Recent advances in the μ-e scattering amplitude at NNLO in QED

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In collaboration with:
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Outline

- Motivations
- μe-scattering at two loops
  — Integrand reduction decomposition
  — UV renormalisation
- Other applications within `Aida`
- Conclusions & Outlook

Motivations

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \ 24.05050988\ (28)\ (\alpha/\pi)^3$

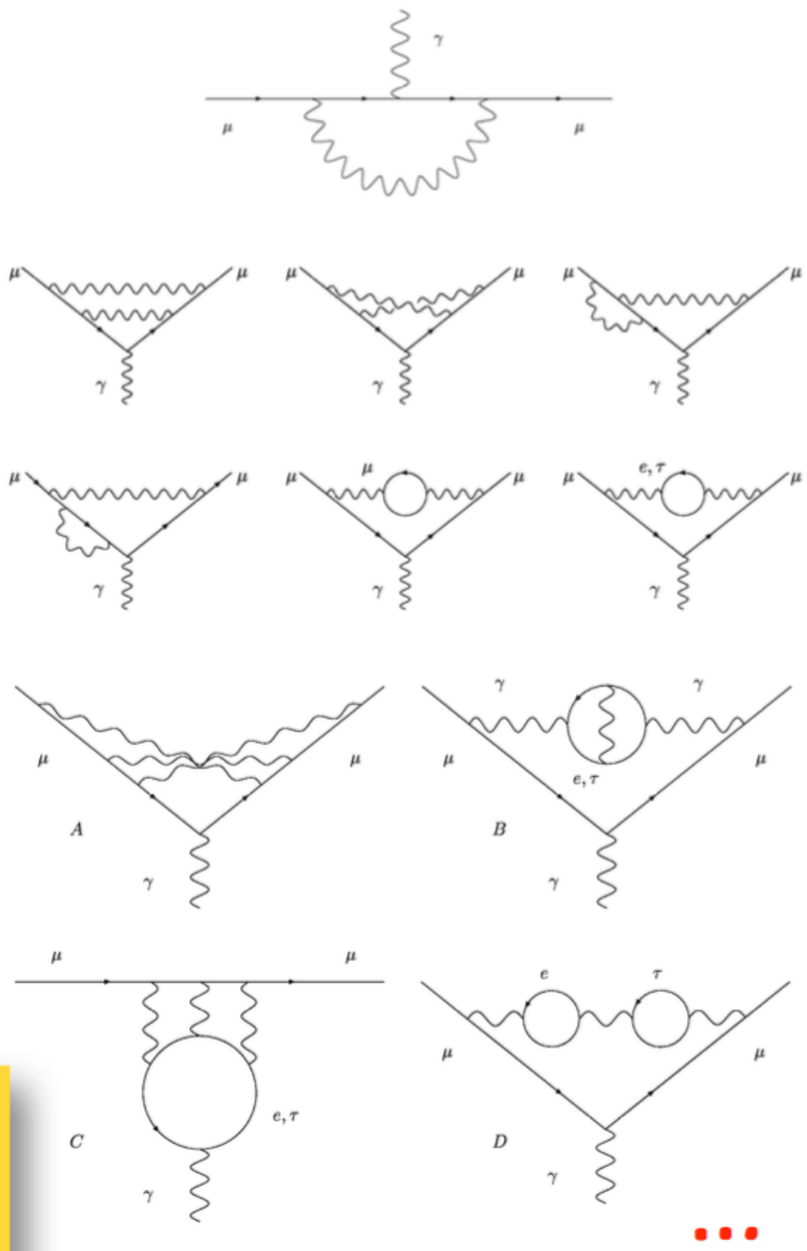
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \ 130.8780\ (60)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
S. Laporta, PLB 2017 (mass independent term). COMPLETED²!

$+ \ 750.80\ (89)\ (\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015 & 2017



Adding up, I get:

$a_\mu^{QED} = 116584718.932\ (20)(23) \times 10^{-11}$
from coeffs, mainly from 4-loop unc ⟵ ⟶ from α (Cs)
with $\alpha = 1/137.035999046(27)$ [0.2ppb] 2018

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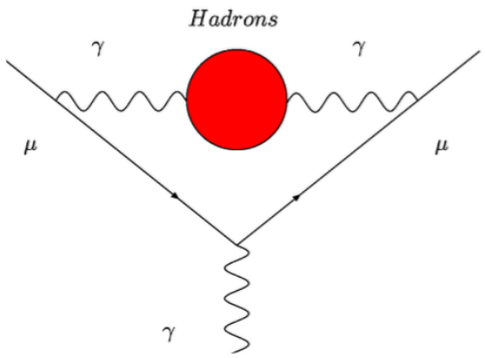
Motivations

- QED provides more than 99.99% of the standard-model value of a_μ
- Simplify calculations in high perturbative orders
- Cross checks from μe scattering
  — One of the cleanest processes
  — Pure t-channel
- Theoretical work needed —> NNLO QED correction unknown

Motivations

New space-like proposal for HLO

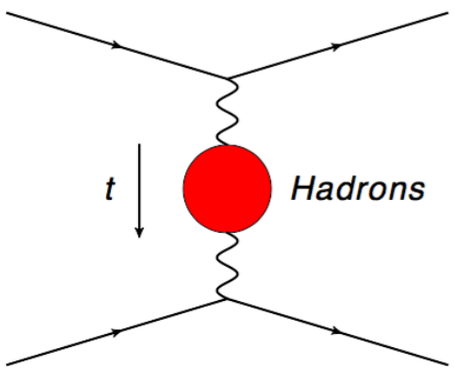
- **At present, the leading hadronic contribution a_μ^{HLO} is computed via the time-like formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\,\sigma_{\rm had}^0(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- **Alternatively, exchanging the x and s integrations in a_μ^{HLO}**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

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>> Venanzoni

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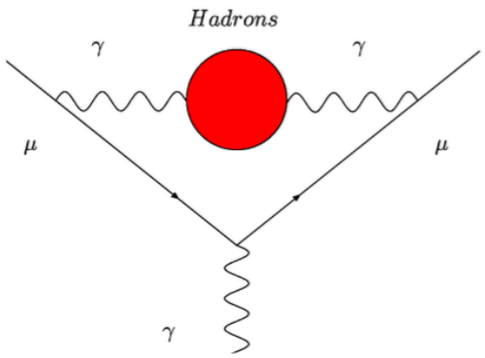

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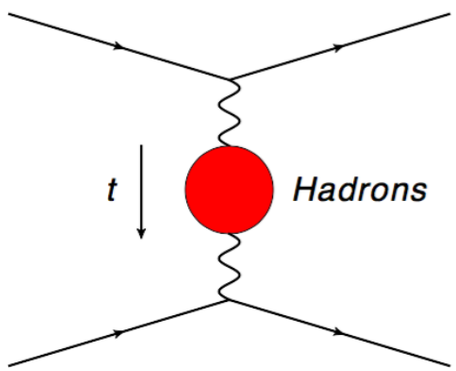

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Muon-electron scattering: The MUonE Project

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

>> Venanzoni

Motivations

Results @NLO

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

Anatomy

- Born matrix element
  tree-level & n-pt process

- Real contribution
  Tree-level (n+1)-particles

- Virtual Contribution
  one-loop n-particles

- No assumption made in the newest result
- QED & EW effects
- Full lepton mass dependence
- Fully differential fixed order MC @ NLO

[Alacevich, Chiesa, Montagna, Nicrosini, Piccinini, Carloni Calame (2018)]

$$\hat{\sigma}_{NLO} \sim \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^R_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\hat{\sigma}^V_{NLO} \quad \text{+ MC integration}$$

- Other checks done in **[Fael, Passera (2019)]**

Motivations

what about @NNLO

Anatomy

$$\mu^+(p_1) + e^-(p_2) \to \mu^+(p_3) + e^-(p_4)$$

- Real-Real contribution
  Tree-level (n+2)-particles

- Real-Virtual Contribution
  one-loop (n+1)-particles

- Virtual-Virtual Contribution
  two-loop n-particles

- GoSam
  Improvements for QED/EW
  Chiesa, Ossola and Tramontano

- ☆ **This talk**

$$\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

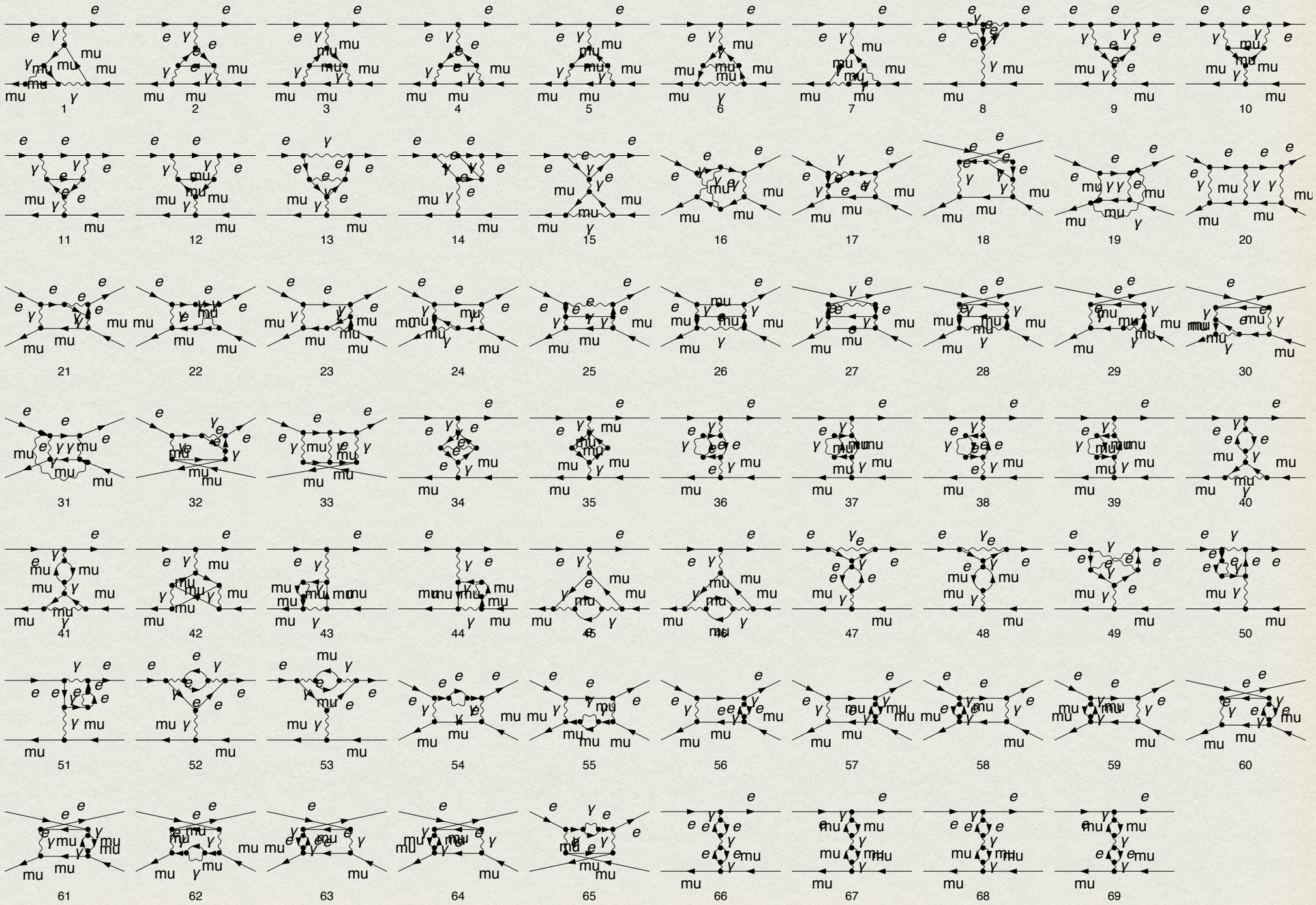
+ Subtractions & MC integrations

μONe @ *two loops*

69 diagrams @ 2-loop

Automatically organised in groups

Integrand reductions by means of **AIDA**



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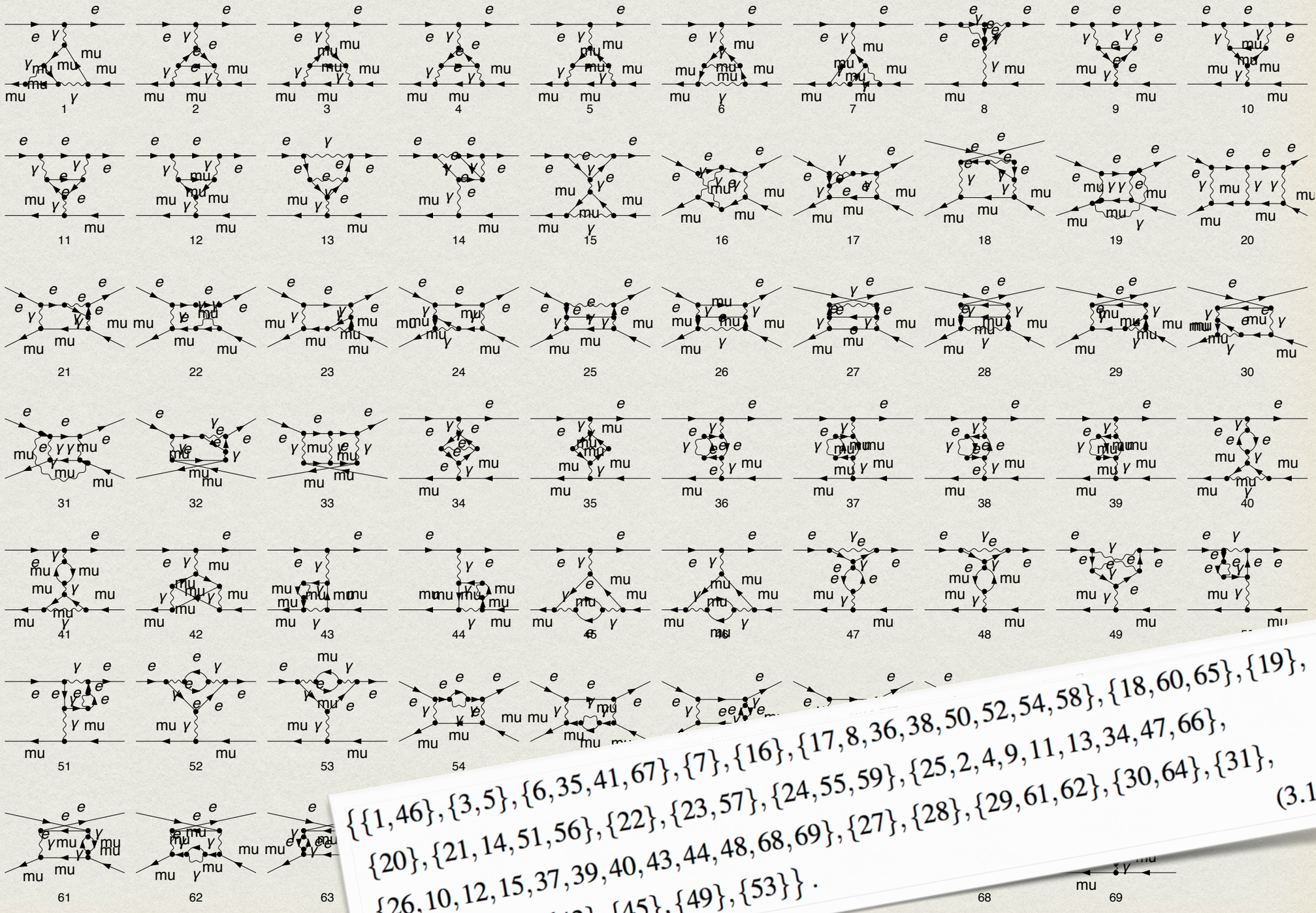
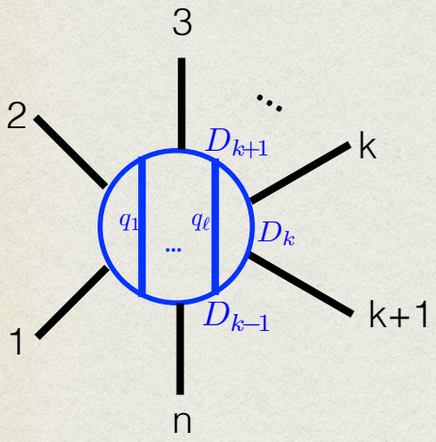


$$\{\{1,46\},\{3,5\},\{6,35,41,67\},\{7\},\{16\},\{17,8,36,38,50,52,54,58\},\{18,60,65\},\{19\},$$
$$\{20\},\{21,14,51,56\},\{22\},\{23,57\},\{24,55,59\},\{25,2,4,9,11,13,34,47,66\},$$
$$\{26,10,12,15,37,39,40,43,44,48,68,69\},\{27\},\{28\},\{29,61,62\},\{30,64\},\{31\},$$
$$\{32,63\},\{33\},\{42\},\{45\},\{49\},\{53\}\}\,. \qquad (3.18)$$

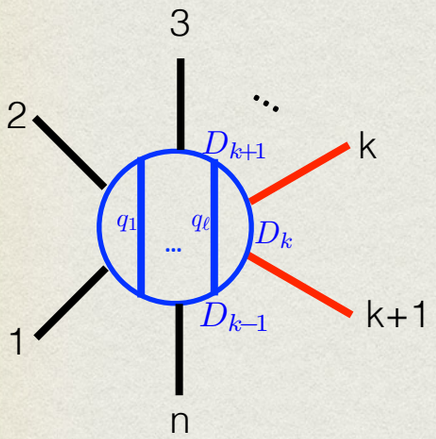
Integrand decomposition method

[Ossola, Papadopoulos, Pittau (2006)]
[Ellis, Giele, Kunszt, Melnikov (2007)]
[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]



$$= \frac{\mathcal{N}_{i_1 \cdots i_m}(q_i)}{D_1 \cdots D_k \cdots D_m}$$

- ☑ Applicable to any theory
- ☑ Ideal for helicity amplitudes
- ☑ Work for any number of external legs
- ☑ Straightforwardly automated



$$= \sum_{k=1}^{m} \frac{\mathcal{N}_{i_1 \cdots i_{k+1} i_{k-1} \cdots i_m}(q_i)\, D_k}{D_1 \cdots D_k \cdots D_m} + \frac{\Delta_{i_1 \cdots i_k \cdots i_m}(q_i)}{D_1 \cdots D_k \cdots D_m}$$

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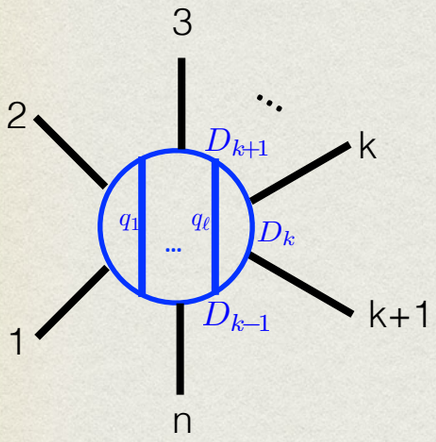

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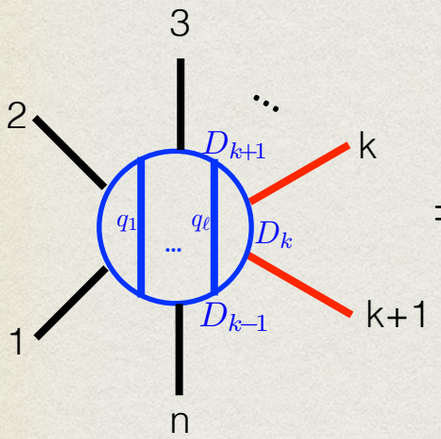

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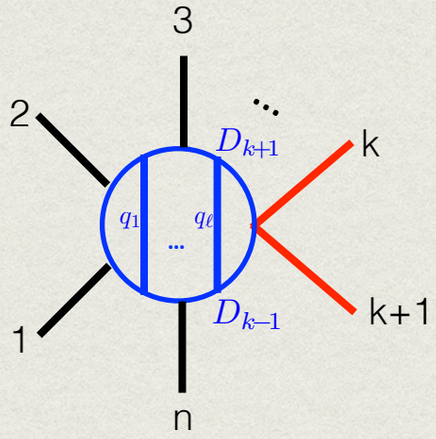

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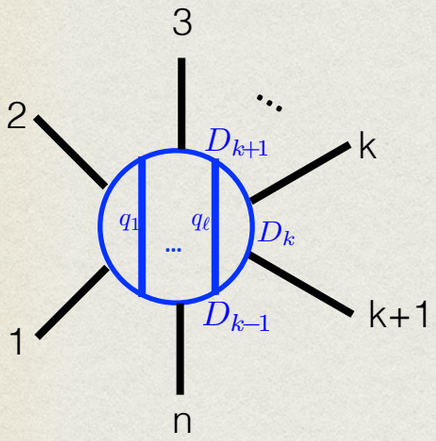

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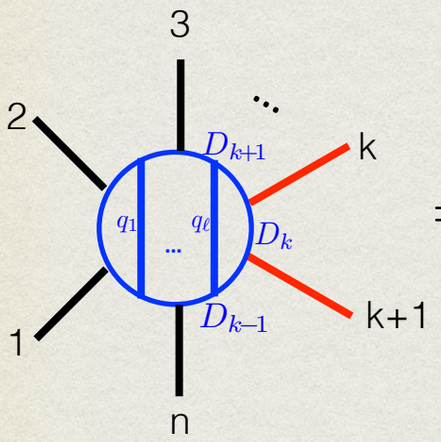

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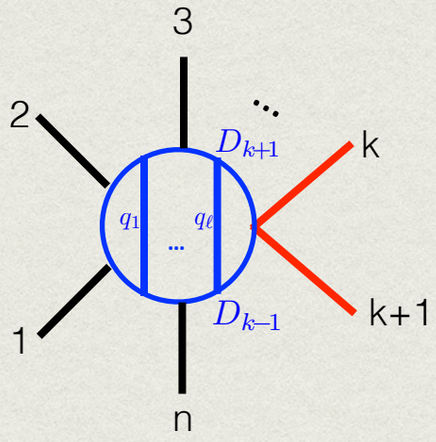
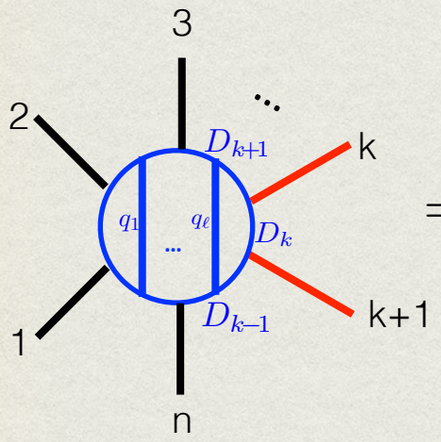


$$+ \frac{\Delta_{i_1 \cdots i_k \cdots i_m}(q_i)}{D_1 \cdots D_k \cdots D_m}$$

Amplitude decomposed into all possible multi-particle cuts



$$= \sum_{k=0}^{m} \sum_{\{1,\cdots,m\}} \frac{\Delta_{i_1 \cdots i_k}(q_i)}{D_1 \cdots D_k}$$

Numerator in terms of
Irreducible polynomials

Polynomial division module Groebner basis

[Mastrolia, Ossola 11]
[Zhang 2012-2016]
[Badger, Frellesvig, Zhang 2012-2013]
[Mastrolia, Mirabella, Ossola, Peraro 2012]

Adaptive integrand decomposition (AID)

$$d = d_{\parallel} + d_{\perp}$$

$$d_{\parallel} = n - 1$$

$$d_{\perp} = (5 - n) - 2\epsilon$$

- Splits d=4-2ε into parallel and orthogonal directions
- Nice properties for less than 5 external legs

[Collins (1984)]
[van Neerven and Vermaseren (1984)]
[Kreimer (1992)]

Loop momenta

$$\bar{l}_i^{\alpha} = \bar{l}_{\parallel i}^{\alpha} + \lambda_i^{\alpha} \longrightarrow \bar{l}_i^{\alpha} = \sum_{j=1}^{d_{\parallel}} x_{ji}\, e_j^{\alpha}\,, \qquad \lambda_i^{\alpha} = \sum_{j=d_{\parallel}+1}^{4} x_{ji}\, e_j^{\alpha} + \mu_i^{\alpha}\,, \qquad \lambda_{ij} = \sum_{l=d_{\parallel}+1}^{4} x_{li} x_{lj} + \mu_{ij}$$

- Numerator and denominators depend on different variables

[Mastrolia, Peraro, Primo (2016)]
[Mastrolia, Peraro, Primo, W.J.T. (2016)]

$$\int \prod_i d^{d_{\parallel}} \bar{l}_{\parallel i} \int \prod_{1 \le i \le j \le \ell} d\lambda_{ij}\, G(\lambda_{ij})^{\frac{d_{\perp}-1-\ell}{2}} \int d\Theta_{\perp} \frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}\Theta_{\perp})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

Straightforward integration of transverse components

Expand in Gegenbauer polynomials

$$\int d\,\mathbf{\Theta}_{\perp} = \int_{-1}^{1} \prod_{i=1}^{4-d_{\parallel}} \prod_{j=1}^{\ell} d\cos\theta_{i+j-1\,j} (\sin\theta_{i+j-1\,j})^{d_{\perp}-i-j-1}$$

$$\int_{-1}^{1} d\cos\theta (\sin\theta)^{2\alpha-1} C_n^{(\alpha)}(\cos\theta) C_m^{(\alpha)}(\cos\theta) = \delta_{mn} \frac{2^{1-2\alpha}\pi\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}$$

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and identification of spurious terms

Adaptive integrand decomposition (AID)

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]

Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in **3** steps

1) Divide and get $\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of λ_{ij}

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

$$\frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

$$\downarrow$$

1)
$$\frac{\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

$$\downarrow$$

2)
$$\frac{\Delta^{\text{int}}(\bar{l}_{\parallel i}, \lambda_{ij})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

$$\downarrow$$

3)
$$\frac{\Delta'(\bar{l}_{\parallel i})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

AIDA: a Mathematica implementation

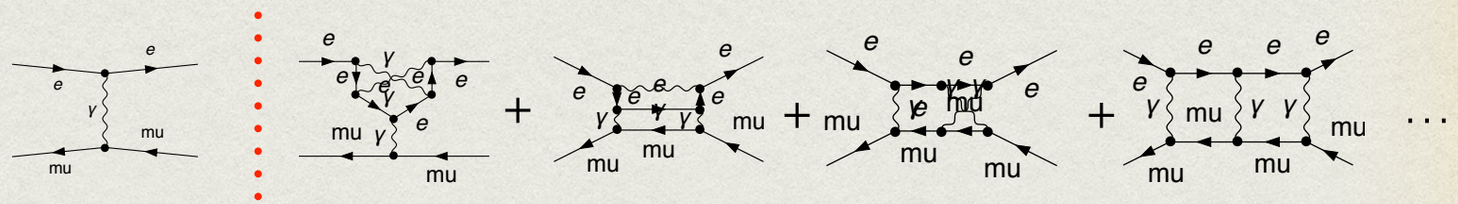
[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

$$\mathcal{M}^{(2)}\left(e\mu \to e\mu\right) = 2\mathrm{Re}\left[\mathcal{A}_4^{(2)} \otimes \left(\mathcal{A}_4^{(0)}\right)^*\right]$$

Amplitude generation

FeynArts-FeynCalc
Qgraph-FormCalc

$\mathcal{M}^{(2)}\left(e\mu \to e\mu\right) =$



Algebraic decomposition

Integrand decomposition
AID

IBP reduction
Reduze, Kira, …

Renormalisation

$$\mathcal{M}^{(2)}\left(e\mu \to e\mu\right) = \sum_k c_k\left(s,t,m^2,\epsilon\right) I_k^{(2)}\left(s,t,m^2,\epsilon\right)$$

$$I_k^{(2)} = \int \mathrm{d}^d q_1 \mathrm{d}^d q_2 \frac{1}{D_1^{\nu_1}\cdots D_{n_k}^{\nu_{n_k}}}$$

Loop integral evaluation

Analytic Continuation

Numeric
Ginac, SecDec, …

Analytic
Master Integrals

[Bonciani, Ferroglia, Gehrmann, von Manteuffel (2008-13)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]

Algebraic decomposition

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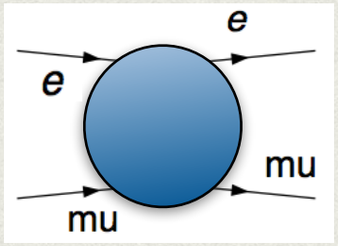
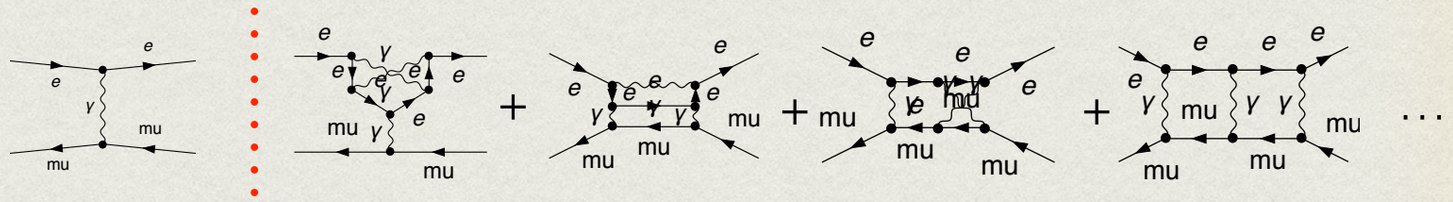




$$e(p_1) + \mu(p_4) \to e(-p_2) + \mu(-p_3)$$

In the massless electron limit,
4-point process depending on 3 scales

$$s = (p_1 + p_2)^2 \qquad t = (p_2 + p_3)^2$$

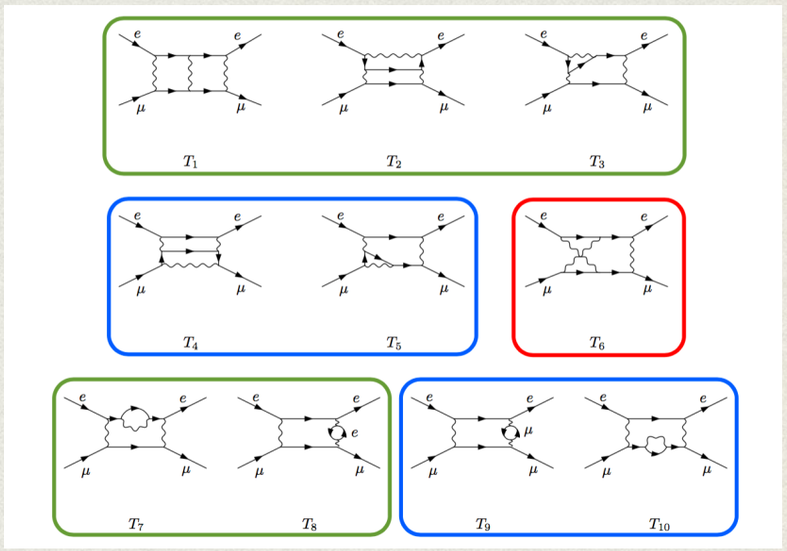
$$m_e^2 \simeq 0 \qquad u = -s - t + 2m^2$$

So far we have obtained

$$\mathcal{M}^{(2)}\left(e\mu \to e\mu\right) = \sum_k c_k\left(s, t, m^2, \epsilon\right) I_k^{(2)}\left(s, t, m^2, \epsilon\right)$$

O(10000) monomials

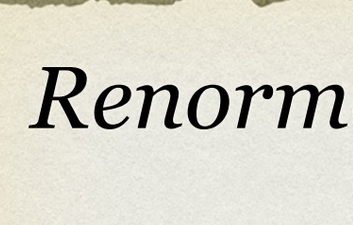
O(100) MIs

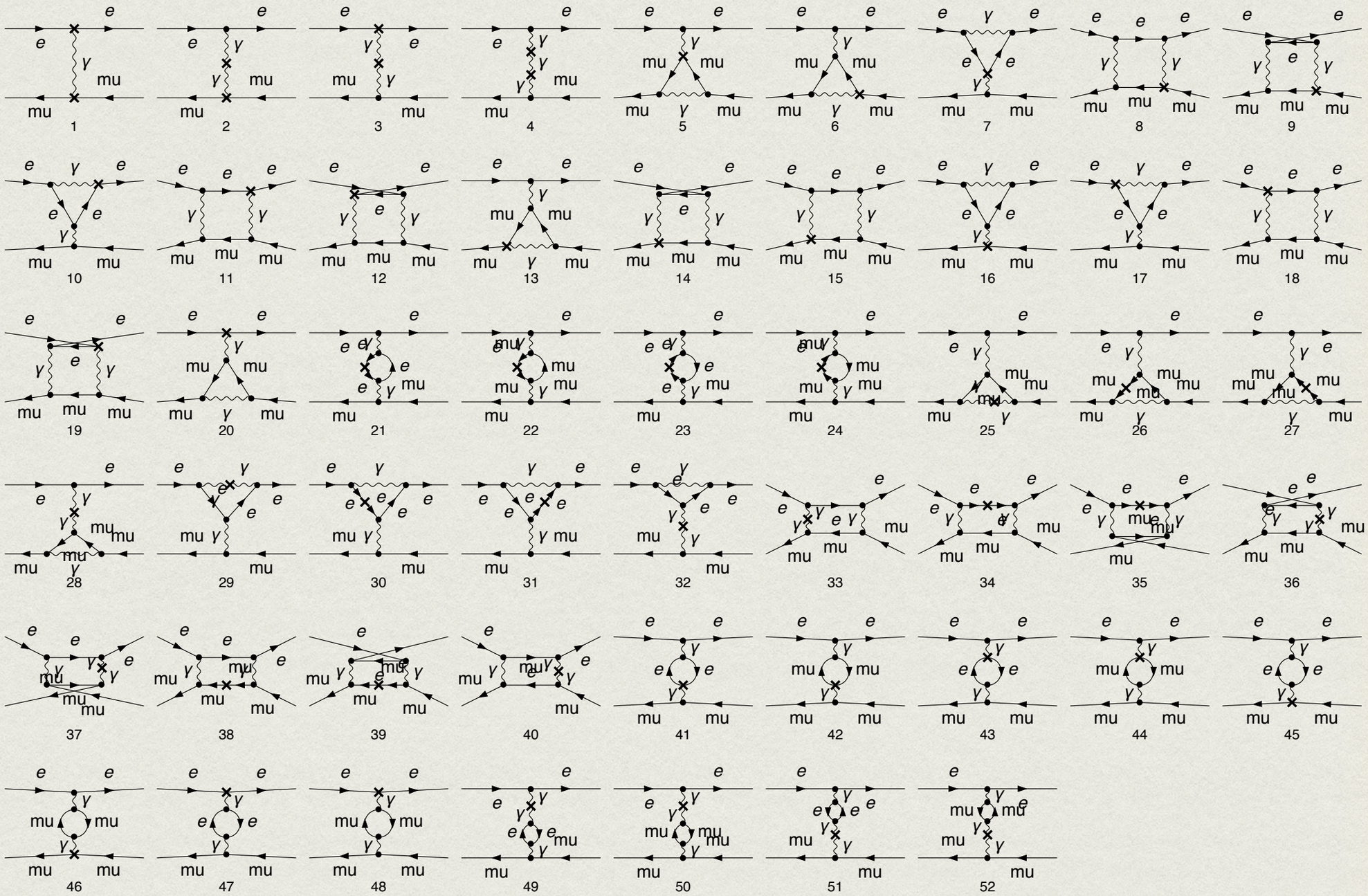


- Full integrand reduction of the amplitude
- Complete IBP reduction for planar topologies
- IBP reduction of non-planar topologies by finite fields (work in progress) [Peraro (2016)]

Renormalisation

$$\mathcal{M}^{(2)}\left(e\mu \to e\mu\right) = 2\mathrm{Re}\left[\mathcal{A}_4^{(2)} \otimes \left(\mathcal{A}_4^{(0)}\right)^*\right]$$





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Recall: amplitude in the massless electron limit

$$\overline{\mathrm{MS}} \qquad \text{vs} \qquad \text{On-shell}$$

Strategy

$$\alpha_{\overline{\mathrm{MS}}} \qquad + \qquad m_{\mathrm{OS}}$$

Massification: get renormalised amplitude in the on-shell scheme

$$\alpha_{\overline{\mathrm{MS}}} \qquad + \qquad m_{\mathrm{OS}} \dashrightarrow \{\alpha, m\}_{\mathrm{OS}}$$

In collaboration w/ ***A. Signer, Y. Ulrich, et al***

Renormalisation

Some comments on $\overline{\text{MS}}$

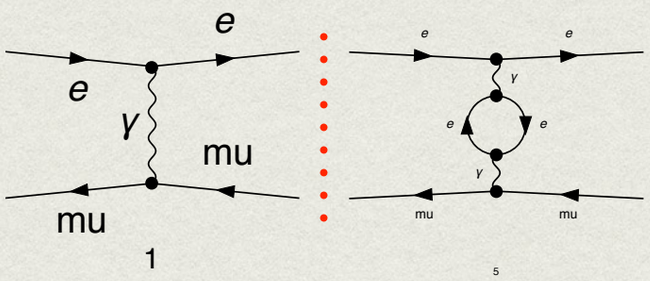
- Expand the integrand in the UV limit

[Becker, Reuschle, Weinzierl (2010)]
[Sborlini, Driencourt-Mangin, Hernandez-Pinto, Rodrigo (2016)]

$$G_F(q_i) = G_F(q_{\text{UV}})\left(1 - \left(2q_{\text{UV}}\cdot k_{i,\text{UV}} + k_{i,\text{UV}}^2 + \mu_{\text{UV}}^2 - m_i^2\right) G_F(q_{\text{UV}}) + \cdots\right)$$

$$G_F(q_{\text{UV}}) = \frac{1}{q_{\text{UV}}^2 - \mu_{\text{UV}}^2 + \imath 0}$$

- Extract the UV c-t of eμ @1-loop, the simplest application



$$= \int \frac{d^d\ell}{(2\pi)^d}\frac{1}{q_{\text{UV}}^2 - \mu_{\text{UV}}^2}\left(\frac{c_1}{\left(q_{\text{UV}}^2 - \mu_{\text{UV}}^2\right)^3} + \frac{\mu_{\text{UV}}^4 c_2 + \mu_{\text{UV}}^2 c_3}{\left(q_{\text{UV}}^2 - \mu_{\text{UV}}^2\right)^2} + \frac{\mu_{\text{UV}}^2 c_4 + c_5}{q_{\text{UV}}^2 - \mu_{\text{UV}}^2} + c_6\right) \overset{\text{IBP}}{=} \int \frac{d^d\ell}{(2\pi)^d}\frac{4}{3}\frac{1}{\left(q_{\text{UV}}^2 - \mu_{\text{UV}}^2\right)^2}$$

$$= -\frac{4}{3\epsilon_{\text{UV}}}\left(\frac{\mu^2}{\mu_{\text{UV}}^2}\right)^{\epsilon} c_\Gamma$$

Compute integral with your favourite method

- Extension @2-loop

[Driencourt-Mangin, Rodrigo, Sborlini, W.J.T. (2019)]

Renormalisation

Some comments on $\overline{\text{MS}}$

ONE-LOOP PROCEDURE

- We consider a Feynman (uncut) integrand $I(\ell, \{p_i\}_N)$, and the replacement

$$S : \begin{cases} \ell^2 \to \lambda^2 \ell^2 + (1-\lambda^2)\mu^2 \\ \ell \cdot p_i \to \lambda \ell \cdot p_i \end{cases}$$

- Computing the **local UV counter-term** C of I is done by
  - Applying the replacement S on I
  - Taking the limit $\lambda \to \infty$
  - Selecting the divergent terms, which gives (unfixed) C
  - Fixing the finite part so C integrates to the desired quantity ($\mathcal{O}(\epsilon^0) = 0$ in $\overline{\text{MS}}$)

- Expand the integrand in the UV limi [Herzog, Ruijl, Ueda, Vermaseren, Vogt (2017)] [Anzierl (2010)] [Rodrigo (2016)]

$$G_F(q_i) = G_F(q_{\text{UV}})\left(1 - \left(2q_{\text{UV}} \cdot k_{i,\text{UV}} + k_{i,\text{UV}}^2\right.\right.$$

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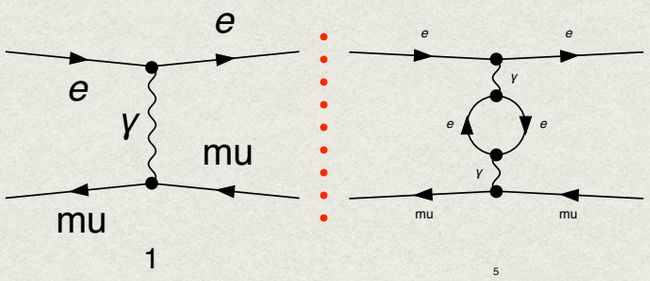

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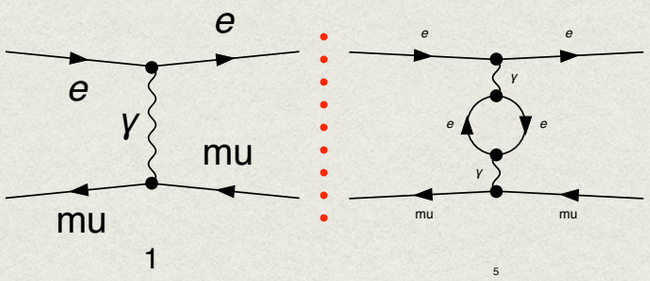

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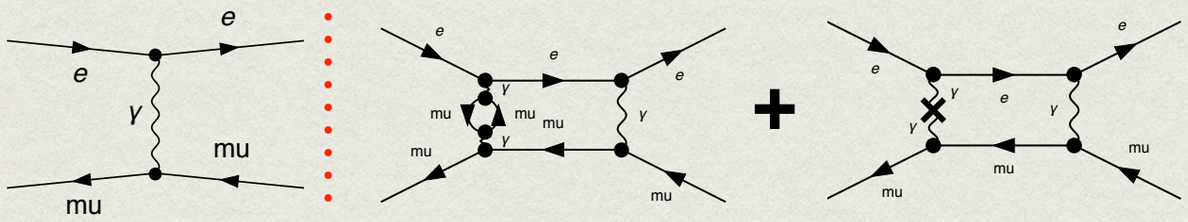
Renormalisation

Preliminary results in **OS**



- ☑ `Aida+Reduze` to end up with MIs
- ☑ Evaluate MIs from **[Mastrolia, Passera, Primo, Schubert (2017)]**
- ☑ Leading pole contribution, ϵ^{-2},

$$\sim |\mathcal{M}_{\rm Born}|^2 \Pi\left(t\right) \qquad \text{Expected factorisation formula}$$

with

$$\Pi\left(\theta\right) = \frac{i\alpha}{3\pi}\left\{\theta - \left(\frac{\theta}{2}+1\right)\sqrt{(1-\theta)}\log\left(\frac{\sqrt{1-\theta}+1}{\sqrt{1-\theta}-1}\right) + \frac{5}{3}\right\}, \qquad \theta = \frac{4m_f^2}{t}$$

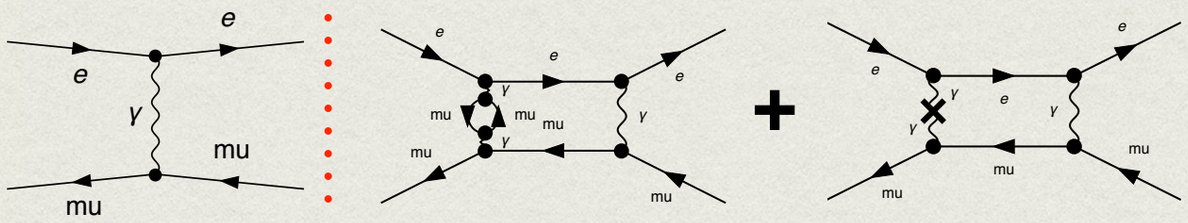
$$|\mathcal{M}_{\rm Born}|^2 = \frac{2\left(2m^4 - 4m^2 s + 2s^2 + 2st + t^2\right)}{t^2}$$

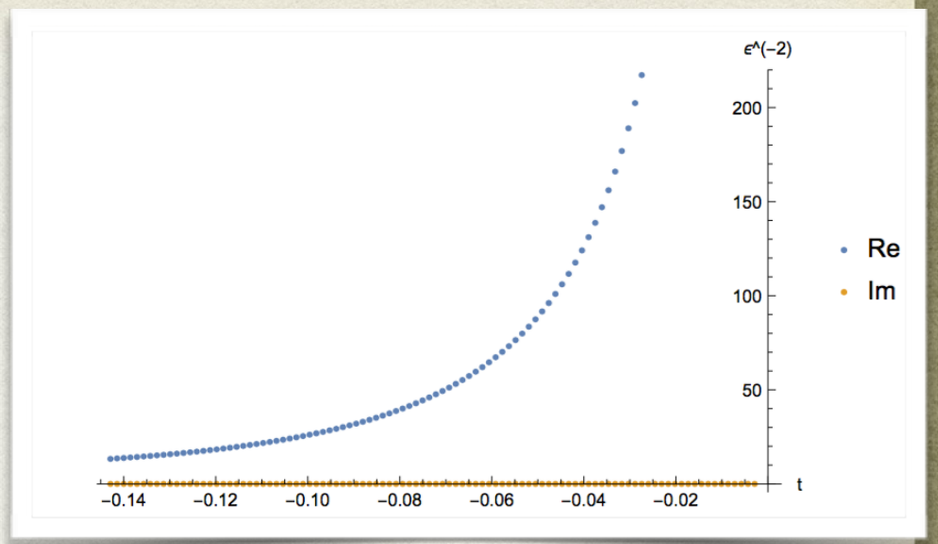
Cross checked results w/ ***M. Fael, M. Passera***

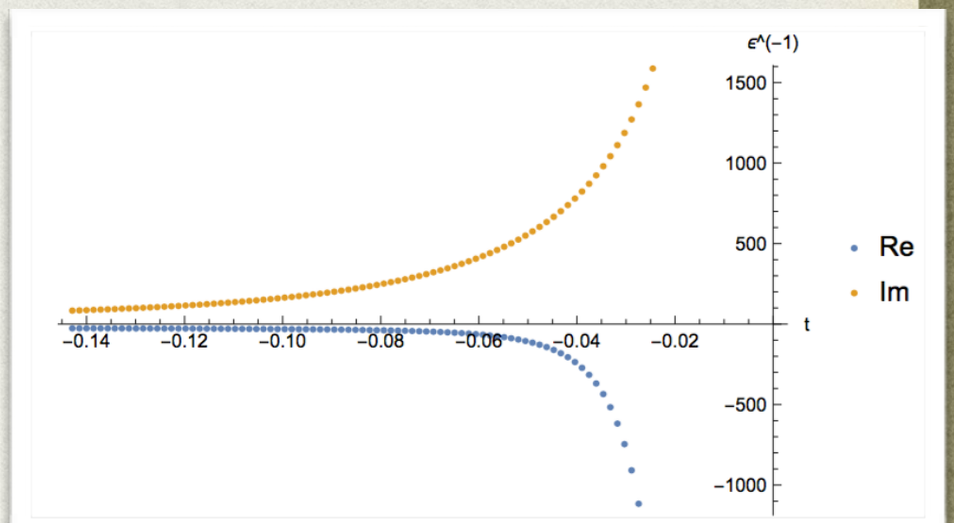
[Fael (2018)]

Renormalisation

Preliminary results in **OS**







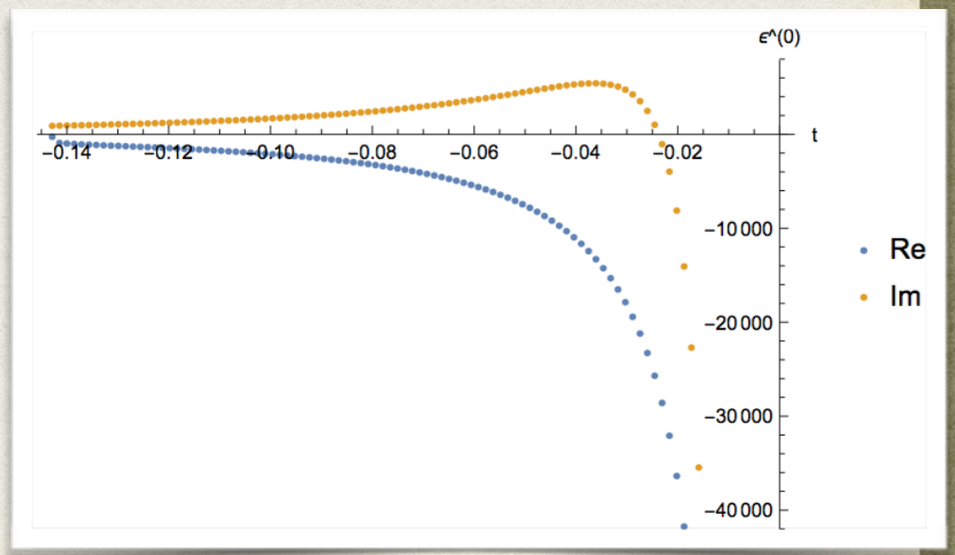
- ☑ `Aida+Reduze` to end up with MIs
- ☑ Evaluate MIs from **[Mastrolia, Passera, Primo, Schubert (2017)]**
- ☑ Numerical evaluation with

$$\begin{aligned}\sqrt{s} &= 0.40554\ \text{GeV}\,,\\ m &= 105.6583745\ \text{MeV}\,,\\ &\text{scan over } t_{\min} \le t < 0\,,\\ t_{\min} &= -0.142893\ \text{GeV}^2\end{aligned}$$

- ☑ t remains negative —> analytic continuation of MIs

$$s \to s + \imath 0^+$$

Use properties of GPLs + `GiNaC`



Applications to two-loop amplitudes

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Process	Scales + d	Analytic	Numeric
$gg \to H$	$s, (m_t)$	✔	✔
$\gamma^* \to e^+e^-$	s, m_e	✔	✔
$gg \to gg$	s, t	✔	✔
$gg \to gh$	s, t	✔	✔
$gg \to gH$	$s, t, m_H, (m_t)$	✔	✔
$gg \to HH$	s, t, m_H, m_t	✔	✔
$e^-\mu^+ \to e^-\mu^+$	$s, t, m_\mu, (m_e)$	✔	✔
$gg \to ggg$	$s_{12}, s_{23}, s_{34}, s_{45}, s_{51}$	✖	✔
$gg \to ggH$	$s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_H, m_t$	✖	✔

Conclusions/Outlook

- ☑ Integrand decomposition methods —> @1 and 2 Loops Automated (AIDA)
- ☑ Analytic decomposition for all 2—>2 processes—> Under control
- ☑ AIDA's output —> Apply IBPs + evaluation of MIs
- ☑ Renormalisation procedure —> Under control

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Thanks