$|V_{us}|$ determinations using kaon decays

▶ $\Gamma(K \to \pi \ell \bar{\nu}_\ell [\gamma]) = \dfrac{G_F^2 m_K^5}{192\pi^3} \, C_K^2 \, S_{\mathsf{EW}}^K \, \left(|V_{us}|f_+^{K\pi}(0)\right)^2 \, I_K^\ell \, \left(1 + \delta_{\mathsf{EM}}^{K\ell} + \delta_{\mathsf{SU}(2)}^{K\pi}\right)^2$ \qquad $\boxed{K_{\ell 3}}$

▶ $\dfrac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \dfrac{|V_{us}|^2}{|V_{ud}|^2} \left(\dfrac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \dfrac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} \, (1 + \delta_{\mathsf{EM}})$ \qquad $\boxed{K_{\ell 2}}$

Notes

▶ Vus calculations require form factors and decay constants from lattice QCD

$|V_{us}|$ determinations using tau decays

▶ $$\frac{R(\tau \to X_{\text{strange}} \, \nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-strange}} \, \nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}},$$

$\boxed{\tau \to X_s \nu}$

Notes

▶ $R(\tau \to X) = \Gamma(\tau \to X) \, / \, \Gamma(\tau \to e\nu\overline{\nu})$

▶ Gamiz, Jamin, Pich, Prades, Schwab, JHEP 01 (2003) 06, PRL 94 (2005) 011803

  ▶ $\delta R_{\tau,\text{SU3 breaking}}$ computed with perturbative QCD (OPE, finite energy sum rules)
  ▶ requires m_s value (lattice QCD)

▶ $$\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2} \frac{R_{\tau/K}}{R_{\tau/\pi}} R_{K/\pi}$$

$\boxed{\tau \to K / \tau \to \pi}$

▶ $$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 R_{\tau/K} R_{K\mu 2}$$

$\boxed{\tau \to K}$

Notes

▶ radiative corrections and other details in HFLAV reports, tau section

Tau BRs best estimated with global fit of all relevant measurements

HFLAV Tau Spring 2017 fit, in HFLAV Summer 2016 report, Eur.Phys.J. C77 (2017)

Tau BRs Measurements

| experiment	number of results
ALEPH	39
CLEO	35
BaBar	23
OPAL	19
Belle	15
DELPHI	14
L3	11
CLEO3	6
TPC	3
ARGUS	2
HRS	2
CELLO	1
total	170

- ▶ 170 measurements, 88 constraint equations
- ▶ fit 135 quantities: 47 BRs, 88 derived quantities (ratios of linear combinations of BRs)
- ▶ $\chi^2/$d.o.f. $= 137/123$, CL $= 17.84\%$
- ▶ consistent with unitarity within 0.1% uncertainty, residual $= (0.0355 \pm 0.1031)\%$

- ▶ since 2016, adopted also for PDG

- ▶ most measurements systematically limited
  - ▶ better experimental conditions at Z^0 peak
  - ▶ moderate progress since ~2000
- ▶ B-factories improved many smaller BRs
- ▶ Belle II may contribute in near future

$\mathcal{B}(\tau \to X_s \nu)$ from HFLAV Spring 2017 fit

Tau decay mode	Branching fraction (%)
$K^- \nu_\tau$	0.6960 ± 0.0096
$K^- \pi^0 \nu_\tau$	0.4327 ± 0.0149
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0640 ± 0.0220
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0428 ± 0.0216
$\pi^- \bar{K}^0 \nu_\tau$	0.8386 ± 0.0141
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3812 ± 0.0129
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0022 ± 0.0008
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9087 ± 0.0482

$|V_{us}|$ from tau BRs using the HFLAV Spring 2017 fit
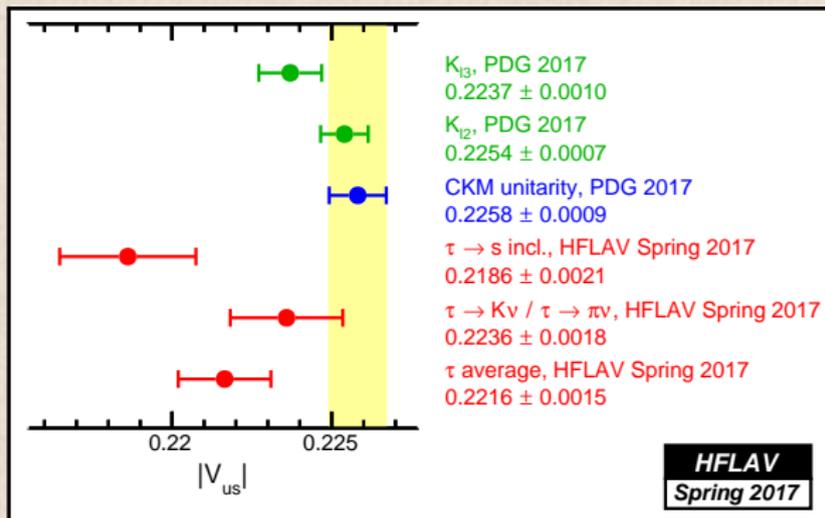
- $|V_{us}|(\tau \to X_s \nu)$ inclusive vs. $|V_{us}|$(CKM) unitarity discrepancy: $-3.1\,\sigma$
  - $m_s = 95.00 \pm 5.00\,\text{MeV}$ (PDG 2015), decay constants from FLAG 2016
  - $\delta R_\tau = 0.242 \pm 0.032$ (E. Gamiz *et al.*, arXiv:hep-ph/0612154v1)
- no calculation done for $|V_{us}|$ with $\tau \to K\nu$ method
- other details in HFLAV Spring 2017 report

Updates since HFLAV Spring 2017 $|V_{us}|$ determinations

- *BABAR* ICHEP 2018 preliminary measurements of
  - $\mathcal{B}(\tau \to K\, n\pi^0 \nu)$, $n = 0, 1, 2, 3$
  - $\mathcal{B}(\tau \to \pi\, n\pi^0 \nu)$, $n = 3, 4$
- *BABAR* Phys. Rev. D 98 (2018) no.3, 032010
  - $\mathcal{B}(\tau^- \to K^- K_S^0 \nu_\tau)$
- added $|V_{us}|$ determination with $\tau \to K\nu$ method
- FLAG 2019 lattice QCD averages
- in the following, preliminary elaborations for incoming HFLAV report

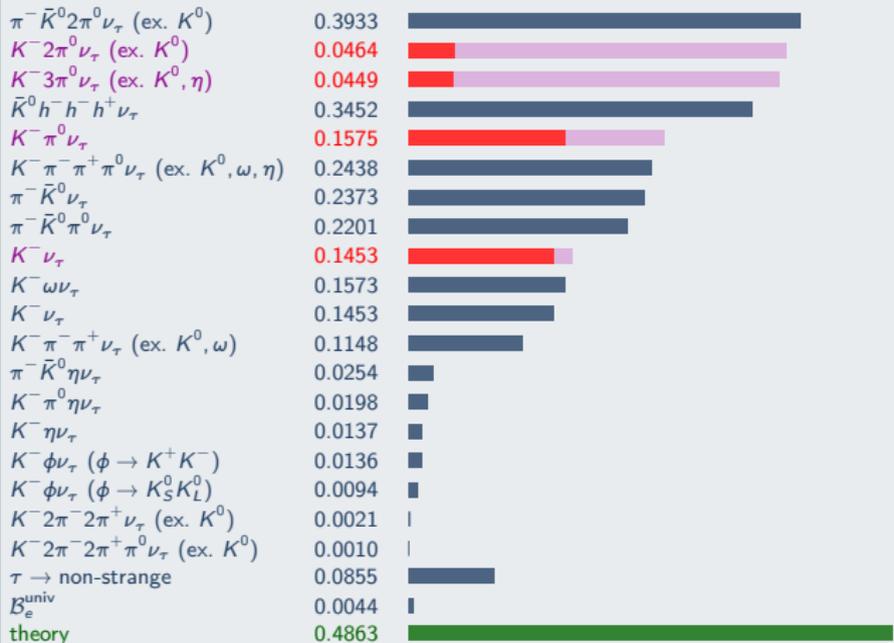## $\mathcal{B}(\tau \to X_s \nu)$, A.L. elaboration, EPS 2019

Tau decay mode	Branching fraction (%)
$K^- \nu_\tau$	0.6986 ± 0.0085
$K^- \pi^0 \nu_\tau$	0.4904 ± 0.0092
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0585 ± 0.0027
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0113 ± 0.0026
$\pi^- \bar{K}^0 \nu_\tau$	0.8378 ± 0.0139
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3807 ± 0.0129
$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.0235 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0154 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0022 ± 0.0008
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9308 ± 0.0412

Updated $|V_{us}|$ determinations with tau leptons
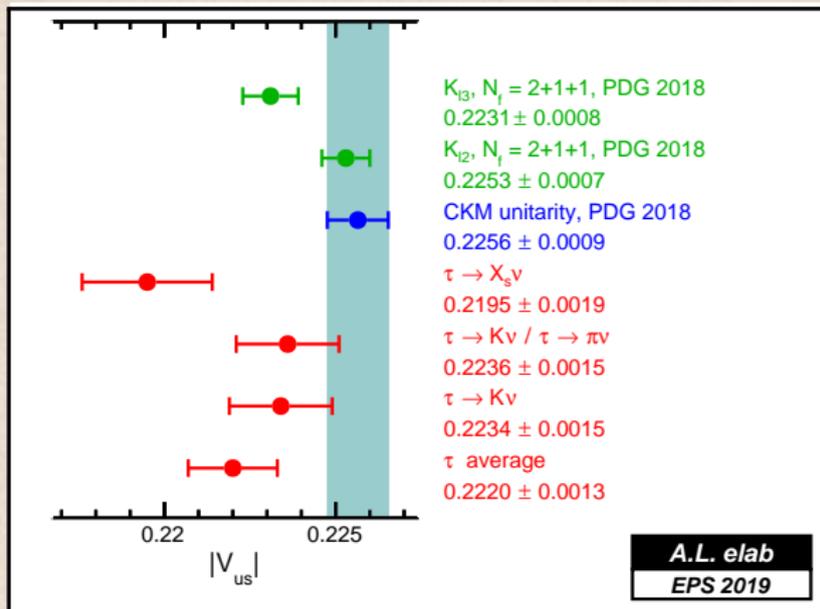
- $|V_{us}|(\tau \to X_s \nu)$ inclusive vs. $|V_{us}|$(CKM unitarity) discrepancy: $-2.9\,\sigma$
  - $m_s = 95.00 \pm 6.70$ MeV (PDG 2018), decay constants from FLAG 2019
  - $\delta R_\tau = 0.242 \pm 0.033$ (negligible variation since HFLAV Spring 2017 due to updated m_s)
- other details in incoming HFLAV report

$|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainties budget: improvements with BABAR 2018 results

Updated $|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainty budget	
$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.3933
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0464
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0449
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3452
$K^- \pi^0 \nu_\tau$	0.1575
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2438
$\pi^- \bar{K}^0 \nu_\tau$	0.2373
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2201
$K^- \nu_\tau$	0.1453
$K^- \omega \nu_\tau$	0.1573
$K^- \nu_\tau$	0.1453
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1148
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0254
$K^- \pi^0 \eta \nu_\tau$	0.0198
$K^- \eta \nu_\tau$	0.0137
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0094
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0855
$\mathcal{B}_e^{univ}$	0.0044
theory	0.4863

▶ shaded magenta bars report HFLAV Spring 2017 uncertainties before BABAR 2018 results

Uncertainty on $|V_{us}|$ from $\tau \to X_s \nu$ over time

Uncertainty on $|V_{us}|$ from $\tau \to X_s \nu_\tau$
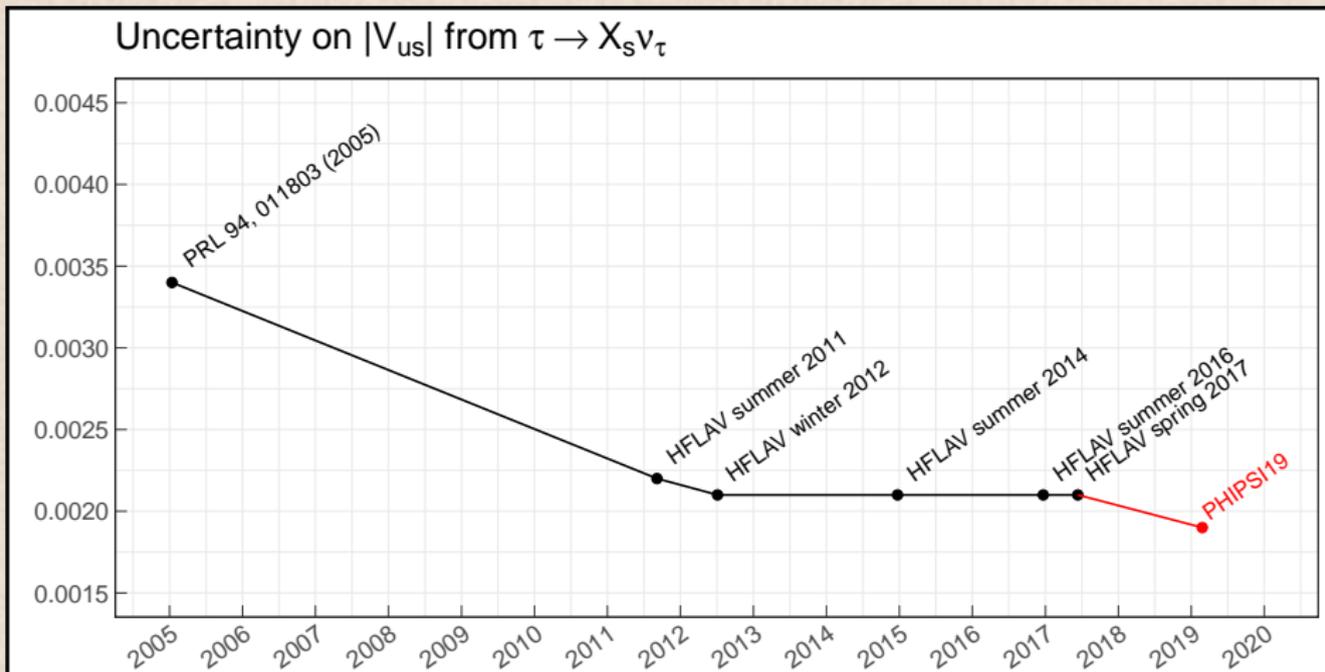
▶ $|V_{us}|$ from $\tau \to X_s \nu$ uncertainty reduced significantly for the 1st time since 2013

Other $|V_{us}|$ from $\tau \to X_s \nu$ determinations

M. Antonelli et al., JHEP 10 (2013) 76

▶ predict tau BRs $\begin{array}{l} \mathcal{B}(\tau \to K\nu) \\ \mathcal{B}(\tau \to K\pi^0\nu) \\ \mathcal{B}(\tau \to K_s^0\pi\nu) \end{array}$ from kaon BRs $\begin{array}{l} \mathcal{B}(K \to \ell\nu) \\ \mathcal{B}(K \to \ell\pi^0\nu) \end{array}$

▶ replace measurements of above tau branching fractions their predictions

▶ compute $|V_{us}|$ with Gamiz et al. technique

▶ other tau branching fractions from HFLAV 2012

A.L., SciPost Phys. Proc. 1 (2019) 1

▶ use Antonelli 2013 predictions of 3 tau branching fractions, but rather than replacing the respective tau measurements, statistically combine predictions and measurements in modified HFLAV tau BRs fit

▶ compute $|V_{us}|$ with Gamiz et al. technique

▶ other tau branching fractions from:
  ▶ HFLAV Spring 2017
  ▶ BABAR ICHEP 2018 results

Other $|V_{us}|$ from $\tau \to X_s \nu$ determinations

J. Hudspith *et al.*, PLB 781 (2018) 206

▶ revised technique, uses also tau spectral functions

"a combination of continuum and lattice results is shown to suggest a new implementation of the flavor-breaking sum rule approach in which not only |Vus|, but also D>4 effective condensates, are fit to data."

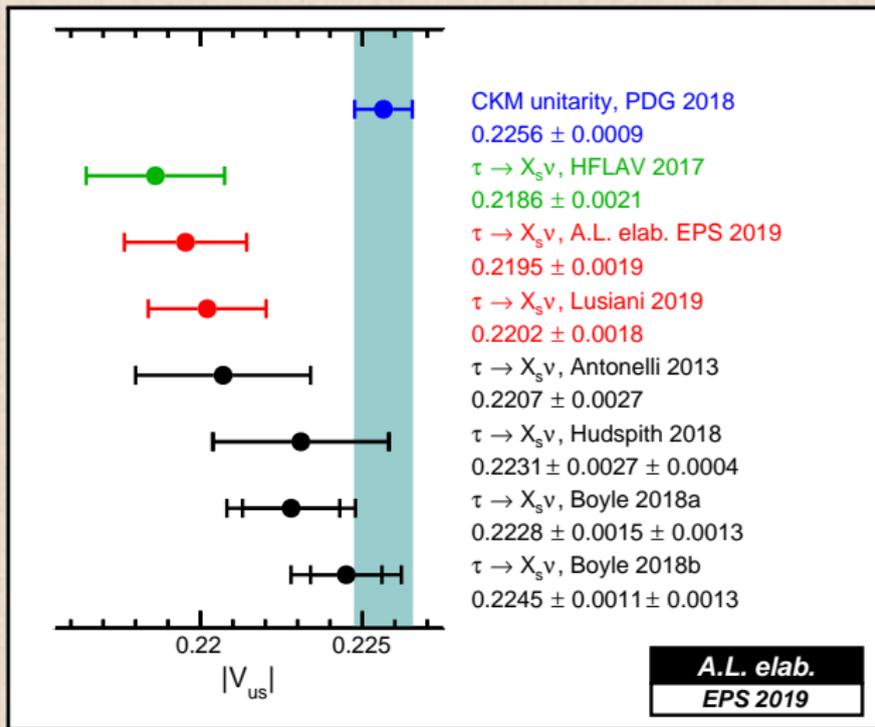▶ replace tau BRs $\quad \dfrac{\mathcal{B}(\tau \to K\pi^0\nu)}{\mathcal{B}(\tau \to K_s^0\pi\nu)} \quad$ with Antonelli 2013 predictions

▶ other tau branching fractions from HFLAV Spring 2017

Other $|V_{us}|$ from $\tau \to X_s \nu$ determinations

P. Boyle *et al.*, PRL 121 (2018) 202003

- compute $|V_{us}|$ from tau inclusive with a nove technique using
  - tau spectral functions
  - lattice QCD
- capitalizes on LQCD work for muon $g-2$ hadronic contribution
- two $|V_{us}|$ results:

  Boyle 2018a: using HFLAV Spring 2017 results

  Boyle 2018b: HFLAV Spring 2017 replacing $\begin{matrix} \mathcal{B}(\tau \to K\pi^0\nu) \\ \mathcal{B}(\tau \to K_s^0\pi\nu) \end{matrix}$ with Antonelli 2013

$|V_{us}|$ from $\tau \to X_s\nu$ determinations

CKM unitarity, PDG 2018
0.2256 ± 0.0009

$\tau \to X_s\nu$, HFLAV 2017
0.2186 ± 0.0021

$\tau \to X_s\nu$, A.L. elab. EPS 2019
0.2195 ± 0.0019

$\tau \to X_s\nu$, Lusiani 2019
0.2202 ± 0.0018

$\tau \to X_s\nu$, Antonelli 2013
0.2207 ± 0.0027

$\tau \to X_s\nu$, Hudspith 2018
$0.2231 \pm 0.0027 \pm 0.0004$

$\tau \to X_s\nu$, Boyle 2018a
$0.2228 \pm 0.0015 \pm 0.0013$

$\tau \to X_s\nu$, Boyle 2018b
$0.2245 \pm 0.0011 \pm 0.0013$

A.L. elab.
EPS 2019

Radiative corrections for $|V_{us}|$ from $\tau \to K\nu$ determination

New in incoming HFLAV report

- $\Gamma(\tau^- \to K^- \nu_\tau) = \dfrac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 m_\tau^3 \left(1 - \dfrac{m_K^2}{m_\tau^2}\right)^2 R_{\tau/K} R_{K\mu2}$

- $R_{\tau/K} = \dfrac{R(\tau \to K\nu)}{R(K \to \mu\nu)} = 1 + (0.90 \pm 0.22)\%$
  - W.J. Marciano, A. Sirlin, Phys. Rev. Lett. 71 (1993) 3629
  - R. Decker and M. Finkemeier, Phys. Lett. B334 (1994) 199
  - R. Decker and M. Finkemeier, Nucl. Phys. B438 (1995) 17
  - R. Decker and M. Finkemeier, Nucl. Phys. Proc. Suppl. 40 (1995) 453

- $R_{K\mu2} = R(K \to \mu\nu) = 1 + (1.07 \pm 0.21)\%$
  - V. Cirigliano and H. Neufeld, Phys. Lett. B700 (2011) 7
    ($R_{K\mu2} = R(\tau \to K\nu)$ computed using the paper formulas and up-to-date experimental inputs by J. L. Rosner, S. Stone, and R. S. Van de Water for the PDG 2015 review on "Leptonic Decays of Charged Pseudoscalar Mesons")

- note:
  - HFLAV Summer 2014: only short distance radiative correction included
  - HFLAV Spring 2017: no determination done

Recent new calculations of $R_{K/\pi}$ and $R_{K\mu2}$

M. Di Carlo *et al.*, arXiv:1904.08731v1 [hep-lat]

- $R_{K/\pi}^{\text{phys}} = R(K \to \mu\nu/\pi \to \mu\nu)^{\text{phys}} = 1 + (-1.29 \pm 0.14)\%$
- $R_{K\mu2}^{\text{phys}} = R(K \to \mu\nu)^{\text{phys}} = 1 + (0.24 \pm 0.10)\%$
- R_x^{phys} includes isospin-breaking correction, must use with isospin-symmetric f_K/f_π, f_K
- R_x does not include isospin-breaking correction, must use with $f_{K\pm}/f_{\pi\pm}$, $f_{K\pm}$
- only products like $f_K^2 R_{K\mu2}$ are well defined physical quantities

paper	$(f_K/f_\pi)^2 \cdot R_{K/\pi}$	$f_K^2 \cdot R_{K\mu2}$
Cirigliano Neufeld 2011 + FLAG 2019	1.4139 ± 0.0051	24502 ± 107
Di Carlo *et al.* 2019 + FLAG 2019	1.4134 ± 0.0037	$24429 \pm\ \ 70$

- new radiative corrections consistent with previous ones
- negligible improvements on $|V_{us}|$ from τ because of larger experimental uncertainties

Conclusions

- *BABAR* 2018 results improve precision on $|V_{us}|$ from $\tau \to X_s \nu$
- persisting $\sim 3\sigma$ discrepancy on CKM unitarity with $|V_{us}|$ from $\tau \to X_s \nu$
- recent preprint on radiative corrections used for $|V_{us}|$ from τ and K decays
- more details will be documented on incoming HFLAV report

Backup Slides

$|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainties bugdet: before and after the BABAR 2018 results

$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.3963
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.3789
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.3715
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3478
$K^- \pi^0 \nu_\tau$	0.2561
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2456
$\pi^- \bar{K}^0 \nu_\tau$	0.2424
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2219
$K^- \nu_\tau$	0.1646
$K^- \omega \nu_\tau$	0.1585
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1157
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0256
$K^- \pi^0 \eta \nu_\tau$	0.0200
$K^- \eta \nu_\tau$	0.0138
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0138
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0096
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0896
$\mathcal{B}_e^{univ}$	0.0045
theory	0.4722

$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.3933
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3452
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2438
$\pi^- \bar{K}^0 \nu_\tau$	0.2373
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2201
$K^- \pi^0 \nu_\tau$	0.1575
$K^- \omega \nu_\tau$	0.1573
$K^- \nu_\tau$	0.1453
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1148
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0464
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0449
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0254
$K^- \pi^0 \eta \nu_\tau$	0.0198
$K^- \eta \nu_\tau$	0.0137
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0094
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0855
$\mathcal{B}_e^{univ}$	0.0044
theory	0.4863