Intrinsic quantum mechanics behind the Standard Model?

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Abstract

We suggest the gauge groups SU(3), SU(2) and U(1) to share a common origin in U(3).

We take the Lie group U(3) to serve as an intrinsic configuration space for baryons. A spontaneous symmetry break in the baryonic state selects a U(2) subgroup for the Higgs mechanism. The Higgs field enters the symmetry break to relate the strong and electroweak energy scales by exchange of one quantum of action between the two sectors. This shapes the Higgs potential to fourth order.

Recently intrinsic quantum mechanics has given a suggestion for the Cabibbo angle from theory (EPL124-2018) and a prediction for the Higgs couplings to gauge bosons (EPL125-2019). Previously it has given the nucleon mass and the parton distribution functions for u and d quarks in the proton (EPL102-2013). It has given a quite accurate equation for the Higgs mass in closed form (IJMPA30-2015) and an N and Delta spectrum essentially without missing resonances (arXiv:1109.4732).

The intrinsic space is to be distinguished from an interior space. The intrinsic space is non-spatial, i.e. no gravity in intrinsic space. The configuration variable is like a generalized spin variable excited from laboratory space by kinematic generators: momentum, spin and Laplace-Runge-Lenz operators.

The baryon dynamics resides in a Hamiltonian on U(3) and projects to laboratory space by the momentum form of the wavefunction. The momentum form generates conjugate quark and gluon fields. Local gauge invariance in laboratory space follows from unitarity of the configuration variable and left-invariance of the coordinate fields in the intrinsic space.

Future work should aim to invoke leptons in the second and third generations and quarks in the third.

Intrinsic space for baryon mass states

We use a reinterpreted Kogut-Susskind Hamiltonian with a Manton potential. We interpret it as describing the intrinsic dynamics for baryon mass states. Thus the Lie group U(3) is treated as intrinsic configuration space

$$\boxed{\frac{\hbar c}{a}[-\tfrac{1}{2}\Delta+\tfrac{1}{2}\mathrm{Tr}\,\chi^2]\Psi(u)=E\Psi(u)} \qquad \boxed{u=e^{i(\theta_j T_j+(\alpha_j S_j+\beta_j M_j)/\hbar)}\equiv e^{i\chi},\quad j=1,2,3} \qquad (1)$$

It is the hypothesis of the present work, that the eigenstates of the above Schrödinger equation describe the baryon mass spectrum with u being the configuration variable of an entire baryonic entity and a is a scale settled by the classical electron radius

$$\pi a = r_e \equiv \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \rightarrow \Lambda \equiv \frac{\hbar c}{a} = \frac{\pi}{\alpha} m_e c^2, \qquad x_j = a\theta_j, \qquad iT_j = \frac{\partial}{\partial\theta_j}$$

The potential is half the squared geodetic distance from the 'point' u to the 'origo' e

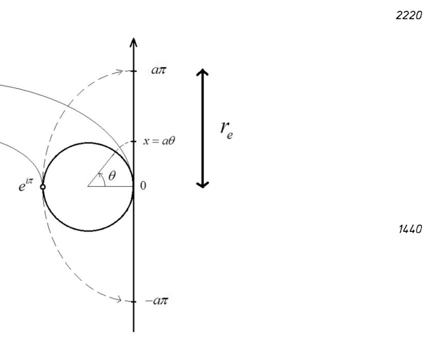
$$\frac{1}{2}\mathrm{Tr}\,\chi^2 = w(\theta_1)+w(\theta_2)+w(\theta_3) = \frac{1}{2}d^2(e,u) \qquad (2)$$

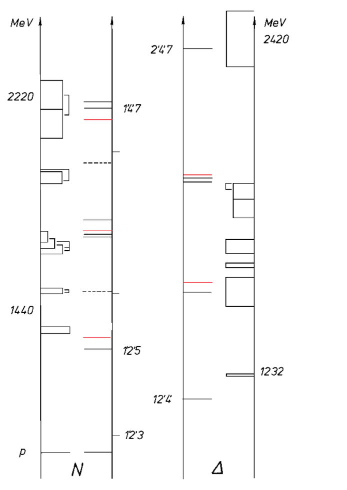
where $e^{i\theta_j}$ are the eigenvalues of u. The potential is periodic in parameter space

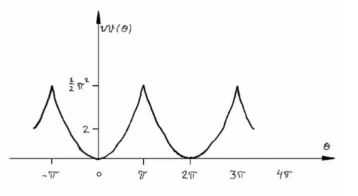
$$w(\theta)=\frac{1}{2}(\theta-q\cdot 2\pi)^2,\ \theta\in[(2q-1)\pi,(2q+1)\pi],\ q\in Z$$

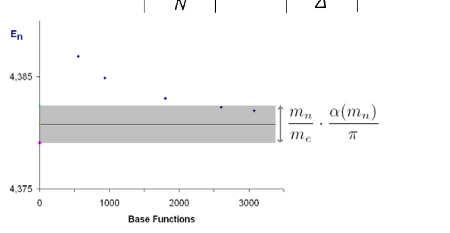
We find exact solutions for alleged N-states and approximate solutions for both alleged N-states and Δ-states. From the ground state eigenvalue $\mathrm{E_n}=E_n/\Lambda$ we get

$$\frac{m_e}{m_n}=\frac{\alpha}{\pi}\frac{1}{\mathrm{E_n}}=\frac{1}{1839(1)}$$









The theory unfolded

The Laplacian in (1) contains off-toroidal derivatives which are represented by the off-diagonal Gell-Mann matrices. We choose three of these to represent spin and group them into $\mathbf{S}=(S_1,S_2,S_3)$. This interpretation is supported by their commutation relations as body fixed angular momentum. The relation between space and intrinsic space is like the relation in nuclear physics between fixed coordinate systems and intrinsic body fixed coordinate systems for the description of rotational degrees of freedom. The remaining three off-toroidal derivatives are grouped into $\mathbf{M}=(M_1,M_2,M_3)$, which is related to hypercharge and isospin. The Laplacian in polar decomposition thus reads

$$\Delta=\sum_{j=1}^{3}\frac{1}{J^2}\frac{\partial}{\partial\theta_j}J^2\frac{\partial}{\partial\theta_j}-\sum_{\substack{i<j\\k\neq i,j}}^{3}\frac{(S_k^2+M_k^2)/\hbar^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}$$

$$J=\prod_{i<j}^{3}2\sin\tfrac{1}{2}(\theta_i-\theta_j)$$

$$\boxed{p_j=-i\hbar\frac{1}{a}\frac{\partial}{\partial\theta_j}=\frac{\hbar}{a}T_j,\qquad [p_j,a\theta_i]=-i\hbar\delta_{ij}}$$

$$\boxed{[M_i,M_j]=[S_i,S_j]=-i\hbar\varepsilon_{ijk}S_k}$$

$$S_1=a\theta_2p_3-a\theta_3p_2=\hbar\lambda_7$$

$$M_1/\hbar=\theta_2\theta_3+\frac{a^2}{\hbar^2}p_2p_3=\lambda_6$$

The off-torus term is analogous to the centrifugal term in the usual treatment of the radial wave function for the hydrogen atom

$$-\frac{\hbar}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}-\frac{1}{r^2}\mathbf{L}^2\right]\psi(r,\theta,\varphi)+V(r)\psi(r,\theta,\varphi)=E\psi(r,\theta,\varphi) \qquad \psi(r,\theta,\varphi)=R(r)Y(\theta,\varphi)$$

With the periodic potential in (2) the complete Schrödinger equation reads with $\mathrm{E}=E/\Lambda$ and $\Lambda\equiv\hbar c/a=214.27\,\mathrm{MeV}$

$$\left[-\tfrac{1}{2}\left(\sum_{j=1}^{3}\frac{1}{J}\frac{\partial^2}{\partial\theta_j^2}J+2-\sum_{\substack{i<j\\k\neq i,j}}^{3}\frac{(S_k^2+M_k^2)/\hbar^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}\right)+w(\theta_1)+w(\theta_2)+w(\theta_3)\right]\Psi(u)=\mathrm{E}\Psi(u). \qquad (3)$$

The constant term in the Laplacian is interpreted as a global curvature potential from differentiating through J.
A factorization of $\Psi(u)=\tau(\theta_1,\theta_2,\theta_3)\cdot\Upsilon(\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9)$ gives for $\Phi(u)=R(\boldsymbol{\theta})\cdot\Upsilon$ with $R(\boldsymbol{\theta})=J(\boldsymbol{\theta})\cdot\tau(\boldsymbol{\theta})$

$$[-\Delta_e+W]R(\theta_1,\theta_2,\theta_3)=2\mathrm{E}\,R(\theta_1,\theta_2,\theta_3)$$

where $\Delta_e=\sum_{j=1}^{3}\frac{\partial^2}{\partial\theta_j^2}$ and $W=-2+\frac{1}{3}(S(S+1)+M^2)\sum_{i<j}^{3}\frac{1}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}+2(w(\theta_1)+w(\theta_2)+w(\theta_3))$. Now R can be expanded on Slater determinants constructed from 1D eigenstates of

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\theta^2}+w(\theta)\right]\varphi_i(\theta)=e_i\varphi_i(\theta)$$

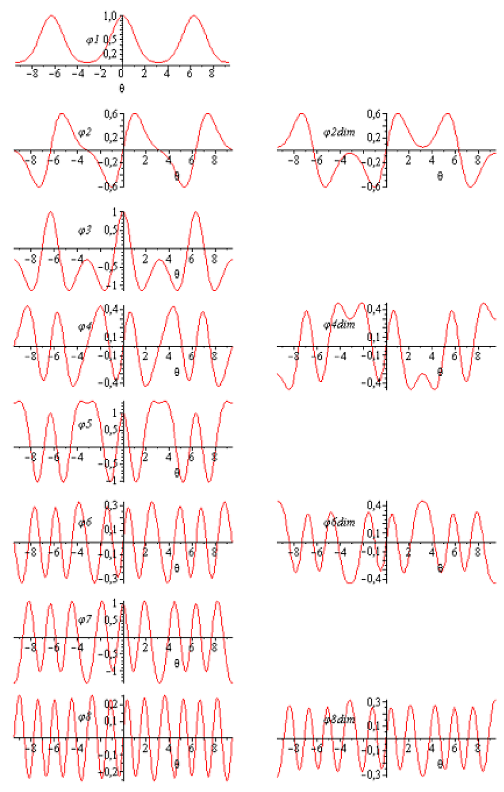
Spin and flavour inherent in the Laplacian

$$S(S+1)+M^2=\frac{4}{3}\left(n+\frac{3}{2}\right)^2-3-\frac{1}{3}y^2-4i_3^2,\ n=0,1,2,\cdots$$

as

$$R=\sum_{lmn}a_{lmn}R_{lmn} \qquad R_{lmn}(\boldsymbol{\theta})=\begin{vmatrix}\varphi_l(\theta_1)&\varphi_l(\theta_2)&\varphi_l(\theta_3)\\\varphi_m(\theta_1)&\varphi_m(\theta_2)&\varphi_m(\theta_3)\\\varphi_n(\theta_1)&\varphi_n(\theta_2)&\varphi_n(\theta_3)\end{vmatrix}$$

The figure shows 1D eigenstates with periodicity 2π to the left and periodicity 4π for diminished states in the right column.
We can couple a diminishing period doubling in level two with an augmenting period doubling in level one. We interpret these coupled period doublings as representing the transformation from a neutral state (e.g. the neutron) to a charged state (e.g. the proton).



$$n\rightarrow p \qquad R_{1'2'3}(\boldsymbol{\theta})=\begin{vmatrix}e^{-i\frac{1}{2}\theta_1}g_{1'}(\theta_1)&e^{-i\frac{1}{2}\theta_2}g_{1'}(\theta_2)&e^{-i\frac{1}{2}\theta_3}g_{1'}(\theta_3)\\e^{i\frac{1}{2}\theta_1}g_{2'}(\theta_1)&e^{i\frac{1}{2}\theta_2}g_{2'}(\theta_2)&e^{i\frac{1}{2}\theta_3}g_{2'}(\theta_3)\\\varphi_3(\theta_1)&\varphi_3(\theta_2)&\varphi_3(\theta_3)\end{vmatrix}$$

The resulting shift in ground state eigenvalue is

$$\frac{E-E''}{E''}=0.13847(14)\,\%\approx 0.1378420(13)\%=\frac{m_n-m_p}{m_p}$$

Conclusions

The Hamiltonian in (1) or (3) may be seen as an effective model or interpreted more radically in an intrinsic conception:
When Resonances - impact momenta act as introtangling operators to generate the maximal torus of U(3).
When Decay, fragmentation - the momentum form induces quark and gluon fields as projections in laboratory space.
Spontaneous symmetry break in baryonic state on U(3) – selects U(2) subspace for Higgs mechanism.
Higgs mass - intrinsic periodic potential shapes the Higgs potential to fourth order.
Electroweak scale - fine structure coupling and strong scale sets electroweak energy scale via Higgs vev.

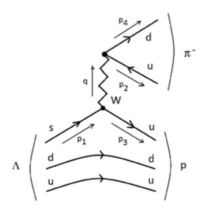
Key predictions:
*accurate Higgs mass, Cabibbo angle, neutral pentaquarks

$$v_{\mathrm{SM}}=\sqrt{2}\varphi_0\sqrt{|V_{ud}|}$$

$$\varphi_0=\frac{2\pi}{\alpha}\Lambda\quad \text{where } \Lambda\equiv\frac{\hbar c}{a}=\frac{\pi}{\alpha}m_ec^2$$

$$^*m_Hc^2=\frac{1}{\sqrt{2}}\frac{2\pi}{\alpha(m_W)}\frac{\pi}{\alpha_e}m_ec^2=125.095\pm0.014\,\mathrm{GeV}$$

$$T_u=\frac{2}{3}T_1-T_3,\qquad T_d=-\frac{1}{3}T_1-T_3,\qquad T_s=-\frac{1}{3}T_1$$



Cabibbo angle

$$\sin\theta_C=\mathrm{Tr}\,T_u^\dagger T_s=-\frac{2}{9}\rightarrow\cos\theta_C=0.974996\cdots\approx|V_{ud}|=0.97420\pm0.00021$$

$$M=\frac{g_W^2}{8(M_Wc)^2}[\bar{u}(3)\gamma^\mu(1-\gamma^5)(\sin\theta_C)u(1)]\cdot[\bar{u}(4)\gamma_\mu(1-\gamma^5)(\cos\theta_C)v(2)]$$

$$\sum_{j=r,b,g}[\bar{u}(3)(T_uc_j)^\dagger\gamma^\mu(1-\gamma^5)T_sc_ju(1)]=[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)]\sum_{j=r,b,g}c_j^\dagger T_u^\dagger T_sc_j$$

Colour states

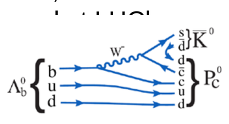
$$c_r^\dagger=(1,0,0),\quad c_b^\dagger=(0,1,0),\quad c_g^\dagger=(0,0,1)$$

The Higgs mass, 125.095+/-0.014 GeV is in excellent agreement with the present world average from a global electroweak fit 125.09+/-0.15 GeV (table 3.1 in ref §) .

Excess Higgs to gauge boson couplings $\frac{\mu_{HVV}}{\mu_{HVV,\,SM}}=\frac{1}{|V_{ud}|}\approx$ 1.03.

Neutral charge, neutral flavour resonances N^0 to be sought at e.g. 2839, … 3206, …, 4499, 4652, 4723, 5103, 5148… MeV. Orange resonances ~ open pentaquark ch

$$\Lambda_b^0\rightarrow\overline{K}^0+P_c^0\rightarrow\overline{K}^0+J/\psi+\Delta^0$$



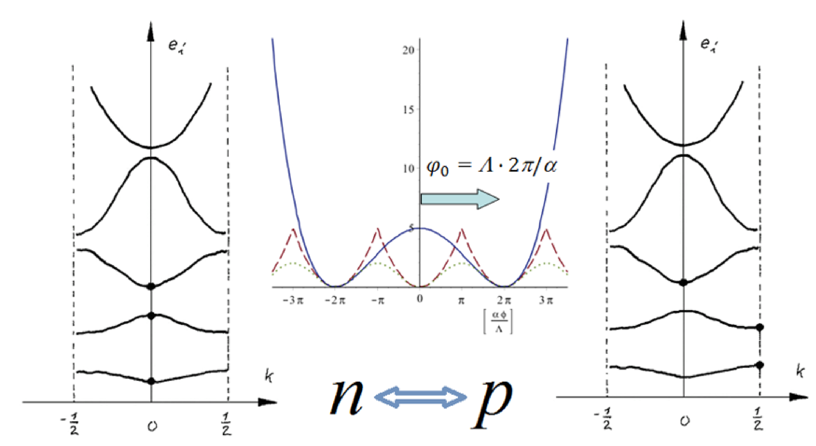
Bloch wave degrees of freedom opened by Higgs mechanism

Approximate energy levels for baryonic states are found by combinations of three 1D eigenstates of the three torus angles. These eigenstates originally have the same periodicity as the potential. However a coupled period doubling can decrease the total energy by introducing Bloch wave degrees of freedom.

Colour quark fields from momentum form
$$c_j=\mathrm{d}R_u(iT_j)$$
In general the momentum form acts as
$$\mathrm{d}R_u(Z)=\frac{\mathrm{d}}{\mathrm{d}t}R(ue^{tZ})\Big|_{t=0}$$
on any Z in the tangent space to the torus

Spontaneous symmetry break in baryonic state.
Exchange minimum quantum of action with electroweak sector:
$$\alpha\varphi_0a=hc,\qquad \Lambda=\frac{\hbar c}{a}$$
$$\varphi_0=\frac{2\pi}{\alpha(m_W)}\Lambda=\frac{v}{\sqrt{2}}$$
$$v_{\mathrm{SM}}=v\sqrt{V_{ud}}$$
Intrinsic potential shapes Higgs potential
$$\frac{1}{2}(\theta\pm2\pi)^2\rightarrow\frac{1}{8}(\phi^2-\phi_0^2)^2=V_{\mathrm{H}}(\phi)$$
$$\Lambda\theta\sim\alpha\varphi \text{ at } \theta=2\pi \text{ determines } \varphi_0$$

$$V_{\mathrm{H}}(\phi)=\frac{1}{2}\delta^2\phi_0^2-\frac{1}{2}\mu^2\phi^2+\frac{1}{4}\lambda^2\phi^4$$
$$\delta^2=\frac{1}{4}\varphi_0^2,\qquad \mu^2=\frac{1}{2}\varphi_0^2,\qquad \lambda^2=\frac{1}{2}$$



We interpret the period doublings as related by the Higgs mechanism to the creation of the proton charge in the neutron decay. Similar states all with one even label give the N resonances. Two even labels give possibilities of double charges which we interpret as Δ resonances.

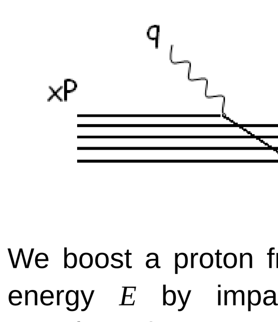
For three even labels the complex phases factorize out and the states may contribute to neutral states.

The black dots in the figure show the Bloch wave number choices for the neutron (left) and the proton state (right).

One may interpret cosmologically the constant delta in the Higgs potential as a contribution to the dark energy content of the universe with one delta for every detained neutron, either primordial or created from the steady transformation of protons into heavier elements during fusion in stars. Accelerating expansion is in accordance with increasing helium and metallicity content Y+Z in stars:

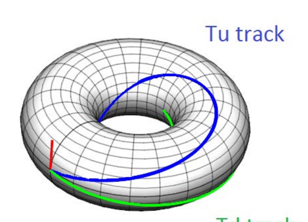
$$\frac{\Omega_\Lambda}{\Omega_b}=\sum_n\delta\Big/\sum_b m_bc^2\approx\frac{(Y+Z)\varphi_0/2}{2m_nc^2}=14.5\pm0.7 \text{ compare with } 14.3\pm0.4 \text{ (observed)}$$

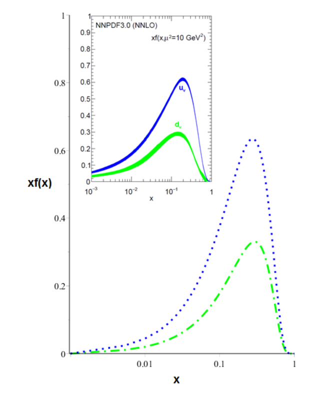
Parton distribution and spin structure functions



We boost a proton from $E_0=m_pc^2$ to energy E by impacting upon it a massless four-momentum q to hit a parton xP. After impact the parton carries a mass xE. Thus
$$(xP_\mu+q_\mu)\cdot(xP^\mu+q^\mu)=x^2E^2$$
which yields for the parton fraction

$$x=\frac{2E_0}{E+E_0} \text{ and boost parameter } \xi\equiv\frac{E-E_0}{E}=\frac{2-2x}{2-x}$$





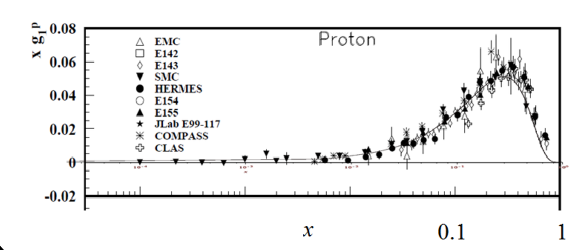
$$f_T(x)dx=\left(\sum_{j=1}^{3}\mathrm{d}R_{u=\exp(\theta iT)}(iT_j)\right)^2d\theta,\quad \text{where } \theta=\pi\xi$$

$$T_u=\tfrac{2}{3}T_1-T_3,\qquad T_d=-\tfrac{1}{3}T_1-T_3=\begin{pmatrix}-\frac{1}{3}&&\\&0&\\&&-1\end{pmatrix}$$

We project from a state constructed from trigonometric functions to mimic approximately the period doublings in the proton state

$$R_{0,\frac{1}{2},1}(\theta_1,\theta_2,\theta_3)=\frac{1}{N}\begin{vmatrix}1&1&1\\\sin\frac{1}{2}\theta_1&\sin\frac{1}{2}\theta_2&\sin\frac{1}{2}\theta_3\\\cos\theta_1&\cos\theta_2&\cos\theta_3\end{vmatrix}$$

We trace the boost with different toroidal generators.
The projection involves the momentum form dR summed over all three colours.



Spin structure function $$g_1^P(x)=\tfrac{1}{2}[e_u^2\tfrac{1}{3}f_{T_u}(x)+e_d^2\tfrac{1}{3}f_{T_d}(x)]$$

Magnetic dipole moment $$m_q=\int_0^1xm_pf_{T_q}(x)\mathrm{d}x,\qquad \mu_q=\frac{1}{2}\frac{e_q\hbar}{m_q}$$

$$\mu_p=\frac{4}{3}\mu_u-\frac{1}{3}\mu_d=2.778\ldots\mu_N,\qquad exp{:}\,2.792847356(23)\mu_N$$

No fitting parameters

Momentum form for quark scattering

Colour quark fields: $\psi_j(u)=\mathrm{d}R_u(iT_j)=(uiT_j)[R]$ - from left-invariant coordinate fields $\boxed{\partial_j|_u=uiT_j}$

$H=\int\psi^\dagger(-i\hbar c\boldsymbol{\alpha}\cdot\nabla+\beta mc^2)\psi d^3x$ (J.J. Sakurai) $\psi^\dagger=(\psi_1^*,\psi_2^*,\psi_3^*)$

Use left-invariance: $\psi'(u')^\dagger\psi'(u')=(u'iT_j[R])^\dagger(u'iT_j[R])=(iT_j[R])^\dagger(u')^\dagger u'(iT_j[R])$
$=\psi(u)^\dagger\psi(u)\rightarrow$ invariant mass term for unitary configuration variable

Impose gauge transformation: $\psi\rightarrow\psi'=g(x)\psi,\quad g(x)\in SU(3),\partial_\mu\rightarrow D_\mu=\partial_\mu+A_\mu$
$A'_\mu=g(x)A_\mu g^{-1}(x)+g(x)\partial_\mu g^{-1}(x),\qquad A_\mu=ig_sA_\mu^k\lambda_k,$
choose $u=g(x)\rightarrow$ gauge invariance from left-invariance

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See also
O. L. Trinhammer, *Intrinsic quantum mechanics behind the Standard Model – From Lie group configurations to Particle physics observations,* ResearchGate, Book, 2018.