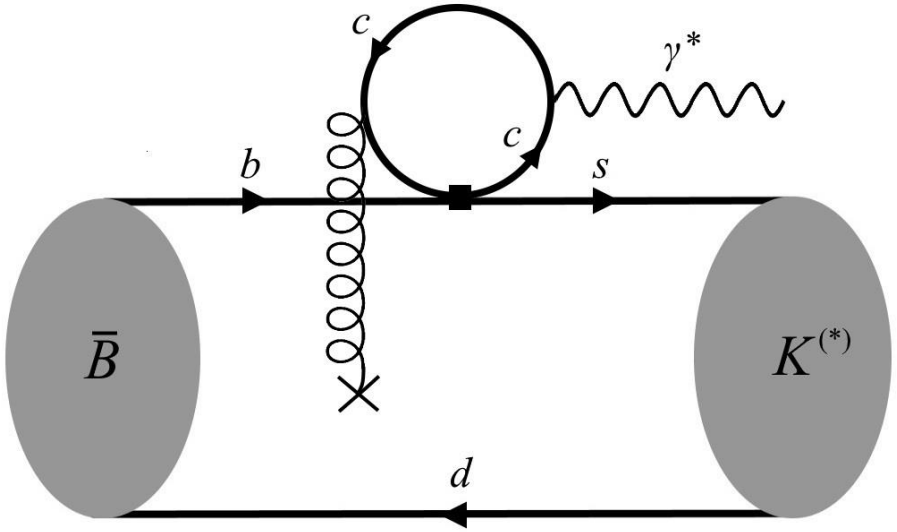


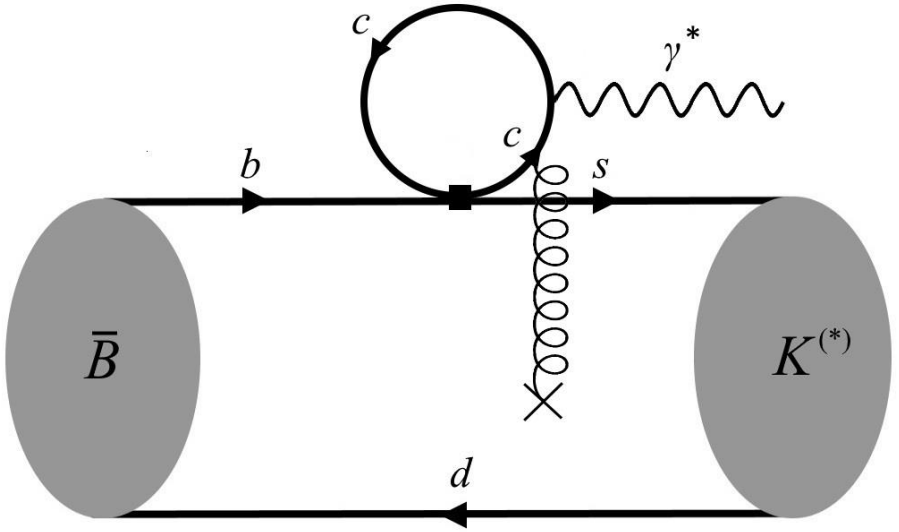
Charm-loop soft gluon emission

two diagrams with a soft-gluon

effects not included in QCD factorization (soft effects that are not included in the FFs)

non-local hadronic matrix element q^2 dependent (q^2 is the dilepton mass squared)





Our approach to the calculation

Methods to compute hadronic matrix elements

QCD perturbation theory breaks down at low energies

non-perturbative techniques are needed

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Light-cone sum rules (LCSRs)

quark-hadron duality approximation

universal B-meson matrix elements

applicable for both local and non-local matrix elements

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Lattice QCD

numerical evaluation of correlators in a finite and discrete space-time

non-local matrix elements still work in progress
($K \to \pi\pi$; not yet possible for heavy mesons)

Light-cone Sum Rules in a nutshell 1

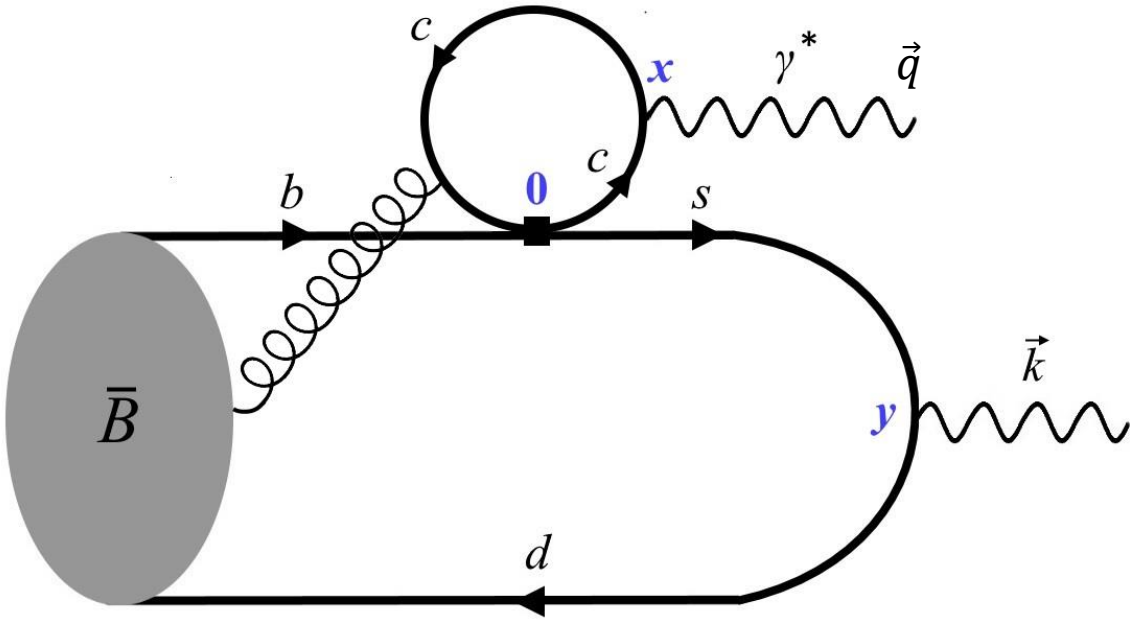
LCSRs are used to determine hadronic matrix elements
from a correlation function $\Pi(k,q)$

$$\Pi(k,q) = i\int \mathrm{d}^4y\, e^{iky}\langle 0|T\{J_{int}(y), \tilde{O}_\mu(0,x)\}|B(q+k)\rangle$$

where

$$J_{int} = \bar{d}\gamma_\nu(\gamma_5)s$$

$$\tilde{O}_\mu(0,x) = I_{\mu\rho\alpha\beta}(\tilde{q})\,\bar{s}_L\gamma_\rho b_L(0)\,G(ux)$$



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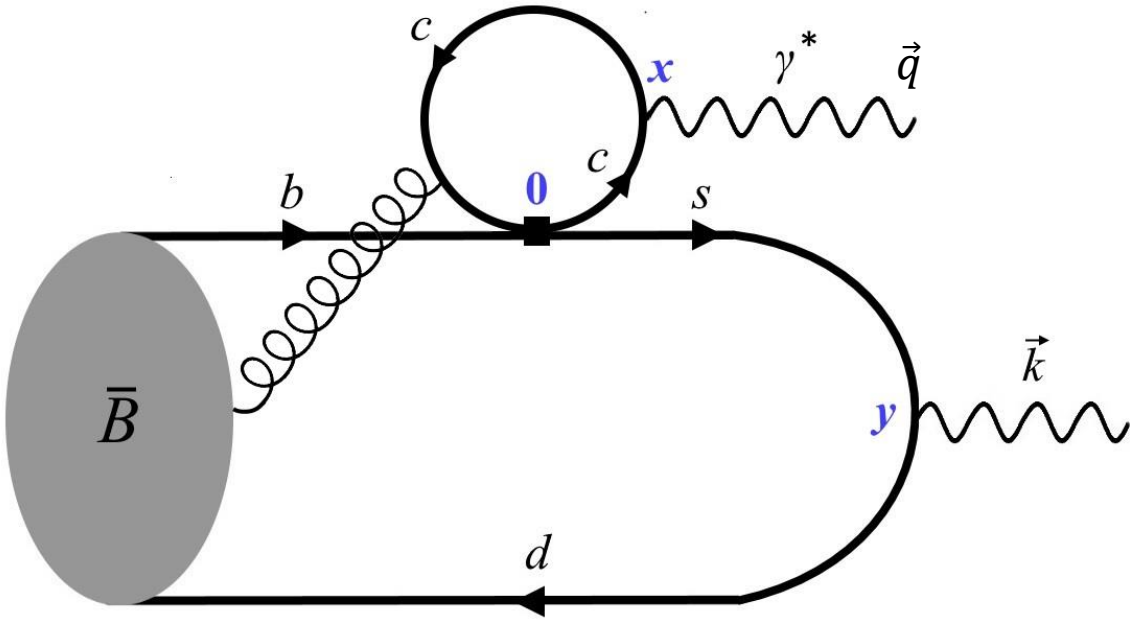
we want to compute the matrix element

$$\boldsymbol{\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle}$$

soft gluon emitted by the B-meson (no need to consider a soft gluon emitted by the Kaon)



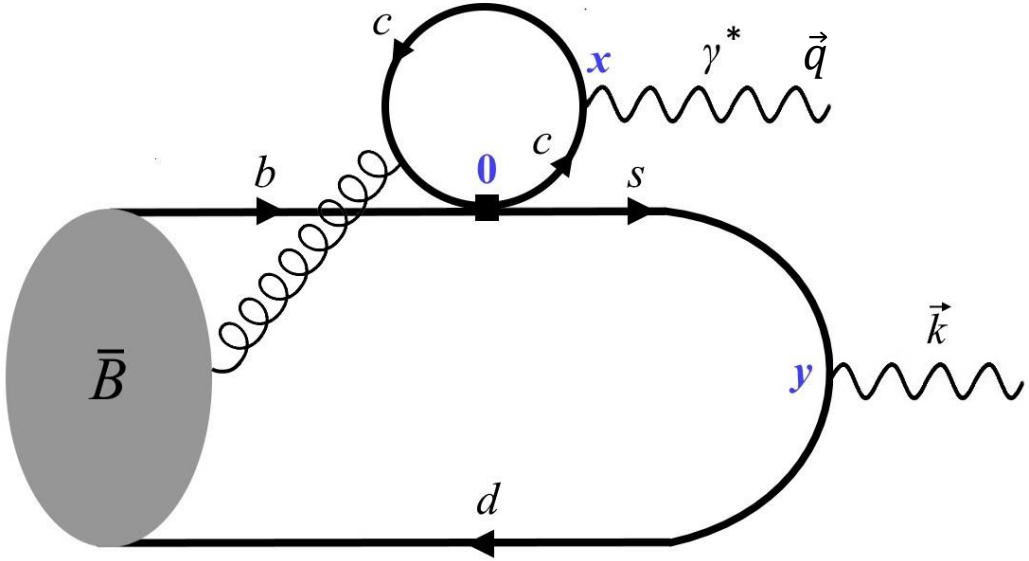
Light-cone Sum Rules in a nutshell 2

two ways to compute the correlator

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{J_{int}(y),\tilde{O}_\mu(0,x)\}|B(q+k)\rangle$$

1

Hadronic representation for positive k^2



2

OPE for large negative k^2 and $q^2 \ll \mathbf{4m_c^2}$

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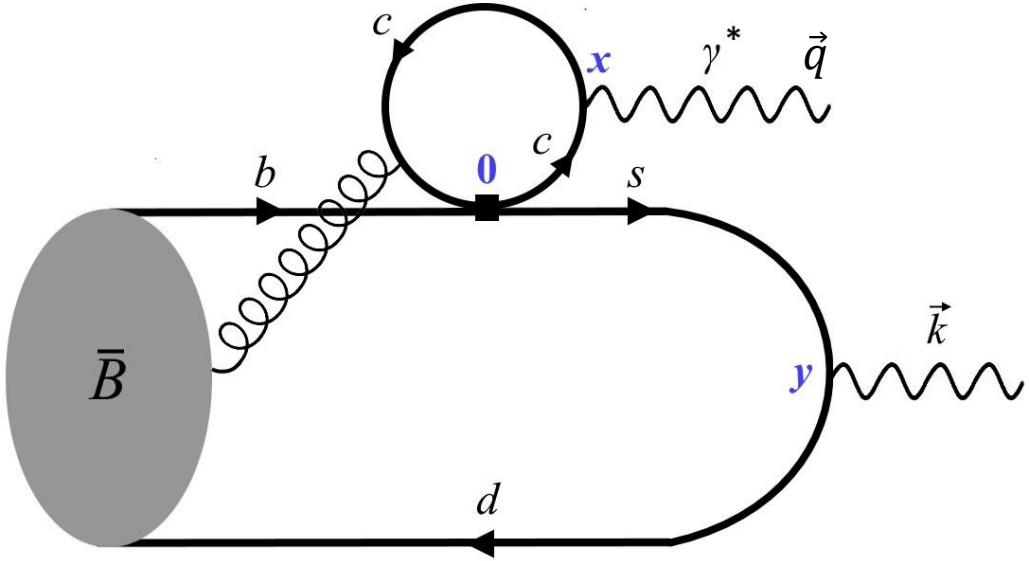


2

OPE for large negative k^2 and $q^2 \ll \mathbf{4m_c^2}$

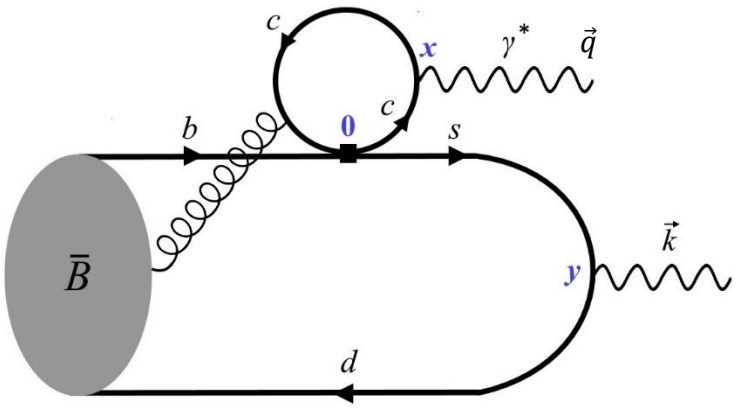
the sum rule is obtained matching the result the two different calculations of $\Pi(k,q)$ and using semi-global quark-hadron duality

Hadronic calculation

for positive k^2

$$\Pi(k,q) = i\int \mathrm{d}^4y\, e^{iky}\langle 0|T\{J_{int}(y), \tilde{O}_\mu(0,x)\}|B(q+k)\rangle$$

insert a complete set
of hadronic states



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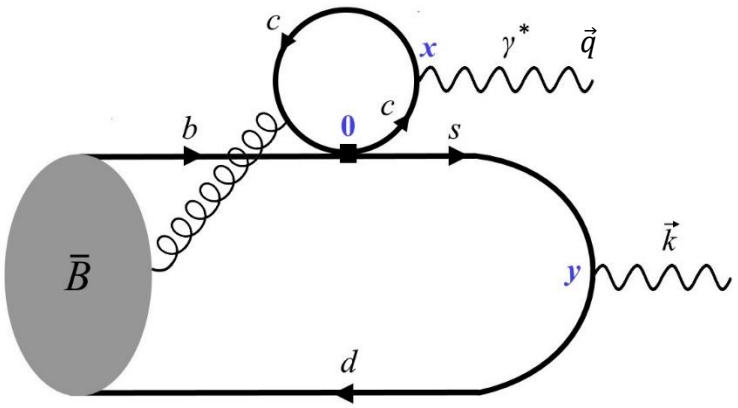
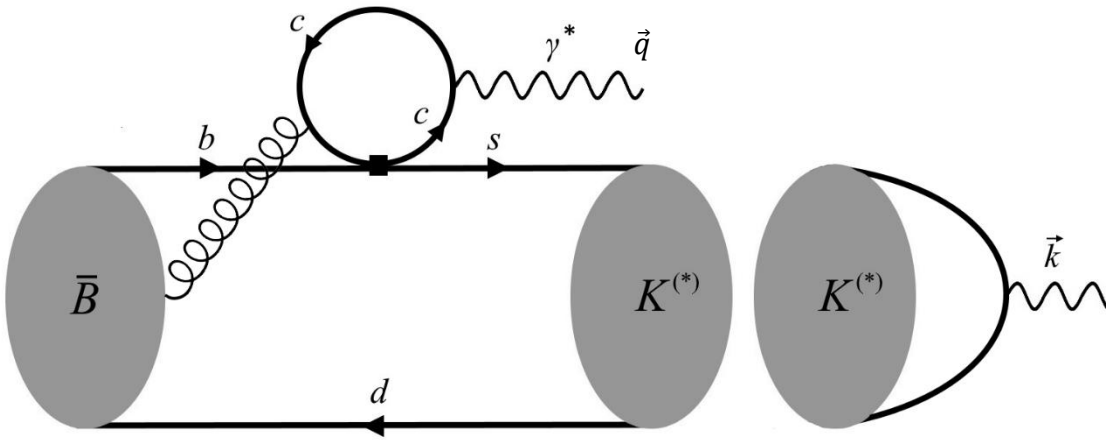
$$\frac{\overbrace{\langle 0|J_{int}|K^{(*)}\rangle}^{f_{K^{(*)}}}\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle}{k^2-m_{K^{(*)}}} + continuum$$

hadronic dispersion relation





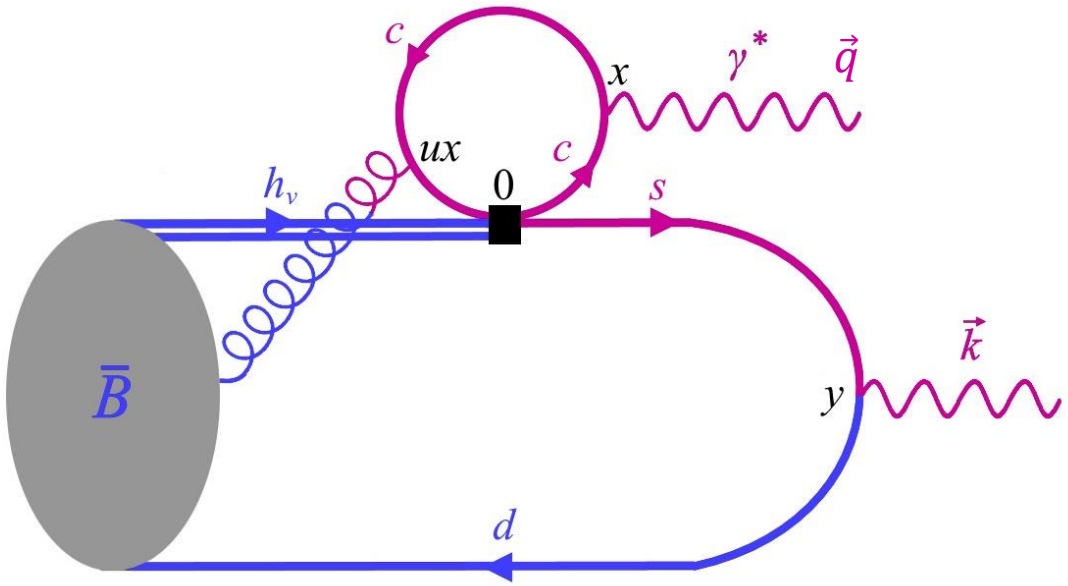
OPE calculation

for large **negative** k^2 and $q^2 \ll \mathbf{4m_c^2}$, the B-meson treated in HQET

$$\Pi(k,q) = i\int \mathrm{d}^4y\, e^{iky}\langle 0|T\{J_{int}(y), \tilde{O}_\mu(0,x)\}|B(v)\rangle$$

factorize hard and soft contributions

$$\Pi(k^2,q^2) = \int_0^\infty \mathrm{d}s \sum_n \frac{I_n^{\alpha\beta}(s,q^2)}{(s-k^2)^n}\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle$$



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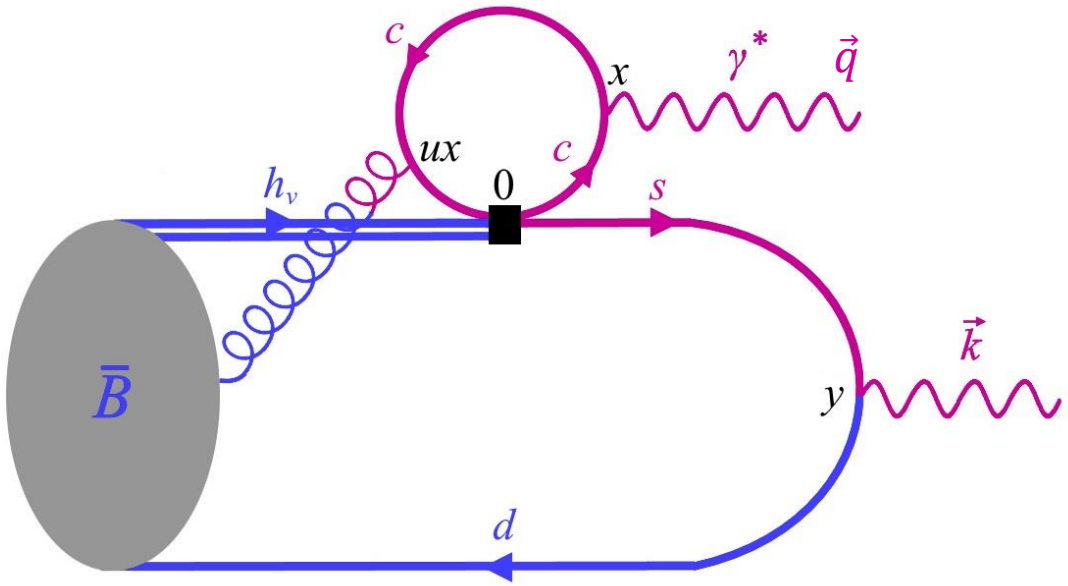
- compute I_n from a **perturbative** hard scattering kernel
- B-to-vacuum non-local matrix element is a necessary **non-perturbative input**



Three-particle distribution amplitudes

traditional set three-particle *B*-meson light-cone distribution amplitudes (*B*-LCDA) [Kawamura et al. '01]

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+\left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right]\right\}(\boldsymbol{y}, \boldsymbol{ux})$$

used in the previous calculation Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMWP2010)

Three-particle distribution amplitudes

basis with all independent Lorentz structures, 8 independent *B*-LCDAs [Braun/Ji/Manashov '17]

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle$$

$$= \frac{f_B m_B}{4}\mathrm{Tr}\Big\{\gamma_5 P_+\Big[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{W} + \boldsymbol{Y_A}}{v\cdot y}$$

$$- i\epsilon_{\alpha\beta\sigma\rho}y^\sigma v^\rho\gamma_5\frac{\boldsymbol{\widetilde{X}_A}}{v\cdot y} + i\epsilon_{\alpha\beta\sigma\rho}y^\sigma\gamma^\rho\gamma_5\frac{\boldsymbol{\widetilde{Y}_A}}{v\cdot y} - (y_\alpha v_\beta - y_\beta v_\alpha)y_\sigma\gamma^\sigma\frac{\boldsymbol{W}}{(v\cdot y)^2} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)y_\sigma\gamma^\sigma\frac{\boldsymbol{Z}}{(v\cdot y)^2}\Big]\Big\}(\boldsymbol{y}, \boldsymbol{ux})$$

the LCDAs $\Psi_A, \Psi_V, X_A, Y_A, \ldots$ have no definite twist (twist = dimension – spin)

higher twists are suppressed by power of $\Lambda_{\mathrm{had}}/m_B$

express the LCDAs in the traditional form in LCDAs with definite twist

three-particle LCDAs twist basis

models given for LCDAs up to twist 4, twist 5 or higher give corrections of the order $1/m_b^2$

[Braun/Ji/Manashov '17]

$$\Psi_A = \frac{1}{2}(\Phi_3 + \Phi_4)$$

$$\Psi_V = \frac{1}{2}(-\Phi_3 + \Phi_4)$$

$$X_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + 2\Psi_4)$$

$$Y_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + \Psi_4 - \Psi_5)$$

$$\tilde{X}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4)$$

$$\tilde{Y}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - \tilde{\Psi}_4 + \tilde{\Psi}_5)$$

$$W = \frac{1}{2}(\Phi_4 - \Psi_4 - \tilde{\Psi}_4 + \Psi_5 + \tilde{\Psi}_5 + \tilde{\Phi}_5)$$

$$Z = \frac{1}{4}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4 + 2\tilde{\Psi}_5 + \tilde{\Phi}_5 + \Phi_6)$$

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$$Z = \frac{1}{4}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4 + 2\tilde{\Psi}_5 + \tilde{\Phi}_5 + \Phi_6)$$

use to compute the sum rule

- all 8 independent Lorentz structures (four of them considered for the first time)
- results using LCDAs up to twist 4
- **new models** for the LCDAs

OPE result

obtaining the OPE result

$$\Pi(k^2, q^2) = \int_0^\infty \mathrm{d}s \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$$

insert the LCDAs in the OPE

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$$\Pi(k^2, q^2) = f_B \int_0^\infty \mathrm{d}s \sum_{n, t=3,4} \frac{I_{n,t}(s, q^2)}{(s - k^2)^n} \Psi_t(y, ux)$$

integrate by parts
to obtain a dispersive integral

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The Sum Rule

matching of the hadronic representation onto the OPE result

$$\frac{\overbrace{\langle 0|J_{int}|K^{(*)}\rangle}^{f_{K^{(*)}}}\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle}{k^2-m_{K^{(*)}}}+continuum$$

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use semi-global **quark-hadron duality**

s_0 = effective threshold

$$\frac{\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle}{k^2-m_{K^{(*)}}} = \frac{f_B}{f_{K^{(*)}}}\int_0^{s_0} \mathrm{d}s \sum_{t=3,4} \frac{I_t(s,q^2)}{s-k^2}\Psi_t(y,ux)$$

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use semi-global **quark-hadron duality**
s_0 = effective threshold

$$\frac{\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle}{k^2-m_{K^{(*)}}}=\frac{f_B}{f_{K^{(*)}}}\int_0^{s_0}\mathrm{d}s\sum_{t=3,4}\frac{I_t(s,q^2)}{s-k^2}\Psi_t(y,ux)$$

apply Borel transformation

Sum Rule

$$\boxed{\langle K^{(*)}(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle=\frac{f_B}{f_{K^{(*)}}}\int_0^{s_0}\mathrm{d}s\,e^{\frac{m_{K^{(*)}}-s}{M^2}}\sum_{t=3,4}I_t(s,q^2)\,\Psi_t(y,ux)}$$

Numerical Results

Preliminary results and comparison

$\Delta C9(q^2)$		KMPW2010	GvDV2019
factorizable contr.		0.27	0.27
$B \to Kll$	$\tilde{\mathcal{A}}(q^2 = 1)$	$-0.09^{+0.06}_{-0.07}$	$(1.9^{+0.6}_{-0.6}) \cdot 10^{-4}$
$B \to K^* ll$	$\tilde{\mathcal{V}}_1(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	$(1.2^{+0.4}_{-0.4}) \cdot 10^{-3}$
	$\tilde{\mathcal{V}}_2(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	$(2.1^{+0.7}_{-0.7}) \cdot 10^{-3}$
	$\tilde{\mathcal{V}}_3(q^2 = 1)$	$1.0^{+1.6}_{-0.8}$	$(3.0^{+1.0}_{-1.0}) \cdot 10^{-3}$
$B_s \to \phi ll$	...	-	???

[q^2 is the dilepton mass square]

results represented as a q^2 dependent correction to $C9$

we fully reproduce the results given in KMWP2010

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matrix elements parametrized analogously to the form factors:

$$\langle K\,(k)|\tilde{O}_\mu(0,x)|B(q+k)\rangle = \left((k\cdot q)q_\mu - q^2 k_\nu\right)\tilde{\mathcal{A}}(q^2) + \ldots$$

$$\langle K^*(k,\eta)|\tilde{O}_\mu(0,x)|B(q+k)\rangle = \epsilon_{\mu\alpha\beta\gamma}\eta^{*\alpha}q^\beta k^\gamma\,\tilde{\mathcal{V}}_1(q^2) + i\left(\left(m_B^2 - m_{K^*}^2\right)\eta^*_\mu - (\eta^*\cdot k)(2k+q)_\mu\right)\tilde{\mathcal{V}}_2(q^2)$$
$$+ i(\eta^*\cdot q)\left(q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(2k+q)_\mu\right)\tilde{\mathcal{V}}_3(q^2) + \ldots$$

Why such different results?

different inputs: B-LCDAs models depend on λ_H^2, λ_E^2 $\rightarrow$

KMPW10:
$\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15\ \mathrm{GeV}^2$
$\Longrightarrow$ twist 3 does not contribute

we use $\lambda_E^2 = 0.03 \pm 0.02\ \mathrm{GeV}^2$
$\lambda_H^2 = 0.06 \pm 0.03\ \mathrm{GeV}^2$
$\Longrightarrow$ ~10 times smaller [Nishikawa/Tanaka '14]

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3-particle *B*-LCDAs heavy quark (twist) expansion $\rightarrow$

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Lorentz structures considered in $\langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$ $\rightarrow$

KMPW10: 4 Lorentz structures

all 8 independent Lorentz structures
$\Longrightarrow$partial cancelation (new structures come with an opposite sign)

Summary

- update the previous calculation to the soft gluon contributions to the charm-loop effect in $B \to K^{(*)}ll$ decays
- add results for $B_s \to \phi ll$
- numerical ~100 smaller comparing with KMPW2010 (inputs, Lorentz structures, twist expansion, new B-LCDAs models)
- results will be easily accessible in the open source software EOS (`https://github.com/eos/eos`)

EOS

Thank you!

Three-particle LCDAs models and $\lambda_{H,E}^2$

KMPW2010

$$\Psi_A(\omega_1,\omega_2) = \Psi_V(\omega_1,\omega_2) = \frac{\lambda_E^2}{6\lambda_B^4}\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$X_A = \frac{\lambda_E^2}{6\lambda_B^4}\omega_2(2\omega_1-\omega_2)e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$Y_A = -\frac{\lambda_E^2}{24\lambda_B^4}\omega_2(7\lambda_B-13\omega_1+3\omega_2)e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

Braun/Ji/Manashov

$$\Phi_3(\omega_1,\omega_2) = \frac{\lambda_E^2-\lambda_H^2}{6\lambda_B^4}\omega_1\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\Phi_4(\omega_1,\omega_2) = \frac{\lambda_E^2+\lambda_H^2}{6\lambda_B^4}\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\Psi_4(\omega_1,\omega_2) = \frac{\lambda_E^2}{3\lambda_B^4}\omega_1\omega_2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\tilde{\Psi}_4(\omega_1,\omega_2) = \frac{\lambda_H^2}{3\lambda_B^4}\omega_1\omega_2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$\lambda_{H,E}^2$ definition

$$\langle 0|\bar{d}(0)G_{\alpha\beta}(0)h_v(0)|B(v)\rangle = -\frac{i}{6}f_B\lambda_H^2\mathrm{Tr}[\gamma_5\Gamma P_+\sigma_{\alpha\beta}] - \frac{i}{6}f_B(\lambda_H^2-\lambda_E^2)\mathrm{Tr}[\gamma_5\Gamma P_+(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)]$$

Threshold s_0 determination

$$f_{K^{(*)}} e^{-\frac{m_{K^{(*)}}}{M^2}} \langle K^{(*)}(k) | \tilde{O}_\mu(0,x) | B(q+k) \rangle = f_B \int_0^{s_0} \mathrm{d}s\, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux) \qquad (\mathbf{1})$$

derive with respect to $1/M^2$
and divide by $(\mathbf{1})$

$$m_{K^{(*)}} = \frac{\int_0^{s_0} \mathrm{d}s\, s\, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux)}{\int_0^{s_0} \mathrm{d}s\, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux)}$$

daughter sum rule to extract s_0

Alignment of the gluon with the $K^{(*)}$ meson

We are interested in the dominant effect of the nonvanishing gluon momenta generated by the exponent in (3.9). Decomposing the covariant derivative in the light-cone vectors

$$\mathcal{D} = (n_+\mathcal{D})\frac{n_-}{2} + (n_-\mathcal{D})\frac{n_+}{2} + \mathcal{D}_\perp, \tag{3.10}$$

we retain only the n_- component, which corresponds to the gluons emitted antiparallel to q, that is, in the same direction as the s-quark in the B-meson rest frame. We then have

$$\begin{aligned} G^{\alpha\beta}(ux) &\simeq \exp[-iu(n_-x)\frac{(in_+\mathcal{D})}{2}]G^{\alpha\beta} \\ &= \int d\omega\, \exp[-iu(n_-x)\omega]\, \delta[\omega - \frac{(in_+\mathcal{D})}{2}]\, G^{\alpha\beta}\,. \end{aligned} \tag{3.11}$$

[...]

is represented in a compact unintegrated form, and we use the notation $\tilde{q} = q - u\omega n_-$, so that $\tilde{q}^2 \simeq q^2 - 2u\omega m_b$. Here we take into account that $\omega \ll m_b$, after the hadronic matrix element is taken. Note that the neglected components of $\mathcal{D}$ in (3.10) produce small, $O(\omega/m_b)$ corrections to $\tilde{q}^2$, hence our approximation is well justified.

[Khodjamirian et al. '10]

Where is the mistake?

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)n^{\beta}h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+\left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right]n^{\beta}\right\}(\boldsymbol{y},\boldsymbol{ux})$$

[Kawamura et al. '01]

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+\left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right]\right\}(\boldsymbol{y},\boldsymbol{ux})$$

[Khodjamirian et al. '06]