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Transverse Momentum Dependent splitting kernels from k_T - factorization

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arXiv:1511.08439, JHEP 1601 (2016) 181.
arXiv:1607.0150, PRD 94 (2016) no.11, 114013.
arXiv:1711.0458, EPJC 78 (2018) no.3, 174.

EPS-HEP2019, July 10-17, Ghent, Belgium

Hannes Jung@ RBRC-BNL workshop June 2017:

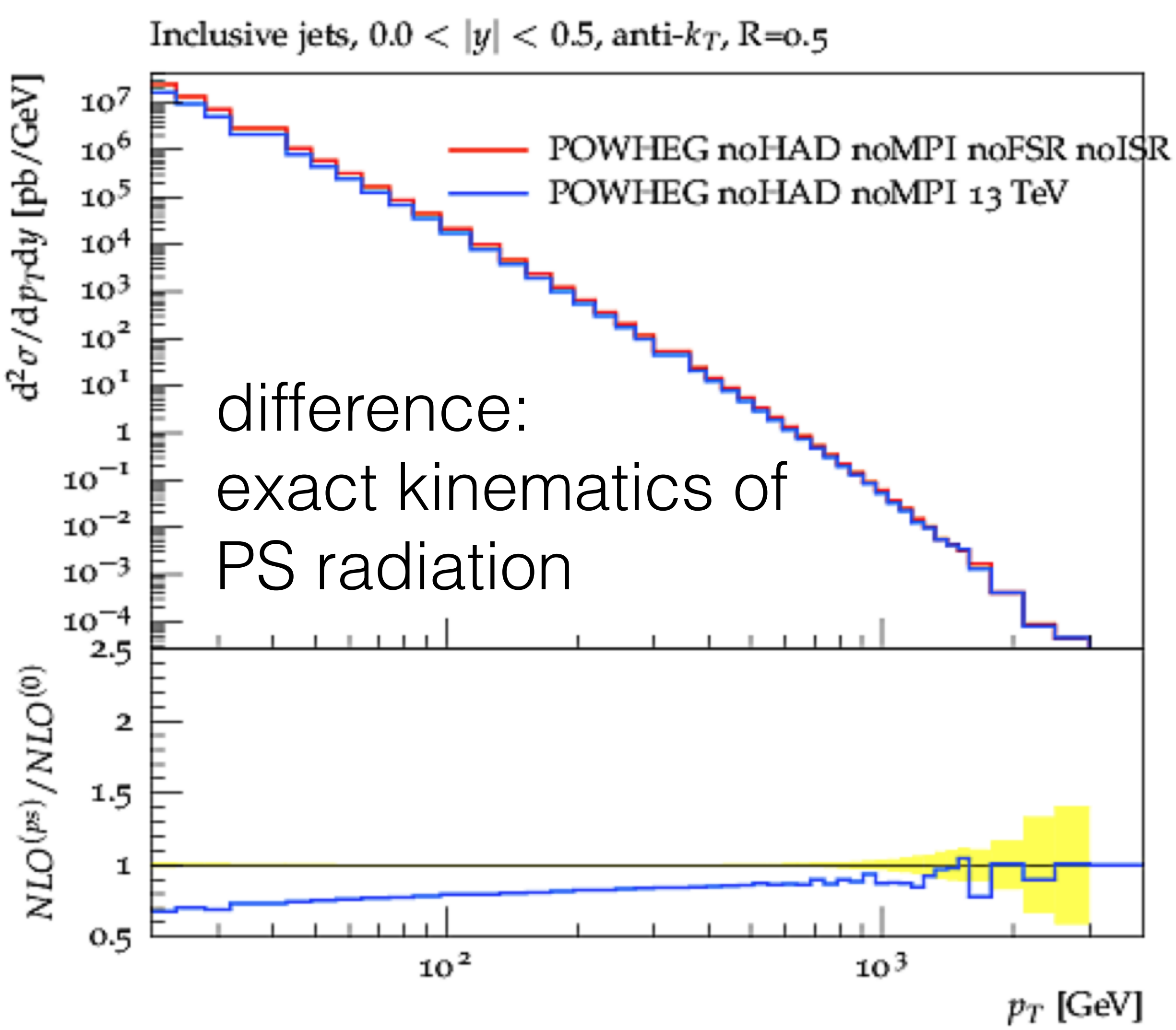
parton shower (MC) vs pure NLO for single inclusive jet → must agree!

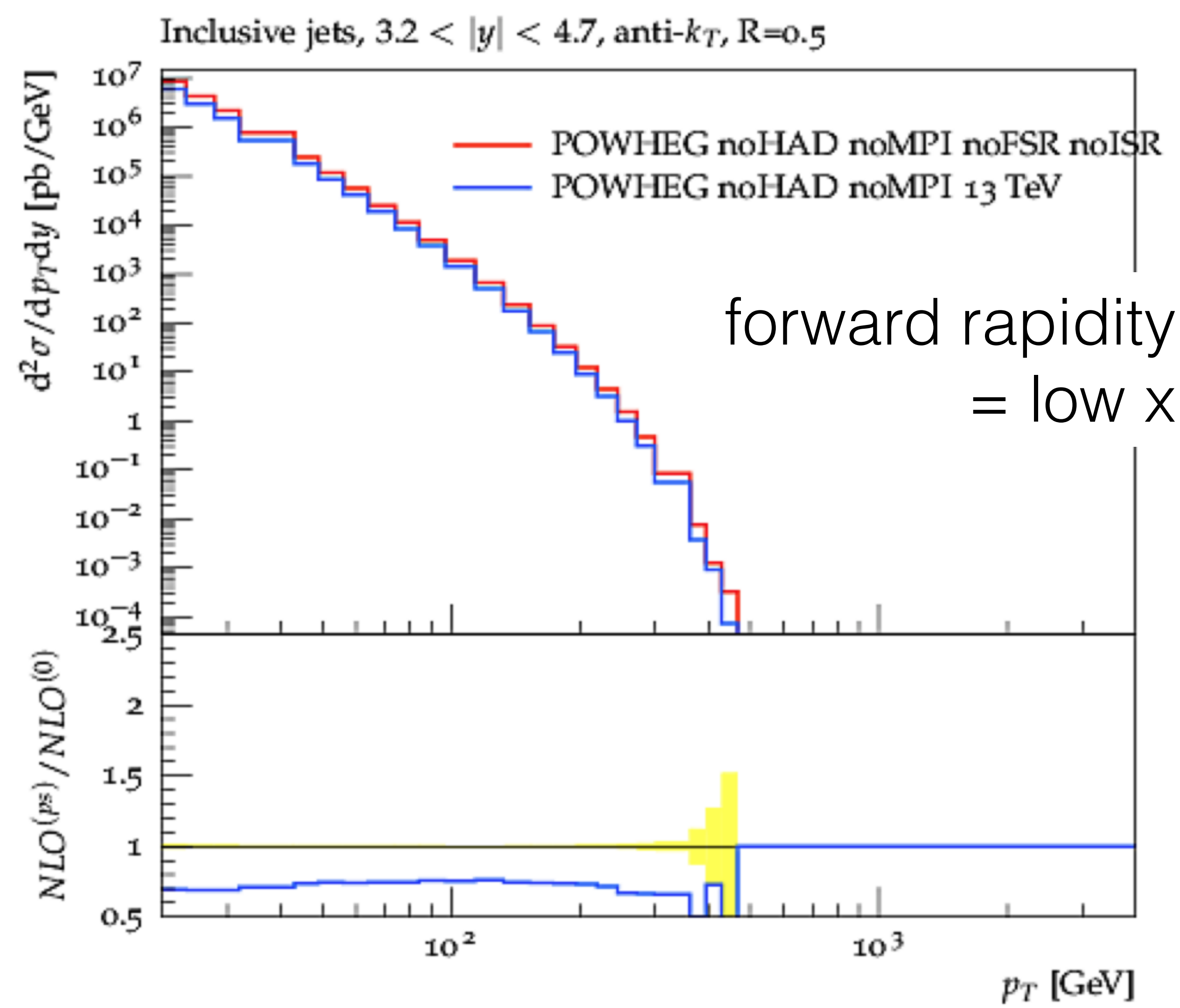
Approach described in: S. Dooling et al Phys.Rev., D87:094009, 2013.

- use NLO+PS to calculate:

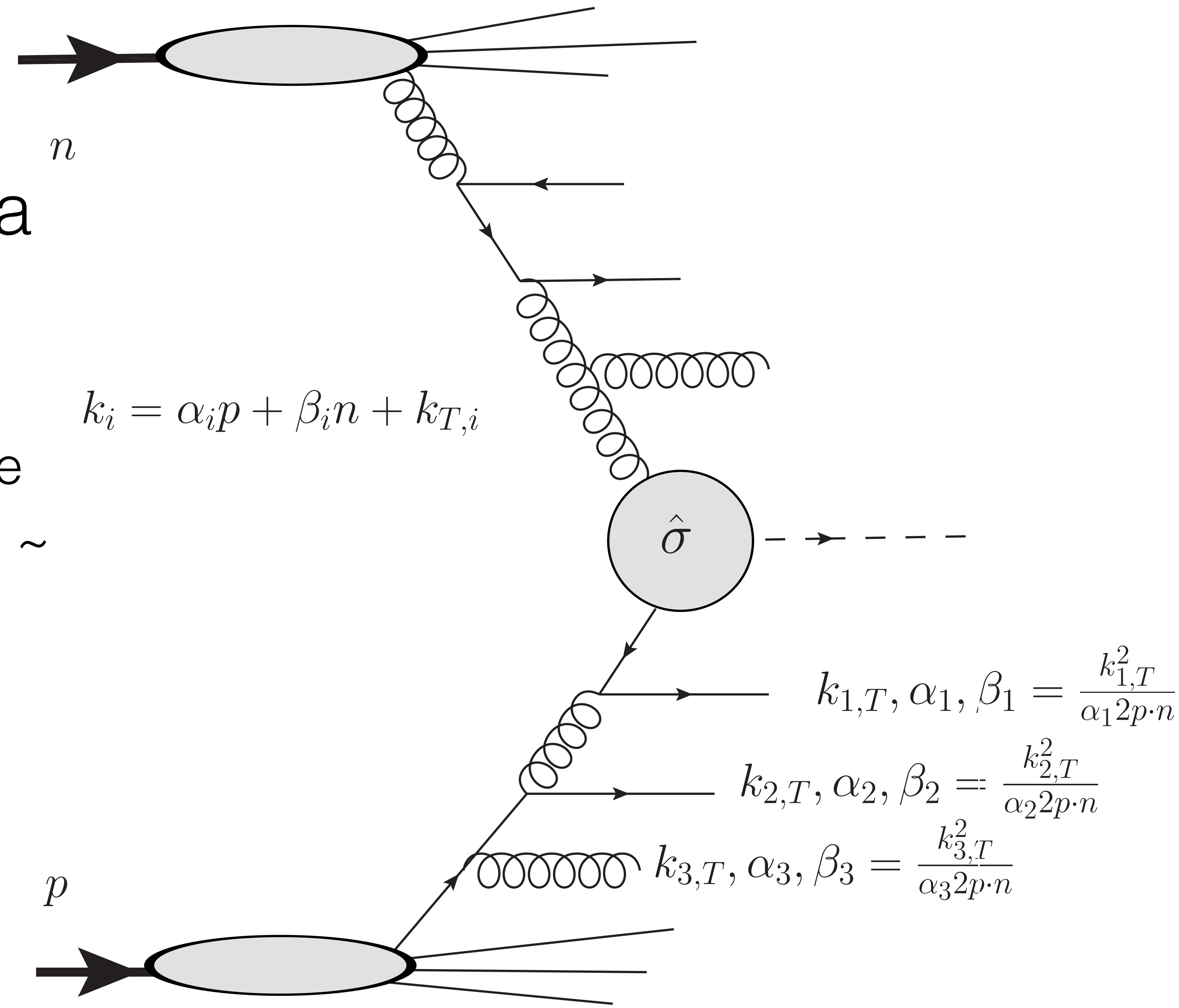
$$K^{PS} = \frac{N^{(ps)}_{NLO-MC}}{N^{(0)}_{NLO-MC}}$$

- Corrections to be applied to fixed order NLO calculations:
  - kinematic effects: TMDs !
  - radiation outside of jet-cone

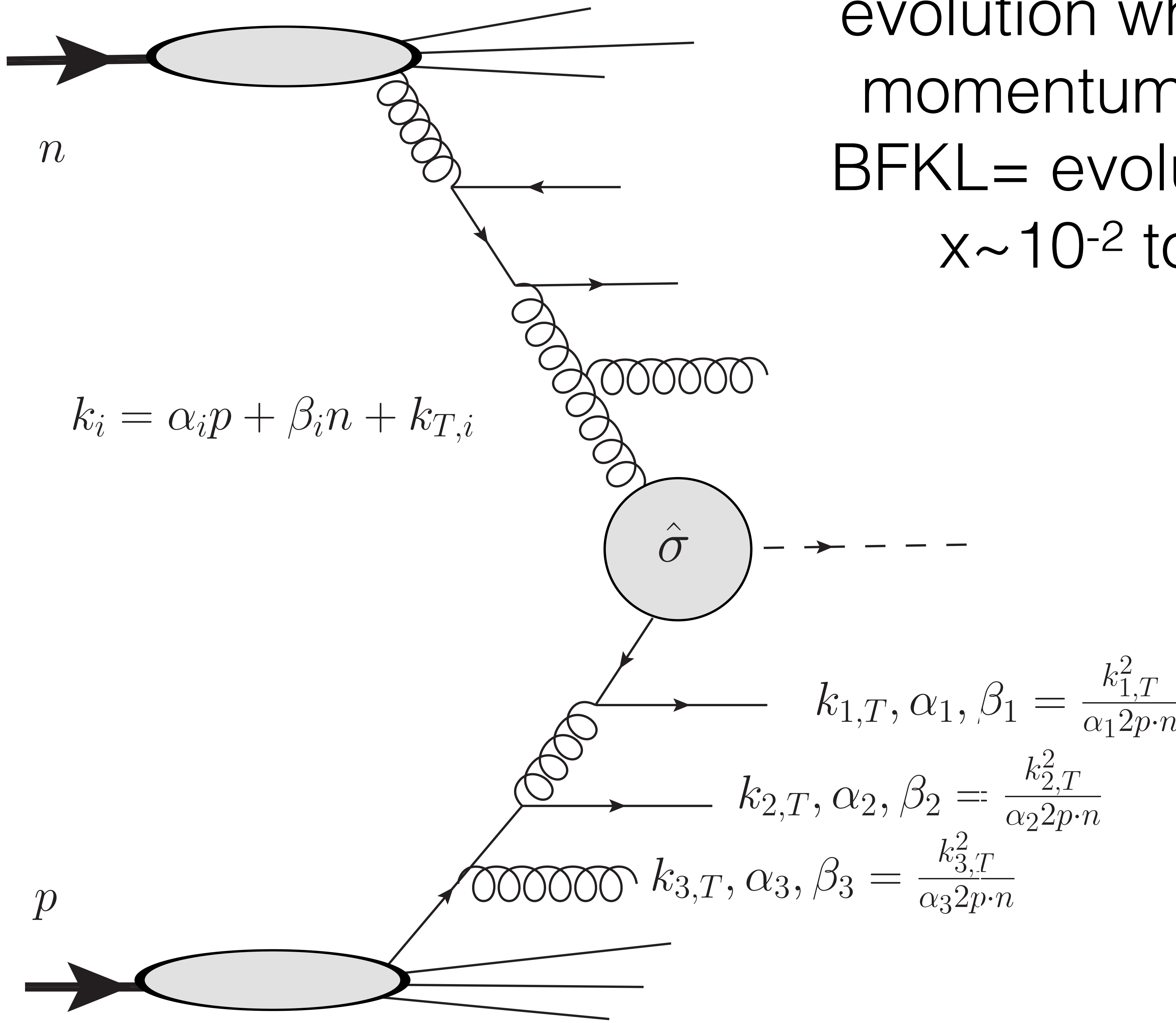




- DGLAP (theory): transverse momenta strongly ordered $\boldsymbol{k}_{i,T} \gg \boldsymbol{k}_{i+1,T}$ (=neglect information on kT⟷ isolate logarithmic enhanced term ~ collinear factorization)
  - Monte Carlo: momentum configuration which obeys exact momentum conservation + order them (no <u>strong ordering</u>) to assign probability weigh



→ mismatch due to exact kinematics in MC shower (ISR & FSR) evolution where transverse momentum is conserved: BFKL= evolution from large $x\sim10^{-2}$ to low $x\sim10^{-6}$

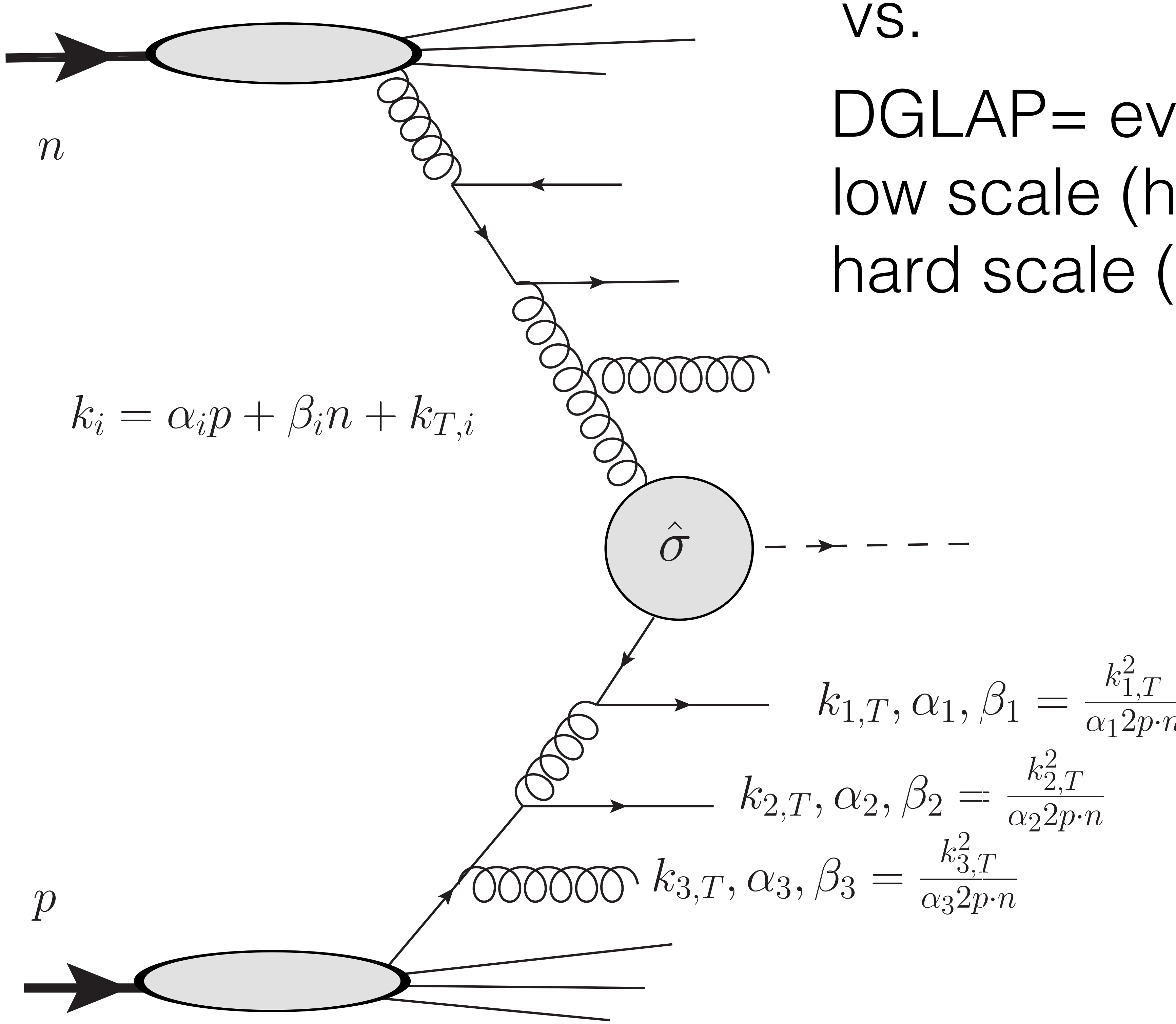


proton momentum fraction $\boldsymbol{\alpha}$ strongly ordered $\boldsymbol{\alpha}_{\mathbf{i}} \gg \boldsymbol{\alpha}_{\mathbf{j}}$ (=neglect information on $\boldsymbol{\alpha}$ $\longleftrightarrow$ isolate logarithmic enhanced term ~ high energy factorization)

transverse momentum treated exactly (no approximation), but implicitly $\boldsymbol{k}_{i,T}\sim\boldsymbol{k}_{i+1,T}$

vs.

DGLAP= evolution from low scale (hadron) to hard scale (process)



transverse momenta strongly ordered $\boldsymbol{k}_{i,T} \gg \boldsymbol{k}_{i+1,T}$ (=neglect information on kT⟷ isolate logarithmic enhanced term ~ collinear factorization)

proton momentum fraction $\boldsymbol{\alpha}$ treated exactly (no approximation), but implicitly $\boldsymbol{\alpha}_i \sim \boldsymbol{\alpha}_j$

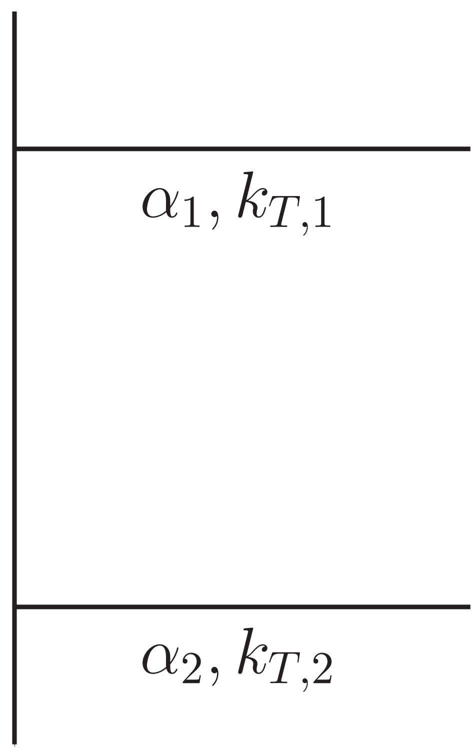
observations:

- (trivial) BFKL limited to low x region → requires extension to treat generic observable
- (NN)LO DLGAP evolution (hard scale) at low x plagued by large low x logs
- NLO BFKL evolution (low x) plagued by large collinear logs
- in both cases: exact kinematics can be a source of large higher order corrections

→ Can we combine both evolutions and take into account kinematics (more) precisely?

A variable that unifies both: rapidity

$$\eta = \frac{1}{2}\ln\frac{k^+}{k^-} = \frac{1}{2}\ln\frac{\alpha}{\beta} = \ln\frac{\alpha\sqrt{2p\cdot n}}{|k_T|} = \ln\frac{k^+}{|k_T|}$$

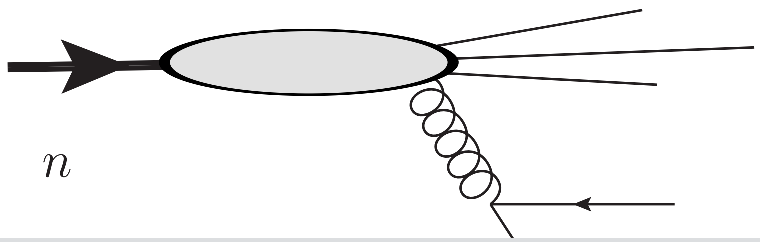


strong ordering in rapidity

$$\Delta\eta_{21} = \eta_2 - \eta_1 = \ln\left(\frac{\alpha_2|k_{T,1}|}{\alpha_1|k_{T,2}|}\right) \gg 1$$

both for

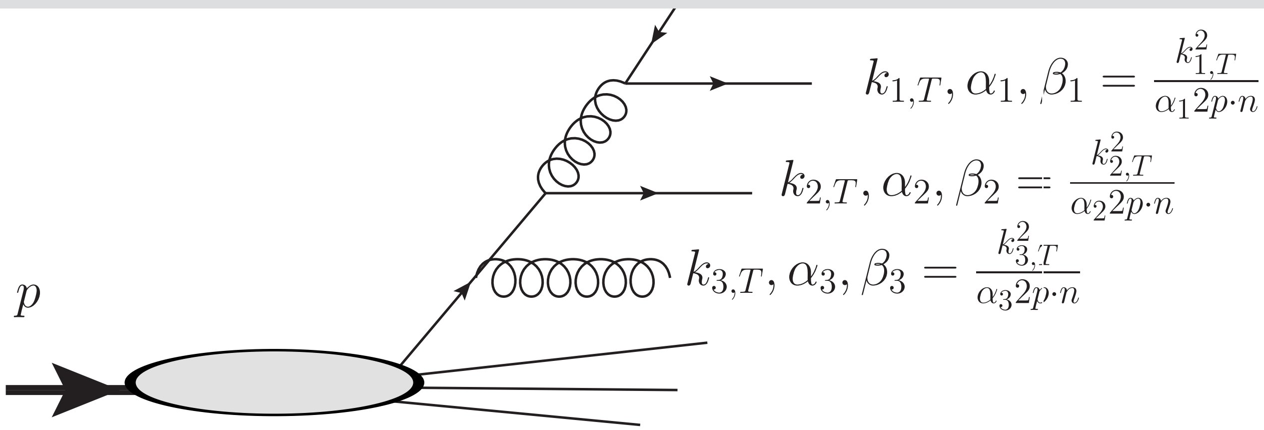
- collinear factorization $k_{T1} \gg k_{T2}$
- high energy factorization $\alpha_2 \gg \alpha_1$



$$k = \alpha p + \beta n + k_T$$

ordering in rapidity vs <u>ordering in β</u>

(momentum fraction w.r.t. collision partner)

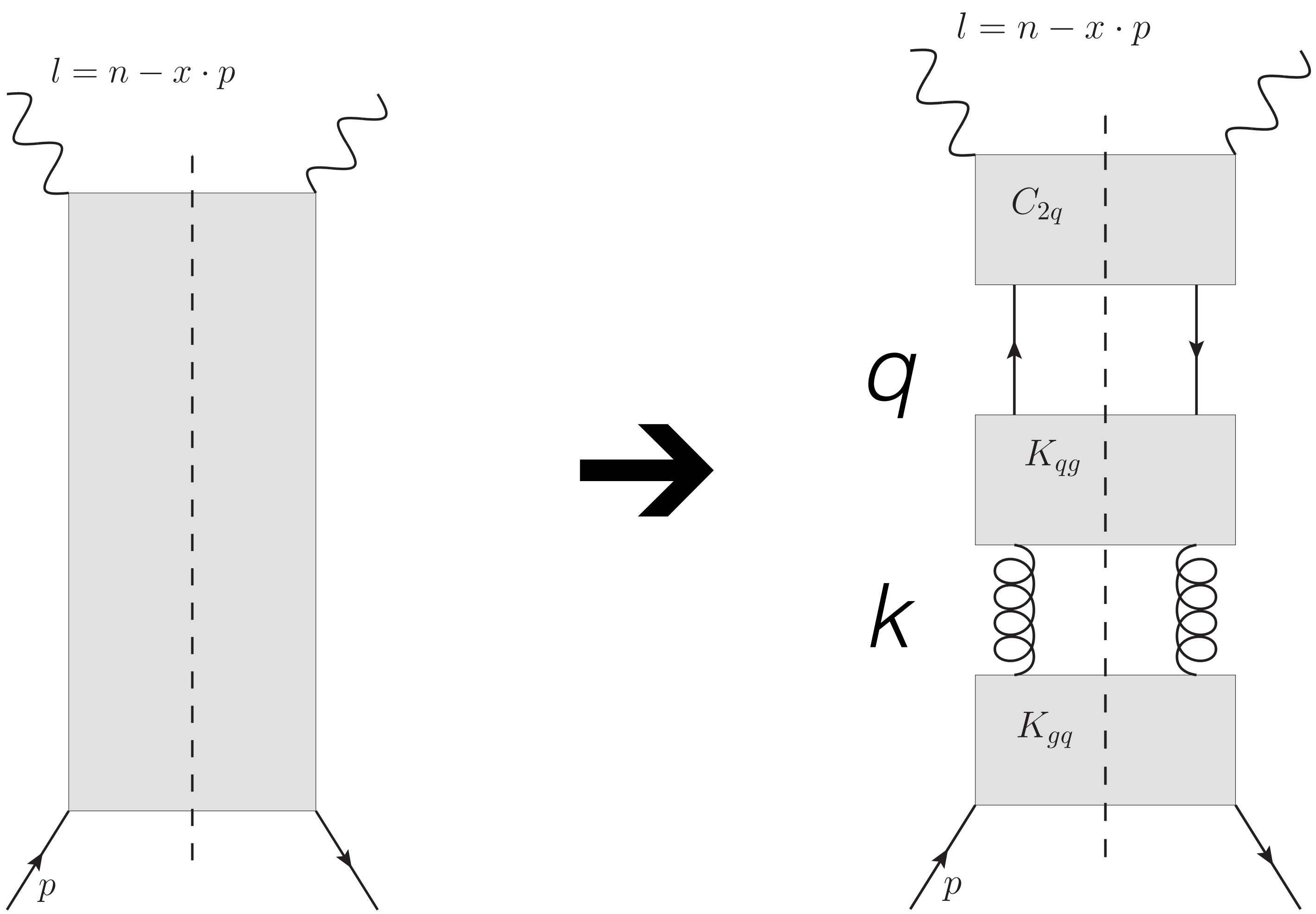


$\beta_1 \gg \beta_2 \gg \beta_3 \gg \ldots$ means $\frac{k_{T,1}^2}{\alpha_1 2n\cdot p} \gg \frac{k_{T,1}^2}{\alpha_1 2n\cdot p} \gg \frac{k_{T,1}^2}{\alpha_1 2n\cdot p} \gg \ldots$

implies:

BFKL/Multi-Regge Kinematics		DGLAP/collinear kinematics
$\alpha_1 \ll \alpha_2 \ll ....$		$\alpha_1 \sim \alpha_2 \sim ....$
$k_{T,1} \sim k_{T,2} \sim ...$	AND	$k_{T,1} \gg k_{T,2} \gg ...$

task: search for factorization of correlators which is only ordered in β & formulate evolution



out-going line

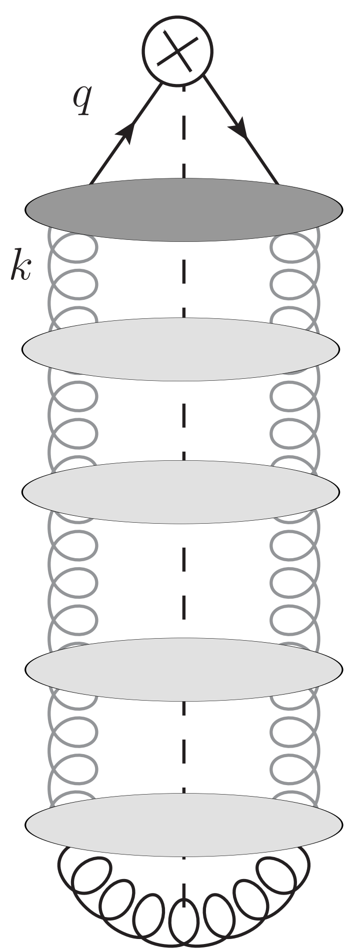
$$q = z \cdot xp + q_T + \beta n$$

$$k = x \cdot p + k_T$$

ingoing line
(“kT-factorization”)
=forget about β

first result in literature: low x resummation of DGLAP splitting functions
[Catani, Hautmann, NPB 427 (1994) 475]

- use diagrammatic definition of collinear factorization
  [Curci, Furmanski, Petronzio; NPB 175 (1980)]
- low x (=BFKL) evolution to resum log1/x to all orders
- TMD splitting function = coefficient for resumed $P_{qg}(z)$



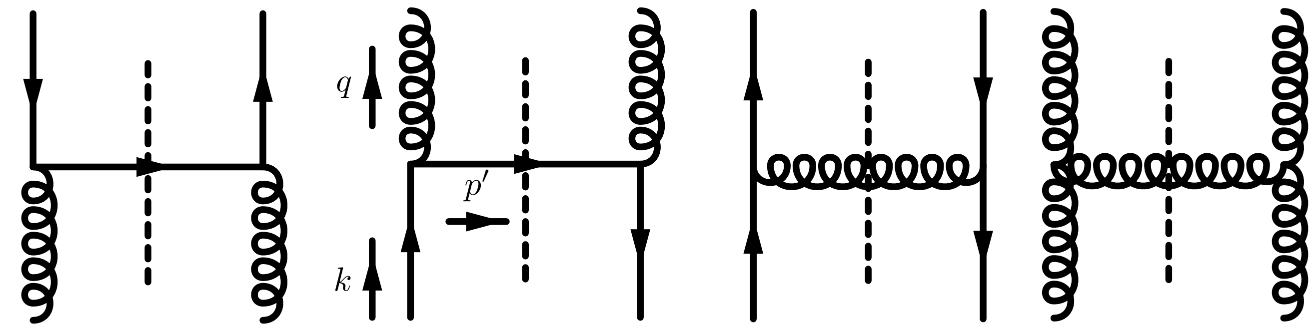
upper blob: no low x logarithm; finite → defines a TMD quark-to-gluon splitting function

$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}, \epsilon\right) = \mathrm{Tr}\left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2 \cdot \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]$$

$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

generalization to off-shell quark [Hautmann, MH, Jung; 1205.1759]

- complete evolution
→ all splittings



(a) $\tilde{P}_{qg}$ (b) $\tilde{P}_{gq}$ (c) $\tilde{P}_{qq}$ (d) $\tilde{P}_{gg}$

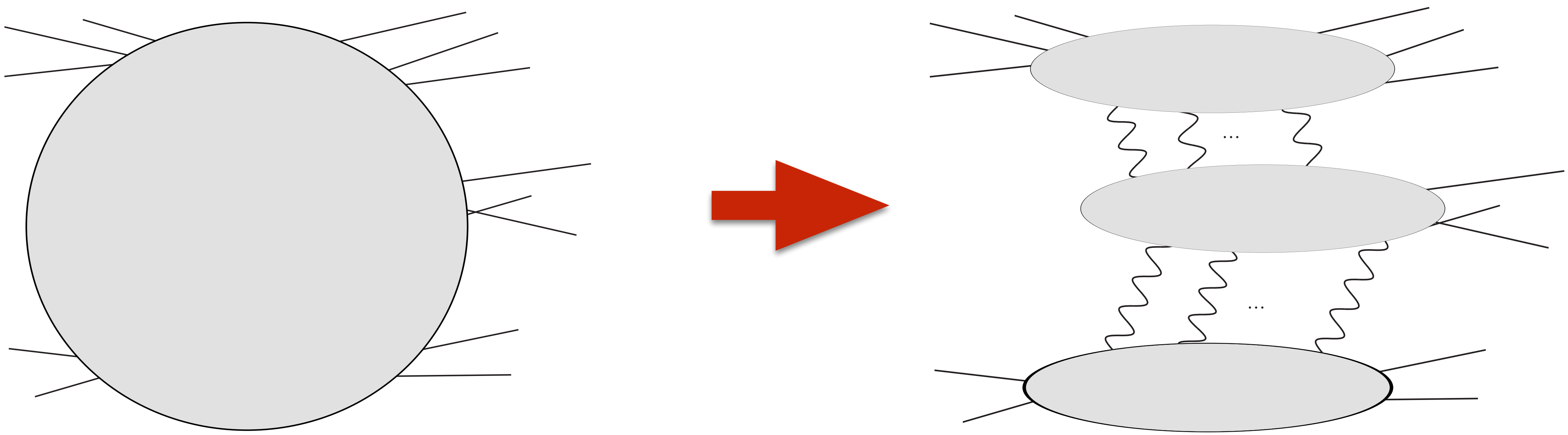
- cannot be defined/determined as coefficient of high energy resummation of DGLAP splitting function

- first attempt construction based on
a) generalization of Curci-Furmanski-Petronzini formalism (light cone gauge)
b) high energy factorization → formalism: high energy effective action [Lipatov; hep-ph/9502308]

an action formalism for high energy factorization: Lipatov's high energy effective action

[Lipatov; hep-ph/9502308]

basic idea:



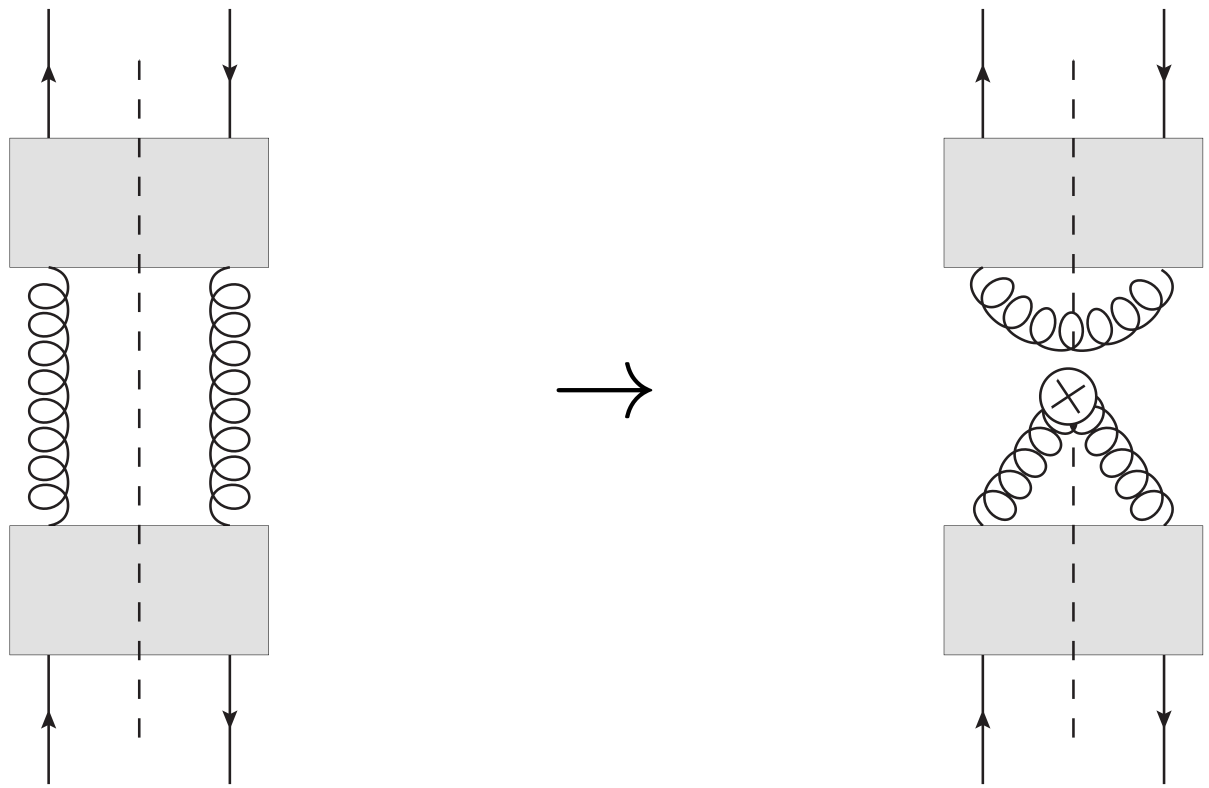
correlator with regions localized in rapidity, significantly separated from each other

factorize using auxiliary degree of freedom = the reggeized gluon

A short appraisal of Lipatov's high energy effective action

- 2 loop gluon trajectory [Chachamis, MH, Madrigal, Sabio Vera, 1307.2591]
- NLO impact factors for jets with and without rapidity gap [MH, Madrigal, Murdaca, Sabio Vera, 1404.2937, 1406.5625, 1409.6704]
- 2 scale process: photon-quark scattering
- description of dilute-dense collision (=Color Glass Condensate formalism) [MH, 1802.06755]
- Complementary (dilute): spinor helicity amplitudes based formalism [van Hameren, Kotko, Kutak; 1211.0961], [van Hameren, Kutak, Salwa; 1308.2861]

→ well tested effective action formalism for high energy factorization



Calculating kernels

= more general kinematics!

- incoming parton (off-shell, from hadron) → high energy factorization + normalization matched to collinear factorization ✓

- out-going parton (off-shell, to partonic process) → <u>generalization of CFP-method</u> + <u>eikonal factors</u> (motivated from high energy factorization) to guarantee gauge invariance → requires generalization

- factorization: generalized projectors → appendix

Quark splittings — relatively straightforward

$$
\begin{aligned}
\Gamma^{\mu}_{q^*g^*q}(q,k,p') &= i\,g\,t^a \left(\gamma^\mu - \frac{n^\mu}{k\cdot n}\,\not{q}\right), \\
\Gamma^{\mu}_{g^*q^*q}(q,k,p') &= i\,g\,t^a \left(\gamma^\mu - \frac{p^\mu}{p\cdot q}\,\not{k}\right), \\
\Gamma^{\mu}_{q^*q^*g}(q,k,p') &= i\,g\,t^a \left(\gamma^\mu - \frac{p^\mu}{p\cdot p'}\,\not{k} + \frac{n^\mu}{n\cdot p'}\,\not{q}\right)
\end{aligned}
$$

- production vertices of high energy factorization can be generalized to TMD kinematics

[Lipatov, Vyazovski, hep-ph/0009340]

- guarantees current conservation for off-shell legs (close relation to Wilson lines) [Gituliar, MH, Kutak, 1511.08439]

Gluon production vertex significantly more complicated

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = \mathcal{V}^{\lambda\kappa\mu_3}(-q,k,-p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}$$

3-gluon vertex

pol. tensor of light-cone gauge

- turns out: Lipatov high energy effective action in light-cone does the job, but does not allow to directly verify current conservation for generalized kinematics

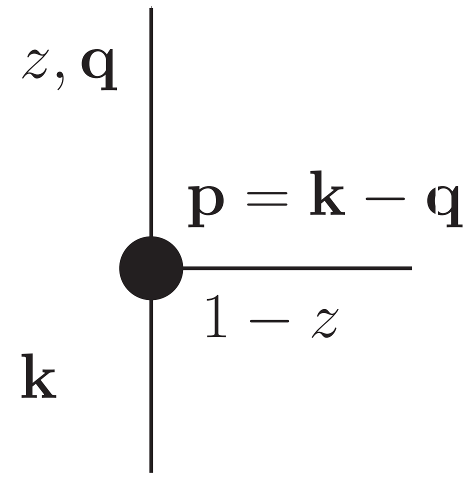
[Lipatov, hep-ph/9502308]

- complete expression: analysis of helicity spinor amplitudes in high energy limit in light-cone gauge

[MH, Kusina, Kutak, Serino; 1711.04587]

Results:

A) quark induced splittings (angular averaged)



$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

their correct collinear limit is verified

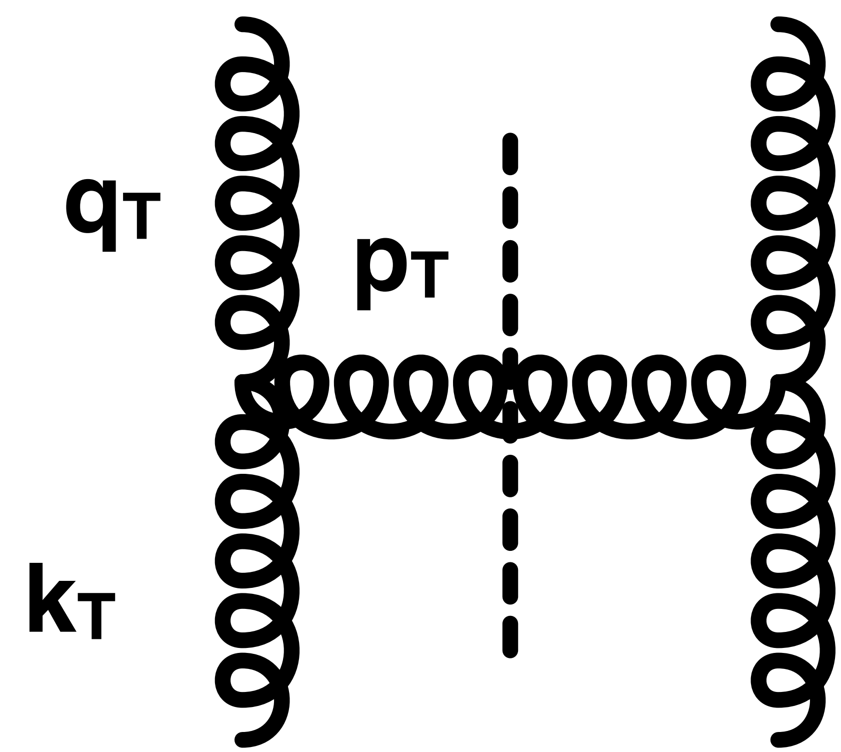
$$P_{gq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right) = C_f \left[\frac{2\boldsymbol{\Delta}^2}{z|\boldsymbol{\Delta}^2 - (1-z)^2\boldsymbol{k}^2|} \right. \\ \left. - \frac{\boldsymbol{\Delta}^2(\boldsymbol{\Delta}^2(2-z) + \boldsymbol{k}^2 z(1-z^2)) - \epsilon z(\boldsymbol{\Delta}^2 + (1-z)^2\boldsymbol{k}^2)}{(\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2)^2} \right],$$

$$P_{qq}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}, \epsilon\right) = C_f \left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2}\right) \left[\frac{\boldsymbol{\Delta}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\boldsymbol{\Delta}^2 - (1-z)^2\boldsymbol{k}^2|} \right. \\ \left. + \frac{z^2\boldsymbol{\Delta}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2 + (1-z)^2\epsilon(\boldsymbol{\Delta}^2 + z^2\boldsymbol{k}^2)}{(1-z)(\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2)} \right].$$

B) P_{gg} splitting

$$\bar{P}_{gg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2}\left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\left|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2\right|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)}\right]$$

- ✓ current conservation
- ✓ collinear limit: DGLAP splitting
- ✓ low x limit: BFKL kernel
- ✓ soft limit $p_T \to 0$: CCFM kernel



- so far: real part of splitting kernels → for P_{gg} and P_{qq} there is also a virtual correction; indeed needed to cancel soft ($p_T \to 0$) singularities
- nice feature of real corrections: follow a relatively simply diagrammatic construction
- open question: how do these kernels relate to operator definitions of TMD parton distribution?
- Turns out: answer to both questions closely related (work in progress)

For definiteness: start with operator definition of unpolarized gluon TMD *eg*. **[Ji, Ma, Yuan; hep-ph/0503015]**

$$xG^{(1)}(x,\boldsymbol{k}) = \int \frac{d\xi^- d^2\boldsymbol{\xi}}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\boldsymbol{k}\cdot\boldsymbol{\xi}} \langle P|F_a^{+i}(\xi^-,\boldsymbol{\xi})\mathcal{L}_\xi^\dagger \mathcal{L}_0 F_a^{+i}(0)|P\rangle$$

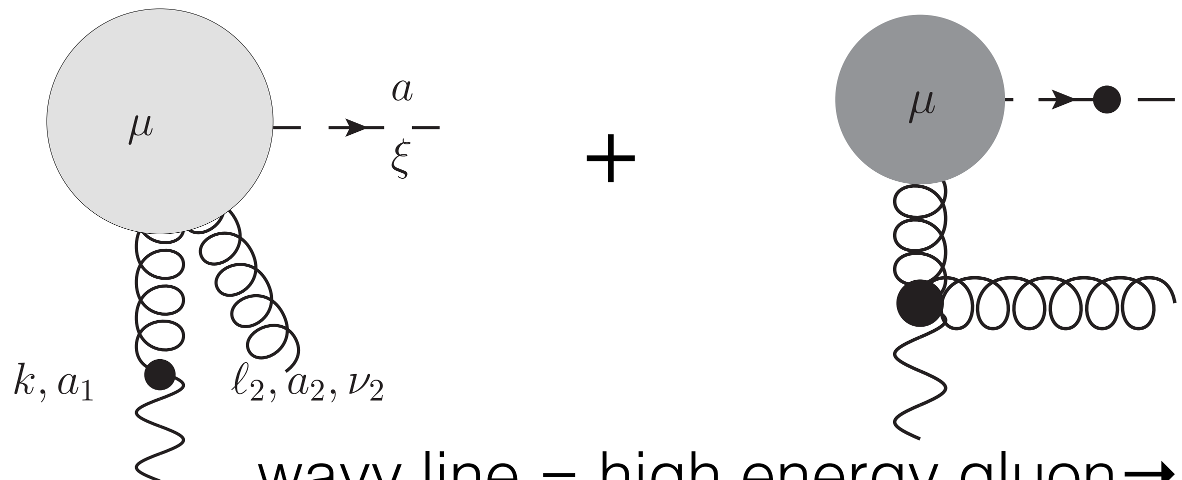
for definiteness: DIS like process
process dependent
(→ loss of universality),
projection on high energy gluon
with definite signature should help
(in progress)

$$\mathcal{L}_\xi = \mathrm{P}\exp\{-ig\int_{\xi^-}^{\infty} d\zeta^- A^+(\zeta,\boldsymbol{\xi})\}$$

to compare with previous result: incoming off-shell high energy gluon at tree-level & one-loop

real correction in covariant gauge:

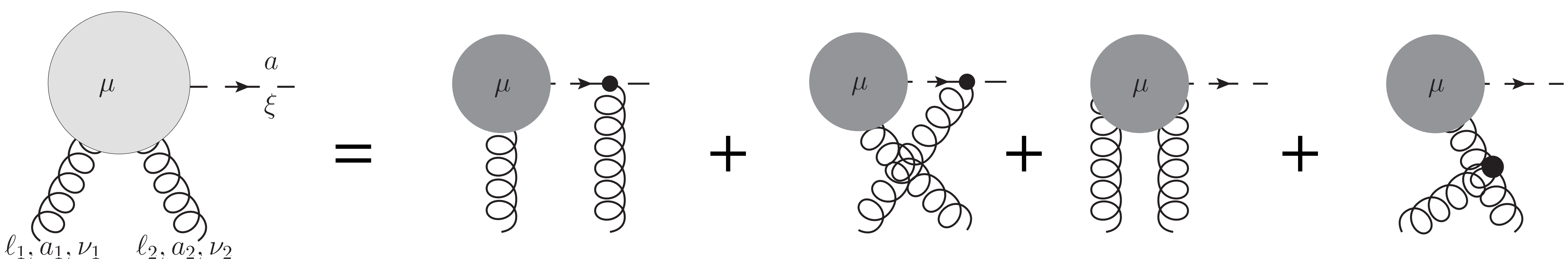
“production vertex”



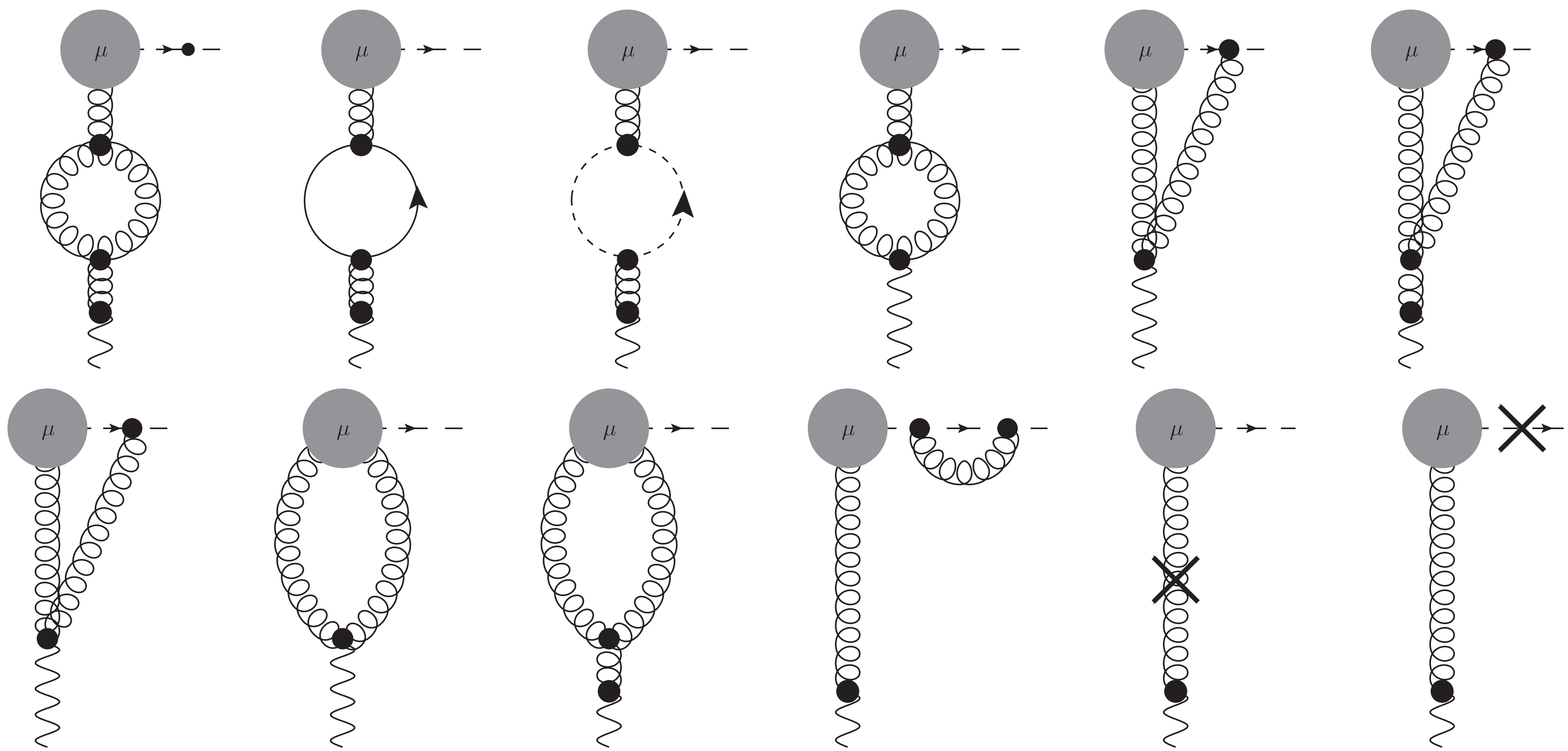
can be shown to agree with previous vertex used for P_{gg}

wavy line = high energy gluon → formalism of high energy effective action

2 gluons in gluon TMD:



Virtual corrections now straightforward and can be now calculated



details: work in progress

Conclusions

- complete set of 4 ***real*** TMD splitting kernels
  → satisfies all necessary constraints so far
- partial evolution equation already formulated **[MH, Kusina, Kutak; 1607.01507]** (not in this talk)
- complete virtual corrections + relation to operator definitions = work in progress; first promising results exist
- in general: there is still a need to properly develop the whole framework; at the very least: a consistent way to combine DGLAP and BFKL

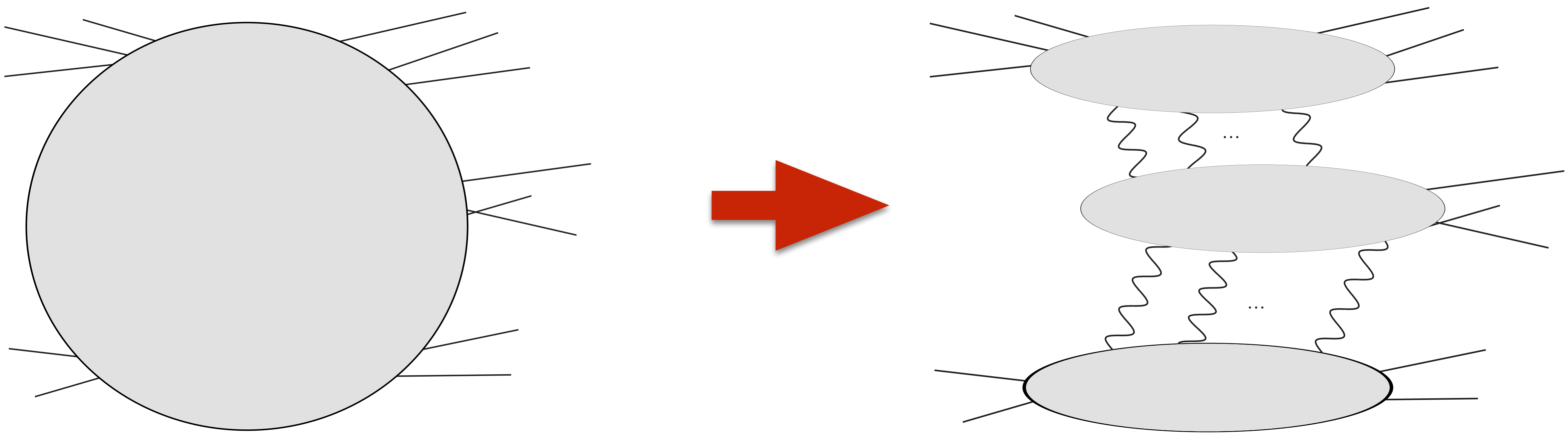
Backup

High energy effective action

an action formalism for high energy factorization: Lipatov's high energy effective action

[Lipatov; hep-ph/9502308]

basic idea:



correlator with regions localized in rapidity, significantly separated from each other

factorize using auxiliary degree of freedom = the reggeized gluon

- idea: factorize QCD amplitudes in the high energy limit through introducing a new kind of field: *the reggeized gluon $A_\pm$* (conventional QCD gluon: v_μ)

kinematics (strong ordering in light-cone momenta between different sectors): $\partial_+ A_-(x) = 0 = \partial_- A_+(x).$

underlying idea:

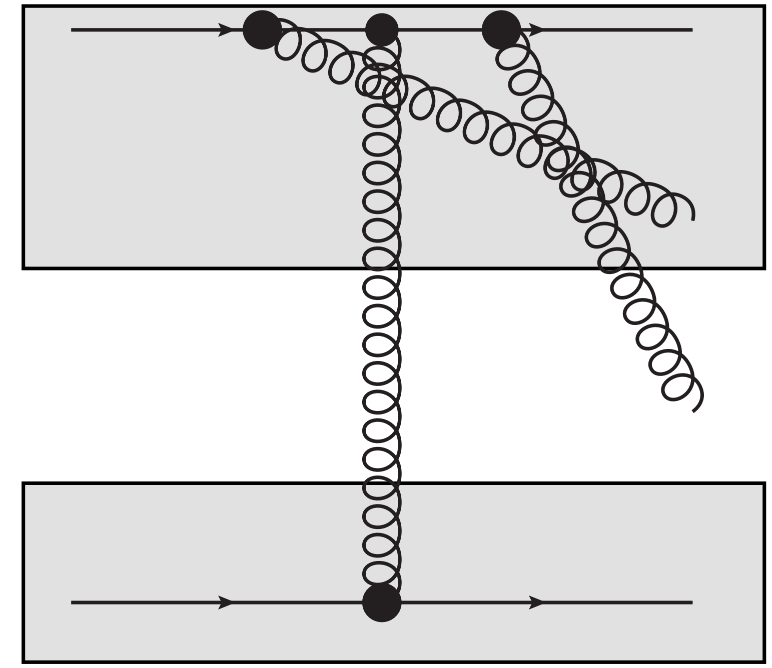
- reggeized gluon globally charged under SU(N_C) $A_\pm(x) = -it^a A^a_\pm(x)$

- but invariant under local gauge transformation

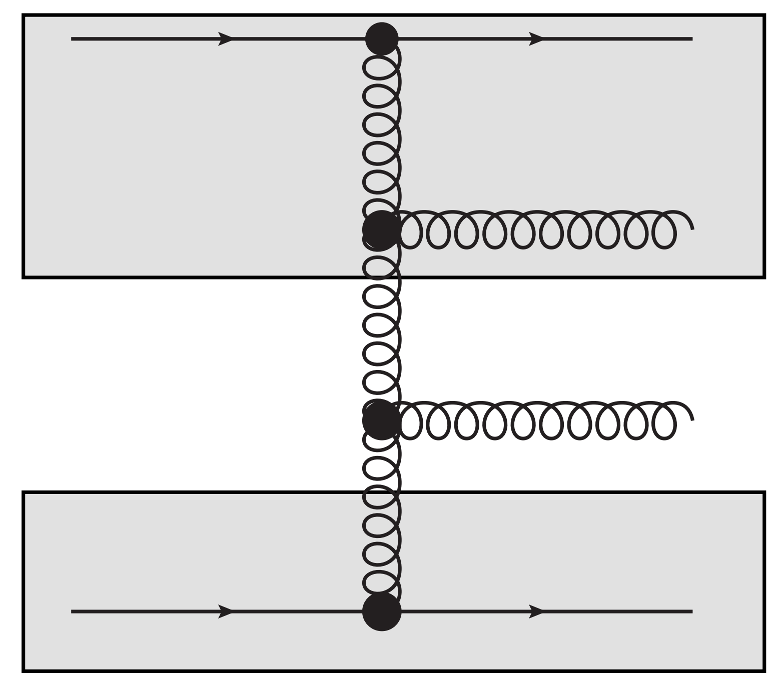
$$\delta_{\mathrm{L}} v\mu = \frac{1}{g}[D_\mu, \chi_L] \qquad \text{vs.} \qquad \delta_{\mathrm{L}} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

→ gauge invariant factorization of QCD correlators underlying idea:

- integrate out specific details of (relatively) fast +/- fields
- description in sub-amplitude local in rapidity: QCD Lagrangian + universal eikonal factor



$$T_\pm[v_\pm] = -\frac{1}{g}\partial_\pm \mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^\pm} dx'^{\pm} v_\pm(x')\right)$$



- effective field theory for *each* local rapidity cluster

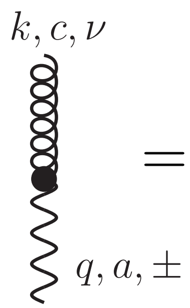
$$\boxed{S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}}$$

$$\boxed{S_{\text{ind.}} = \int \mathrm{d}^4 x \left\{ \mathrm{tr}\left[\left(T_-[v(x)] - A_-(x)\right)\partial_\perp^2 A_+(x)\right] + \left[\text{" + "} \leftrightarrow \text{" − "}\right]\right\}.}$$

eikonal factor = special

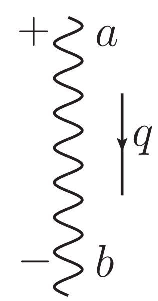
$$T_{\pm}[v_{\pm}] = -\frac{1}{g}\partial_{\pm}\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^{\pm}} dx'^{\pm} v_{\pm}(x')\right)$$

an ∞ # of gluons in the reggeized gluon



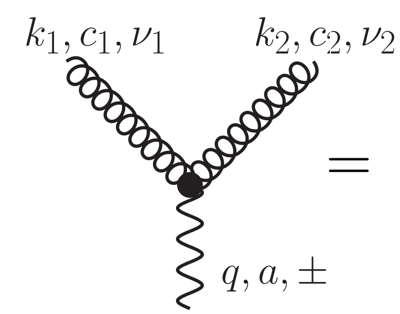
$$= \quad \frac{-i}{2}\boldsymbol{q}^2\delta^{ac}(n^{\pm})^{\nu}, \qquad k^{\pm} = 0.$$

(a)



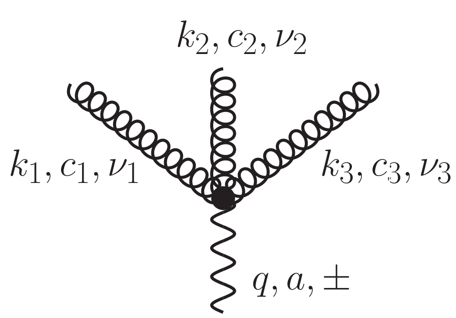
$$= \quad \delta^{ab}\frac{2i}{\boldsymbol{q}^2} \ ,$$

(b)



$$= \quad \frac{g}{2}f^{c_1c_2a}\frac{\boldsymbol{q}^2}{k_1^{\pm}}(n^{\pm})^{\nu_1}(n^{\pm})^{\nu_2}, \qquad k_1^{\pm} + k_2^{\pm} = 0,$$

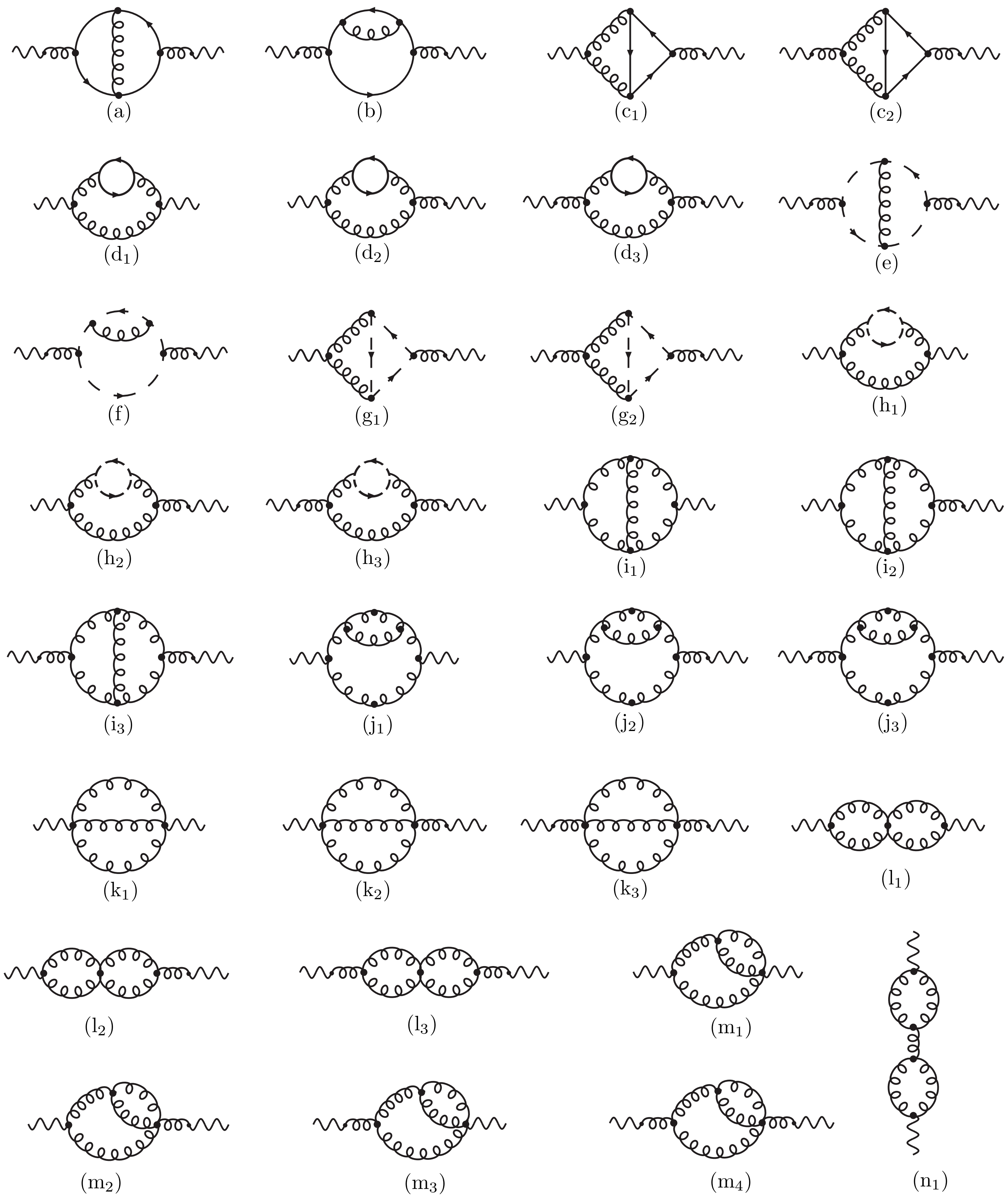
(c)



$$= \frac{ig^2}{2}\boldsymbol{q}^2\left(\frac{f^{a_3a_2e}f^{a_1ea}}{k_3^{\pm}k_1^{\pm}} + \frac{f^{a_3a_1e}f^{a_2ea}}{k_3^{\pm}k_2^{\pm}}\right)(n^{\pm})^{\nu_1}(n^{\pm})^{\nu_2}(n^{\pm})^{\nu_3}, \qquad k_1^{\pm} + k_2^{\pm} + k_3^{\pm} = 0.$$

“induced” vertices

(d)



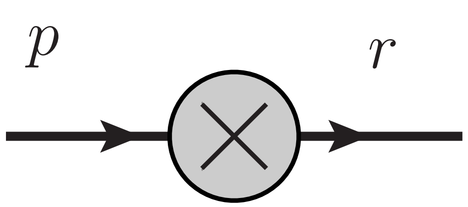
allows for (highly non-trivial) calculation of the gluon Regge trajectory up to 2 loops

[Chachamis, MH, Madrigal, Sabio Vera, 1307.2591]

+ NLO impact factor for forward jets with rapidity gap

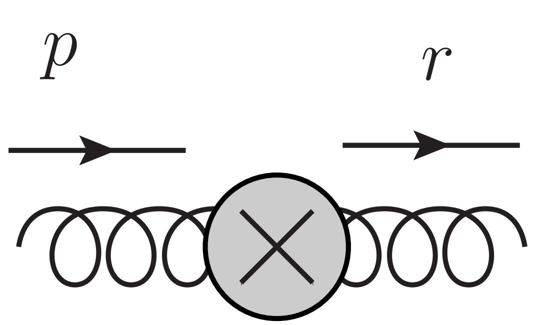
[MH, Madrigal, Murdaca, Sabio Vera, 1404.2937, 1406.5625, 1409.6704]

more recent: description of dilute-dense collision (=Color Glass Condensate formalism) from high energy effective action → confirms previous light-cone gauge results **[MH, 1802.06755]**



$$= \tau_F(q,-r) = 2\pi\delta(p^+ - r^+)\not{n}^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})}$$

$$\cdot \left[\theta(p^+)\left[W(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[[W(\boldsymbol{z})]^\dagger - 1\right]\right].$$



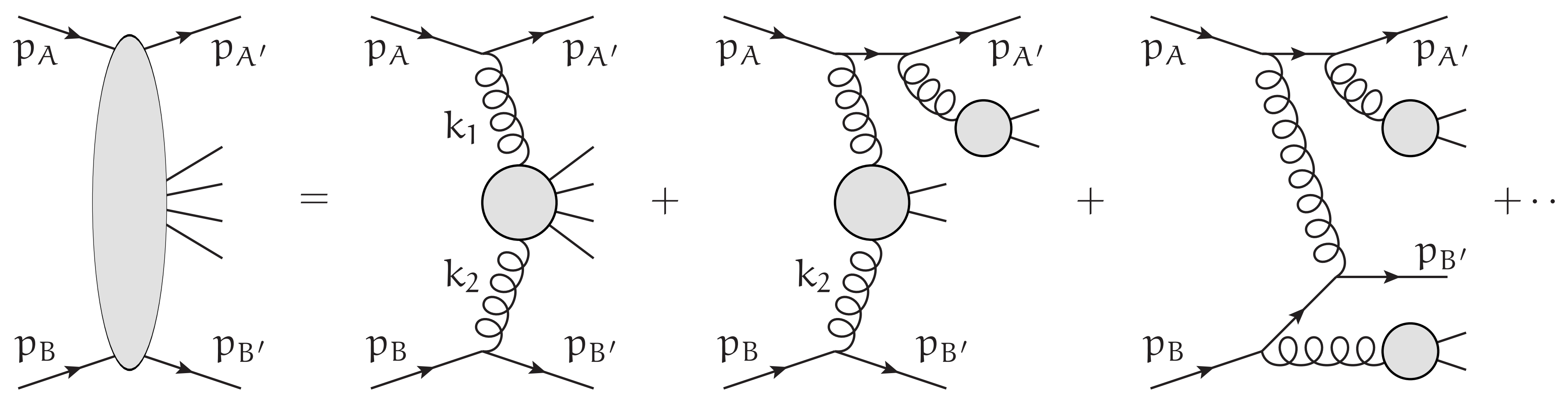
$$= \tau^{ab}_{G,\nu\mu}(p,-r) = -4\pi\delta(p^+ - r^+)\Gamma_{\nu\mu}(r,p)\int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})}$$

$$\cdot \left[\theta(p^+)\left[U^{ba}(\boldsymbol{z}) - \delta^{ab}\right] - \theta(-p^+)\left[[U^{ba}(\boldsymbol{z})]^\dagger - \delta^{ab}\right]\right].$$

Complementary formalism (for dilute collisions)

same amplitude (with 1 initial reggeized gluon per scattering hadron) can be directly calculated from spinor-helicity amplitudes

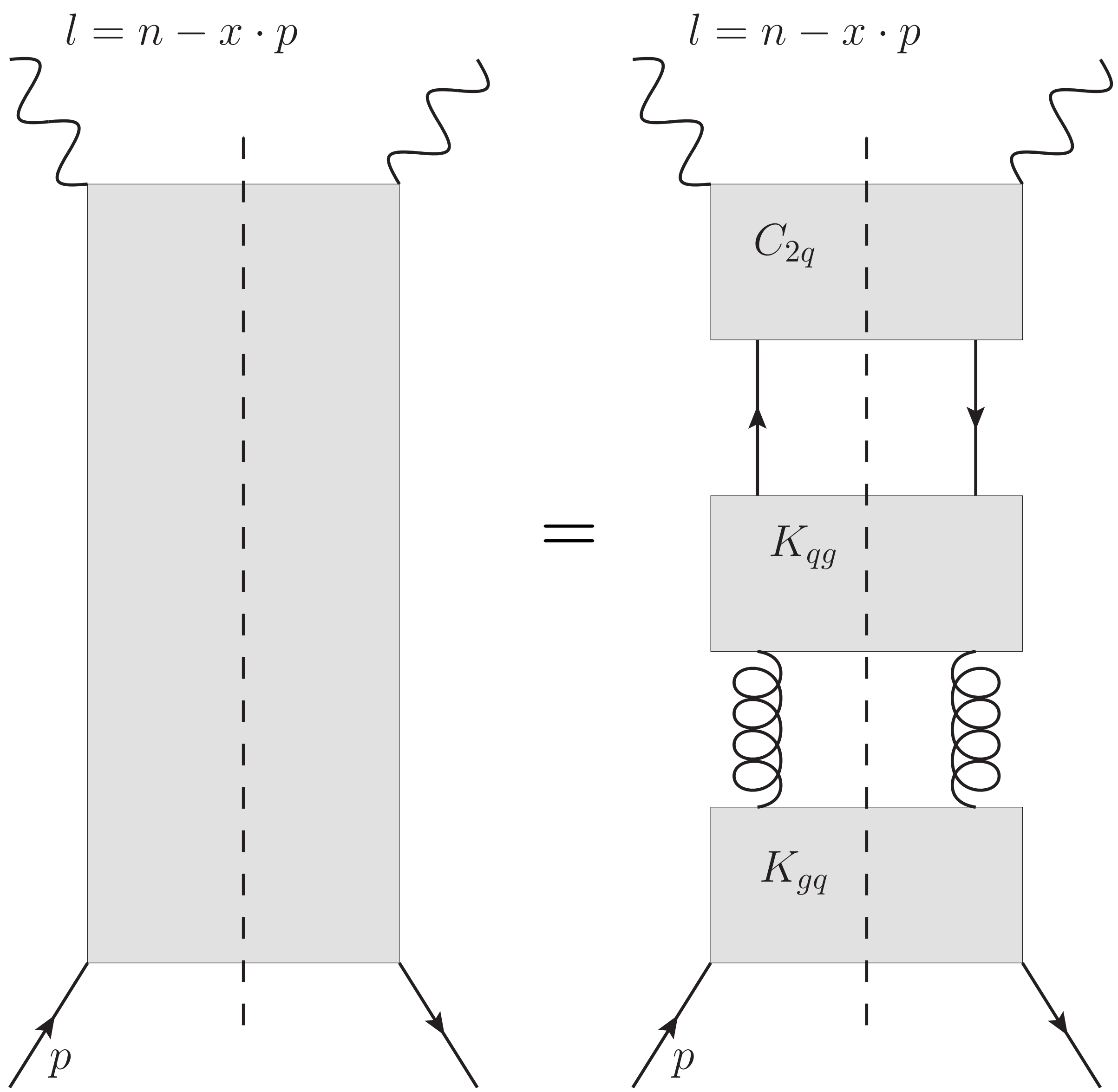
[van Hameren, Kotko, Kutak; 1211.0961], [van Hameren, Kutak, Salwa; 1308.2861]



CFP-formalism

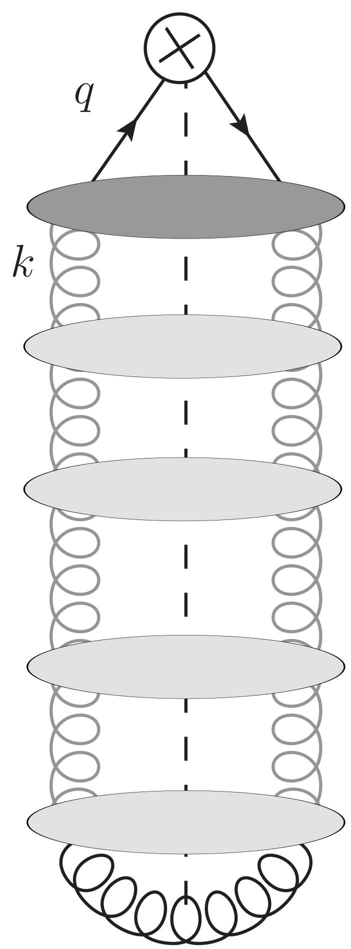
starting point: diagrammatic definition of collinear factorization

[Curci, Furmanski, Petronzio, Nucl.Phys. B 175 (1980) 27]



- axial, light-cone gauge: collinear singularities only form propagator which connect sub-amplitudes
- to isolate coefficient of collinear singularities use projectors in spinor/Lorentz space
- calculate DGLAP splitting functions as expansion in α_s

[Catani, Hautmann, NPB 427 (1994) 475] : TMD splitting function $P_{qg}(z,k_T)$ as coefficient for all order resumed $P_{qg}(z)$



upper blob: no low x logarithm; finite → defines a TMD quark-to-gluon splitting function

$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}, \epsilon\right) = \mathrm{Tr}\left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2 \cdot \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]$$

$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

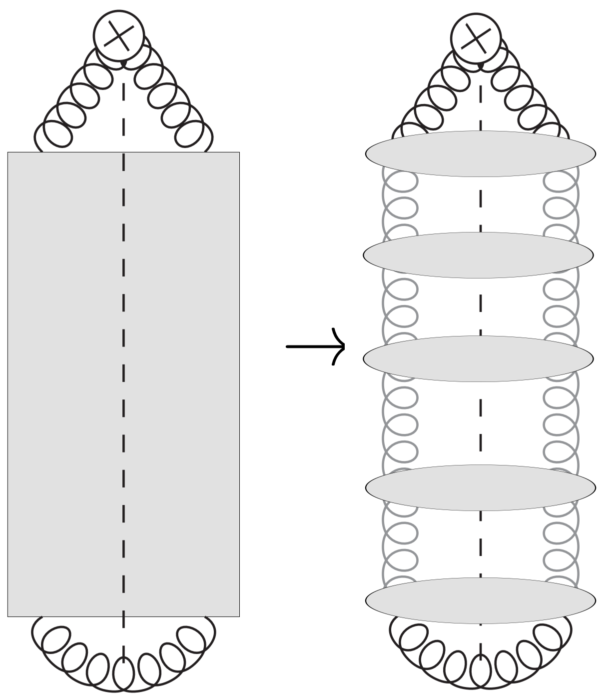
later on: use of high energy factorization for quarks

→ off-shell factorization "*splitting function*" ⊗ "*coefficient*" **[Hautmann, MH, Jung; 1205.1759]**

first generalization to finite TM

High energy/low x resummation of splitting functions

[Catani, Hautmann; NPB 427 (1994) 475]



- essentially the BFKL Green's function ➡ low x resummation of gluon splitting function
- use off-shell extension of incoming projector $\mathbb{P}^{\mu\nu}_{\text{gluon, in}} \to \frac{\boldsymbol{k}^\mu \boldsymbol{k}^\nu}{\boldsymbol{k}^2}$
- derived within high energy factorization + reduces to conventional projector in on-shell limit

obtain: all order P_{gg} with $(\alpha_s \ln 1/x)^n$
however:
all order P_{qg} requires $\alpha_s\,(\alpha_s \ln 1/x)^n$ (starts at NLL ➡ finite coefficient)

Generalized projectors

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{in}} = \frac{1}{d-2}\left(-g^{\mu\nu} + \frac{l^\mu n^\nu + n^\mu l^\nu}{l\cdot n}\right), \quad \mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{out}} = -g^{\mu\nu},$$

$$\mathbb{P}^{s}_{q,\,\mathrm{in}} = \frac{\not{l}}{2}, \quad \mathbb{P}^{s}_{q,\,\mathrm{out}} = \frac{\not{n}}{2\,n\cdot l}.$$

collinear splitting functions

[Curci, Furmanski, Petronzini; NPB 175 (1980)]

generalization:

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{in}} = -y^2\,\frac{p^\mu p^\nu}{k_\perp^2}, \quad \mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} - k^2\,\frac{n_\mu n_\nu}{(k\cdot n)^2},$$

$$\mathbb{P}^{s}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}, \quad \mathbb{P}^{s}_{q,\,\mathrm{out}} = \frac{\not{n}}{2\,n\cdot l}.$$

worked out in:

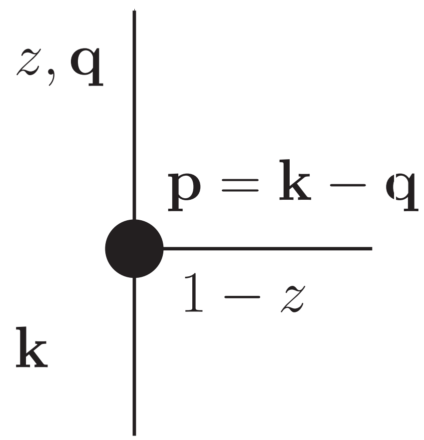
[Catani, Hautmann, NPB 427 (1994) 475]

[Gituliar, MH, Kutak; 1511.08439]

[MH, Kusina, Kutak, Serino; 1711.0458]

Evolution equation and soft limit

Soft singularities $\boldsymbol{p} \to 0$



$\tilde{\boldsymbol{p}} = \frac{\boldsymbol{p}}{1-z}$

- Appears for P_{qq}, P_{qg}, P_{gg}
- q-q, g-g: regularized by virtual corrections (gg essentially verified)

$$\hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu_F^2}, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \int \frac{d^{2+2\epsilon}\tilde{\boldsymbol{p}}}{\pi^{1+\epsilon}(\mu^2 e^{\gamma_E})^{\epsilon}} \Theta\left(\mu_F^2 - q^2\right) \frac{\tilde{P}_{ij}\left(z, \tilde{\boldsymbol{p}}, \boldsymbol{k}, \epsilon\right)}{\tilde{\boldsymbol{p}}^2}$$

the coefficients of the singularities:

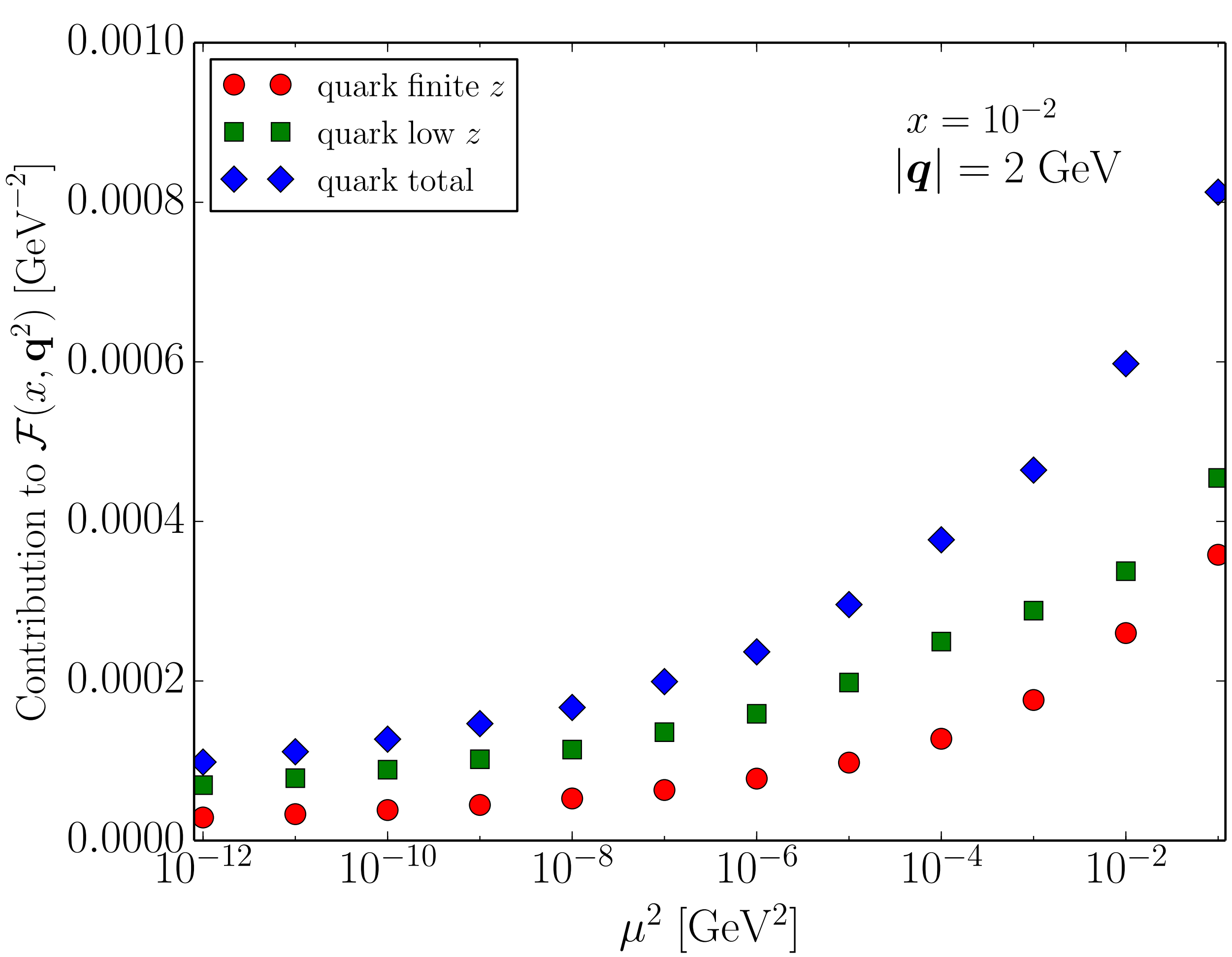
$$\lim_{\tilde{\boldsymbol{p}}^2 \to 0} \tilde{P}_{qq} = \frac{2 \cdot C_f}{(1-z)^{1-2\epsilon}} \qquad \lim_{\tilde{\boldsymbol{p}}^2 \to 0} \tilde{P}_{gq} = \frac{2 \cdot C_f (1-z)^{2\epsilon}}{z}$$

$$\lim_{\tilde{\boldsymbol{p}}^2 \to 0} \tilde{P}_{gg} = 2N_c \left[\frac{1}{z} + \frac{1}{1-z}\right]$$

- what about q→g?

Can study these effects already the pure high energy limit

$$\mathcal{F}(x,\boldsymbol{q}^2) = \tilde{\mathcal{F}}^0(x,\boldsymbol{q}^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\int \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\theta(\boldsymbol{p}^2-\mu^2)\Bigg[\Delta_R(z,\boldsymbol{q}^2,\mu^2)$$
$$\left(2C_A\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right) + C_F\mathcal{Q}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)\right)$$
$$-\int_z^1 \frac{dz_1}{z_1}\Delta_R(z_1,\boldsymbol{q}^2,\mu^2)\left[\tilde{P}'_{gq}\left(\frac{z}{z_1},\boldsymbol{p},\boldsymbol{q}\right)\frac{z}{z_1}\right]\mathcal{Q}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)\Bigg]$$



Find:
Resummation of (finite) virtual result (IR poles cancelled) regularizes the singularity of the P_{gq} splitting

[MH, Kusina, Kutak; 1607.01507]