Mathematical aspects of the scattering amplitude for $H \to gg$ within the loop-tree duality

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Outline

- Motivations
- Loop-tree duality @1L
  — H->gg @1L
  — UV local renormalisation
  — IR local subtraction
- Some remarks
- Outlook & conclusions

Introduction

- Standard Model (SM) of Particle Physics —> best Quantum Field Theory
- SM leaves to much physics without descriptions —> Physics Beyond Standard Model (BSM)
- LHC results demand a refinement of our understanding of the SM physics
  High precision predictions in background processes —> New physics at the TeV scale
- Relevant observables
  —> computation of Quantum Chromodynamics (QCD) **Scattering Amplitudes**

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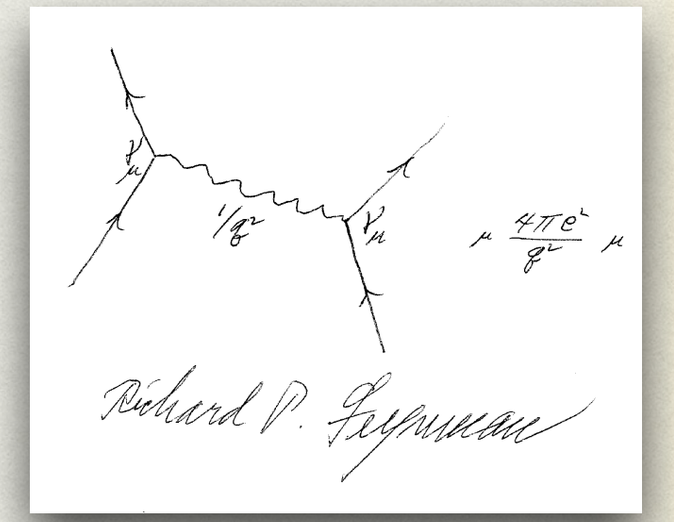
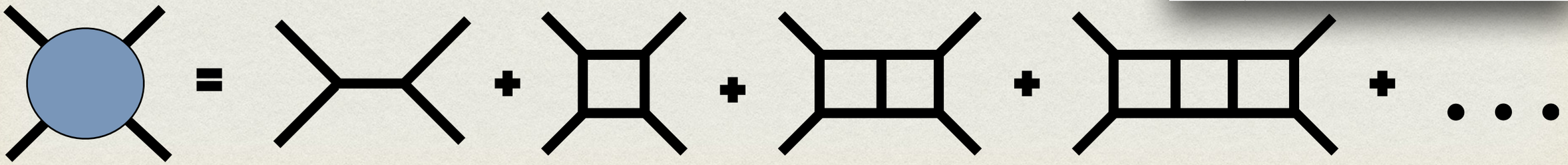
  — Practical applications in particle physics
  — Mathematical elegance
  — Gauge invariant objects
- Perturbative expansion





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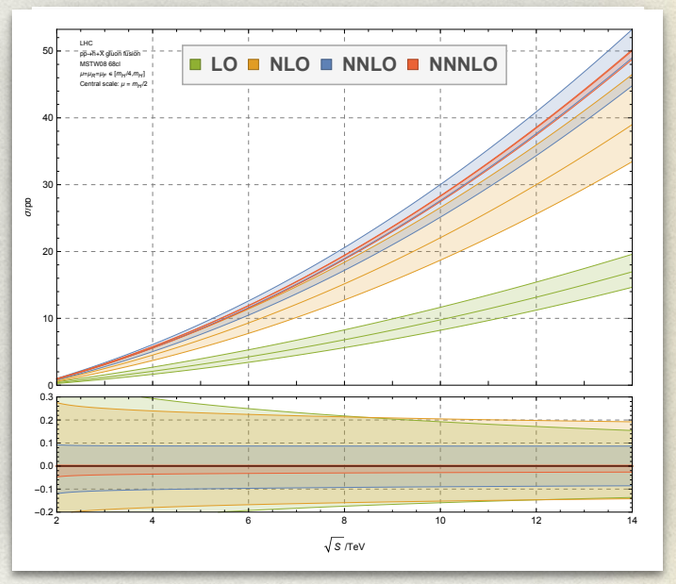

Motivation

- ☑ Simplify the calculations in High-Energy Physics.
- ☑ Discover hidden properties of Quantum Field Theories
- ☑ Towards NNLO is the **Next Frontier.**

Dimensional regularisation schemes

Before computing multi-loop amplitudes...

Consider

$$I_0 = \int_0^\infty \frac{dx}{x}$$

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Tweak the integrand

$$I_\epsilon = \int_0^\infty \frac{dx}{x^{1+\epsilon}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} + \int_1^\infty \frac{dx}{x^{1+\epsilon}} \qquad (\text{with } \epsilon \in \mathbb{C})$$

well defined ☺

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$$-\frac{1}{\epsilon} \qquad \Re(\epsilon) < 0$$

$$+\frac{1}{\epsilon} \qquad \Re(\epsilon) > 0$$

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$$-\frac{1}{\epsilon} \qquad\qquad +\frac{1}{\epsilon}$$

$$\cancel{\Re(\epsilon) < 0} \qquad\qquad \cancel{\Re(\epsilon) > 0}$$

$$I_\epsilon = 0\,, \forall \epsilon \in \mathbb{C} \longrightarrow I_0 = 0$$ (analytical continuation)

Dimensional regularisation schemes

Before computing multi-loop amplitudes...

— All computations made in d=4-2ε
— Singularities manifest as poles in ε
— physical observables don't depend on ε

Consider

$$I_0 = \int_0^\infty \frac{dx}{x}$$

does not exist ☹

Tweak the integrand

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This was **dimensional regularisation**

Motivation

We are used to

- Modify dimension of the space-time $\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \to \mu_{\mathrm{DS}}^{4-d} \int \frac{\mathrm{d}^d \bar{l}}{(2\pi)^d}$
- IR and UV singularities appear after integrations
- Virtual and real correction are considered separately

Several approaches to deal with infinities

To d,	or not to d ?
<u>traditional dimensional schemes</u> • 't Hooft / Veltman (HV) '72 • conventional dim. reg. (CDR) '73 • dim. reduction (DRED) '79 • four-dim. helicity (FDH) '92 <u>reformulations of dimensional schemes</u> • six-dim. formalism (SDF) '09 • four-dim. formalism (FDF) '14	<u>non-dimensional schemes</u> • implicit reg. (IREG) '98 • loop regularization (LORE) '03 • four-dim. reg. / ren. (FDR) '12 • four-dim. unsubtraction (FDU) '16

[Gnendiger, *et al* (W.J.T.) (2017)]

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Motivation

In this talk **Loop-tree duality + Four-dimensional Unsubtraction scheme**

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[Gnendiger, *et al* (W.J.T.) (2017)]

One loop

The loop-tree duality theorem

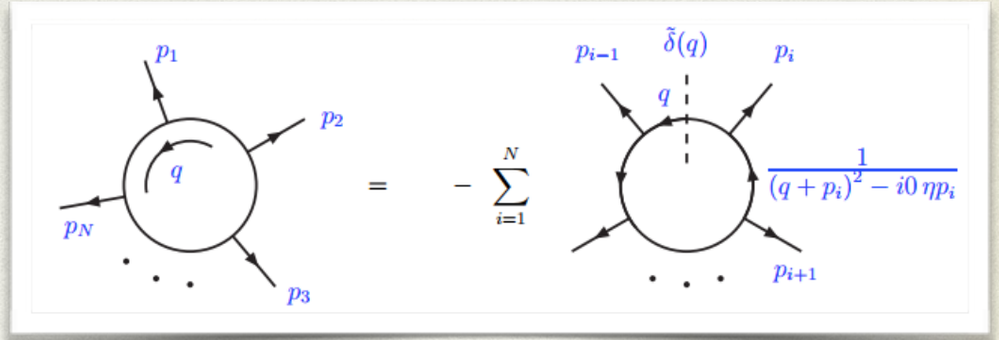
[Catani, Gleisberg, Krauss, Rodrigo, Winter (2008)]

One-loop integrals decomposes as a linear combination of N single-cut phase-space integrals

$$\int_\ell \prod G_F(q_i) = -\sum \int_\ell \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

$$\tilde{\delta}(q_i) \equiv 2\pi\imath\,\delta^{(+)}\left(q_i^2 - m_i^2\right)$$

$$G_D(q_i;q_j) = \frac{1}{q_j^2 - m_j^2 - i0\,\eta\,k_{ji}}$$

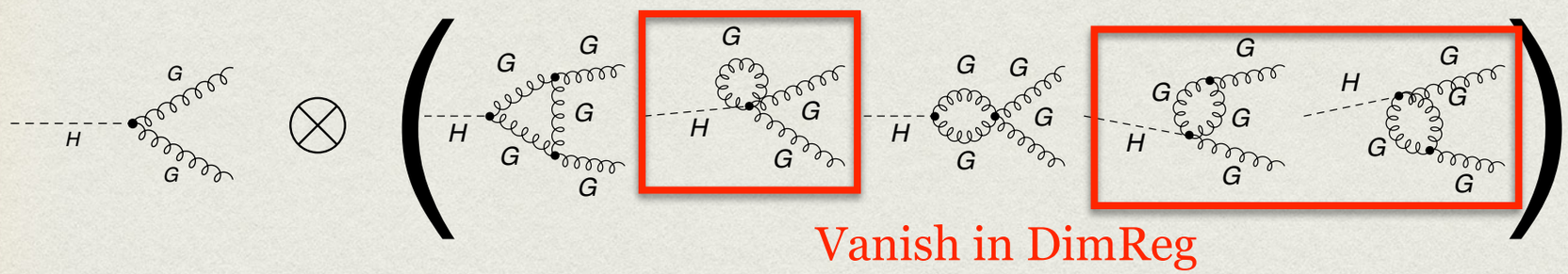


- Modify +i0 prescription of the Feynman props.
  It compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem
- Lorentz-covariant dual prescription —> η a **future-like** vector

- Number of single cut dual contributions = the number of legs.
- Singularities of the loop diagram —> singularities of the dual integrals.
- Loop-Tree Duality works only on propagators.
  Same procedure for Tensor loop integrals and scattering amplitudes.

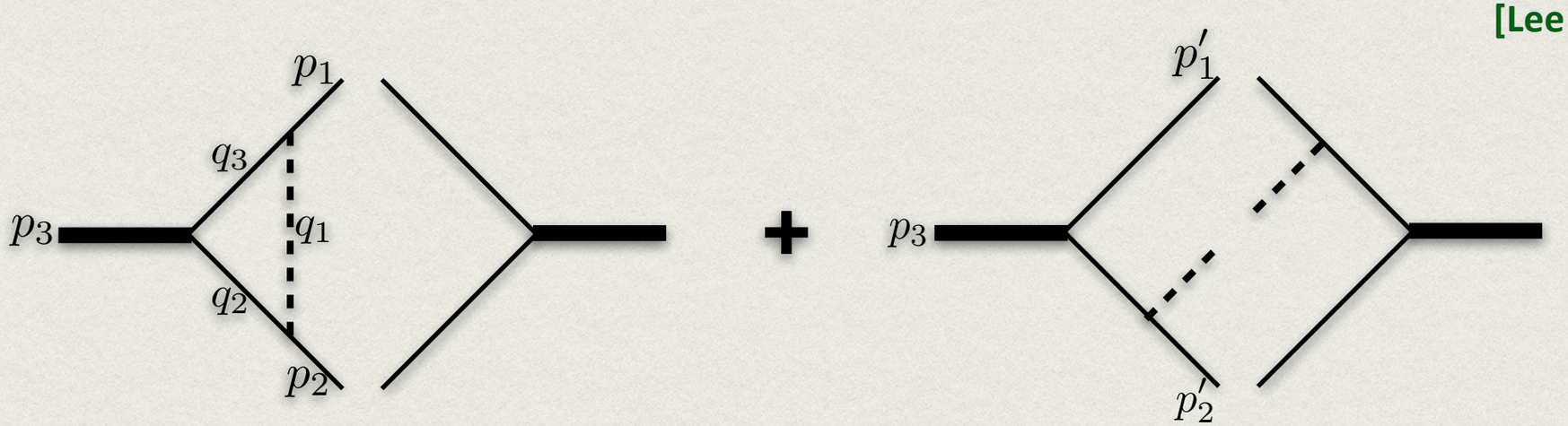
H->gg @1L

In principle, one has



How to deal with infinities?

[Kinoshita (1962)]
[Lee and Nauenberg (1964)]



- ☆ Traditional approach: singularities cancel **after** integration
- ☆ Work out the Four-dimensional Unsubtracted (FDU) scheme cancellations occur **before** integration
- ☆ Cancellations are locally performed by means of LTD

H->gg @1L

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]

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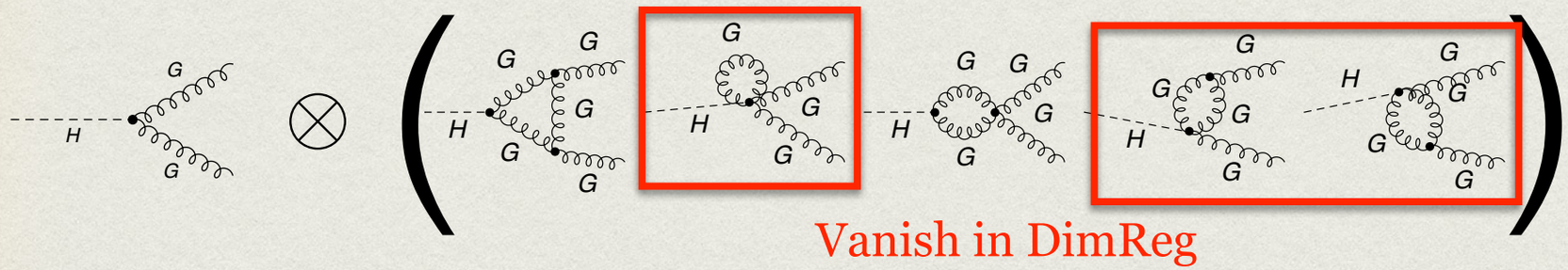

Vanish in DimReg

Four-dimensional Unsubtraction scheme: keep track of IR and UV singularities

- Local UV renormalisation -> study UV behaviour @ integrand level
- Local IR subtraction -> match real contributions with virtual ones

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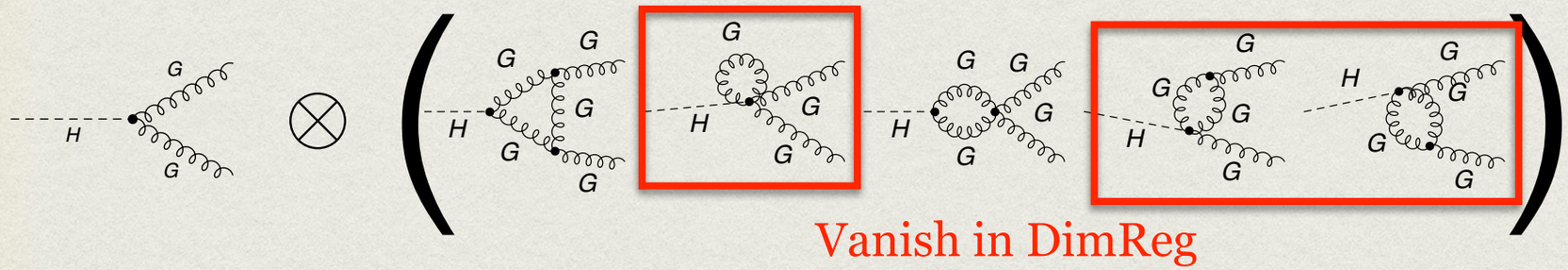

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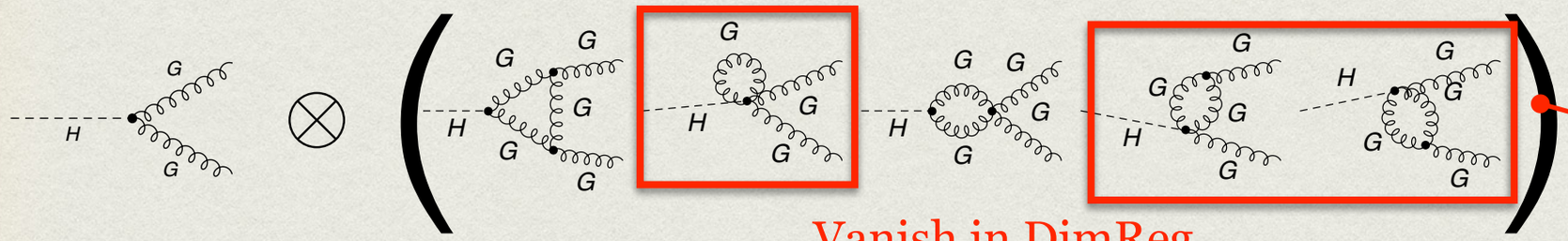

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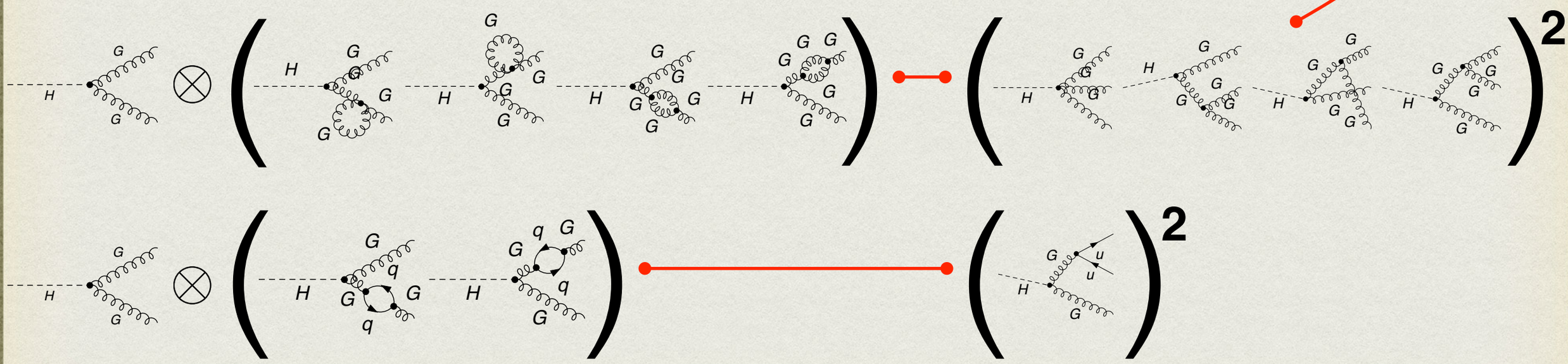

IR local behaviour completely cancelled

H->gg @1L

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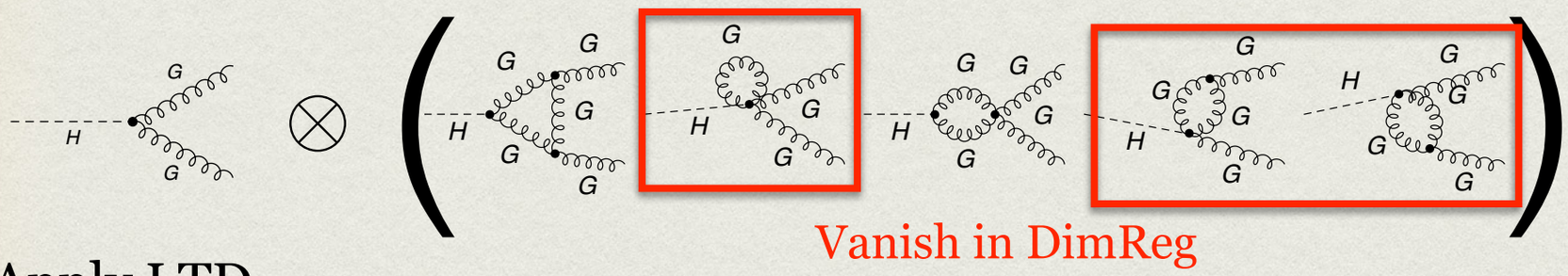

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Apply LTD

- Amplitude can be decomposed in terms of form factors $$\mathcal{A}^{(1,f)}_{\mu\nu} = \sum_{i=1}^{5} \mathcal{A}^{(1,f)}_i T^i_{\mu\nu} \,,$$
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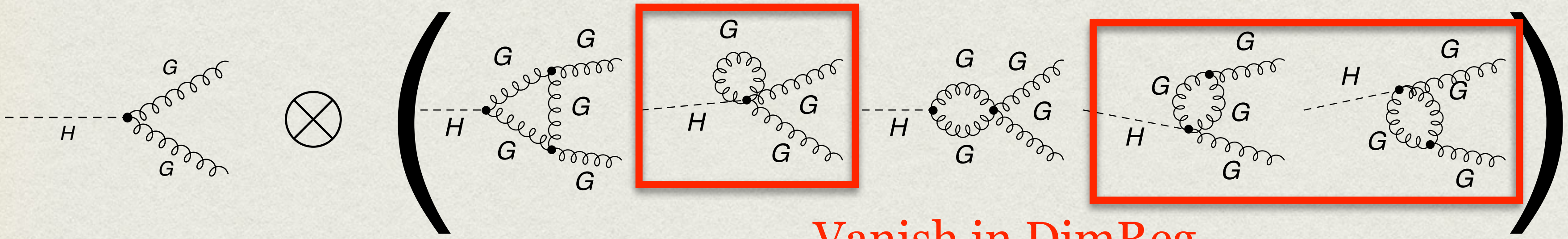

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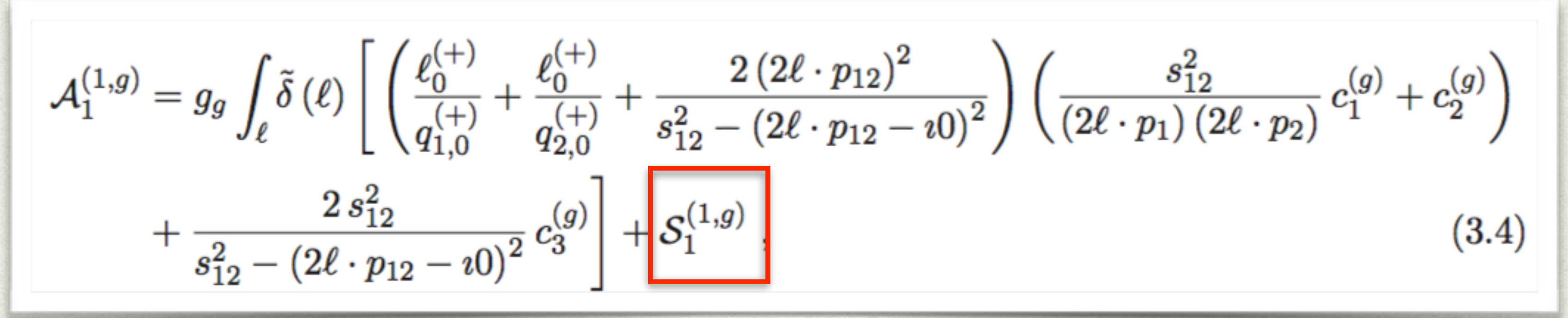
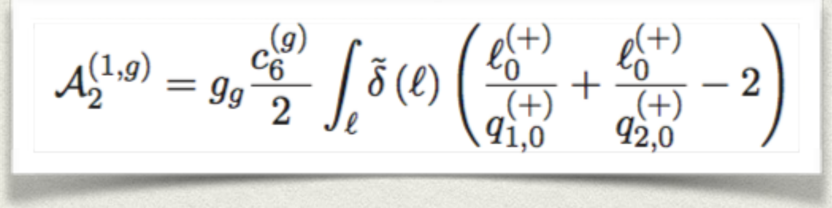


$$\mathcal{A}_1^{(1,g)} = g_g \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12}^2}{(2\ell\cdot p_1)\,(2\ell\cdot p_2)}\, c_1^{(g)} + c_2^{(g)} \right) + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_3^{(g)} \right] + \boxed{\mathcal{S}_1^{(1,g)}} \tag{3.4}$$

[Driencourt-Mangin, Rodrigo, Sborlini (2018)]



$$\mathcal{A}_2^{(1,g)} = g_g \frac{c_6^{(g)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} - 2 \right)$$

Same structure of universal dual amplitudes
+ additional contribution S1

H->gg @1L

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]

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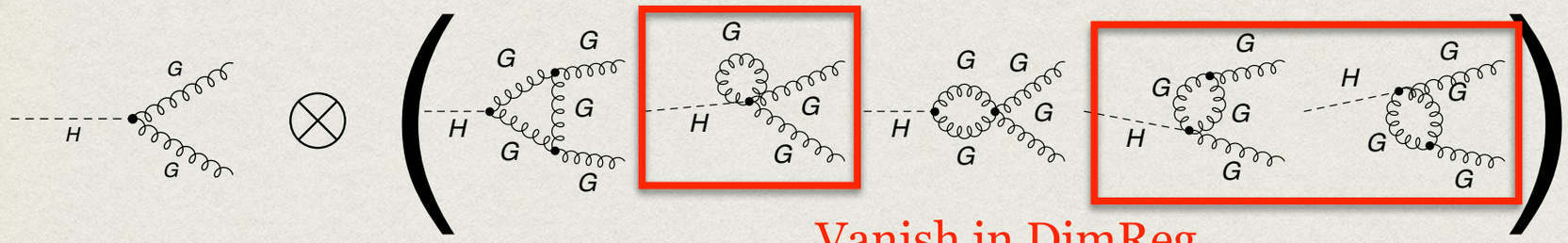
$$\text{Massless bubbles} \sim \frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}$$



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Same structure of universal dual amplitudes
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- Recall A2 vanishes upon integration (gauge invariance)
- LTD works @integrand level -> A2 helps to remove "spurious" terms

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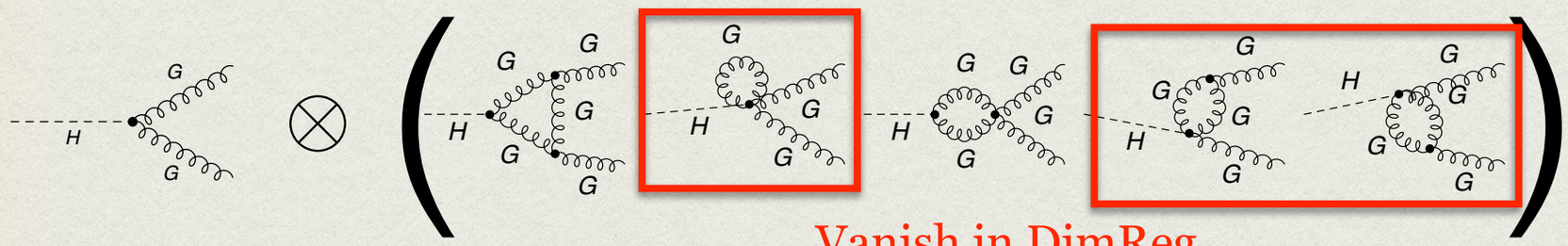

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S1 vanishes in DimReg

c1 & c23 contain the full dependence of the amplitude

c23 ~ **(d-4)** ⟶ needs to be UV renormalised @ integrand level

H->gg @1L

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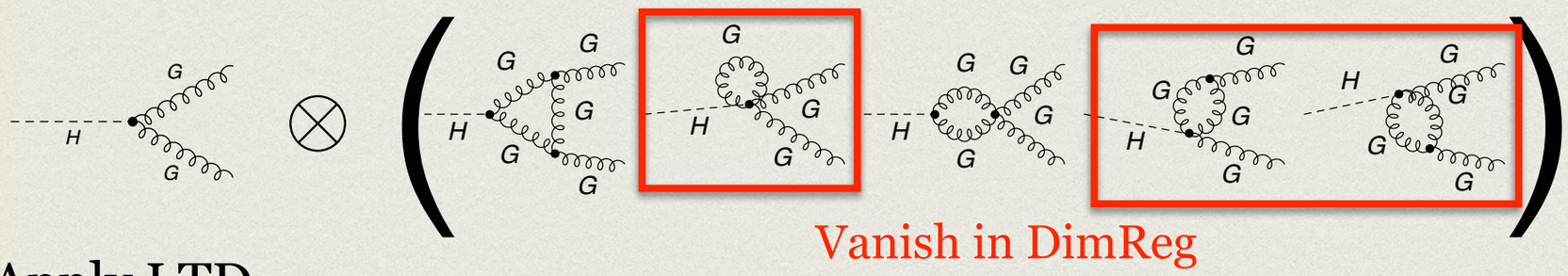

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c23 ~ **(d-4)** → needs to be UV renormalised @ integrand level

$$\mathcal{A}_{1,\rm UV}^{(1,f)} = -g_f\, s_{12} \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2(q_{\rm UV,0}^{(+)})^2} \left(1 + \frac{1}{(q_{\rm UV,0}^{(+)})^2} \frac{3\,\mu_{\rm UV}^2}{d-4} \right) c_{23}^{(f)} = 0$$

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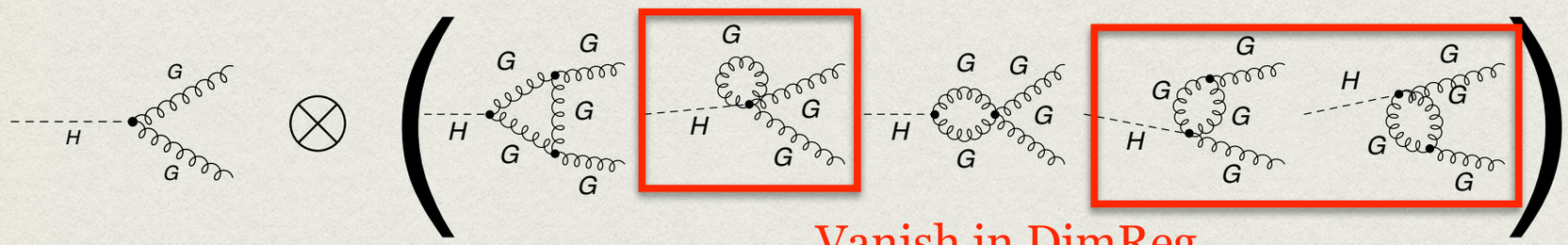

Vanish in DimReg

Apply LTD

- Amplitude can be decomposed in terms of form factors $\mathcal{A}^{(1,f)}_{\mu\nu} = \sum_{i=1}^{5} \mathcal{A}^{(1,f)}_{i} T^{i}_{\mu\nu}\,,$
- "Relevant" form factors: A1 and A2 $T^{\mu\nu}_{i} = \left\{ g^{\mu\nu} - \frac{2p_1^\nu\, p_2^\mu}{s_{12}}\,,\, g^{\mu\nu}\,,\, \frac{2p_1^\mu\, p_2^\nu}{s_{12}}\,,\, \frac{2p_1^\mu\, p_1^\nu}{s_{12}}\,,\, \frac{2p_2^\mu\, p_2^\nu}{s_{12}} \right\}$

$$\mathcal{A}_1^{(1,g)} = g_g \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \frac{s_{12}^2}{(2\ell\cdot p_1)\,(2\ell\cdot p_2)} \boxed{c_1^{(g)}} + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \boxed{c_{23}^{(g)}} \right] + \boxed{\mathcal{S}_1^{(1,g)}}\,, \tag{3.8}$$

S1 vanishes in DimReg

c1 & c23 contain the full dependence of the amplitude

c23 ~ **(d-4)** → needs to be UV renormalised @ integrand level

$$\mathcal{A}_{1,\rm R}^{(1,f)} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\rm UV}^{(1,f)} \right)\Big|_{d=4}$$

$$\mathcal{A}_{1,\rm UV}^{(1,f)} = -g_f\, s_{12} \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2(q_{\rm UV,0}^{(+)})^2} \left(1 + \frac{1}{(q_{\rm UV,0}^{(+)})^2} \frac{3\,\mu_{\rm UV}^2}{d-4} \right) c_{23}^{(f)} = 0$$

Previous results for the full theory

[Driencourt-Mangin, Rodrigo, Sborlini (2018)]

UV local renormalisation @1L

What in practice do we do (independently of LTD)?

- Expand the integrand in the UV limit

[Becker, Reuschle, Weinzierl (2010)]
[Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]

$$G_F\left(q_i\right) = G_F\left(q_{\mathrm{UV}}\right)\left(1 - \left(2q_{\mathrm{UV}} \cdot k_{i,\mathrm{UV}} + k_{i,\mathrm{UV}}^2 + \mu_{\mathrm{UV}}^2 - m_i^2\right) G_F\left(q_{\mathrm{UV}}\right) + \cdots\right)$$

$$G_F\left(q_{\mathrm{UV}}\right) = \frac{1}{q_{\mathrm{UV}}^2 - \mu_{\mathrm{UV}}^2 + \imath 0}$$

- Consider $I_{\mathrm{N}}(\ell; \{p_i\})$ and the replacement

$$S : \begin{cases} \ell^2 \to \lambda^2\, \ell^2 + (1-\lambda^2)\mu^2 \\ \ell \cdot p_i \to \lambda\, \ell \cdot p_i \end{cases}$$

  - Apply the replacements S on I
  - Take the limit $\lambda \to \infty$
  - Select the divergent parts —> build a counter-term **C**
  - Fix C according to the renormalisation scheme —> In this talk $\overline{\mathrm{MS}}$

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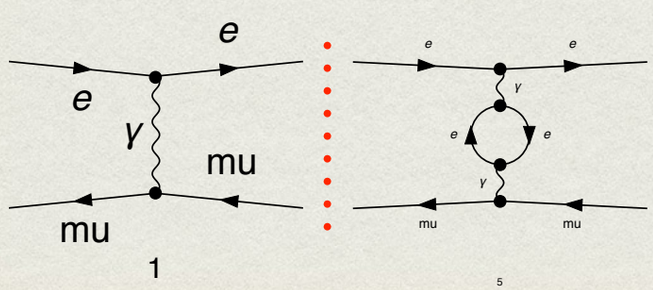
- E.g. eμ @1-loop, the simplest application,



$$= \int \frac{d^d\ell}{(2\pi)^d}\frac{1}{q_{\mathrm{UV}}^2-\mu_{\mathrm{UV}}^2}\left(\frac{c_1}{\left(q_{\mathrm{UV}}^2-\mu_{\mathrm{UV}}^2\right)^3} + \frac{\mu_{\mathrm{UV}}^4 c_2 + \mu_{\mathrm{UV}}^2 c_3}{\left(q_{\mathrm{UV}}^2-\mu_{\mathrm{UV}}^2\right)^2} + \frac{\mu_{\mathrm{UV}}^2 c_4 + c_5}{q_{\mathrm{UV}}^2-\mu_{\mathrm{UV}}^2} + c_6\right) \overset{\mathrm{IBP}}{=} \int \frac{d^d\ell}{(2\pi)^d}\frac{4}{3}\frac{1}{\left(q_{\mathrm{UV}}^2-\mu_{\mathrm{UV}}^2\right)^2}$$

$$= -\frac{4}{3\epsilon_{\mathrm{UV}}}\left(\frac{\mu^2}{\mu_{\mathrm{UV}}^2}\right)^{\epsilon} c_\Gamma$$

In practice —> compute tadpoles with raised powers

Decay width of H->gg @1L

[Hernandez, Renteria, Rodrigo, W.J.T. (to appear)]

Local UV renormalisation

Parametric form of the local UV counter-term

$$\imath\, C_A\, g_s^2 \int_{\ell_{\rm UV}} G_F^2\left(q_{3;\rm UV}\right)\left[\frac{\left(p_1\cdot q_{3;\rm UV}\right)^2\left(p_2\cdot q_{3;\rm UV}\right)^2}{s_{12}^2} G_F^2\left(q_{3;\rm UV}\right)\, c_4^{(x)} + \frac{\left(p_1\cdot q_{3;\rm UV}\right)\left(p_2\cdot q_{3;\rm UV}\right)}{s_{12}} G_F\left(q_{3;\rm UV}\right)\, c_3^{(x)} + c_2^{(x)} + \mu_{\rm UV}^2 G_F\left(q_{3;\rm UV}\right)\, c_3^{(x,\rm sub)}\right]$$

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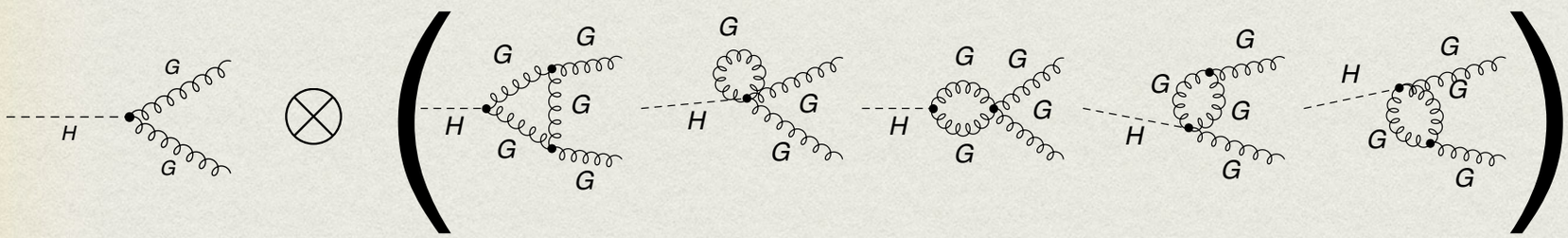

$$c_4^{(g)} = c_3^{(g,\rm sub)} = 0\,,$$
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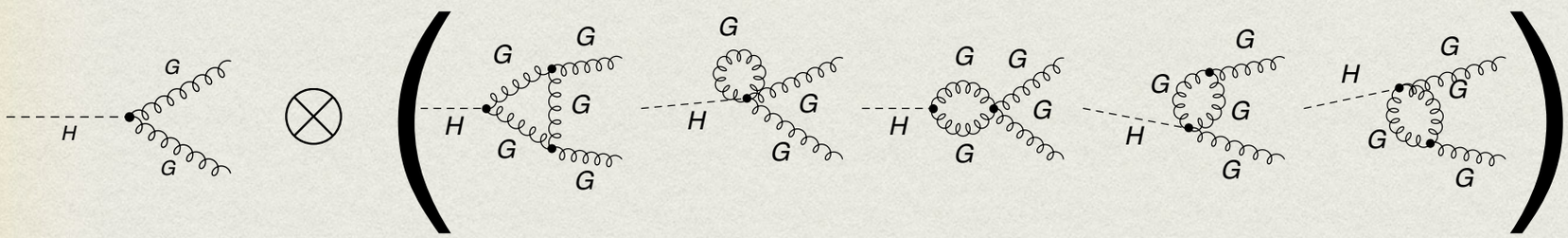

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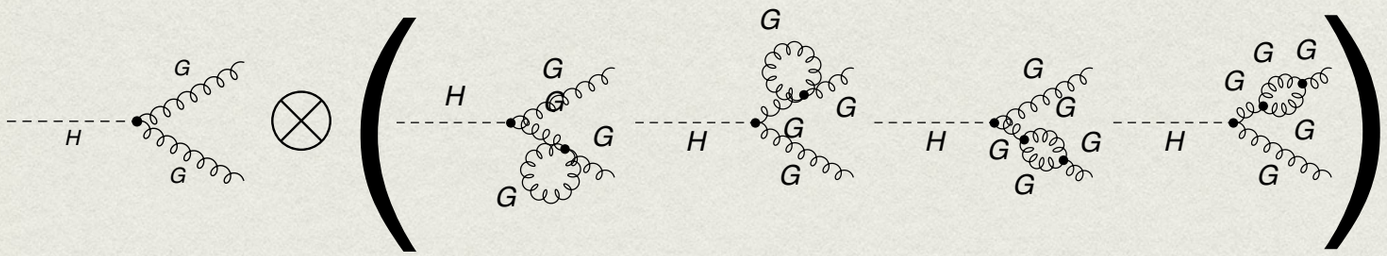
Renormalisation of massless bubbles



$$c_4^{(f)} = -64\,,$$
$$c_3^{(f)} = 16\,,$$
$$c_2^{(f)} = 0\,,$$
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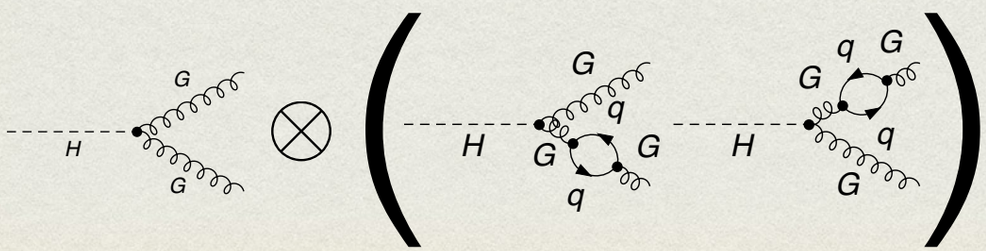

$$c_4^{(g)} = 32(d-2)\,,$$
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Separately integration recovers expected results

$$p_1^\alpha = \frac{\sqrt{s_{12}}}{2}\left(1, \mathbf{0}_\perp, 1\right), \qquad p_2^\alpha = \frac{\sqrt{s_{12}}}{2}\left(1, \mathbf{0}_\perp, -1\right),$$
$$q_i^\alpha = \frac{\sqrt{s_{12}}}{2}\left(\sqrt{\xi_{i,0}^2 + \mu_{\mathrm{UV}}^2}, 2\xi_{i,0}\sqrt{v_i\left(1-v_i\right)}\mathbf{e}_{i,\perp}, \xi_{i,0}\left(1-2v_i\right)\right)$$

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Parametric form of the local UV counter-term

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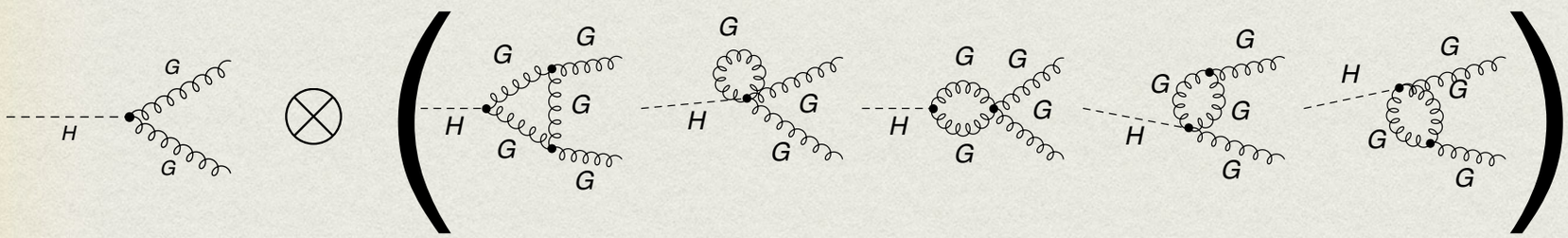

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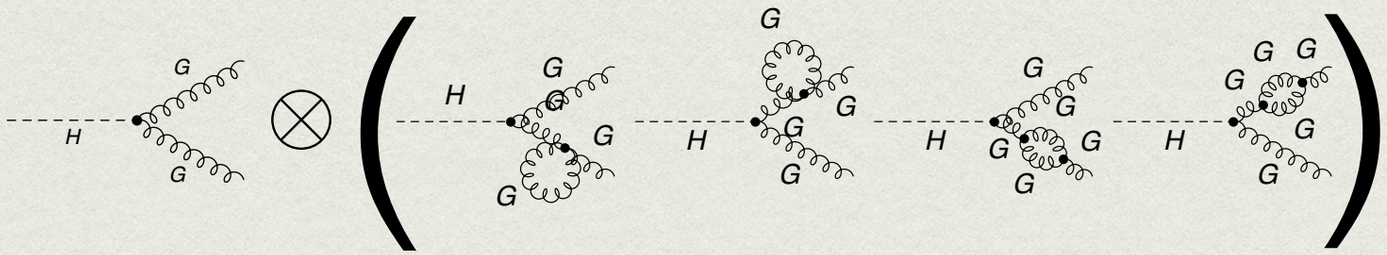

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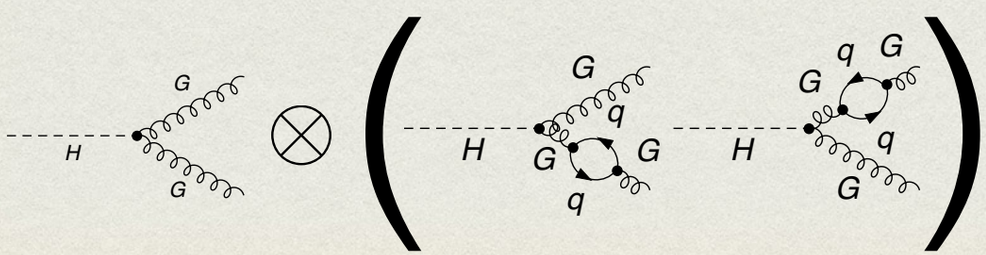

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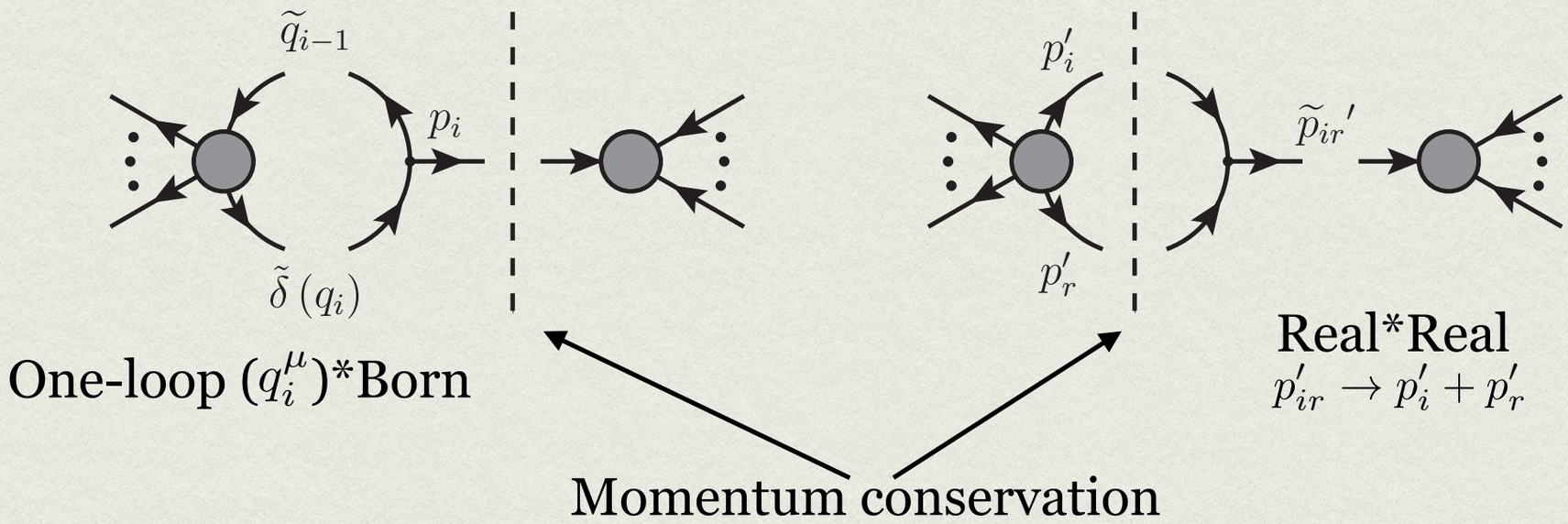

- Motivated by the factorisation properties of QCD

[Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]

$$p_r'^\mu = q_i^\mu \ ,$$

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$$p_j'^\mu = (1 - \alpha_i) \, p_j^\mu \ , \qquad p_k'^\mu = p_k^\mu \ , \qquad k \neq i, j$$

- Match 1—> 3 kinematics from 1—> 2 kinematics. Take into account loop three-momentum
- Work by region to separate collinear and soft divergencies

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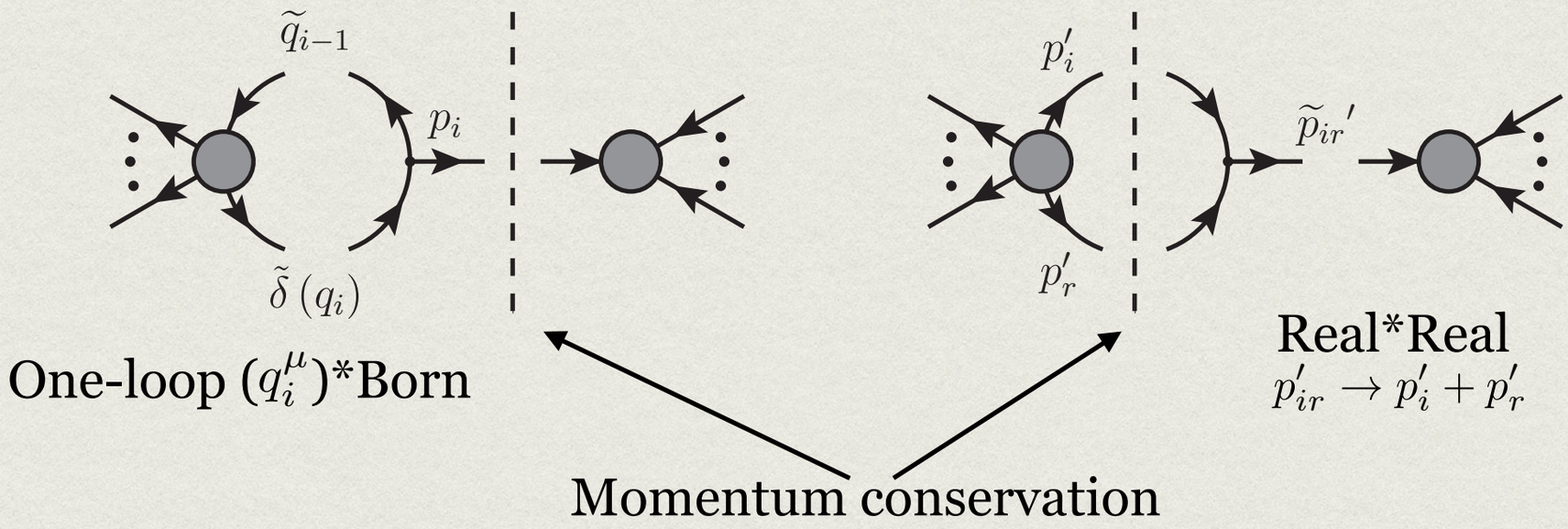

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Some remarks about the one- and two-loop calculations

Remarks

Literature

Traditional methods

integrand decomposition

IBP reduction

- Cross check of calculations
- Adaptive integrand decomposition

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]

- Splits d=4-2ε into parallel and orthogonal directions
- Nice properties for less than 5 external legs
- Numerator and denominators depend on different variables

$$d = d_{\parallel} + d_{\perp}$$

$$d_{\parallel} = n - 1$$

$$d_{\perp} = (5 - n) - 2\epsilon$$

$$\int \prod_i d^{d_\parallel} \bar{l}_{\parallel i} \int \prod_{1 \le i \le j \le \ell} d\lambda_{ij}\, G(\lambda_{ij})^{\frac{d_\perp - 1 - \ell}{2}} \int d\Theta_\perp \frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}\Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

Straightforward integration of transverse components

Expand in Gegenbauer polynomials

$$\int d\,\Theta_\perp = \int_{-1}^{1} \prod_{i=1}^{4-d_\parallel} \prod_{j=1}^{\ell} d\cos\theta_{i+j-1\,j} (\sin\theta_{i+j-1\,j})^{d_\perp - i - j - 1}$$

$$\int_{-1}^{1} d\cos\theta (\sin\theta)^{2\alpha-1} C_n^{(\alpha)}(\cos\theta) C_m^{(\alpha)}(\cos\theta) = \delta_{mn} \frac{2^{1-2\alpha}\pi\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}$$

and identification of spurious terms

Adaptive integrand decomposition (AID)

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]

Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in 3 steps

1) Divide and get $\Delta(\bar{l}_{\|i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of λ_{ij}

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

$$\frac{\mathcal{N}(\bar{l}_{\|i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

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$$\frac{\Delta'(\bar{l}_{\|i})}{D_1(\bar{l}_{\|i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\|i}, \lambda_{ij})}$$

→ `AIDA`

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

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Algorithm already **automated** → `AIDA`

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

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We have reached:

- ☑ Straightforward application of LTD @ **1-L**
- ☑ Matched virtual and real correction by means of FDU
- ☑ Application of LTD in effective field theoris
- ☑ Renormalisation @ **1-L** (at integrand level)-> completely under control

We are working on:

- ☐ Deal with processes at two loops that contain
  - — Threshold singularities (contour deformation)
  - — IR singularities
- ☐ Extend FDU to two loops

To be continued…

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Thanks