# Flavor – exotic tetraquarks in large – Nc QCD : do they exist ?

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**Properties of Green functions of four bilinear quark currents made up of quarks of four different flavours, $\bar{a}, b, \bar{c}, d$ at large $N_c$ require two narrow tetraquark states of the same flavour content. On the other hand, we have at hand only one diquark-antidiquark flavor structure $(\bar{a}\bar{c})(bd)$ that might produce a compact tetraquark bound state. This contradiction suggests that large-$N_c$ QCD does not support the existence of narrow flavor-exotic tetraquarks.**

**Based on W.Lucha, D.M., H.Sazdjian,**

*Narrow exotic tetraquark mesons in large-$N_c$ QCD*, **PRD96, 014022, 2017;**

*Tetraquark and two-meson states at large-$N_c$*, **EPJC77, 866, 2017;**

*Are there narrow fravour-exotic tetraquarks in large-$N_c$ QCD?*, **PRD98, 094011, 2018.**

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# Four – point Green functions of bilinear currents and tetraquark poles

• **We study four-point Green functions of bilinear color-singlet quark currents of the form** $j_{ab} = \bar{q}_a q_b$**. Depend on 6 variables** $p_1^2$, $p_2^2$, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, **and the two Mandelstam variables** $s = p^2$ **and** $t = (p_1 - p_1')^2$**.**

**Criteria for selecting diagrams which potentially contribute to the tetraquark pole at** $s = M_T^2$**:**

1. **The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable** $s$**.**
2. **The diagram should have a four-particle cut (i.e. threshold at** $s = (m_a + m_b + m_c + m_d)^2$**), where** $m_i$ **are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

**Diagrams satisfying these criteria are "T-phile" diagrams.**

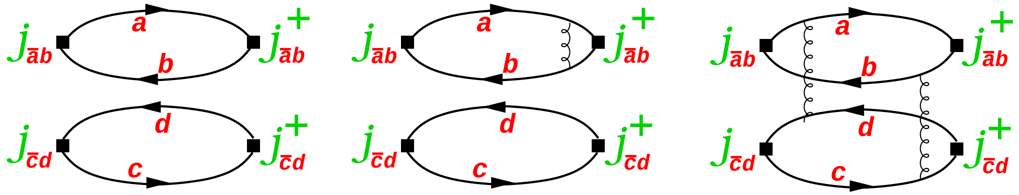
• **We consider "direct" and "recombination" Green functions:**

$$(dir) : \langle j_{ab} j_{cd} j^\dagger_{ab} j^\dagger_{cd}\rangle, \langle j_{ad} j_{cb} j^\dagger_{ad} j^\dagger_{cb}\rangle \qquad\qquad (rec) : \langle j_{ab} j_{cd} j^\dagger_{ad} j^\dagger_{cb}\rangle$$

• **We study these Green functions in large** $N_c$ **QCD: expansion in** $1/N_c$ **keeping** $\alpha_s = 0(1/N_c)$**.**

# Direct Green functions

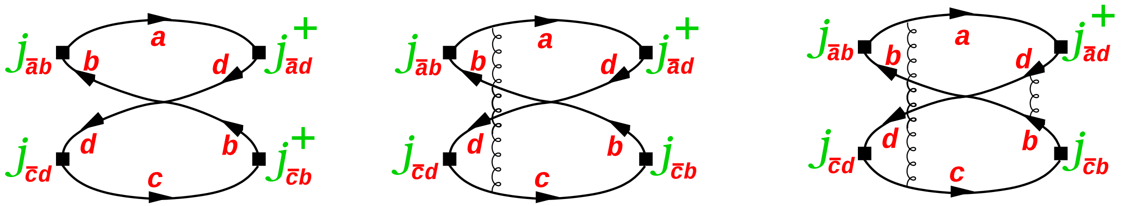
**$\langle j_{ab} j_{cd} j^{\dagger}_{ab} j^{\dagger}_{cd} \rangle$ (similar diagrams for $\langle j_{ad} j_{cb} j^{\dagger}_{ad} j^{\dagger}_{cb} \rangle$):**




(a) ~ $N_c^2$ (b) ~ $N_c^3 \alpha_s$ (c) ~ $N_c^2 \alpha_s^2$

**T-phile diagrams appear at order $O(\alpha_s^2)$ and higher and behave as $O(N_c^0)$.**

# Recombination Green functions




(a) ~ $N_c$ (b) ~ $N_c^2 \alpha_s$ (c) ~ $N_c\, \alpha_s^2$

**Which QCD diagrams contain four-quark $s$-channel cut? Result: (a) and (b) do not have 4-quark cut; one needs two gluon exchanges $O(\alpha_s^2)$ [diagram (c)], order $O(1/N_c)$.**

**Direct and recombination $T$-phile diagrams have different large-$N_c$ behavior.**

• **If $T$-pole appears in direct Green functions it appears also in recombination Green function (and Vice Versa).**

• **One $T$ is not sufficient to saturate both dir and rec Green functions. One needs two different poles**

$$T_A \to M_{\bar{a}b}M_{\bar{c}d} = O(1/N_c), \qquad T_A \to M_{\bar{a}d}M_{\bar{c}b} = O(1/N_c^2)$$
$$T_B \to M_{\bar{a}b}M_{\bar{c}d} = O(1/N_c^2) \qquad T_B \to M_{\bar{a}d}M_{\bar{c}b} = O(1/N_c).$$

• **The diquark-antidiquark structure is the only viable candidate for a compact tetraquark state. We have one diquark-antidiquark flavor structure $(\bar{a}\bar{c})(bd)$. So it is impossible to obtain two different compact tetraquarks decaying dominantly into distinct two-meson channels.**

• **This contradiction suggests that**

**Large-$N_c$ QCD does not support the existence of narrow flavor-exotic tetraquarks.**

# Conclusions

**• A salient feature shared by all tetraquark candidates observed in experiment is the absence of flavor-exotic states of the type $\bar{a}b\bar{c}d$, with four different quark flavors. This phenomenon may be understood from the properties of large-$N_c$ QCD:**

**<u>On the one hand</u>, consistency conditions for flavor-exotic Green functions, potentially containing these tetraquark poles, require the existence of two tetraquarks $T_A$ and $T_B$: each of them should decay dominantly via a single two-meson channel.**

**<u>On the other hand</u>, we have at hand only one diquark-antidiquark flavor structure $(\bar{a}\bar{c})(bd)$ that might produce a compact tetraquark bound state. Taking into account that the diquark-antidiquark structure is the only viable candidate for a compact tetraquark state, one concludes that it is impossible to obtain two different compact tetraquarks decaying dominantly into distinct two-meson channels.**

**We conclude that**

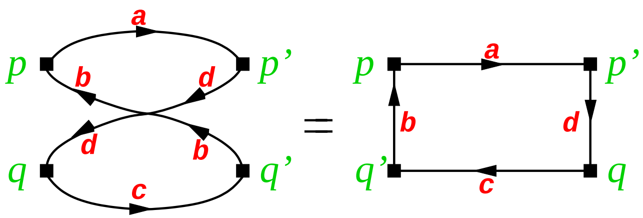
**Large-$N_c$ QCD does not support the existence of narrow flavor-exotic tetraquarks.**

**• The arguments above do not rule out the possible existence of broad molecular-type flavor-exotic states (Green functions require two different tetraquark states, we may easily have two different tetraquark molecular states), or of molecular-type bound states lying very close to the two-meson thresholds.**

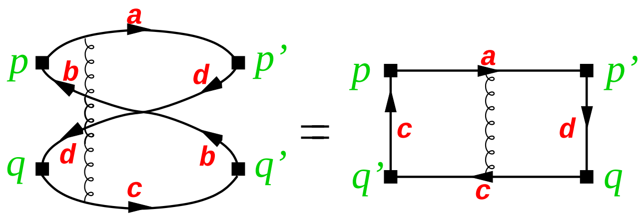
**Further details see W.L., D.M., H.S., PRD98, 094011 (2018).**

**Which recombination QCD diagrams contain four-quark *s*-channel cut?**

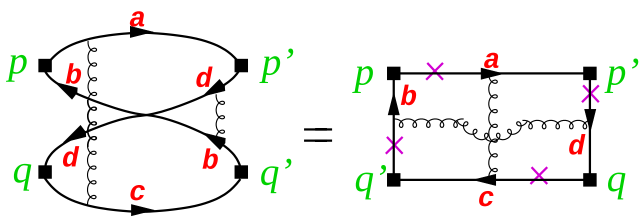
**To answer this question (i) unfold diagrams (ii) solve Landau equations**




*(a)*




*(b)*




*(c)*