Mathematical aspects of the scattering amplitude for $H \rightarrow gg$ within the loop-tree duality

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Outline

- Motivations
- Loop-tree duality @1L
  - H->gg @1L
  - UV local renormalisation
  - IR local subtraction
- Some remarks
- Outlook & conclusions
Introduction

- Standard Model (SM) of Particle Physics \(\rightarrow\) best Quantum Field Theory

- SM leaves to much physics without descriptions \(\rightarrow\) Physics Beyond Standard Model (BSM)

- LHC results demand a refinement of our understanding of the SM physics
  High precision predictions in background processes \(\rightarrow\) New physics at the TeV scale

- Relevant observables
  \(\rightarrow\) computation of Quantum Chromodynamics (QCD) Scattering Amplitudes
**Introduction**

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High precision predictions in background processes $\rightarrow$ New physics at the TeV scale

Relevant observables
$\rightarrow$ computation of Quantum Chromodynamics (QCD) **Scattering Amplitudes**

**Scattering Amplitudes**

- Practical applications in particle physics
- Mathematical elegance
- Gauge invariant objects

Perturbative expansion
**Introduction**

- Standard Model (SM) of Particle Physics —> best Quantum Field Theory
- SM leaves too much physics without descriptions —> Physics Beyond Standard Model (BSM)
- LHC results demand a refinement of our understanding of the SM physics
  - High precision predictions in background processes —> New physics at the TeV scale
- Relevant observables
  - computation of Quantum Chromodynamics (QCD) **Scattering Amplitudes**

**Motivation**

- **Scattering Amplitudes** — Practical applications in particle physics
  - Mathematical elegance
  - Gauge invariant objects

- Simplify the calculations in High-Energy Physics.
- Discover hidden properties of Quantum Field Theories
- Towards NNLO is the **Next Frontier**.

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger]
Dimensional regularisation schemes

Before computing multi-loop amplitudes...

Consider

\[ I_0 = \int_0^\infty \frac{dx}{x} \]
Dimensional regularisation schemes

Before computing multi-loop amplitudes...

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does not exist 😞
Dimensional regularisation schemes

Before computing multi-loop amplitudes...

Consider

\[ I_0 = \int_0^\infty \frac{dx}{x} \quad \text{does not exist} \]

Tweak the integrand

\[ I_\varepsilon = \int_0^\infty \frac{dx}{x^{1+\varepsilon}} = \int_0^1 \frac{dx}{x^{1+\varepsilon}} + \int_1^\infty \frac{dx}{x^{1+\varepsilon}} \quad (\text{with } \varepsilon \in \mathbb{C}) \]

well defined 😊
Dimensional regularisation schemes

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well defined

\[ \frac{1}{\varepsilon} \lesssim \begin{cases} \varepsilon & \Re (\varepsilon) < 0 \\ -\frac{1}{\varepsilon} & \Re (\varepsilon) > 0 \end{cases} \]
**Dimensional regularisation schemes**

Before computing multi-loop amplitudes...

Consider

$$ I_0 = \int_0^\infty \frac{dx}{x} \quad \text{does not exist} \quad \frown \frown $$

Tweak the integrand

$$ I_\epsilon = \int_0^\infty \frac{dx}{x^{1+\epsilon}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} + \int_1^\infty \frac{dx}{x^{1+\epsilon}} \quad \text{(with } \epsilon \in \mathbb{C}) $$

well defined 😊

$$ \Re (\epsilon) \leq 0 $$

$$ \Re (\epsilon) > 0 $$

$$ I_\epsilon = 0 \ , \ \forall \epsilon \in \mathbb{C} \quad \rightarrow \quad I_0 = 0 \quad \text{(analytical continuation)} $$
**Dimensional regularisation schemes**

Before computing multi-loop amplitudes...

Consider

\[ I_0 = \int_0^\infty \frac{dx}{x} \]

does not exist

Tweak the integrand

\[ I_\varepsilon = \int_0^\infty \frac{dx}{x^{1+\varepsilon}} = \int_0^1 \frac{dx}{x^{1+\varepsilon}} + \int_1^\infty \frac{dx}{x^{1+\varepsilon}} \quad \text{(with } \varepsilon \in \mathbb{C}) \]

well defined

\[ -\frac{1}{\varepsilon} \quad \Re(\varepsilon) < 0 \]

\[ +\frac{1}{\varepsilon} \quad \Re(\varepsilon) > 0 \]

\[ I_\varepsilon = 0 \text{, } \forall \varepsilon \in \mathbb{C} \rightarrow I_0 = 0 \]

(analytical continuation)

This was **dimensional regularisation**

---

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4
Motivation

We are used to

- Modify dimension of the space-time

\[ \int \frac{d^4l}{(2\pi)^4} \xrightarrow{\mu_{\text{DS}}^{4-d}} \int \frac{d^d\vec{l}}{(2\pi)^d} \]

- IR and UV singularities appear after integrations

- Virtual and real correction are considered separately

Several approaches to deal with infinities

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<th>To (d),</th>
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Motivation

In this talk

We are used to

don’t

- Modify dimension of the space-time
- IR and UV singularities appear before integrations
- Virtual and real correction are considered simultaneously

Several approaches to deal with infinities

\[ \int \frac{d^4l}{(2\pi)^4} \rightarrow \mu_{\text{DS}}^4 \int \frac{d^{d-\epsilon}l}{(2\pi)^d} \]

To \( d \),

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Motivation

In this talk

- Loop-tree duality + Four-dimensional Unsubtraction scheme

We are used to

don’t

- Modify dimension of the space-time
- IR and UV singularities appear after integrations
- Virtual and real correction are considered separately

Several approaches to deal with infinities

\[
\int \frac{d^4 l}{(2\pi)^4} \rightarrow \mu_{DS}^4 \int d^{d-\varepsilon} l
\]

[GNENDIGER, ET AL (W.J.T.) (2017)]
One loop
The loop-tree duality theorem

One-loop integrals decomposes as a linear combination of \( N \) single-cut phase-space integrals

\[
\int \ell \prod G_F(q_i) = - \sum \int \ell \prod \tilde{\delta}(q_i) G_D(q_i; q_j)
\]

\[
\tilde{\delta}(q_i) = 2\pi i \delta^{(+)} (q_i^2 - m_i^2)
\]

\[
G_D(q_i; q_j) = - \frac{1}{q_j^2 - m_j^2 - i0} \eta k_{ji}
\]

- Modify +i0 prescription of the Feynman props.
  - It compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem

- Lorentz-covariant dual prescription \( \rightarrow \eta \) a future-like vector

- Number of single cut dual contributions = the number of legs.

- Singularities of the loop diagram \( \rightarrow \) singularities of the dual integrals.

- Loop-Tree Duality works only on propagators.
  - Same procedure for Tensor loop integrals and scattering amplitudes.
**H→gg @1L**

In principle, one has

\[
\begin{array}{c}
\text{Vanish in DimReg}
\end{array}
\]

How to deal with infinities?

☆ Traditional approach: singularities cancel **after** integration
☆ Work out the Four-dimensional Unsubtracted (FDU) scheme cancellations occur **before** integration
☆ Cancellations are locally performed by means of LTD

[Kinoshita (1962)]
[Lee and Nauenberg (1964)]
In principle, one has

\[ H \rightarrow gg \ @1L \]

Four-dimensional Unsubtraction scheme: keep track of IR and UV singularities

- Local UV renormalisation -> study UV behaviour @ integrand level
- Local IR subtraction -> match real contributions with virtual ones

[Vanish in DimReg]
\[ H \rightarrow gg \; @1L \]

In principle, one has

\[
\text{Massless bubbles} \sim \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}}
\]

Four-dimensional Unsubtraction scheme: keep track of IR and UV singularities

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Four-dimensional Unsubtraction scheme: keep track of IR and UV singularities

- Local UV renormalisation -> study UV behaviour @ integrand level
- Local IR subtraction -> match real contributions with virtual ones

Vanish in DimReg

IR local behaviour completely cancelled

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]
$H \rightarrow gg @1L$

In principle, one has

$$\text{Massless bubbles} \sim \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}}$$

Apply LTD

- Amplitude can be decomposed in terms of form factors
  $$A_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} A_{i}^{(1,f)} T_{\mu\nu}^{i},$$

- “Relevant” form factors: A1 and A2
  $$T_{i}^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2p_{1}^\nu p_{2}^\mu}{s_{12}}, g^{\mu\nu}, \frac{2p_{1}^\mu p_{2}^\nu}{s_{12}}, \frac{2p_{1}^\nu p_{2}^\mu}{s_{12}}, \frac{2p_{1}^\mu p_{2}^\nu}{s_{12}} \right\}$$
$H \rightarrow gg \ @ 1L$

In principle, one has

Amplitude can be decomposed in terms of form factors

“Relevant” form factors: $A_1$ and $A_2$

$T^i_{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2p_1^\mu p_2^\nu}{s_{12}}, g^{\mu\nu}, \frac{2p_1^\mu p_2^\nu}{s_{12}}, \frac{2p_1^\mu p_1^\nu}{s_{12}}, \frac{2p_2^\mu p_2^\nu}{s_{12}} \right\}$

Same structure of universal dual amplitudes + additional contribution $S_1$

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]

Vanish in DimReg

Massless bubbles $\sim \frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}$

[Diencourt-Mangin, Rodrigo, Sborlini (2018)]
In principle, one has

\[ Massless\ \text{bubbles} \sim \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}} \]

Apply LTD

- Amplitude can be decomposed in terms of form factors

\[ A_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} A_{i}^{(1,f)} T_{i}^{\mu\nu}, \]

- “Relevant” form factors: A1 and A2

\[ T_{i}^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2p_{1}^{\nu} p_{2}^{\mu}}{s_{12}}, \quad g^{\mu\nu}, \quad \frac{2p_{1}^{\mu} p_{2}^{\nu}}{s_{12}}, \quad \frac{2p_{1}^{\mu} p_{1}^{\nu}}{s_{12}}, \quad \frac{2p_{2}^{\mu} p_{2}^{\nu}}{s_{12}} \right\} \]

\[ A_{1}^{(1,g)} = g_{g} \int_{\ell} \delta(\ell) \left[ \left( \frac{\ell_{0}^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_{0}^{(+)}}{q_{2,0}^{(+)}} \right) + \frac{2(2\ell \cdot p_{12})^{2}}{s_{12}^{2} - (2\ell \cdot p_{12} - i0)^{2}} \right] \left( \frac{s_{12}^{2}}{(2\ell \cdot p_{1})(2\ell \cdot p_{2})} c_{1}^{(g)} + c_{2}^{(g)} \right) \]

\[ + \frac{2s_{12}^{2}}{s_{12}^{2} - (2\ell \cdot p_{12} - i0)^{2}} c_{3}^{(g)} \]  

\[ A_{2}^{(1,g)} = g_{g} \frac{c_{6}^{(g)}}{2} \int_{\ell} \delta(\ell) \left( \frac{\ell_{0}^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_{0}^{(+)}}{q_{2,0}^{(+)}} - 2 \right) \]

Same structure of universal dual amplitudes + additional contribution \( S_{1} \)

Recall A2 vanishes upon integration (gauge invariance)

LTD works @ integrand level -> A2 helps to remove “spurious” terms

[Driencourt-Mangin, Rodrigo, Sborlini (2018)]
\textbf{H}\textrightarrow gg @1L

In principle, one has

Amplitude can be decomposed in terms of form factors

\begin{align*}
A_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} A_{i}^{(1,f)} T_{\mu\nu}^{i} ,
\end{align*}

“Relevant” form factors: A1 and A2

\begin{align*}
T_{i}^{\mu\nu} = \left\{ g_{\mu\nu} - \frac{2p_{1}^{\nu} p_{2}^{\mu}}{s_{12}} , g_{\mu\nu} , \frac{2p_{1}^{\mu} p_{2}^{\nu}}{s_{12}} , \frac{2p_{1}^{\mu} p_{1}^{\nu}}{s_{12}} , \frac{2p_{2}^{\mu} p_{2}^{\nu}}{s_{12}} \right\}
\end{align*}

S1 vanishes in DimReg

c1 & c23 contain the full dependence of the amplitude

c23 \sim (d-4) \quad \text{needs to be UV renormalised @ integrand level}
$H \rightarrow gg \ @1L$

In principle, one has

$$A_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} A_{i}^{(1,f)} T^{i}_{\mu\nu},$$

"Relevant" form factors: $A_1$ and $A_2$

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$c_1$ & $c_{23}$ contain the full dependence of the amplitude

$c_{23} \sim (d-4)$ needs to be UV renormalised @ integrand level

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[ Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]
**$H \rightarrow gg @1L$**

In principle, one has

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\text{Massless bubbles} \sim \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}}
\]

Apply LTD

- Amplitude can be decomposed in terms of form factors

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A_{\mu
u}^{(1,f)} = \sum_{i=1}^{5} A_{i}^{(1,f)} T_{i}^{\mu \nu},
\]

- "Relevant" form factors: $A_1$ and $A_2$

\[
T_{i}^{\mu \nu} = \left\{ g^{\mu \nu} - \frac{2 p_{1}^{\nu} p_{2}^{\mu}}{s_{12}}, g^{\mu \nu}, \frac{2 p_{1}^{\mu} p_{1}^{\nu}}{s_{12}}, \frac{2 p_{2}^{\mu} p_{1}^{\nu}}{s_{12}}, \frac{2 p_{2}^{\mu} p_{2}^{\nu}}{s_{12}} \right\}
\]

$c_1$ & $c_{23}$ contain the full dependence of the amplitude

$c_{23} \sim (d-4)$ needs to be UV renormalised @ integrand level

Previous results for the full theory

[William J. Torres Bobadilla]

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]

[Driencourt-Mangin, Rodrigo, Sborlini (2018)]
**UV local renormalisation @1L**

What in practice do we do (independently of LTD)?

- **Expand the integrand in the UV limit**
  
  \[
  G_F (q_i) = G_F (q_{UV}) \left(1 - \left(2q_{UV} \cdot k_{i,UV} + k_{i,UV}^2 + \mu_{UV}^2 - m_i^2\right)G_F (q_{UV}) + \cdots\right)
  \]

  \[
  G_F (q_{UV}) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i0}
  \]

- **Consider** \(I_N (\ell; \{p_i\})\) and the replacement
  
  - Apply the replacements \(S\) on \(I\)
  - Take the limit \(\lambda \to \infty\)
  - Select the divergent parts \(\to\) build a counter-term \(C\)
  - Fix \(C\) according to the renormalisation scheme \(\to\) In this talk \(\overline{\text{MS}}\)

[Becker, Reuschle, Weinzierl (2010)]
[Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]
**UV local renormalisation @1L**

What in practice do we do (independently of LTD)?

- Expand the integrand in the UV limit

\[
G_F(q_i) = G_F(q_{UV})\left(1 - (2q_{UV} \cdot k_{i,UV} + k_{i,UV}^2 + \mu_{UV}^2 - m_i^2) G_F(q_{UV}) + \cdots\right)
\]

\[
G_F(q_{UV}) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i\epsilon}
\]

- Consider \( I_N(\ell; \{p_i\}) \) and the replacement

\[
S: \begin{cases} 
\ell^2 \rightarrow \lambda^2 \ell^2 + (1 - \lambda^2)\mu^2 \\
\ell \cdot p_i \rightarrow \lambda \ell \cdot p_i
\end{cases}
\]

- Apply the replacements \( S \) on \( I \)
- Take the limit \( \lambda \rightarrow \infty \)
- Select the divergent parts \( \rightarrow \) build a counter-term \( C \)
- Fix \( C \) according to the renormalisation scheme \( \rightarrow \) In this talk \( \overline{\text{MS}} \)

E.g. \( e\mu @1\text{-loop}, \) the simplest application,

\[
\int \frac{d^d \ell}{(2\pi)^d} \frac{1}{q_{UV}^2 - \mu_{UV}^2} \left( \frac{c_1}{(q_{UV}^2 - \mu_{UV}^2)^3} + \frac{\mu_{UV}^4 c_2 + \mu_{UV}^2 c_3}{(q_{UV}^2 - \mu_{UV}^2)^2} + \frac{\mu_{UV}^2 c_4 + c_5}{q_{UV}^2 - \mu_{UV}^2} + c_6 \right) = \int \frac{d^d \ell}{(2\pi)^d} \frac{4}{3\epsilon_{UV}} \left( \frac{\mu_{UV}^2}{\mu_{UV}^2} \right)^\epsilon c_\Gamma
\]

In practice \( \rightarrow \) compute tadpoles with raised powers
Decay width of $H \rightarrow gg$ @1L

Local UV renormalisation

Parametric form of the local UV counter-term

$$\frac{\alpha}{\pi} C_A g_s^2 \int_{\text{UV}} G_F^2(q_{3;\text{UV}}) \left[ \frac{(p_1 \cdot q_{3;\text{UV}})^2 (p_2 \cdot q_{3;\text{UV}})^2}{s_{12}} G_F^2(q_{3;\text{UV}}) c_4^{(x)} + \frac{(p_1 \cdot q_{3;\text{UV}})(p_2 \cdot q_{3;\text{UV}})}{s_{12}} G_F(q_{3;\text{UV}}) c_3^{(x)} \right]$$

$$+ c_2^{(x)} + \mu_{\text{UV}}^2 G_F(q_{3;\text{UV}}) c_3^{(x,\text{sub})}$$
Decay width of $H \to gg$ @1L

Local UV renormalisation

Parametric form of the local UV counter-term

$$\frac{\int_{UV} C_A g_s^2}{\int_{UV}} \left[ \frac{(p_1 \cdot q_3;UV)^2}{s_{12}^2} \frac{(p_2 \cdot q_3;UV)^2}{s_{12}} G_F^2(q_3;UV) c_4^{(x)} + \frac{(p_1 \cdot q_3;UV)(p_2 \cdot q_3;UV)}{s_{12}} G_F(q_3;UV) c_3^{(x)} \right]$$

$$+ c_2^{(x)} + \mu_{UV}^2 G_F(q_3;UV) c_3^{(x,sub)}$$

$$c_4^{(g)} = c_3^{(g,sub)} = 0,$$

$$c_3^{(g)} = \frac{8(3d - 10)}{d - 2},$$

$$c_2^{(g)} = \frac{(d - 10)(d - 3)}{d - 2}.$$
Decay width of $H\rightarrow gg$ @1L

Local UV renormalisation

Parametric form of the local UV counter-term

\[ i C_A g_s^2 \int_{QE} G_F^2 (q_3; UV) \left[ \frac{(p_1 \cdot q_3; UV)^2 (p_2 \cdot q_3; UV)^2}{s_{12}^2} G_F^2 (q_3; UV) c_4^{(x)} + \frac{(p_1 \cdot q_3; UV)(p_2 \cdot q_3; UV)}{s_{12}} G_F (q_3; UV) c_3^{(x)} \right] + c_2^{(x)} + \mu_{UV}^2 G_F (q_3; UV) c_3^{(x, sub)} \]

Renormalisation of massless bubbles

$C_4^{(g)} = C_3^{(g, sub)} = 0$, $C_4^{(f)} = \frac{8(3d - 10)}{d - 2}$, $C_2^{(f)} = 0$, $C_3^{(f, sub)} = 0$

$C_4^{(g)} = 32(d - 2)$, $C_3^{(g)} = -8(d - 2)$, $C_2^{(g)} = \frac{(d - 6)}{2}$, $C_3^{(g, sub)} = -\frac{(d - 2)}{3}$
Decay width of $H \rightarrow gg \ @ 1L$

Local UV renormalisation

Parametric form of the local UV counter-term

\[
\frac{\alpha}{\pi} C_A g_s^2 \int_{\mu} \frac{G_F^2(q_{3;UV})}{s_{12}^2} \left[ \frac{(p_1 \cdot q_{3;UV})^2}{s_{12}^2} G_F^2(q_{3;UV}) c_4^{(g)} + \frac{(p_1 \cdot q_{3;UV})(p_2 \cdot q_{3;UV})}{s_{12}} G_F(q_{3;UV}) c_3^{(g)} \right] + c_2^{(g)} + \mu_{UV}^2 G_F(q_{3;UV}) c_3^{(g,sub)}
\]

Renormalisation of massless bubbles

Separately integration recovers expected results

\[
p_1^g = \frac{\sqrt{s_{12}}}{2} (1, 0, 1), \quad p_2^g = \frac{\sqrt{s_{12}}}{2} (1, 0, -1), \\
q_i^g = \frac{\sqrt{s_{12}}}{2} \left( \sqrt{\xi_{i,0}^2 + \mu_{UV}^2}, 2\xi_{i,0}\sqrt{v_t(1 - v_t)e_{i,\perp, \xi_{i,0} (1 - 2v_t)} \right)
\]

\[
c_4^{(g)} = c_3^{(g,sub)} = 0, \\
c_3^{(g)} = \frac{8(3d - 10)}{d - 2}, \\
c_2^{(g)} = \frac{(d - 10)(d - 3)}{d - 2}
\]

\[
c_4^{(f)} = -64, \\
c_3^{(f)} = 16, \\
c_2^{(f)} = 0, \\
c_3^{(f,sub)} = 0, \\
c_4^{(g)} = 32(d - 2), \\
c_3^{(g)} = -8(d - 2), \\
c_2^{(g)} = \frac{d - 6}{2}, \\
c_3^{(g,sub)} = -\frac{(d - 2)}{3}
\]
Decay width of $H\rightarrow gg$ @1L

Local IR subtraction

One-loop $(q_i^\mu)^\ast$Born

Momentum conservation

Real*Real

$p_{ir}^\prime \rightarrow p_i^\prime + p_r^\prime$

Motivated by the factorisation properties of QCD [Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]

$$p_i^\prime\mu = (1 - \alpha_i) p_j^\prime\mu,$$

$$\alpha_i = \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)},$$

$$p_k^\prime\mu = p_k^\mu,$$

$k \neq i, j$

Match 1$\rightarrow$3 kinematics from 1$\rightarrow$2 kinematics. Take into account loop three-momentum

Work by region to separate collinear and soft divergencies
Decay width of $H\rightarrow gg @1L$

Local IR subtraction

\[ \tilde{g}_{i-1} \]
\[ (q_i) \]
\[ p_i \]
\[ \tilde{p}_{ir} \]
\[ p_i' \]
\[ p_r' \]
\[ \tilde{p}_{ir}' \]

One-loop $(q_i^\mu)^*\text{Born}$

Momentum conservation

Real*Real

Motivated by the factorisation properties of QCD

\[ p_r'^\mu = q_i^\mu, \]
\[ p_i'^\mu = p_i'^\mu - q_i^\mu + \alpha_i p_j^\mu, \quad \alpha_i = \frac{(q_i - p_i)^2}{2p_j \cdot (q_i - p_i)}, \]
\[ p_j'^\mu = (1 - \alpha_i) p_j^\mu, \quad p_k'^\mu = p_k^\mu, \quad k \neq i, j \]

Match 1$\rightarrow$ 3 kinematics from 1$\rightarrow$ 2 kinematics. Take into account loop three-momentum

Work by region to separate collinear and soft divergencies

\[ \text{[Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]} \]
Some remarks about the one- and two-loop calculations
Remarks

Cross check of calculations

Adaptive integrand decomposition

- Splits $d=4-2\varepsilon$ into parallel and orthogonal directions
- Nice properties for less than 5 external legs
- Numerator and denominators depend on different variables

\[
\int \prod_i d^d \bar{l}_{i\parallel} \int \prod_{1\leq i \leq j \leq \ell} d\lambda_{ij} \, G(\lambda_{ij}) \frac{d^d - 1 - \ell}{2} \int d\Theta_\perp \frac{\mathcal{N}(\bar{l}_{i\parallel}, \lambda_{ij} \Theta_\perp)}{D_1(\bar{l}_{i\parallel}, \lambda_{ij}) \cdots D_m(\bar{l}_{i\parallel}, \lambda_{ij})}
\]

Expand in Gegenbauer polynomials

\[
\int d\Theta_\perp = \int_{-1}^1 \prod_{i=1}^{4-d} \prod_{j=1}^{\ell} d\cos \theta_i + j_{-1,j} (\sin \theta_i + j_{-1,j})^{d-1-i-j}
\]

Straightforward integration of transverse components

and identification of spurious terms

\[
d = d_{\parallel} + d_\perp
\]

\[
d_{\parallel} = n - 1
\]

\[
d_\perp = (5 - n) - 2\varepsilon
\]

[I.B.P. reduction

Literature

Traditional methods

Integrand decomposition

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]
Adaptive integrand decomposition (AID)

Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in 3 steps

1) Divide and get \( \Delta(\bar{l}_i, \lambda_{ij}, \Theta_{\perp}) \)
2) Integrate out transverse variables \( \Theta_{\perp} \)
3) Divide again to get rid of \( \lambda_{ij} \)

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

\[ \mathcal{N}(l_i, \lambda_{ij}, \Theta_{\perp}) \]
\[ D_1(\bar{l}_i, \lambda_{ij}) \cdots D_m(\bar{l}_i, \lambda_{ij}) \]

AIDA

William J. Torres Bobadilla

[Mastrolia, Peraro, Primo (2016)]
[Mastrolia, Peraro, Primo, W.J.T. (2016)]
[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]
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Recipe in 3 steps

1) Divide and get $\Delta (\bar{l}_{||i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of $\lambda_{ij}$

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

Algorithm already **automated** AIDA

[Adaptive integrand decomposition (AID) by William J. Torres Bobadilla] [Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]
Conclusions/Outlook
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We have reached:

- Straightforward application of LTD @ 1-L
- Matched virtual and real correction by means of FDU
- Application of LTD in effective field theoris
- Renormalisation @ 1-L (at integrand level) -> completely under control

We are working on:

- Deal with processes at two loops that contain
  - Threshold singularities (contour deformation)
  - IR singularities
- Extend FDU to two loops

To be continued...

>> Driencourt-Mangin
>> Sborlini

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