Combination and QCD analysis of charm and beauty production cross sections in DIS at HERA

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for the H1 and ZEUS collaborations

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- Introduction: heavy flavours in DIS
- Combination of charm and beauty data
- Comparison with QCD predictions
- QCD fit and determination of $m_c(m_c)$ and $m_b(m_b)$
- Discussion
The HERA ep collider and experiments

HERA I: $\sim 130 \text{ pb}^{-1}$ (physics)
HERA II: $\sim 380 \text{ pb}^{-1}$ (physics)
combined: $\sim 2 \times 0.5 \text{ fb}^{-1}$

up to 30% of cross section
Deep Inelastic ep Scattering at HERA

HERA:

(ℓ) Electron
γ, Z
q

Proton (P)

(ℓ′) Electron
q = ℓ − ℓ′

kinematic variables:

\[ Q^2 = -q^2 \]
photon (or Z) virtuality, squared momentum transfer

\[ \chi_{Bj} = \frac{Q^2}{2Pq} \]
Bjorken scaling variable, for \( Q^2 \gg (2m_q)^2 \):

\[ \gamma = \frac{q_P}{\ell_P} \]
inelasticity, \( \gamma \) momentum fraction (of e)

\[ Q^2 \lesssim 1 \text{ GeV}^2; \text{ photoproduction} \]

\[ Q^2 \gtrsim 1 \text{ GeV}^2; \text{ DIS} \]
Heavy flavour contributions to $\sigma_r$

Measure cross section

$$\frac{d^2\sigma}{dx \, dQ^2} \approx \frac{2\pi\alpha_s^2}{Q^4 x_{Bj}} \left[ 1 + (1 - y)^2 \right] \sigma_r(x_{Bj}, Q^2)$$

**Diagram:**

1. $e^+$ detected
2. $Q^2, x_{Bj} = Q^2/2pq$
3. **anything**
4. $b\bar{b}$ or $c\bar{c}$
5. $\sigma_r b\bar{b}$, $\sigma_r c\bar{c}$
6. QCD
7. $e^+$
8. $\gamma$
9. $Q^2$
10. $\sqrt{\alpha_s}$
11. $g(x)$
12. $27.6$ GeV
13. 920 GeV
14. $b, c$
15. $\bar{b}, \bar{c}$
16. $p$
Combination of 13 charm+beauty data sets

final summary from HERA on c, b in DIS:

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Tagging</th>
<th>$Q^2$ range [GeV$^2$]</th>
<th>$L$ [pb$^{-1}$]</th>
<th>$\sqrt{s}$ [GeV]</th>
<th>$N_c$</th>
<th>$N_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 H1 VTX [14]</td>
<td>VTX</td>
<td>5 – 2000</td>
<td>245</td>
<td>318</td>
<td>29</td>
<td>12</td>
</tr>
<tr>
<td>2 H1 $D^{*\pm}$ HERA-I [10]</td>
<td>$D^{*+}$</td>
<td>2 – 100</td>
<td>47</td>
<td>318</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>3 H1 $D^{*\pm}$ HERA-II (medium $Q^2$) [20]</td>
<td>$D^{*+}$</td>
<td>5 – 100</td>
<td>348</td>
<td>318</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4 H1 $D^{*\pm}$ HERA-II (high $Q^2$) [15]</td>
<td>$D^{*+}$</td>
<td>100 – 1000</td>
<td>351</td>
<td>318</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>5 ZEUS $D^{*+}$ 96-97 [4]</td>
<td>$D^{*+}$</td>
<td>1 – 200</td>
<td>37</td>
<td>300</td>
<td>21</td>
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</tr>
<tr>
<td>6 ZEUS $D^{*+}$ 98-00 [6]</td>
<td>$D^{*+}$</td>
<td>1.5 – 1000</td>
<td>82</td>
<td>318</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>7 ZEUS $D^0$ 2005 [12]</td>
<td>$D^0$</td>
<td>5 – 1000</td>
<td>134</td>
<td>318</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>8 ZEUS $\mu$ 2005 [13]</td>
<td>$\mu$</td>
<td>20 – 10000</td>
<td>126</td>
<td>318</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9 ZEUS $D^+$ HERA-II [21]</td>
<td>$D^+$</td>
<td>5 – 1000</td>
<td>354</td>
<td>318</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>10 ZEUS $D^{*+}$ HERA-II [22]</td>
<td>$D^{*+}$</td>
<td>5 – 1000</td>
<td>363</td>
<td>318</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>11 ZEUS VTX HERA-II [23]</td>
<td>VTX</td>
<td>5 – 1000</td>
<td>354</td>
<td>318</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>12 ZEUS $e$ HERA-II [19]</td>
<td>$e$</td>
<td>10 – 1000</td>
<td>363</td>
<td>318</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>13 ZEUS $\mu$ + jet HERA-I [16]</td>
<td>$\mu$</td>
<td>2 – 3000</td>
<td>114</td>
<td>318</td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

3 additional charm datasets w.r.t. EPJ C73 (2013) 2311

beauty combined for the first time

account for all systematic correlations between data points, data sets, and between charm and beauty

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Charm combination

209 -> 52 data points

3 HERA II data sets added
Beauty combination

57 -> 27 data points

combined for the first time

H1 and ZEUS

Q^2 = 2.5 GeV^2
Q^2 = 5 GeV^2
Q^2 = 7 GeV^2
Q^2 = 12 GeV^2
Q^2 = 18 GeV^2
Q^2 = 32 GeV^2
Q^2 = 60 GeV^2
Q^2 = 120 GeV^2
Q^2 = 200 GeV^2
Q^2 = 350 GeV^2
Q^2 = 650 GeV^2
Q^2 = 2000 GeV^2

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**Fixed Flavour Number Scheme (FFNS)**

**example: charm**

- no charm in proton

- full kinematical treatment of charm mass

  (multi-scale problem: $Q^2, p_T, m_c \rightarrow \text{logs of ratios}$)

- no resummation of logs

- no extra matching parameters

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+ NLO (+partial NNLO) corrections,

"natural" scale: $\mu^2 = Q^2 + 4m_c^2$

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13.07.19

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Comparison to FFNS QCD predictions

- Data reasonably described
- Best: HERAPDF2.0 FF and ABKM09NLO
- ~3σ tension with $x_{Bj}$ slope
- Appr. NNLO does not improve
Comparison to VFNS QCD predictions

Data description reasonable but not better than FF overall, NLO better than appr. NNLO

Beauty in backup: larger uncertainties -> all consistent

arXiv:1804.01019
simultaneous NLO QCD fit of
- combined inclusive DIS data (arXiv:1506.06042), $Q^2_{\text{min}} = 3.5$ GeV$^2$
- new combined charm and beauty DIS data (this work)

simultaneously fit PDF’s (a la HERAPDF FF) in FFNS at NLO and charm quark and beauty quark “running” masses in MSbar scheme
- using xFitter [www.xfitter.org], 14 parameters ($\pm 1$)
- NLO DGLAP [QCDNUM] and matrix elements [OPENQCDRAD], $n_f = 3$
- $\mu_F = \mu_R = \sqrt{Q^2 + 4m_Q^2}$, varied by factor 2 (for heavy flavour part only)
- free $m_c(m_c)$, $m_b(m_b)$
- $\alpha_s(M_Z)^{n_f=3} = 0.106$, equivalent to $\alpha_s(M_Z)^{n_f=5} = 0.118 \pm 0.002$
- fit uncertainty using $\Delta \chi^2 = 1$

--> HERAPDF-HQMASS
QCD fit: charm subset

fully consistent with HERAPDF2.0 FF3A

uncertainty breakdown in backup

$$m_c(m_c) = 1.29^{+0.05}_{-0.04}\ \text{exp/fit} +0.06^{+0.06}_{-0.01}\ \text{mod/scale} +0.00^{+0.00}_{-0.03}\ \text{par} \ \text{GeV}$$

PDG: $1.27 \pm 0.03 \ \text{GeV}$ (lattice QCD + time-like processes)
Comparison with other $m_c(m_c)$ determinations

\textbf{this work:} \hfill
\begin{align*}
&m_c(m_c) = 1.29 \pm 0.05_{\text{exp/fit}} ^{+0.06}_{-0.01 \text{ mod/scale}} \pm 0.00_{\text{par}} ^{+0.06}_{-0.03 \text{ GeV}} \\
\end{align*}

latest ABMP16 result: $m_c(m_c) = 1.252 \pm 0.018 \pm 0.032$ GeV

previous results summarized in V. Bertone et al., arXiv:1605.01946, JHEP 1608 (2016) 050:

\begin{table}[h]
\centering
\begin{tabular}{|l|c|}
\hline
scheme & \quad m_c(m_c) \quad [\text{GeV}] \\
\hline
FONLL (this work) & 1.335 \pm 0.043(\text{exp}) ^{+0.019}_{-0.009}(\text{param}) ^{+0.011}_{-0.008}(\text{mod}) ^{+0.003}_{-0.005}(\text{th}) \\
FFN (this work) & 1.318 \pm 0.054(\text{exp}) ^{+0.011}_{-0.010}(\text{param}) ^{+0.015}_{-0.019}(\text{mod}) ^{+0.006}_{-0.006}(\text{th}) \\
FFN (HERA) [9] & 1.26 \pm 0.05(\text{exp}) \pm 0.03(\text{mod}) \pm 0.02(\alpha_s) \\
FFN (Alekhin et al.) [24] & 1.24 \pm 0.03(\text{exp}) ^{+0.03}_{-0.02}(\text{scale}) ^{+0.09}_{-0.07}(\text{th}) \text{ (approx. NNLO)} \\
& 1.15 \pm 0.04(\text{exp}) ^{+0.04}_{-0.00}(\text{scale}) \text{ (NLO)} \\
S-ACOT-$\chi$ (CT10) [29] & 1.12 ^{+0.05}_{-0.01} \text{ (strategy 1)} \\
& 1.18 ^{+0.05}_{-0.11} \text{ (strategy 2)} \\
& 1.19 ^{+0.06}_{-0.15} \text{ (strategy 3)} \\
& 1.24 ^{+0.06}_{-0.15} \text{ (strategy 4)} \\
\hline
\text{World average} [53] & 1.275 \pm 0.025 \\
\end{tabular}
\end{table}
QCD fit: beauty subset

fully consistent with HERAPDF FF3A

new: \( m_b(m_b) = 4.05^{+0.10}_{-0.11} \) GeV

ZEUS: \( m_b(m_b) = 4.07^{+0.14}_{-0.14} \) GeV

PDG: \( 4.18 \pm 0.03 \) GeV (lattice QCD + time-like processes)
QCD fit: charm x slope

plot data/fit vs. $<x>$ of incoming partons (rather than $x_{Bj}$) for each data point

LO:

$$x = x_{Bj} \cdot \left(1 + \frac{s}{Q^2}\right)$$

$x$ calculated at NLO using HVQDIS

$->$ common $<x>$ trend for all $Q^2$

further discussion (gluon shape (?), low x resummation (?), ...) see backup
Conclusions

Final HERA charm and beauty data in DIS have been combined including all correlations; charm precision improved by ~20%, beauty combined for the first time.

Data are reasonably described by FFNS (best) and VFNS predictions (NLO better than approx. NNLO), but show ~3σ tension in x slope w.r.t inclusive.

QCD fit of inclusive, charm and beauty data (simultaneous fit of PDFs, \(m_c\) and \(m_b\) in FFNS at NLO) yields

\[
\begin{align*}
    m_c(m_c) &= 1290^{+46}_{-41}\text{(exp/fit)}^{+62}_{-14}\text{(mod)}^{+3}_{-31}\text{(par)}\text{ MeV} \\
    m_b(m_b) &= 4049^{+104}_{-109}\text{(exp/fit)}^{+90}_{-32}\text{(mod)}^{+1}_{-31}\text{(par)}\text{ MeV}
\end{align*}
\]

in agreement with world average and previous measurements (not affected by x slope tension within uncertainties).

More detailed studies of x slope tension -> can not be solved by varying the gluon density, or adding higher orders, or resumming log 1/x terms, within the respective current pQCD frameworks -> further investigations useful.
Backup
Comparison to previous charm combination

H1 and ZEUS

- HERA
- HERA 2012

\[ Q^2 = \text{2.5 GeV}^2 \]
\[ Q^2 = \text{5 GeV}^2 \]
\[ Q^2 = \text{7 GeV}^2 \]
\[ Q^2 = \text{12 GeV}^2 \]
\[ Q^2 = \text{18 GeV}^2 \]
\[ Q^2 = \text{32 GeV}^2 \]
\[ Q^2 = \text{60 GeV}^2 \]
\[ Q^2 = \text{120 GeV}^2 \]
\[ Q^2 = \text{200 GeV}^2 \]
\[ Q^2 = \text{350 GeV}^2 \]
\[ Q^2 = \text{650 GeV}^2 \]
\[ Q^2 = \text{2000 GeV}^2 \]
Comparison to FFNS predictions

beauty:
Comparison to FFNS and VFNS predictions

Beauty:

[Graphs showing data points and theoretical predictions for H1 and ZEUS for different values of $Q^2$ and $x_{Bj}$]
Predictions w/o and with log 1/x resummation

NLL resummation of log 1/x terms improves $x_{Bj}$ slope but deteriorates normalisation

overall, NNPDF3.1sx (fitted charm, arXiv:1710.05935) either with or w/o log 1/x resummation not better than HERAPDF (FONLL-C + NLLx see below)
$\chi^2$ and p-values for various QCD predictions

<table>
<thead>
<tr>
<th>Dataset</th>
<th>PDF (scheme)</th>
<th>$\chi^2$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>charm [38]</td>
<td>HERAPDF20_NLO_FF3A (FFNS)</td>
<td>59</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>ABKM09 (FFNS)</td>
<td>59</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>ABMP16_3_nlo (FFNS)</td>
<td>61</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>ABMP16_3_nlnlo (FFNS)</td>
<td>70</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>HERAPDF20_NLO_EIG (RTOPT)</td>
<td>71</td>
<td>0.04</td>
</tr>
<tr>
<td>(N_{data} = 52)</td>
<td>NNPDF31sx NNLO (FONLL-C)</td>
<td>106</td>
<td>1.5 $\times$ 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>NNPDF31sx NNLO+NLLX (FONLL-C)</td>
<td>71</td>
<td>0.013</td>
</tr>
<tr>
<td>charm, this analysis</td>
<td>HERAPDF20_NLO_FF3A (FFNS)</td>
<td>86</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>ABKM09 (FFNS)</td>
<td>82</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>ABMP16_3_nlo (FFNS)</td>
<td>90</td>
<td>0.00008</td>
</tr>
<tr>
<td></td>
<td>ABMP16_3_nlnlo (FFNS)</td>
<td>109</td>
<td>6.1 $\times$ 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>HERAPDF20_NLO_EIG (RTOPT)</td>
<td>99</td>
<td>9.1 $\times$ 10^{-5}</td>
</tr>
<tr>
<td>(N_{data} = 52)</td>
<td>NNPDF31sx NNLO (FONLL-C)</td>
<td>102</td>
<td>4.1 $\times$ 10^{-5}</td>
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<tr>
<td></td>
<td>NNPDF31sx NNLO+NLLX (FONLL-C)</td>
<td>114</td>
<td>5 $\times$ 10^{-7}</td>
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<tr>
<td>beauty, this analysis</td>
<td>HERAPDF20_NLO_FF3A (FFNS)</td>
<td>33</td>
<td>0.20</td>
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<tr>
<td></td>
<td>ABMP16_3_nlo (FFNS)</td>
<td>37</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>ABMP16_3_nlnlo (FFNS)</td>
<td>41</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>HERAPDF20_NLO_EIG (RTOPT)</td>
<td>45</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 4: The $\chi^2$, p-values and number of data points of the charm and beauty data with respect to the NLO and approximate NNLO calculations using various PDFs as described in the text. The measurements at $Q^2 = 2.5$ GeV$^2$ are excluded in the calculations of the $\chi^2$ values for the NNPDF3.1sx predictions, by which the number of data points is reduced to 47, as detailed in the caption of figure 12.
## QCD fit: systematic uncertainties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>$m_c(m_c)$ uncertainty</th>
<th>$m_b(m_b)$ uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[GeV]</td>
<td>[GeV]</td>
</tr>
<tr>
<td><strong>Experimental / Fit uncertainty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$\Delta \chi^2 = 1$</td>
<td>$+0.046$</td>
<td>$-0.041$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+0.104$</td>
<td>$-0.109$</td>
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<tr>
<td><strong>Model uncertainty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_s$</td>
<td>$0.4^{+0.1}_{-0.1}$</td>
<td>$-0.003$</td>
<td>$-0.001$</td>
</tr>
<tr>
<td>$Q_{\text{min}}^2$</td>
<td>$3.5^{+1.5}_{-1.0}$ GeV$^2$</td>
<td>$-0.001$</td>
<td>$-0.005$</td>
</tr>
<tr>
<td>$\mu_{r,f}$</td>
<td>$\times 2.0$</td>
<td>$+0.030$</td>
<td>$+0.007$</td>
</tr>
<tr>
<td>$\mu_{r,f}$</td>
<td>$\times 0.5$</td>
<td>$+0.066$</td>
<td>$+0.007$</td>
</tr>
<tr>
<td>$\alpha_s^{n_f=3}(M_Z)$</td>
<td>$0.1060^{+0.0015}_{-0.0015}$</td>
<td>$-0.014$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>$+0.062$</td>
<td>$+0.002$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.014$</td>
<td>$-0.005$</td>
</tr>
<tr>
<td><strong>PDF parameterisation uncertainty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{f,0}^2$</td>
<td>$1.9 \pm 0.3$ GeV$^2$</td>
<td>$+0.003$</td>
<td>$-0.001$</td>
</tr>
<tr>
<td>$E_{w_i}$</td>
<td>set to 0</td>
<td>$-0.031$</td>
<td>$-0.031$</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>$+0.003$</td>
<td>$+0.001$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.031$</td>
<td>$-0.031$</td>
</tr>
</tbody>
</table>

Table 5: List of uncertainties for the charm- and beauty-quark mass determination. The PDF parameterisation uncertainties not shown have no effect on $m_c(m_c)$ and $m_b(m_b)$. 

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arXiv:1804.01019
QCD fit: charm

fully consistent with HERAPDF2.0FF3A
QCD fit: beauty

fully consistent with HERAPDF2.0FF3A
PDFs consistent with those of inclusive data only (and c, b masses fixed to PDG)

$\rightarrow$ inclusive data (and c,b mass values) dominate in fixing PDF
QCD fit: inclusive data, parametrisation uncert.

Reminder, full fit: $\Delta \chi^2 = 1$

- $m_c(m_c) = 1290^{+46}_{-41}(\text{exp/fit})^{+62}_{-14}(\text{mod})^{+3}_{-31}(\text{par})$ MeV
- $m_b(m_b) = 4049^{+104}_{-109}(\text{exp/fit})^{+90}_{-32}(\text{mod})^{+1}_{-31}(\text{par})$ MeV

Using inclusive HERA data only (14p):

- $m_c(m_c) = 1798^{+144}_{-134}(\text{exp/fit})$ MeV
- $m_b(m_b) = 8450^{+2280}_{-1810}(\text{fit})$ MeV

no full uncertainty evaluation, but large sensitivity to PDF parametrisation (→ 13p):

- $m_c(m_c) = 1798 \rightarrow 1450$ MeV,
- $m_b(m_b) = 8450 \rightarrow 3995$ MeV

→ inclusive HERA data alone cannot constrain HQ masses reliably
→ interplay of PDFs and HQ masses needs careful treatment
plot data/fit vs. $<x>$ of incoming partons (rather than $x_{Bj}$) for each data point

LO: $x = x_{Bj} \left( 1 + \frac{s}{Q^2} \right)$

$<x>$ calculated at NLO using HVQDIS

$\rightarrow$ beauty consistent with charm but does not add information
$\chi^2$ as function of min. $x_{Bj}$ cut

H1 and ZEUS

$\chi^2$/d.o.f.

- DIS+c+b
- c+b

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arXiv:1804.01019
QCD fit with $x_{Bj} > 0.01$ for inclusive data

can improve low $x$ charm slope
(no longer constrained by inclusive)

but fails to describe low $x$ inclusive data

$\rightarrow$ not a solution (but hint)
QCD fit with $x_{\text{Bj}} > 0.01$ for inclusive data

charm and beauty mass floating

gluon at $x < 0.01$ inconsistent with inclusive fit
FONLL-C fit of inclusive data

arXiv:1802.00064 (XFitter team):
FONLL-C inclusive fit (no charm) with and without NLLx resummation

personal remark:
FONLL-C inclusive fit with NLLx qualitatively consistent with FF charm
+ x > 0.01 inclusive fit (compare previous slide)

-> combine both worlds by applying NLLx to light flavours only in FF scheme?

Figure 3 The up valence PDF $xu_u$, the gluon PDF $xg$ and the total singlet PDF $x\Sigma$ for the final fits with (NNLO+NLLx) and without (NNLO) $\ln(1/x)$ resummation.
Comparison of HERAPDF with FONLL-C + NLLx

for inclusive data only

from

arXiv:1802.00064:

inclusion of
NLLx resummation with
FONLL-C achieves
similar performance
as HERAPDF2.0 FF3B

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Table 1: The $\chi^2$ per degree of freedom (d.o.f.) for the PDF fits under different conditions, starting from the settings for the HERAPDF2.0 NNLO.

<table>
<thead>
<tr>
<th>Step-1</th>
<th>Step-2</th>
<th>Step-3</th>
<th>Step-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>HERAPDF2.0</td>
<td>FONLL-C</td>
<td>Move $Q_0^2$ and charm threshold</td>
<td>include NLLx resummation</td>
</tr>
<tr>
<td>HERA $\chi^2$/d.o.f</td>
<td>1363/1131</td>
<td>1387/1131</td>
<td>1389/1131</td>
</tr>
</tbody>
</table>

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Table 4: The values of $\chi^2$ per degree of freedom for HERAPDF2.0 and its variants.

<table>
<thead>
<tr>
<th>HERAPDF</th>
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