Implications for New Physics in $b \rightarrow s \mu \mu$ transitions after recent measurements by Belle and LHCb

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based on the work with Kamila Kowalska, Enrico Maria Sessolo
arXiv : 1903.10932

July 13, 2019
Deviations from SM in $b \to s$ sector

\[ R_K = \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)} \]

\[ R_{K^*} = \frac{\mathcal{B}(B \to K^* \mu^+ \mu^-)}{\mathcal{B}(B \to K^* e^+ e^-)} \]


$\sim 2.5\sigma$

$(\text{JHEP 09 (2015) 179})$

$\sim 3.1\sigma$

$LHCb$

$(\text{JHEP 1708 (2017) 055})$

$\sim 2.5\sigma$

$(\text{ATLAS (JHEP 10 (2018) 047)})$

$\sim 3.0\sigma$

$\sim 3.0\sigma$

$LHCb$

$LHCb$
New measurement of $R_{K^*}$ by Belle

Consistent with SM with large uncertainties.  Belle arXiv 1904.02440
Effective field theory analysis

The effective Hamiltonian for $b \to sll$ transitions

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum (C_i^l O_i^l + C_i^{l'} O_i^{l'}) + h.c.$$ 

We assume the presence of NP in semileptonic operators:

$$O_9^l = (\bar{s}_L \gamma^\mu b_L)(\bar{l}_\gamma \gamma^\mu l), \quad O_9^{l'} = (\bar{s}_R \gamma^\mu b_R)(\bar{l}_\gamma \gamma_{55} l)$$

$$O_{10}^l = (\bar{s}_L \gamma^\mu b_L)(\bar{l}_\gamma \gamma_{55} l), \quad O_{10}^{l'} = (\bar{s}_R \gamma^\mu b_R)(\bar{l}_\gamma \gamma_{55} l)$$

NP in scalar and pseudoscalar operators $O_S^{(')}$ and $O_{P}^{(')}$ are severely contrained by the $B_s \to \mu^+ \mu^-$ measurements. R. Alonso et al. PRL 113(2014) 241802, W. Altmannshofer et al. JHEP 05 (2017) 076.

NP in electromagnetic dipole operator $O_7^{(')}$ is tightly constrained by radiative decays. A. Paul et al. JHEP 04 (2017) 027
Several fits after the measurement of $R_K$ and $R_{K^*}$


- We consider the NP in muon or muon and electron.
- Global fit with all the relevant data in $b \rightarrow s\mu\mu$ and $b \rightarrow see$.
- We performed global fits with 1, 2, 4 and 8 independent input parameters, plus a nuisance parameter, $V_{cb}$.
- also make contact with frequentist approach with best fit values and pull from SM.

see also talk by J. Aebischer
Fit methodology

- Bayes’s theorem: \[ p(m|d) = \frac{p(d|\xi(m))\pi(m)}{p(d)} \]
- The Likelihood function is defined as
  \[ L(m) = \exp \left\{ -\frac{1}{2} \left[ O_{\text{th}}(m) - O_{\exp} \right]^T (C_{\exp} + C^{\text{th}})^{-1} \left[ O_{\text{th}}(m) - O_{\exp} \right] \right\} \]
- evidence: comparison of different models by computing the Bayes factor \( p(d)_{\mathcal{M}_1}/p(d)_{\mathcal{M}_2} \).
- We estimate the significance of Bayes factors according to Jeffery’s scale.
140 observables included in the likelihood function \((B^0 \to K^{*0}l^+l^-, B^+ \to K^0\mu^+\mu^-, B^0 \to K^+l^+l^-, B^+_s \to \phi\mu^+\mu^-, \Lambda_b \to \Lambda\mu^+\mu^-, BR(B_s \to X_s l^+l^-), BR(B_s \to \mu^+\mu^-)\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_9^\mu)</td>
<td>((-3, 3))</td>
<td>Flat</td>
</tr>
<tr>
<td>(C_9^\mu = -C_{10}^\mu)</td>
<td>((-3, 3))</td>
<td>Flat</td>
</tr>
<tr>
<td>(C_9^\mu, C_{10}^\mu)</td>
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<tr>
<td>(C_9^{\prime\mu}, C_{10}^{\prime\mu}, C_9^{\prime\epsilon}, C_{10}^{\prime\epsilon})</td>
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<td>Flat</td>
</tr>
<tr>
<td>(m_{Z'}/g_X, M_Q/\lambda_Q, M_D/\lambda_D)</td>
<td>(500–5000\text{GeV})</td>
<td>Log</td>
</tr>
<tr>
<td>(m_{Z'}/g_X, M_Q/\lambda_Q, M_E/\lambda_E, 2)</td>
<td>(500–5000\text{GeV})</td>
<td>Log</td>
</tr>
<tr>
<td>Nuisance parameter</td>
<td>Central value, error ((\times10^{-2}))</td>
<td></td>
</tr>
<tr>
<td>CKM matrix element (V_{cb})</td>
<td>((4.22, 0.08))</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>
$C_{9}^{\mu} = -C_{10}^{\mu}$

$$\begin{align*}
\text{Posterior pdf} & \quad \begin{cases} 
-\ln Z = 77.3, \text{ pull } = 4.7\sigma(5.0\sigma) \\
-\ln Z = 77.5, \text{ pull } = 4.8\sigma(5.3\sigma)
\end{cases} \\
\text{Bayes factor} & \quad \frac{Z_{C_{9}^{\mu}}}{Z_{C_{9}^{\mu} = -C_{10}^{\mu}}} = 1.2(1/4).
\end{align*}$$
-\ln Z = 77.6, \text{ pull} = 4.7\sigma (5.3\sigma)

\[ R_K = R_{K*} \approx 1 + 0.24(C_9^\mu - C_{10}^\mu) \]

tension arises between these two measurements.

Bayes factor

\[ \frac{Z(C_9^\mu, C_9'^\mu)}{Z(C_9^\mu, C_{10}^\mu)} = 6 \text{ (positive)} \]

-\ln Z = 75.8, \text{ pull} = 5.0\sigma (5.2\sigma)

\[ R_K \approx 1 + 0.24(C_9^\mu + C_9'^\mu) \]

\[ R_{K*} \approx 1 + 0.24C_9^\mu - 0.17C_9'^\mu \]
4 vs. 2 wilson coefficients

\[-\ln Z = 76.0, \text{ pull} = 5.1\sigma\]

\[R_K \approx 1 + 0.24(C_9^\mu - C_{10}^\mu + C_9'^\mu - C_{10}'^\mu)\]  
\[R_K^* \approx 1 + 0.24(C_9^\mu - C_{10}^\mu) - 0.17(C_9'^\mu - C_{10}'^\mu)\]

larger negative \(C_9^\mu\) w.r.t. \((C_9^\mu, C_{10}^\mu)\)  
\(C_9'^\mu \leq 0\) is allowed w.r.t. \((C_9^\mu, C_{10}'^\mu)\)

\[\frac{Z(C_9^\mu, C_{10}^\mu, C_9'^\mu, C_{10}'^\mu)}{Z(C_9^\mu, C_{10}^\mu)} = 5.0\ (\text{positive}).\]

\[\frac{Z(C_9^\mu, C_9'^\mu)}{Z(C_9^\mu, C_{10}^\mu, C_9'^\mu, C_{10}'^\mu)} = 1.2\ (\text{barely worth mentioning}).\]
4 vs. 2 wilson coefficients

\[ (C_{9}^{\mu}, C_{10}^{\mu}) \text{ & } (C_{9}^{\mu}, C_{10}^{\mu}, C_{9}^{e}, C_{10}^{e}) \]

\[ (C_{9}^{\mu}, C_{9}^{\prime \mu}) \text{ & } (C_{9}^{\mu}, C_{9}^{\prime \mu}, C_{9}^{e}, C_{9}^{\prime e}) \]
pdf of the Wilson coefficients of the electron sector remain consistent with zero at 2σ.

The global data set can be easily explained by the presence of NP in the muon sector only.
limited impact the Wilson coefficients of the electron sector bring to the fit.
Heavy $Z'$

- The most generic Lagrangian, parametrizing LFUV couplings of $Z'$ to the $b$-$s$ current and the muons

$$\mathcal{L} \supset Z'_\alpha \left( \Delta^{sb}_L \bar{s}_L \gamma^\alpha b_L + \Delta^{sb}_R \bar{s}_R \gamma^\alpha b_R + \text{H.c.} \right) + Z'_\alpha \left( \Delta^{\mu\mu}_L \bar{\mu}_L \gamma^\alpha \mu_L + \Delta^{\mu\mu}_R \bar{\mu}_R \gamma^\alpha \mu_R \right).$$

- The relevant Wilson coefficients are then given by

$$C^{\mu}_{9,\text{NP}} = -\frac{\Delta^{sb}_L (\Delta^{\mu\mu}_L + \Delta^{\mu\mu}_R)}{V_{tb} V_{ts}^*} \left( \frac{\Lambda_v}{m_{Z'}} \right)^2,$$

$$C^{\mu}_{10,\text{NP}} = -\frac{\Delta^{sb}_R (\Delta^{\mu\mu}_L - \Delta^{\mu\mu}_R)}{V_{tb} V_{ts}^*} \left( \frac{\Lambda_v}{m_{Z'}} \right)^2,$$

$$C'_{9,\text{NP}} = -\frac{\Delta^{sb}_L (\Delta^{\mu\mu}_L + \Delta^{\mu\mu}_R)}{V_{tb} V_{ts}^*} \left( \frac{\Lambda_v}{m_{Z'}} \right)^2,$$

$$C'_{10,\text{NP}} = -\frac{\Delta^{sb}_R (\Delta^{\mu\mu}_L - \Delta^{\mu\mu}_R)}{V_{tb} V_{ts}^*} \left( \frac{\Lambda_v}{m_{Z'}} \right)^2.$$

- If the heavy $Z'$ is the gauge boson of a new $U(1)_X$ gauge group, its couplings to the gauge eigenstates must be flavor-conserving, and an additional structure is required to generate $\Delta^{sb}_L$ and $\Delta^{sb}_R$.

- we also consider the impact of the new LHCb and Belle data on the masses and couplings of a few simplified but UV complete models.
Heavy $Z'$ with $L_\mu - L_\tau$ symmetry


- SM leptons carry an additional charge in $L_\mu - L_\tau$ model $(SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X)$

\[
\begin{align*}
  l_1 & : (1, 2, -1/2, 0) \\
  l_2 & : (1, 2, -1/2, 1) \\
  l_3 & : (1, 2, -1/2, -1) \\
  e_R & : (1, 1, 1, 0) \\
  \mu_R & : (1, 1, 1, -1) \\
  \tau_R & : (1, 1, 1, 1).
\end{align*}
\]


\[
\begin{align*}
  S & : (1, 1, 0, -1), \\
  Q & : (3, 2, 1/6, -1) \\
  Q' & : (\bar{3}, 2, -1/6, 1), \\
  D & : (\bar{3}, 1, 1/3, -1) \\
  D' & : (3, 1, -1/3, 1).
\end{align*}
\]

\[
\mathcal{L} \supset (-\lambda_{Q,i}SQ'q_i - \lambda_{D,i}SD'd_{R,i} + \text{H.c.}) - M_Q Q' Q - M_D D' D,
\]

\[
C^\mu_{g,\text{NP}} = -\frac{2\Lambda_v^2}{V_{tb}V_{ts}^*} \frac{\lambda_{Q,2}\lambda_{Q,3}}{2M_Q^2 + \left(\lambda_{Q,2}^2 + \lambda_{Q,3}^2\right) v_S^2}, \\
C'_{g,\text{NP}} = -\frac{2\Lambda_v^2}{V_{tb}V_{ts}^*} \frac{\lambda_{D,2}\lambda_{D,3}}{2M_D^2 + \left(\lambda_{D,2}^2 + \lambda_{D,3}^2\right) v_S^2},
\]

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Model 2: $Z'$ + a scalar singlet $S$ + one pair of VL quarks + one pair of VL leptons

$S : (1, 1, 0, -1)$, $Q : (3, 2, 1/6, -1)$, $Q' : (\bar{3}, 2, -1/6, 1)$, $E : (1, 1, 1, 0)$, $E' : (1, 1, -1, 0)$.

\[
\mathcal{L} \supset \left( -\lambda_{E,2} S^* E' \mu_R - \lambda_{E,3} S E' \tau_R - \tilde{Y}_E \phi^\dagger l_1 E + \text{H.c.} \right) - M_E E' E ,
\]

\[
C_{9,\text{NP}}^\mu = \frac{\Lambda_v^2}{V_{tb} V_{ts}^*} \left( \frac{\lambda_{Q,2} \lambda_{Q,3}}{2 M_Q^2 + (\lambda_{Q,2}^2 + \lambda_{Q,3}^2) v_S^2} \right) \left( 1 + \frac{2 M_E^2}{2 M_E^2 + \lambda_{E,2}^2 v_S^2} \right) ,
\]

\[
C_{10,\text{NP}}^\mu = \frac{\Lambda_v^2}{V_{tb} V_{ts}^*} \left( \frac{\lambda_{Q,2} \lambda_{Q,3}}{2 M_Q^2 + (\lambda_{Q,2}^2 + \lambda_{Q,3}^2) v_S^2} \right) \left( -1 + \frac{2 M_E^2}{2 M_E^2 + \lambda_{E,2}^2 v_S^2} \right) .
\]
 VLQ mass range is determined by the 2σ range in $C_{9, NP}^{\mu}$.

 VLD and VLE masses are unbounded from the above at the 2σ level ($C_{9, NP}^{\prime \mu}$ in Model 1 and especially $C_{10, NP}^{\mu}$ in Model 2 are consistent with the zero at the 2σ level).

 $m_{Z'}/g_X$ is limited to values below 5 TeV, as a result of the $B_s$ mixing constraint.
Results of $Z'$ model

<table>
<thead>
<tr>
<th>$Z'$ + VL</th>
<th>$-\ln \mathcal{Z}$</th>
<th>Pull</th>
<th>$m_{Z'}/g_X$ (TeV)</th>
<th>$M_Q/\lambda_Q$ (TeV)</th>
<th>$M_{VL}/\lambda_{VL}$ (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>78.4</td>
<td>4.5 $\sigma$</td>
<td>0.7 TeV</td>
<td>24.4 TeV</td>
<td>34.2 TeV</td>
</tr>
<tr>
<td>Model 2</td>
<td>80.0</td>
<td>4.1 $\sigma$</td>
<td>0.7 TeV</td>
<td>24.7 TeV</td>
<td>0.5 TeV</td>
</tr>
</tbody>
</table>

$\frac{\mathcal{Z}_{model1}}{\mathcal{Z}_{model2}} = 5.0$ (positive)
Global Bayesian analysis of NP effects with new measurements of $R_K$ and $R_K^{(*)}$ in Morionod 2019.

$R_K$ shifts closer to SM predictions hence $(C_9, C_{10})$ shifts slightly towards zero.

The impact of the Wilson coefficients of the electron sector on the data is negligible w.r.t. to muon sector.

$(C_9^\mu, C_9'^\mu)$ and $(C_9^\mu, C_{10}^\mu, C_9'^\mu, C_{10}'^\mu)$ are slightly favored by the data.
Thank You
<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>$-\ln \mathcal{Z}$</th>
<th>Best fit</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SM</strong></td>
<td>88.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>88.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$C_9^{\mu}$</td>
<td>75.8</td>
<td>-0.90</td>
<td>5.0$\sigma$</td>
</tr>
<tr>
<td></td>
<td>77.3</td>
<td>-0.64</td>
<td>4.3$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu} = -C_{10}^{\mu}$</td>
<td>74.4</td>
<td>-0.64</td>
<td>5.3$\sigma$</td>
</tr>
<tr>
<td></td>
<td>77.5</td>
<td>-0.48</td>
<td>4.8$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_{10}^{\mu}$</td>
<td>74.5</td>
<td>-0.91</td>
<td>5.3$\sigma$</td>
</tr>
<tr>
<td></td>
<td>77.6</td>
<td>-0.78</td>
<td>4.7$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_9^{\tau\mu}$</td>
<td>75.1</td>
<td>(-1.08,0.49)</td>
<td>5.2$\sigma$</td>
</tr>
<tr>
<td></td>
<td>75.8</td>
<td>(-1.03,0.53)</td>
<td>5.0$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_{10}^{\mu}, C_9^{\tau\mu}, C_{10}^{\tau\mu}$</td>
<td>74.0</td>
<td>(-1.14,0.28,0.21,-0.31)</td>
<td>5.4$\sigma$</td>
</tr>
<tr>
<td></td>
<td>76.0</td>
<td>(-1.06,0.18,0.18,-0.34)</td>
<td>5.2$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_{10}^{\mu}, C_9^{e}, C_{10}^{e}$</td>
<td>75.6</td>
<td>(-0.92,0.40,-1.50,-0.90)</td>
<td>4.9$\sigma$</td>
</tr>
<tr>
<td></td>
<td>78.0</td>
<td>(-0.88,0.34,-1.69,-0.71)</td>
<td>4.5$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_9^{\tau\mu}, C_9^{e}, C_9^{\tau\mu}$</td>
<td>75.8</td>
<td>(-1.02,0.54,0.58,-0.17)</td>
<td>4.9$\sigma$</td>
</tr>
<tr>
<td></td>
<td>77.7</td>
<td>(-0.97,0.55,0.34,-0.17)</td>
<td>4.6$\sigma$</td>
</tr>
<tr>
<td>$C_9^{\mu}, C_9^{e}, C_{10}^{\mu}, C_{10}^{e}$</td>
<td>76.2</td>
<td>(-1.10,0.21,0.21,-0.30,-0.63,-0.73,-0.57)</td>
<td>4.7$\sigma$</td>
</tr>
<tr>
<td></td>
<td>78.3</td>
<td>(-1.05,0.13,0.10,-0.38,-2.18,-0.07,-2.73,-1.34)</td>
<td>4.4$\sigma$</td>
</tr>
</tbody>
</table>

**Table:** Evidence, best fit and pull from the SM of the considered scenarios.
The approximate formulae for $R_K$ and $R_{K^*}$ with real Wilson coefficients and the polarization fraction of the $K^*$ meson set at $p = 0.86$

\[
R_K \approx 1 + 0.24 \left( C_9^\mu - C_10^\mu + C_9'^\mu - C_10'^\mu \right) - (\mu \rightarrow e),
\]
\[
R_{K^*} \approx 1 + 0.24 \left( C_9^\mu - C_10^\mu \right) - 0.17 \left( C_9'^\mu - C_10'^\mu \right) - (\mu \rightarrow e).
\]