ANALYSIS OF SINGULARITIES AND THE 4D REPRESENTATION OF PHYSICAL OBSERVABLES WITHIN THE LTD FORMALISM

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Content

- Characterization of singularities with LTD
  - Location of IR singularities
  - Threshold singularities @ 1-loop
- FDU approach @ NLO
  - Real-dual mappings
  - Applications (toy model & boson decays)
- Conclusions and perspectives

Basic references for FDU/LTD:

1. *Catani et al, JHEP 09 (2008) 065*
2. *Rodrigo et al, Nucl.Phys.Proc.Suppl. 183:262-267 (2008)*
3. **Rodrigo et al, JHEP 02 (2016) 044; JHEP 08 (2016) 160; JHEP 10 (2016) 162; Eur.Phys.J. C78 (2018) n°3, 231; JHEP 02 (2019) 143; arXiv:1904.08389 [hep-ph]**

See talks by William and Felix in this session!

Introduction to Loop-tree duality

Dual representation of one-loop integrals

Loop Feynman integral

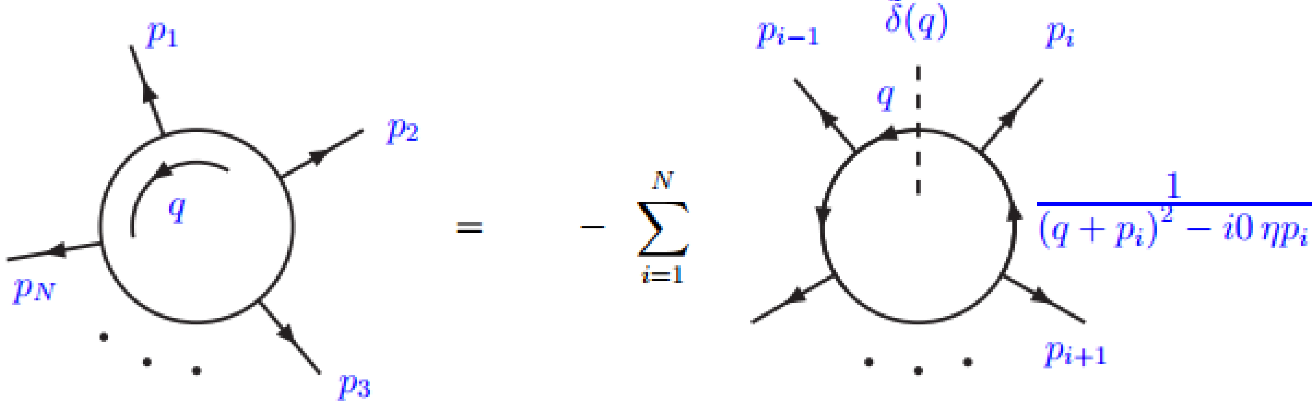
$$L^{(1)}(p_1,\dots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$



Even at higher-orders, the number of cuts is equal the number of loops

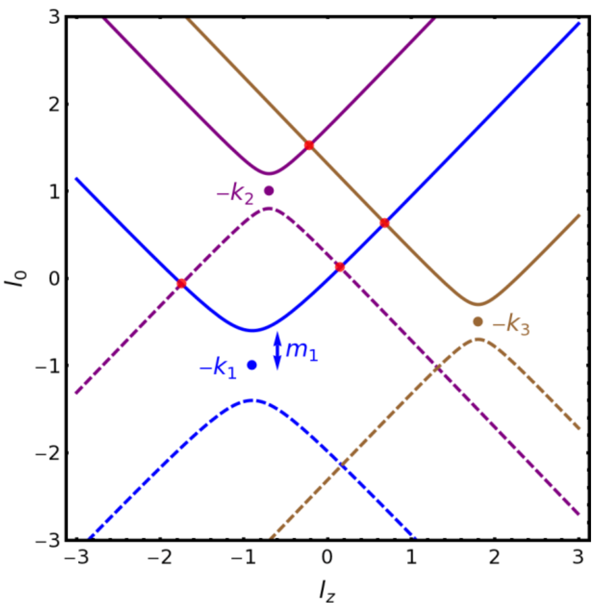
Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Characterization of singularities with LTD

Location of IR singularities in the dual-space

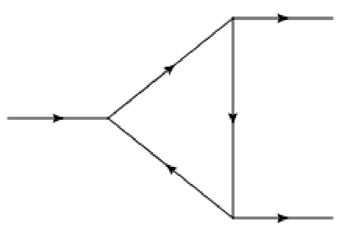
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition:

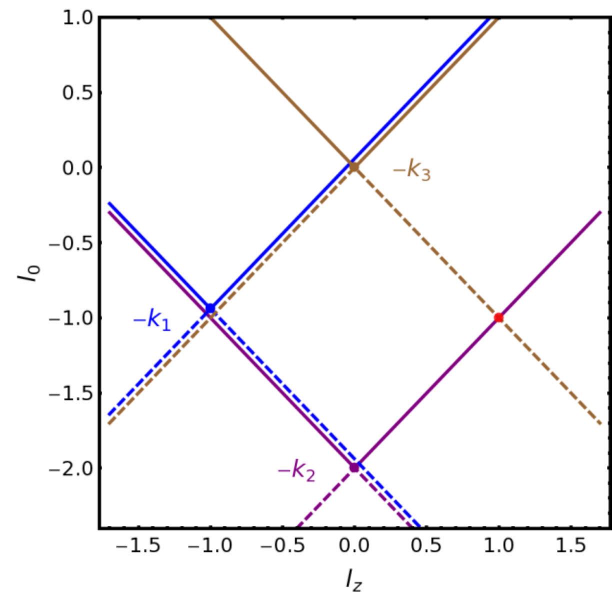
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy solutions.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)





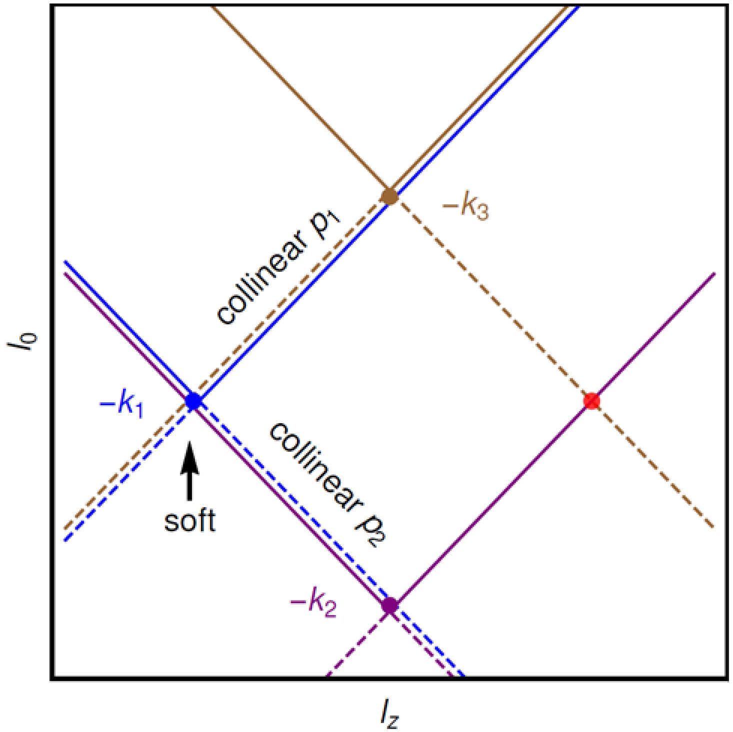
Massless case: light-cones

Rodrigo et al, JHEP11(2014)014, JHEP02(2016)044, JHEP08(2016)160, arXiv:1904.08389

Characterization of singularities with LTD

Location of IR singularities in the dual-space

- The application of LTD converts loop-integrals into PS ones: **integration over forward light-cones**.



- Only **forward-backward** interferences originate **threshold or IR poles** (*other propagators become singular in the integration domain*)
- **Forward-forward** singularities cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- **No threshold or IR singularity at large loop momentum**

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically unstable)

Rodrigo et al, JHEP11(2014)014, JHEP02(2016)044, JHEP08(2016)160, arXiv:1904.08389

Characterization of singularities with LTD

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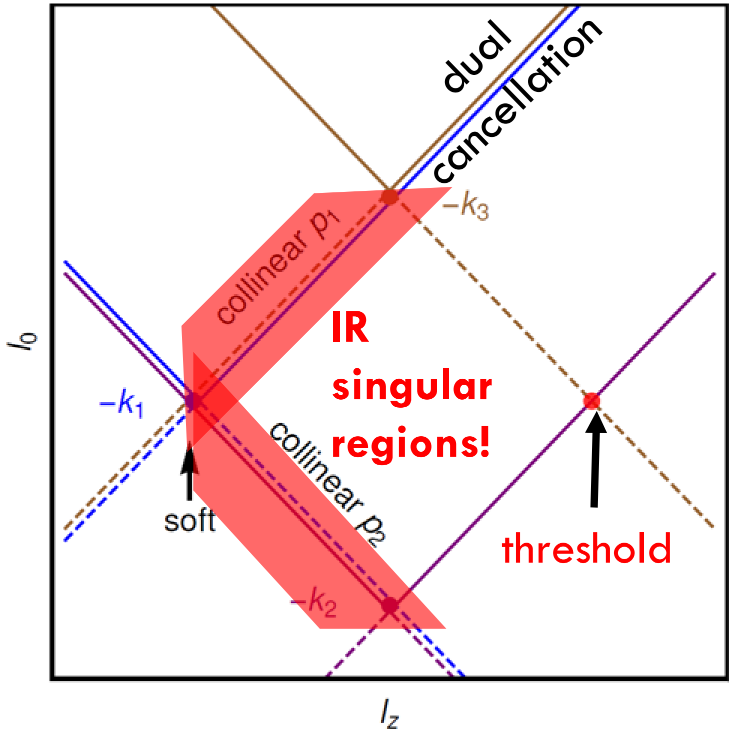

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Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- In general, the location of the singularities is given by the solutions of

$$\lambda_{ij}^{\pm\pm} = \pm q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0} = 0$$

with q_i on-shell and $k_{ji} = q_j - q_i$.

- We consider the following test functions

$$\mathcal{S}_{ij}^{(1)} = (2\pi \imath)^{-1}\, G_D(q_i; q_j)\, \tilde{\delta}\,(q_i) + (i \leftrightarrow j)$$

Up to 2 on-shell states (standard thresholds)

$$\mathcal{S}_{ijk}^{(1)} = (2\pi \imath)^{-1}\, G_D(q_i; q_k)\, G_D(q_i; q_j)\, \tilde{\delta}\,(q_i) + \text{perm.}$$

Up to 3 on-shell states (*anomalous thresholds*)

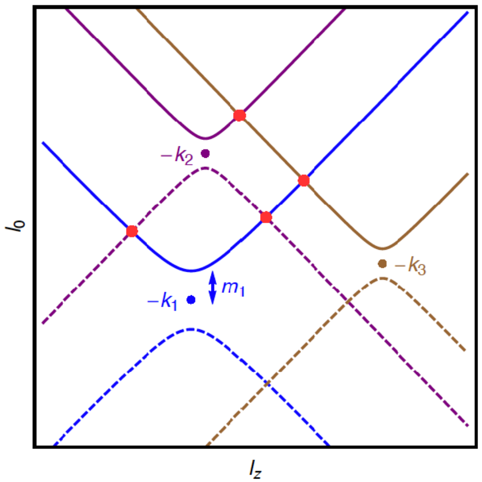
- **IMPORTANT:** *The singular structure of scattering amplitudes is dictated by their propagators. So, the proposed test functions are general enough to do a proper analysis of threshold singularities.*

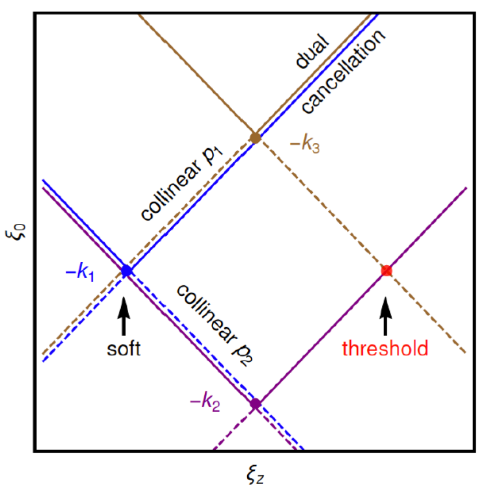
Rodrigo et al, arXiv:1904.08389 [hep-ph]

Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- The singular structure depends on the separation among momenta:





- **Time-like separation (causal connection):**

$$k_{ji}^2 - (m_j + m_i)^2 \geq 0$$

Physical threshold singularities are originated.

$$\lim_{\lambda_{ij}^{++} \to 0} \mathcal{S}_{ij}^{(1)} = \frac{\theta(-k_{ji,0})\theta(k_{ji}^2 - (m_i + m_j)^2)}{x_{ij}(-\lambda_{ij}^{++} - \imath 0 k_{ji,0})} + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$

Always +i0 !!!

$$x_{ij} = 4\, q_{i,0}^{(+)} q_{j,0}^{(+)}$$

The prescription is crucial to determine the imaginary part: it is always **+i0** and corresponds to the usual Feynman prescription! For this configuration, LTD and FTT give equivalent descriptions!

Rodrigo et al, arXiv:1904.08389 [hep-ph]

Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

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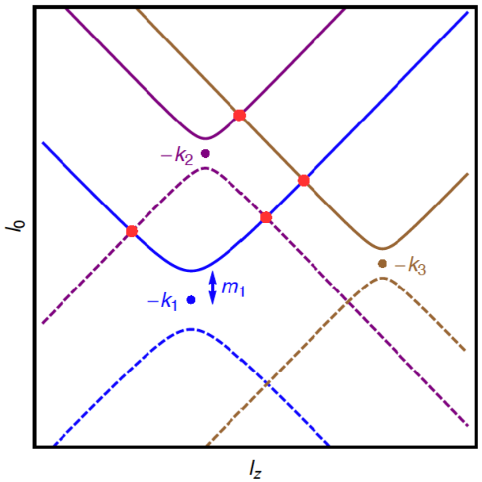
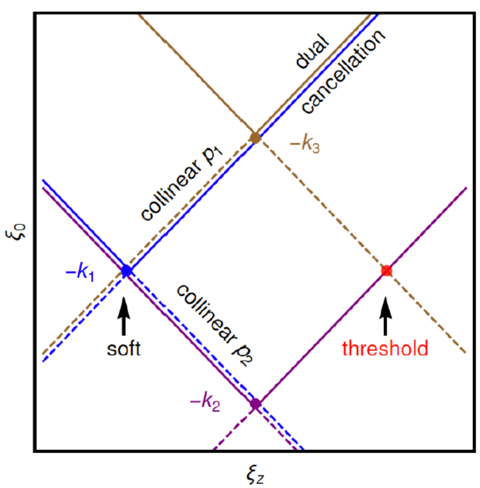




- **Space-like separation:**

$$k_{ji}^2 - (m_j - m_i)^2 \leq 0$$

The dual-prescription changes sign within the different contributions, which allows a perfect cancellation of any singular behaviour.

$$q_{j,0}^{(+)} \, G_D(q_i; q_j)|_{\lambda_{ij}^{+-} \to 0} = -q_{i,0}^{(+)} \, G_D(q_j; q_i)|_{\lambda_{ij}^{+-} \to 0}$$

$$\lim_{\lambda_{ij}^{+-} \to 0} \mathcal{S}_{ij}^{(1)} = \mathcal{O}\left((\lambda_{ij}^{+-})^0\right)$$

Cancellation codified by multiple-cuts in FTT!!

- **Light-like separation:**

It originates IR and threshold singularities that remain in a compact region of the integration domain. There is a partial cancellation among dual contributions, *but IR might remain!*

Rodrigo et al, arXiv:1904.08389 [hep-ph]

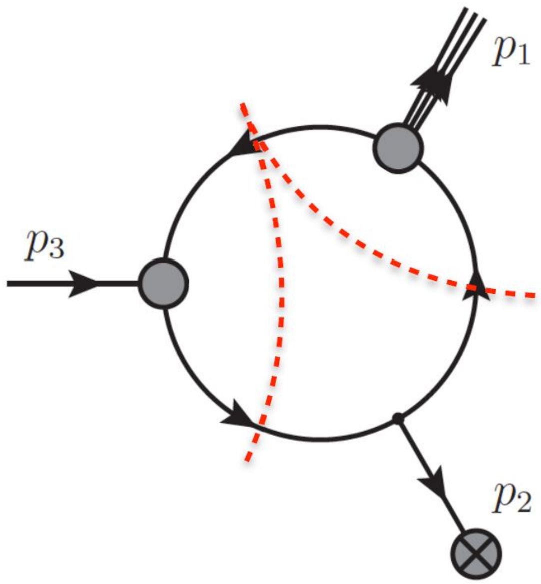
Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- **Anomalous thresholds:** *causal (i.e. time-like separated) singularities originated by multiple propagators going on-shell.*

$$\lim_{\lambda_{ij}^{++},\lambda_{ik}^{++}\to 0} \mathcal{S}_{ijk}^{(1)} = \frac{1}{x_{ijk}} \prod_{r=j,k} \frac{\theta(-k_{ri,0})\,\theta(k_{ri}^2-(m_i+m_r)^2)}{(-\lambda_{ir}^{++} - \imath 0 k_{ri,0})} + \mathcal{O}\left((\lambda_{ij}^{++})^{-1}, (\lambda_{ik}^{++})^{-1}\right)$$

$$x_{ijk} = 8\, q_{i,0}^{(+)}\, q_{j,0}^{(+)}\, q_{k,0}^{(+)}$$



- Intersections of two hyperboloids lead to the standard IR and threshold singularities.
- Anomalous thresholds are originated from the intersection of **two** forward (backward) and **one** backward (forward) hyperboloids.
- **There are not singularities for** $\lambda_{jk}^{-+} = \lambda_{ik}^{++} - \lambda_{ij}^{++} \to 0$ **!!!**

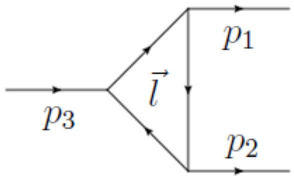
Rodrigo et al, arXiv:1904.08389 [hep-ph]

FDU approach @ NLO

Real-virtual momentum mapping

- Suppose **one-loop** scalar scattering amplitude given by the triangle (scalar toy-model!):

Virtual

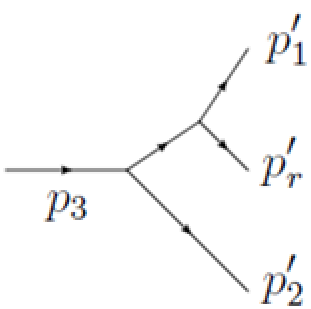


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

FDU approach @ NLO

Real-virtual momentum mapping

- **Mapping of momenta:** generate **1->3 real** emission kinematics (**3 external on-shell momenta**) starting from the variables available in the dual description of **1->2 virtual** contributions (**2 external on-shell momenta and 1 free three-momentum**)
  - ✓ Split the real phase space into two regions, i.e. $y'_{1r}<y'_{2r}$ and $y'_{2r}<y'_{1r}$, to separate the possible collinear singularities
  - ✓ Implement an optimized mapping in each region, to allow a fully local cancellation of IR singularities with those present in the dual terms

REGION 1:

$$p_r'^{\mu} = q_1^{\mu}\,, \qquad p_1'^{\mu} = p_1^{\mu} - q_1^{\mu} + \alpha_1\, p_2^{\mu}\,,$$
$$p_2'^{\mu} = (1-\alpha_1)\, p_2^{\mu}\,, \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2}\,,$$

$$\Rightarrow \quad y'_{1r} = \frac{v_1\, \xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}} \qquad y'_{12} = 1-\xi_{1,0}$$
$$y'_{2r} = \frac{(1-v_1)(1-\xi_{1,0})\,\xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}}$$

REGION 2:

$$p_2'^{\mu} = q_2^{\mu}\,, \qquad p_r'^{\mu} = p_2^{\mu} - q_2^{\mu} + \alpha_2\, p_1^{\mu}\,,$$
$$p_1'^{\mu} = (1-\alpha_2)\, p_1^{\mu}\,, \qquad \alpha_2 = \frac{q_1^2}{2 q_1 \cdot p_1}\,,$$

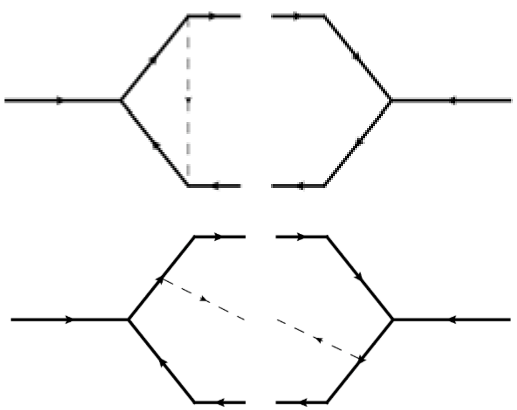
$$\Rightarrow \quad y'_{1r} = 1-\xi_{2,0} \qquad y'_{2r} = \frac{(1-v_2)\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$
$$y'_{12} = \frac{v_2\,(1-\xi_{2,0})\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$

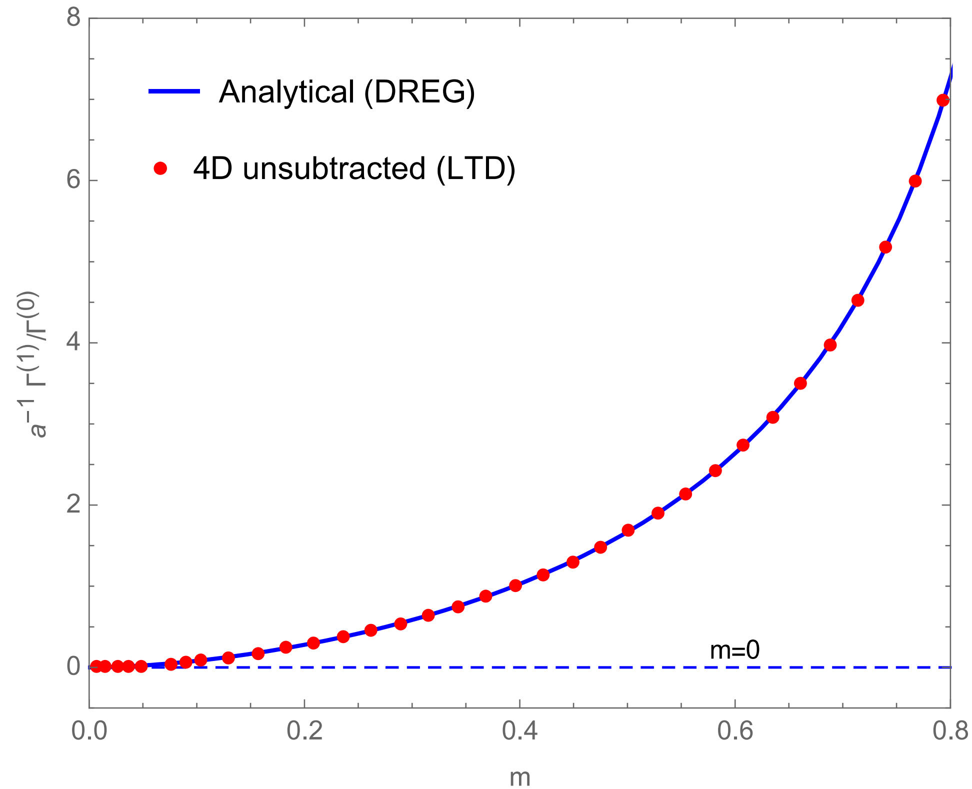
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

FDU approach @ NLO

Example: massive scalar three-point function

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**

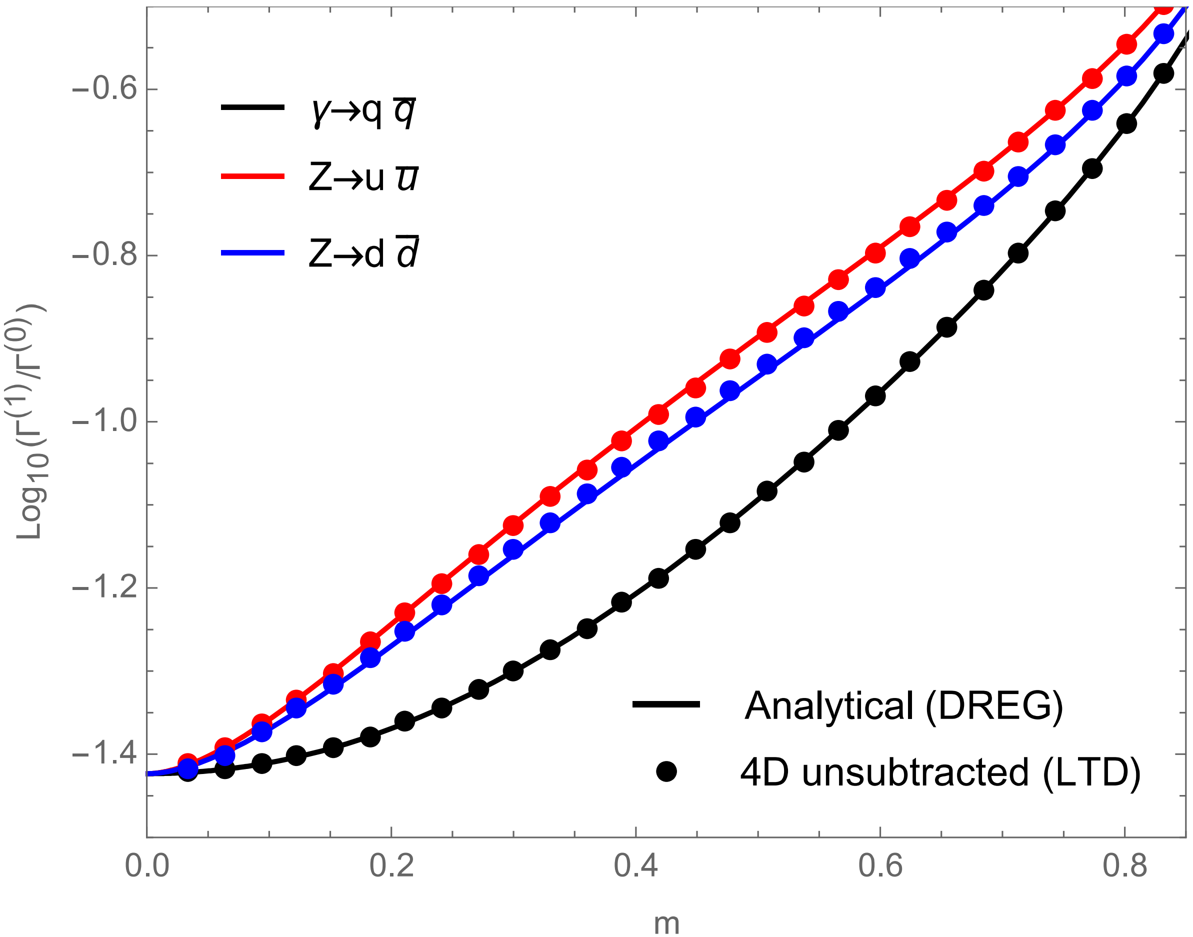




Rodrigo et al, JHEP10(2016)162

FDU approach @ NLO

Example: vector boson decays

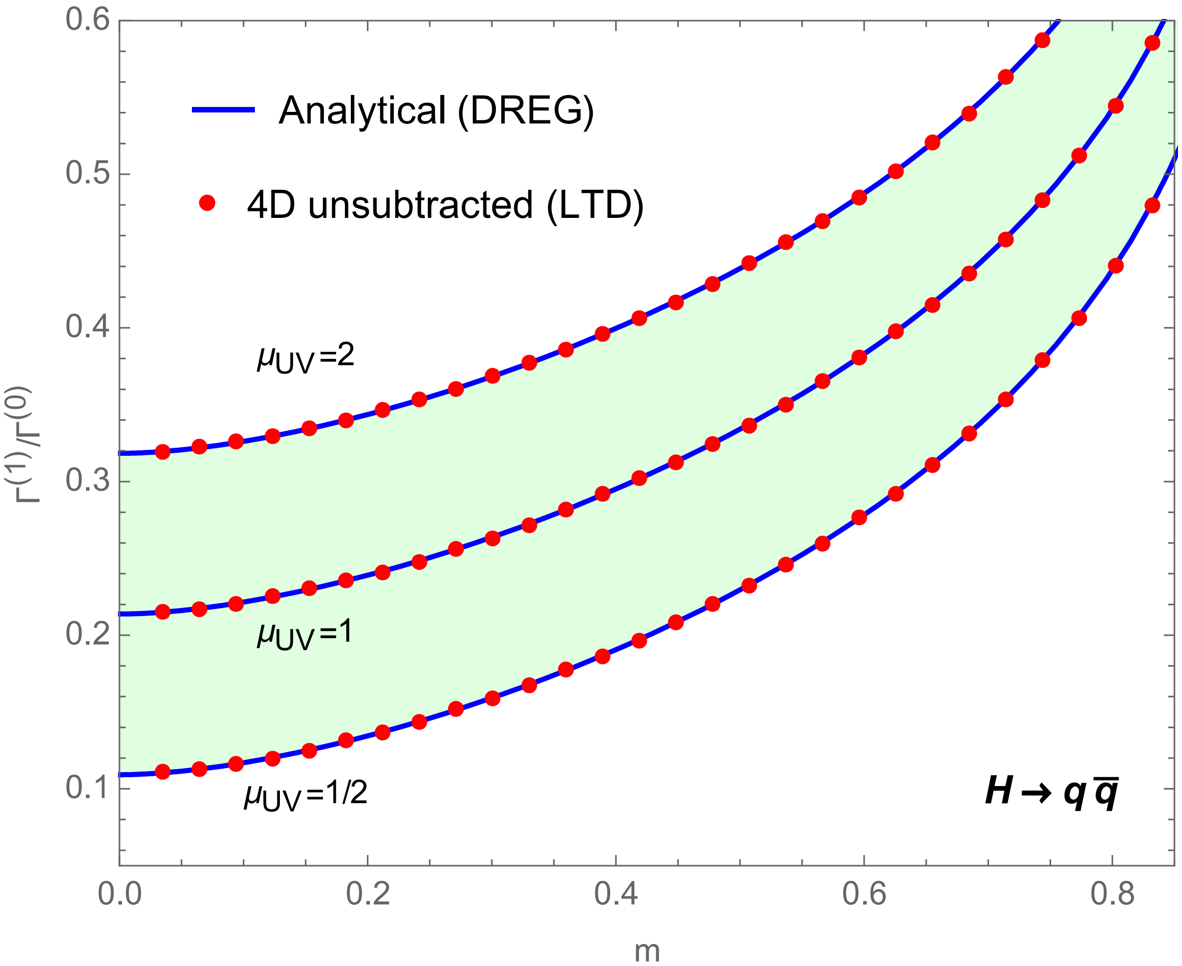


- *Total decay rate for a vector particle into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Efficient numerical implementation
  - Cancellation of UV log's (as in DREG…)

Rodrigo et al, JHEP10(2016)162

FDU approach @ NLO

Example: Higgs decay at NLO



- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

FDU approach @ NLO

Final remarks

- The total decay-rate can be expressed using purely **four-dimensional integrands**
- We recover the total NLO correction, while **avoiding dealing with DREG**

- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for **ε=0**)

    Finite integral for ε=0 $\not\Rightarrow$ Integrability with ε=0

    With FDU is true!
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants,** which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)
  - ✓ **Mapped real-contribution used as a fully local IR counter-term for the dual contribution!**

Rodrigo et al, JHEP10(2016)162

Conclusions and perspectives

- ✓ Loop-tree duality allows to treat **virtual and real** contributions **simultaneously** (*loop measure expressed in Euclidean space*)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals
- ✓ More transparent description of **thresholds**
- ✓ **Combined virtual-real terms are integrable in four space-time dimensions!! FDU**

- **Perspectives:**
  - Automation of multileg processes @ NLO (ongoing)
  - Extension of the local IR formalism to NNLO → 2-loop examples available!!! (Felix's talk)
  - Exploit simplifications due to easier asymptotic expansions @ NNLO (ongoing)
  - *Carefull comparison with other schemes* → "Workstop-Thinkstart meeting" ***UZH, Zurich, Sep. 2016*** ***Eur.Phys.J. C77 (2017) no.7, 471***

Thanks for the attention!!!

LTD/FDU approach: multileg

Real-virtual momentum mapping (GENERAL)

- **Real-virtual momentum mapping with massive particles:**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\begin{aligned}
p'^{\mu}_r &= q^{\mu}_1 \\
p'^{\mu}_1 &= (1-\alpha_1)\,\hat{p}^{\mu}_1 + (1-\gamma_1)\,\hat{p}^{\mu}_2 - q^{\mu}_1 \\
p'^{\mu}_2 &= \alpha_1\,\hat{p}^{\mu}_1 + \gamma_1\,\hat{p}^{\mu}_2
\end{aligned}
$$

Impose on-shell conditions to determine mapping parameters

with $\hat{p}_i$ massless four-vectors build using p_i (simplify the expressions)

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

Repeat in each region of the partition…

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- LTD must be applied to deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2 q_{\rm UV} \cdot k_{i,{\rm UV}} + k_{i,{\rm UV}}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2 q_{\rm UV} \cdot k_{i,{\rm UV}})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2}\right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2}\right)$$

Becker, Reuschle, Weinzierl, JHEP12(2010)013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

LTD extended to deal with multiple poles

(use residue formula to obtain the dual representation)

- **3:** Take into account **wave-function and vertex renormalization** constants (not trivial in the massive case!)

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not{p}_1 = M) = 0 \qquad \left.\frac{d\Sigma_R(\not{p}_1)}{d\not{p}_1}\right|_{\not{p}_1=M} = 0$$

- ***Wave-function renormalization constant*** (**both IR and UV poles**):

$$\Delta Z_2(p_1) \;=\; -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} + 4M^2 \left(1 - \frac{q_1\cdot p_2}{p_1\cdot p_2}\right) G_F(q_3)\right)$$

- ***Vertex renormalization (only UV):***

$$\mathbf{\Gamma}^{(1)}_{A,\mathrm{UV}} = g_S^2\, C_F \int_\ell \left(G_F(q_{\mathrm{UV}})\right)^3 \left[\gamma^\nu\, \not{q}_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A\, \not{q}_{\mathrm{UV}}\, \gamma_\nu - d_{A,\mathrm{UV}}\, \mu^2_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A\right]$$

- **Important features:**
  - Integrated results agrees with standard UV counter-terms!
  - **Smooth massless limit!**

Rodrigo et al, JHEP10(2016)162