New HVP Calculation for the Muon g-2

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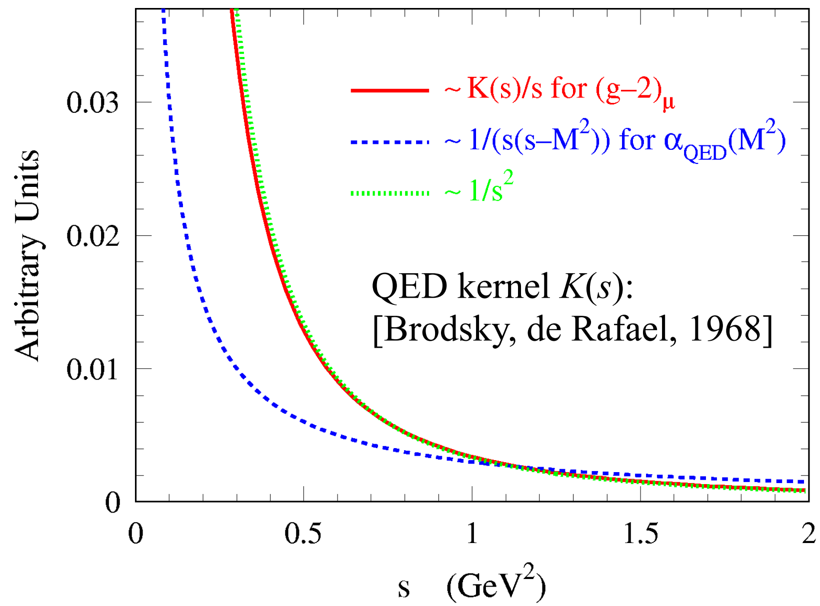
Dispersion relation: [Bouchiat and Michel, 1961]

$$a_\mu^{\mathrm{had}} = \frac{\alpha^2(0)}{3\pi^2}\int_{4\pi_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)\,, \qquad R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Outline of the talk

- What's new?
- Results (preliminary)
- Summary

Based on arXiv:1706.09436
+ new updates (arXiv:1908.00921)



What's new since [1706.09436]?

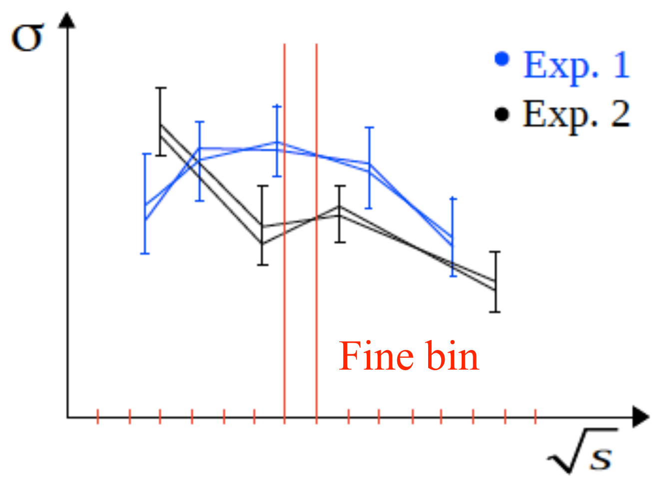
- For the dominant $\pi^+\pi^-$ channel
  - Updated measurements from KLOE [1711.03085]
  - New measurement from CLEO [1712.04530]
  - Fit based on analyticity and unitarity to all data below 1 GeV
    A. Fit result used [threshold, 0.6 GeV]
    B. Data integration using combined data from HVPTools [0.6, 1.8] GeV
    C. Take into account the correlation between the two regions

- For K^+K^-
  - New measurement from CMD-3 [1710.02989]

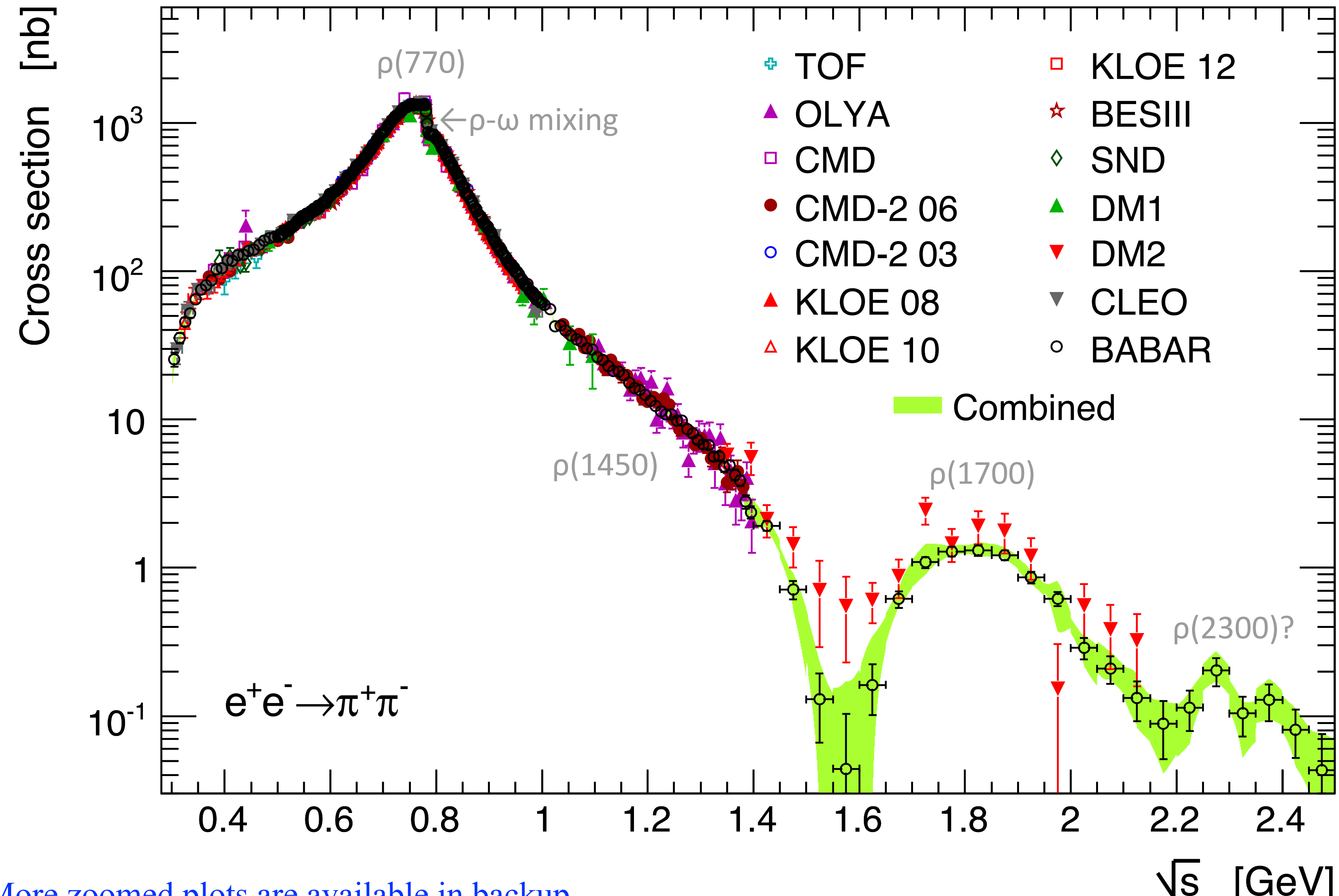
- More conservative treatment of discrepancy in the $\pi^+\pi^-$ channel

Data Combination using HVPTools [0908.4300]

- Combine experimental spectra with arbitrary spacing/binning
- Linear/quadratic splines to interpolate between points/bins of each experiment
  - For binned measurements: preserve integral inside each bin
- Fluctuate data points taking into account correlations and redo the splines for each (pseudo)experiment
  - Each uncertainty fluctuated coherently for all points/bins that it impacts
  - Eigenvector decomposition for (stat & syst) covariance matrices
- Resulting combination shown in fine binning
  - Local error inflation following PDG prescription ($\sqrt{\chi^2/\text{dof}}$) to take better into account data tension

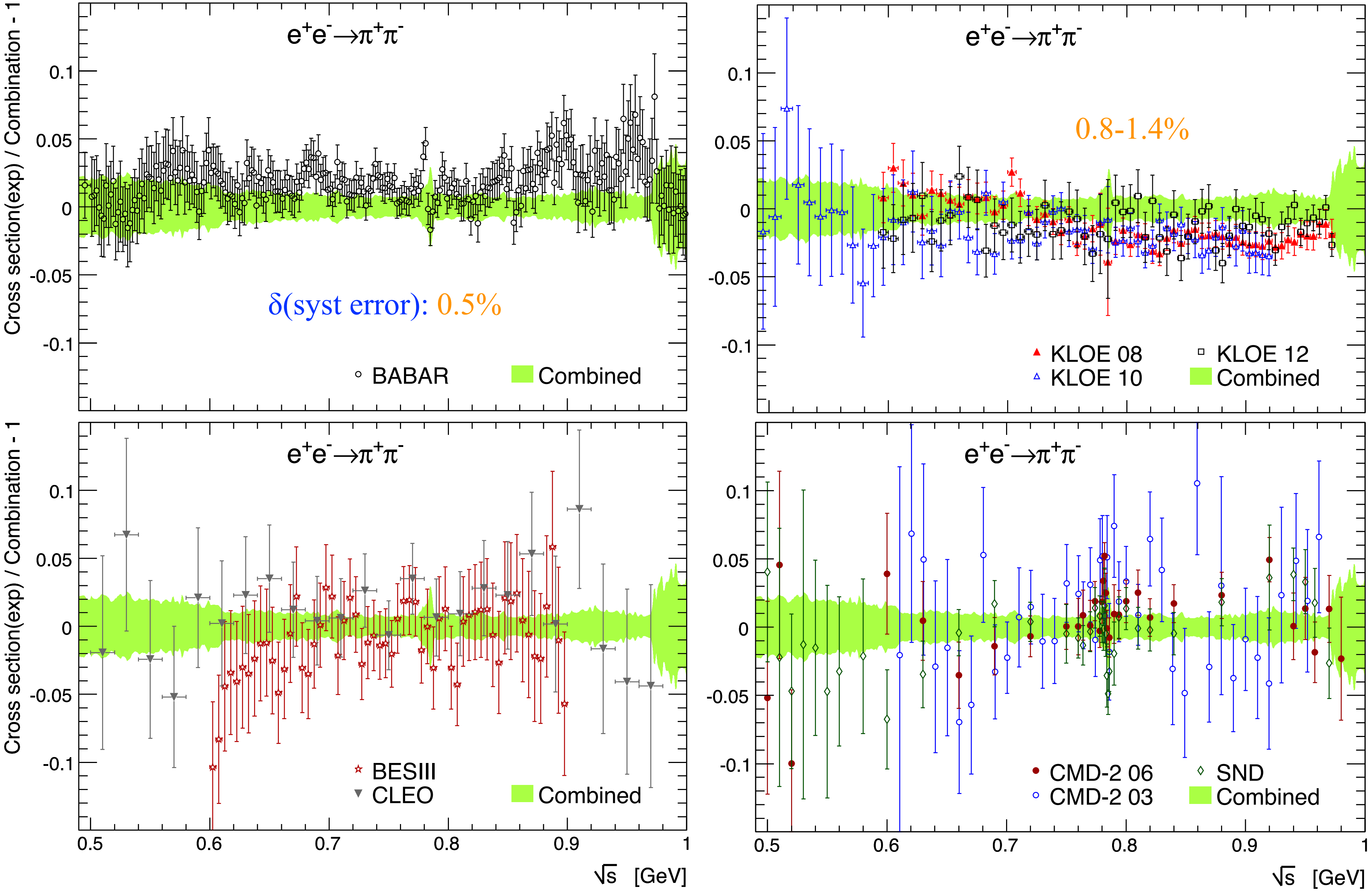


The Dominant $\pi^+\pi^-$ Channel (1)

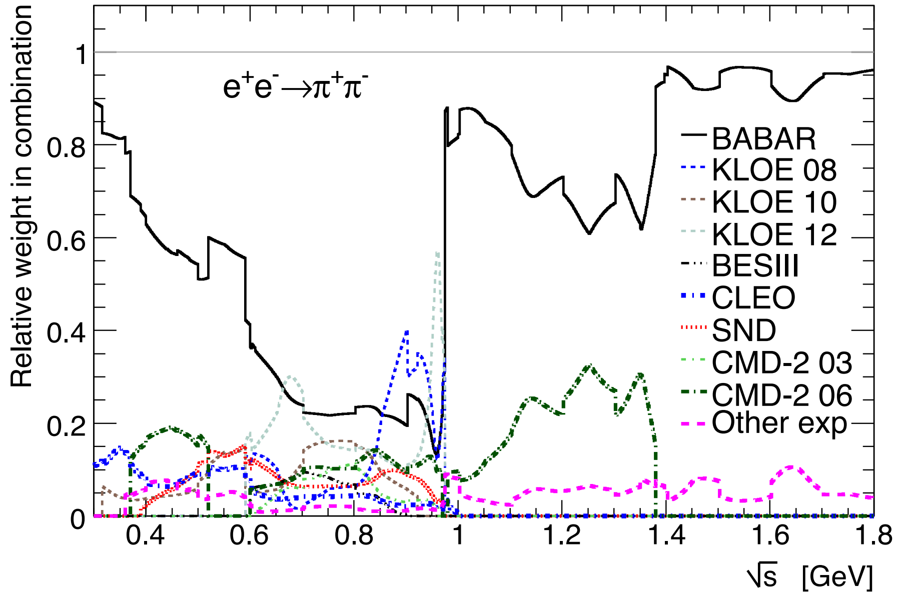


More zoomed plots are available in backup

The Dominant $\pi^+\pi^-$ Channel (2)

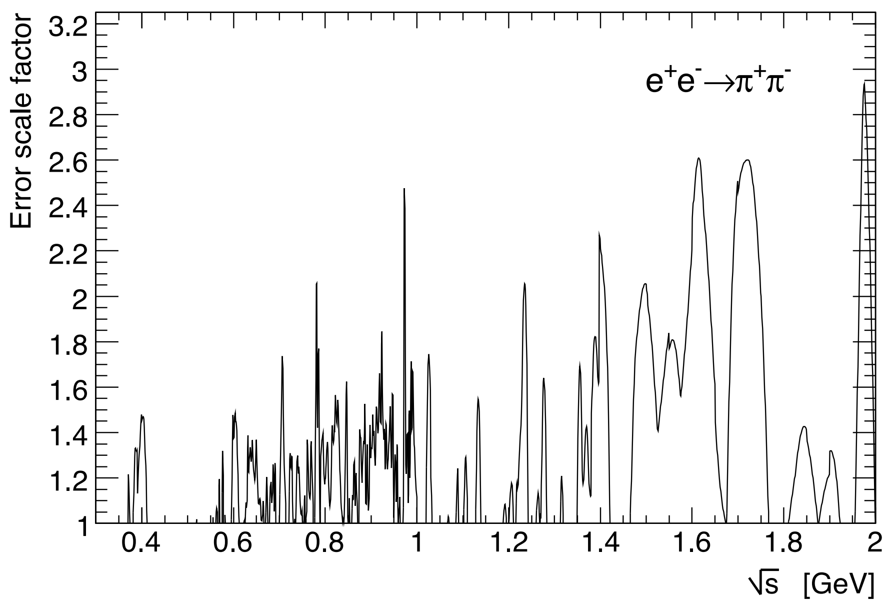


The Dominant $\pi^+\pi^-$ Channel (3)



In the energy range of [0.6-0.9] GeV, BABAR and KLOE dominate, elsewhere BABAR has better precision

Inconsistency between measurements reflected by large local scale factor (SF) defined by $\sqrt{\chi^2/\mathrm{dof}}$ a la PDG (~15% increase on $\delta a_\mu^{\mathrm{had}}$)



Fit Based on Analyticity and Unitarity

Pion form factor F_π^0 extracted from $\pi^+\pi^-$ bare cross sections as in [1810.00007]

$$|F_\pi^0|^2 = |R(s) \times J(s)|^2$$

$$R(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}$$

$$J(s) = e^{1-\frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1-\frac{\delta_1(s_0)}{\pi}\right]\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi}\int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}}$$

$$\cot\delta_1(s) = \frac{\sqrt{s}}{2k^3}\left(m_\rho^2 - s\right)\left[\frac{2m_\pi^3}{m_\rho^2\sqrt{s}} + B_0 + B_1\omega(s)\right]$$

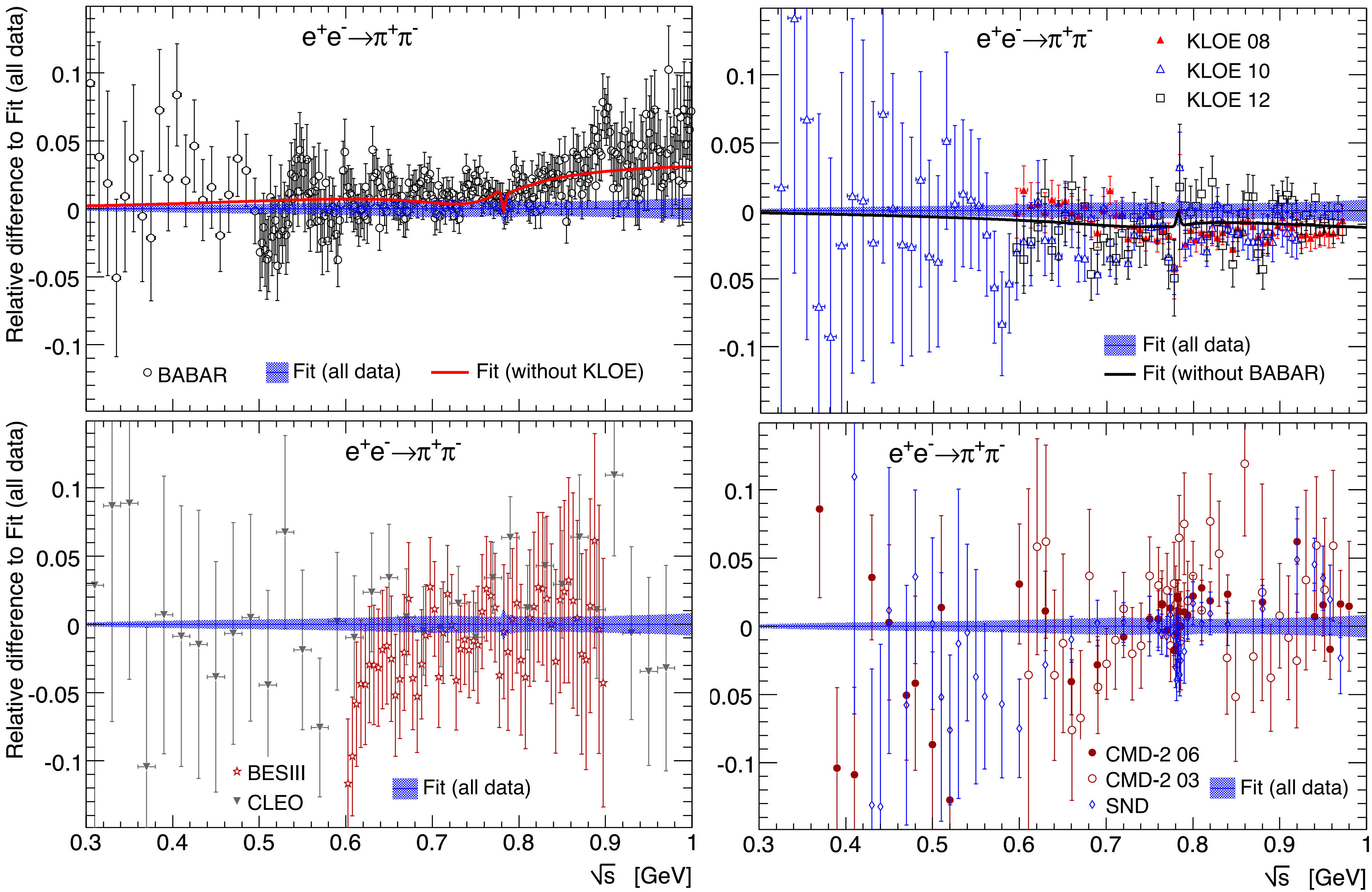
$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}$$

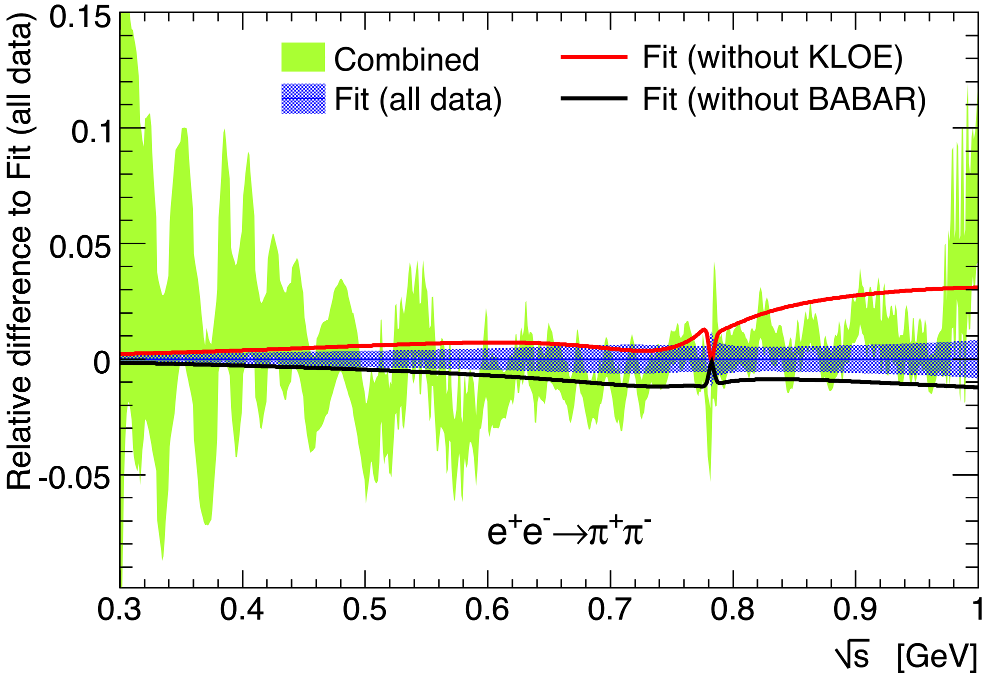
$$\sqrt{s_0} = 1.05 \text{ GeV}$$

$R(s)$ from [1611.09359]
$J(s)$ from [hep-ph/0106025, 0402285]
$\cot\delta_1(s)$ from [1102.2183]

Fit Performed to 1 GeV, Results Used to 0.6 GeV



Comparison Fit and Data Integration



- ❒ Use fit only below 0.6 GeV for a_μ^{had} integral
  - ➤ Where the data are less precise and scarce
  - ➤ Less affected by potential uncertainties of inelastic

$\sqrt{s}$ range [GeV]	a_μ^{had} [10^{-10}] Fit	a_μ^{had} [10^{-10}] Data Integration
0.3 - 0.6	$109.80 \pm 0.37_{exp} \pm 0.19_{para*}$	$109.6 \pm 1.0_{exp}$

⇒ The difference 0.2 ± 0.8 (correlation accounted for)

⇒ The fit improves the precision by 60%

* Parameter uncertainty corresponds to variations with an additional B_2 term in the phase shift formula and $\sqrt{s_0}$ varied from 1.05 GeV to 1.3 GeV

Combined Results Fit [<0.6 GeV] + Data [0.6-1.8 GeV]

$\sqrt{s}$ range [GeV]	a_μ^{had} [10^{-10}] All data	a_μ^{had} [10^{-10}] All but BABAR	a_μ^{had} [10^{-10}] All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

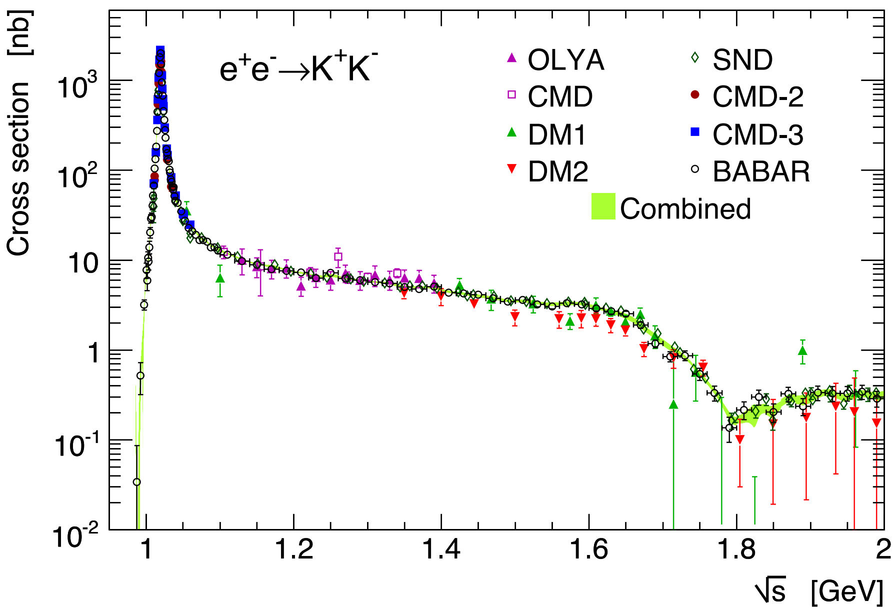
⇒ The difference "All but BABAR" and "All but KLOE" = 5.6 to be compared with 1.9 uncertainty with "All data"

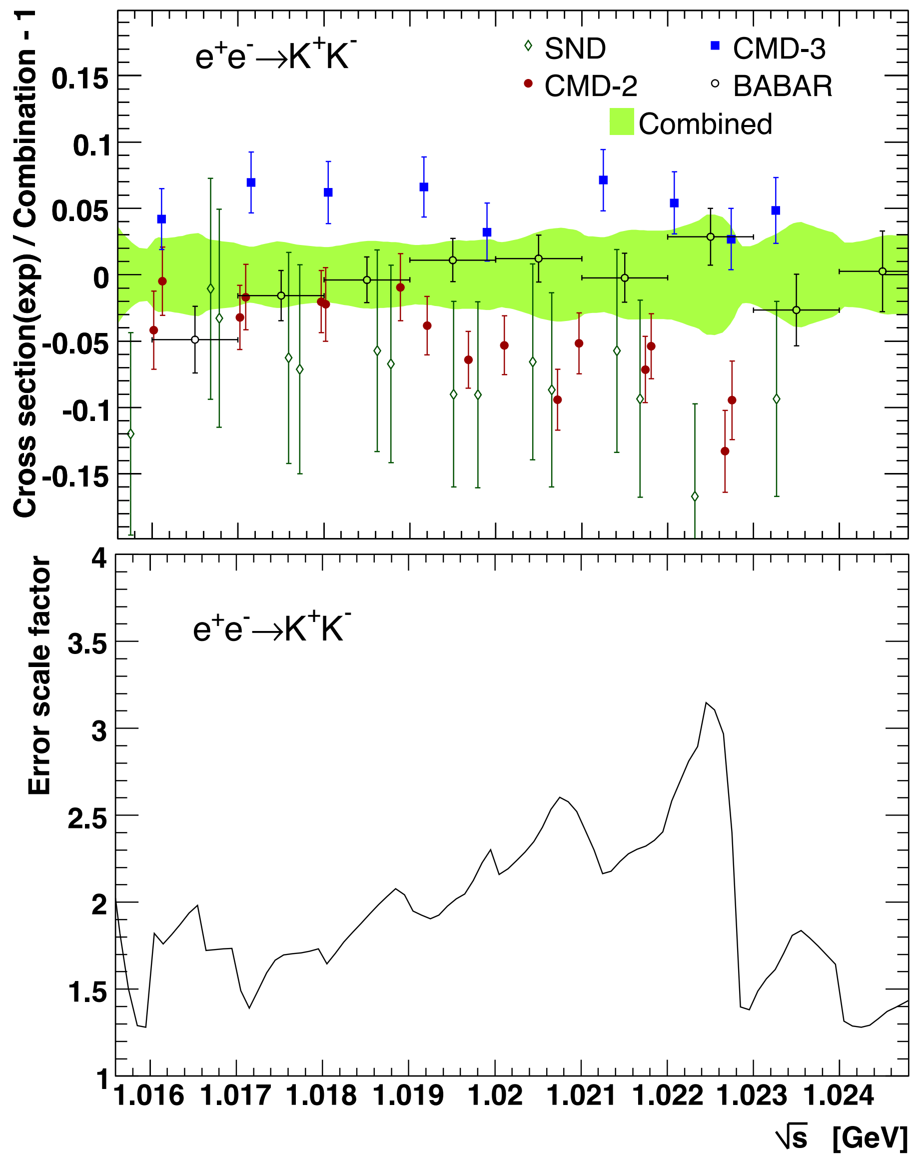
- ➤ The local error inflation is not sufficient to amplify the uncertainty
- ➤ Global tension (normalisation/shape) not previously accounted for
- ➤ Potential underestimated uncertainty in at least one of the measurements?
- ➤ Other measurements not precise enough and are in agreement with BABAR or KLOE

⇒ Given the fact we do not know which dataset is problematic, we decide to

- ➤ Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- ➤ Take the mean value "All but BABAR" and "All but KLOE" as our central value

New Results for K^+K^- Channel





Large discrepancy among measurements

Difficult to determine kaon detection efficiency with precision at CMD-2/3, SND

Easier for BABAR with large ISR photons and strongly boosted kaons

Overall Results

Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$	$\Delta\alpha(m_Z^2)[10^{-4}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.17 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\text{QCD}}\ [1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
$R_{\text{QCD}}\ [5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
$R_{\text{QCD}}\ [9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
$R_{\text{QCD}}\ [12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
$R_{\text{QCD}}\ [> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
$R_{\text{QCD}}\ [> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

Essentially all exclusive channels (>30) below 1.8 GeV are included thanks mainly to measurements in many modes from BABAR (including the recent $\pi^+\pi^-3\pi^0$)

Estimation for missing modes based on isospin constraints becomes negligible (0.016%)

Comparison with Other Determinations

Include other contributions in unit of 10^{-10}:

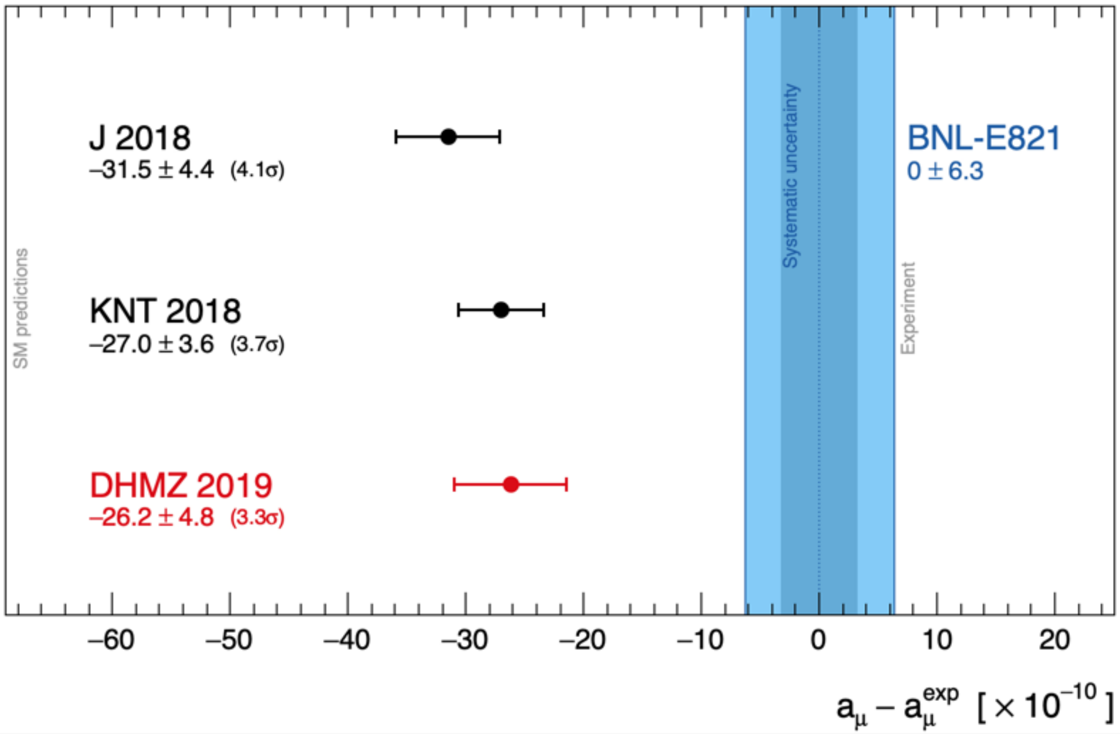
QCD NLO: -9.87 ± 0.07; NNLO: 1.24 ± 0.01; LBL: 10.5 ± 2.6

EW: 15.29 ± 0.10; QED: $11\,658\,471.895 \pm 0.008$

$\Rightarrow a_\mu = 11\,659\,182.9 \pm 4.8_{\text{total}}$

In comparison with the direct measurement: $11\,659\,209.1 \pm 6.3_{\text{total}}$

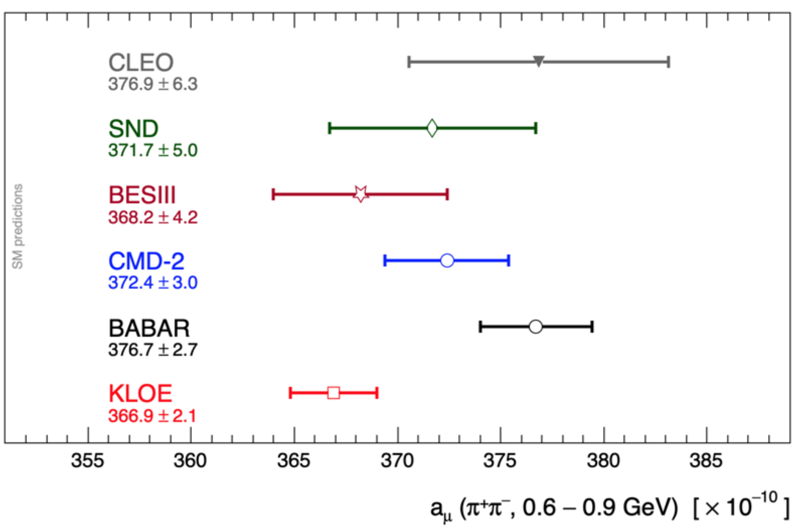
$\Rightarrow 26.2 \pm 7.9\ (3.3\sigma)$

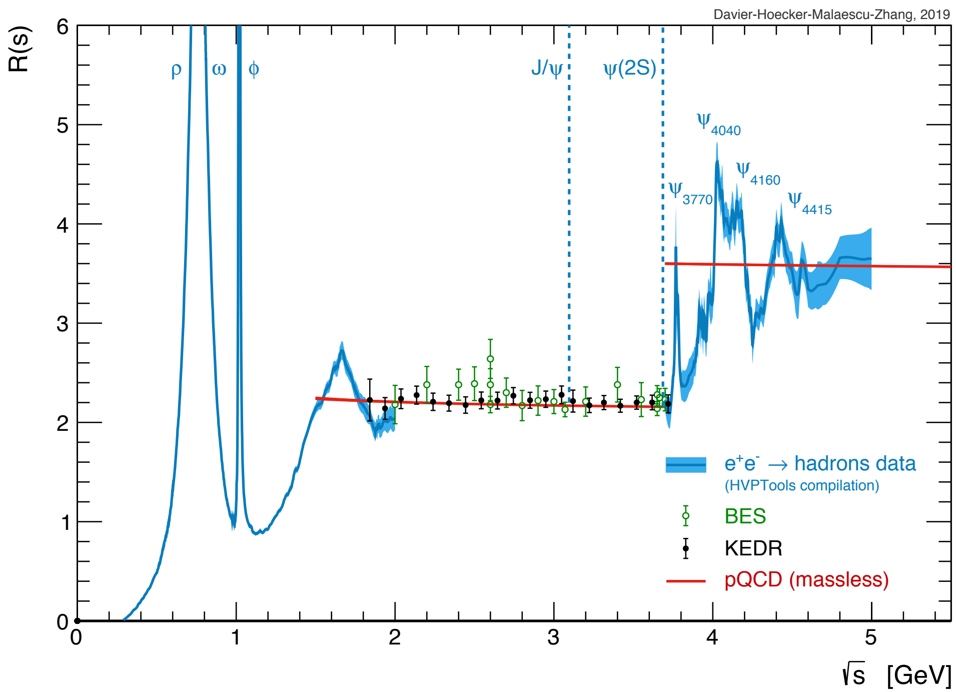


NLO/NNLO: Kurz et al. [1511.08222]
LBL: Prades-de Rafael-Vainshtein [0901.0306]
EW: Tadashi-Nakazawa-Yasui [1810.13445]
QED: Anyama-Hayakawa-Kinoshita-Nio [1205.5370]

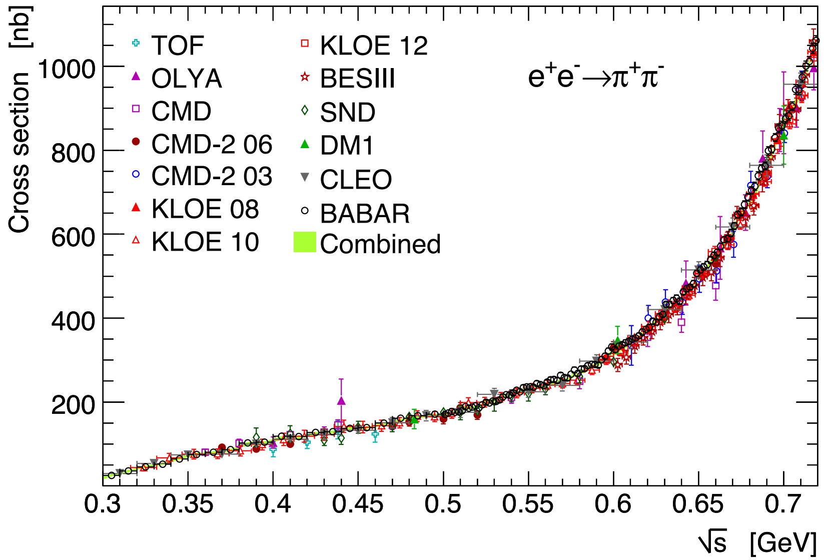
Summary

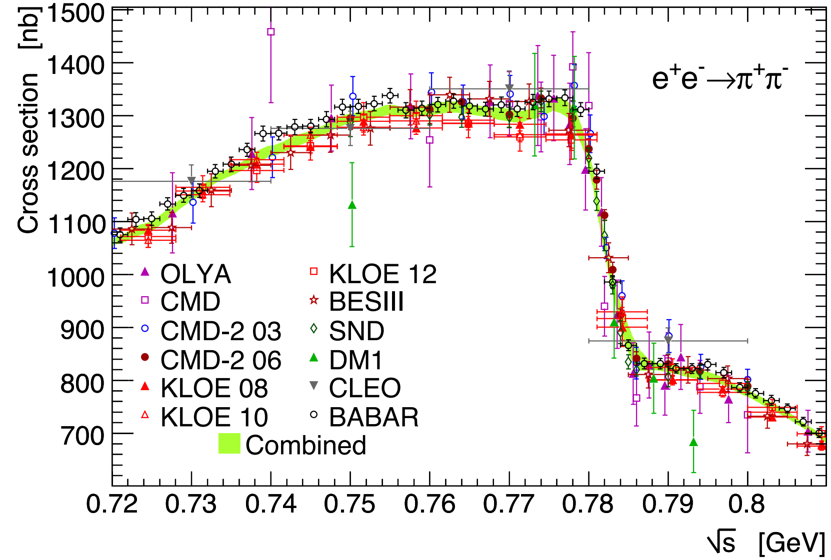
- ❐ A few new measurements/updates included
- ❐ The fit based on analyticity and unitarity improves the precision by 60% for energy range below 0.6 GeV
- ❐ The large discrepancy between BABAR and KLOE in the $\pi^+\pi^-$ channel is not covered by the usual uncertainty estimation (even when local error inflation is applied), we quote this discrepancy as an additional (dominant) uncertainty in our new evaluation
- ❐ We need much more precise and independent measurements to resolve the discrepancy

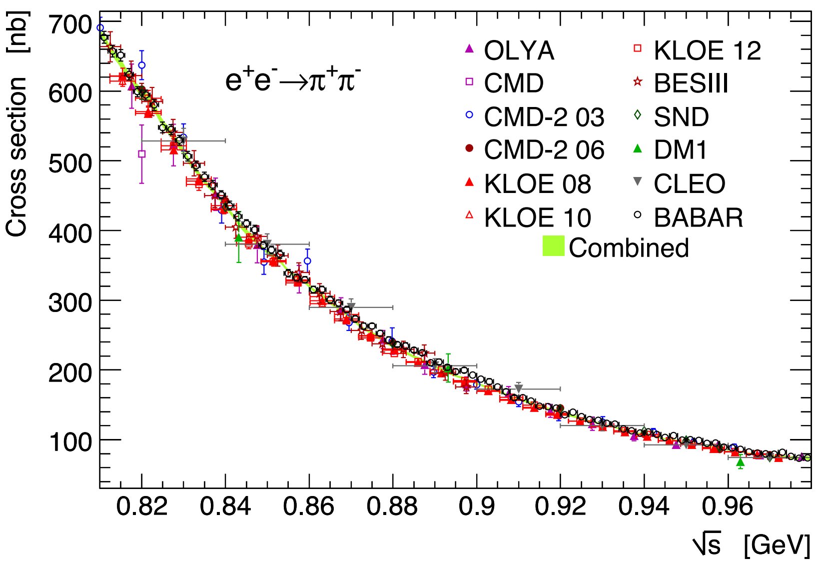


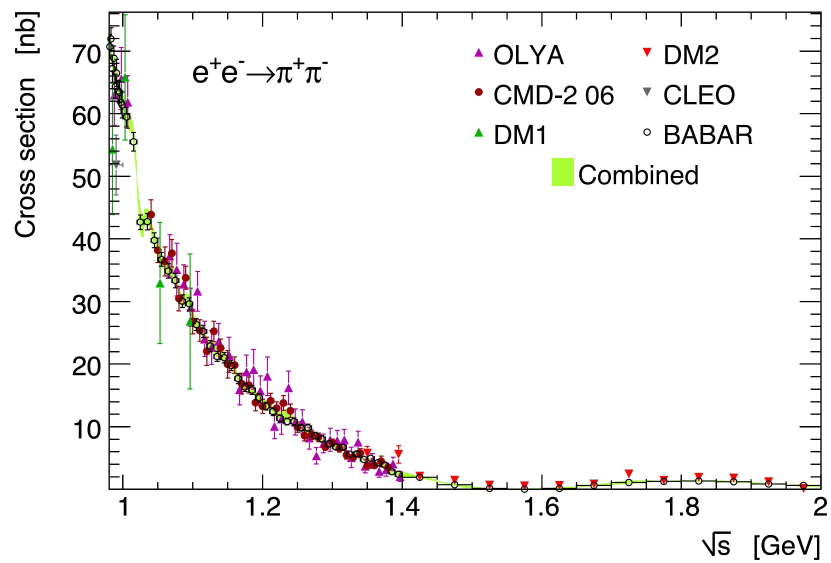


Other Zoomed Plots









KLOE Uncombined Datasets vs Combined One

❒ By default, we use 3 uncombined datasets as provided in [1711.03085]

- ➤ Namely the same inputs that were used for the combination in the above ref.
- ➤ The 3 datasets have different ISR topologies and normalisations
- ➤ But we do compare the difference when the combined dataset is used

$\sqrt{s}$ range [Threshold - 1.8 GeV]	a_μ^{had} [10^{-10}] Uncombined datasets	a_μ^{had} [10^{-10}] Combined one
Fit + data integration	506.9 ± 1.9	506.6 ± 2.0
Data integration only	506.8 ± 2.3	506.6 ± 2.4

Fit Results Compared with Other Determinations

$\sqrt{s}$ range [GeV]	a_μ^{had} [10^{-10}] Our fit	a_μ^{had} [10^{-10}] [1810.00007]	a_μ^{had} [10^{-10}] [1907.01767]
Threshold - 0.63	$133.37 \pm 0.48 \pm 0.23$	$132.5 \pm 1.0 \pm 0.4$	132.91 ± 0.76