Three-loop soft anomalous dimensions for top-quark production

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- **Higher-order soft-gluon corrections**
- **Three-loop calculations**
- **Top-quark production**

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Soft-gluon corrections

partonic processes

$$f_1(p_1) + f_2(p_2) \to t(p_t) + X$$

define $s=(p_1+p_2)^2$, $t=(p_1-p_t)^2$, $u=(p_2-p_t)^2$ **and** $s_4=s+t+u-\sum m^2$

At partonic threshold $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \le 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

At NNLL accuracy we need two-loop soft anomalous dimensions

At N^{3}LL accuracy we need three-loop soft anomalous dimensions

Finite-order expansions-no prescription needed

Approximate NNLO (aNNLO) and N^{3}LO (aN^{3}LO) predictions

for cross sections and differential distributions

Soft-gluon Resummation

moments of the partonic cross section with moment variable N:
$\hat{\sigma}(N) = \int (ds_4/s)\, e^{-N s_4/s} \hat{\sigma}(s_4)$

factorized expression for the cross section in $4-\epsilon$ dimensions

$$\sigma^{f_1 f_2 \to tX}(N,\epsilon) = H_{IL}^{f_1 f_2 \to tX}\left(\alpha_s(\mu_R)\right)\, S_{LI}^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F},\alpha_s(\mu_R)\right)$$
$$\times \prod J_{\rm in}\left(N,\mu_F,\epsilon\right) \prod J_{\rm out}\left(N,\mu_F,\epsilon\right)$$

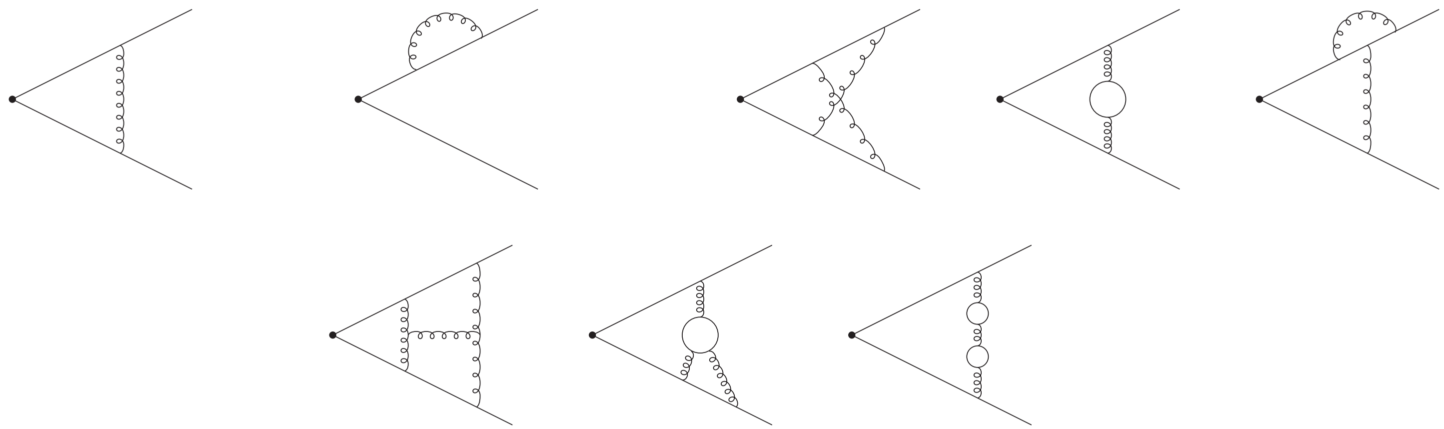
$H_{IL}^{f_1 f_2 \to tX}$ **is hard function and** $S_{LI}^{f_1 f_2 \to tX}$ **is soft function**

S_{LI} **satisfies the renormalization group equation**

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LK} S_{KI} - S_{LK}(\Gamma_S)_{KI}$$

Soft anomalous dimension Γ_S controls the evolution of the soft function which gives the exponentiation of logarithms of N

Cusp anomalous dimension



A basic ingredient of soft anomalous dimensions

cusp angle $\theta = \cosh^{-1}(p_i \cdot p_j / \sqrt{p_i^2 p_j^2})$ **and** $\Gamma_{\rm cusp} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \Gamma_{\rm cusp}^{(n)}$

One loop

$$\Gamma_{\rm cusp}^{(1)} = C_F (\theta \coth\theta - 1)$$

In terms of $\beta = \tanh(\theta/2) = \sqrt{1 - \frac{4m^2}{s}}$ **we have** $\theta = \ln\left[\frac{(1+\beta)}{(1-\beta)}\right]$ **and**

$$\Gamma_{\rm cusp}^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln\frac{(1-\beta)}{(1+\beta)} - 1 \right] = -C_F \left(L_\beta + 1\right)$$

Two loops

$$\Gamma_{\rm cusp}^{(2)} = \frac{K}{2}\,\Gamma_{\rm cusp}^{(1)} + \frac{1}{2}C_F C_A \left\{1+\zeta_2+\theta^2-\coth\theta\left[\zeta_2\theta+\theta^2+\frac{\theta^3}{3}+{\rm Li}_2\left(1-e^{-2\theta}\right)\right]\right.$$
$$\left. +\coth^2\theta\left[-\zeta_3+\zeta_2\theta+\frac{\theta^3}{3}+\theta\,{\rm Li}_2\left(e^{-2\theta}\right)+{\rm Li}_3\left(e^{-2\theta}\right)\right]\right\}$$

where $K = C_A(67/18-\zeta_2)-5n_f/9$

Three loops (very long expression for $C^{(3)}$)

$$\Gamma_{\rm cusp}^{(3)} = C^{(3)} + K'^{(3)}\Gamma_{\rm cusp}^{(1)} + K\left[\Gamma_{\rm cusp}^{(2)} - \frac{K}{2}\Gamma_{\rm cusp}^{(1)}\right]$$

For $n_f = 5$

$$\Gamma_{\rm cusp}^{(3)\,{\rm approx}}(\beta) = 0.092\,\beta^2 + 2.803\,\Gamma_{\rm cusp}^{(1)}(\beta)$$

Cusp anomalous dimension

Case where one line is massive and one is massless: simpler expressions

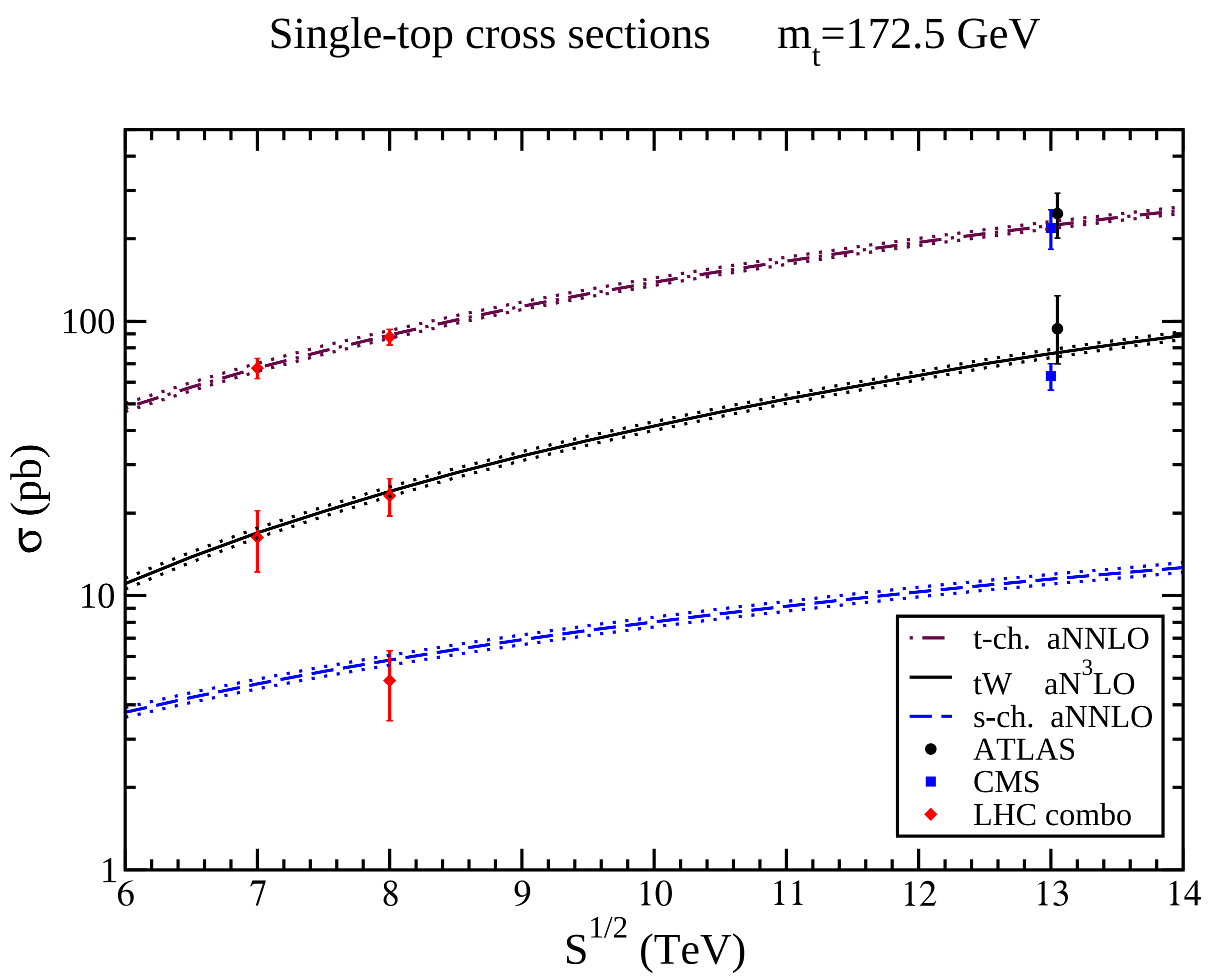
If eikonal line i represents a massive quark and eikonal line j a massless quark, then

$$\Gamma_c^{(1)} = C_F \left[\ln\left(\frac{2p_i \cdot p_j}{m_i\sqrt{s}}\right) - \frac{1}{2}\right]$$

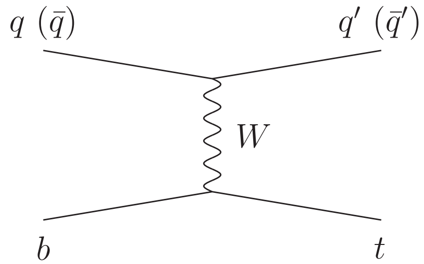
$$\Gamma_c^{(2)} = C_F \frac{K}{2}\left[\ln\left(\frac{2p_i \cdot p_j}{m_i\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{1}{4} C_F C_A (1-\zeta_3)$$

$$\begin{aligned}\Gamma_c^{(3)} = &\; C_F K^{'(3)} \left[\ln\left(\frac{2p_i \cdot p_j}{m_i\sqrt{s}}\right) - \frac{1}{2}\right] + C_F C_A \frac{K}{4}(1-\zeta_3) \\ &+ C_F C_A^2 \left[-\frac{1}{4} + \frac{3}{8}\zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8}\zeta_2\zeta_3 + \frac{9}{16}\zeta_5\right]\end{aligned}$$

Single-top production



Single-top t-channel production



At one loop

$$\Gamma_{S\,11}^{t\,(1)} = C_F \left[\ln\left(\frac{t(t-m_t^2)}{m_t s^{3/2}}\right) - \frac{1}{2}\right], \qquad \Gamma_{S\,12}^{t\,(1)} = \frac{C_F}{2N} \ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right), \qquad \Gamma_{S\,21}^{t\,(1)} = \ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{t\,(1)} = C_F \left[\ln\left(\frac{t(t-m_t^2)}{m_t s^{3/2}}\right) - \frac{1}{2}\right] - \frac{1}{N}\ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right) + \frac{N}{2}\ln\left(\frac{u(u-m_t^2)}{t(t-m_t^2)}\right)$$

At two loops

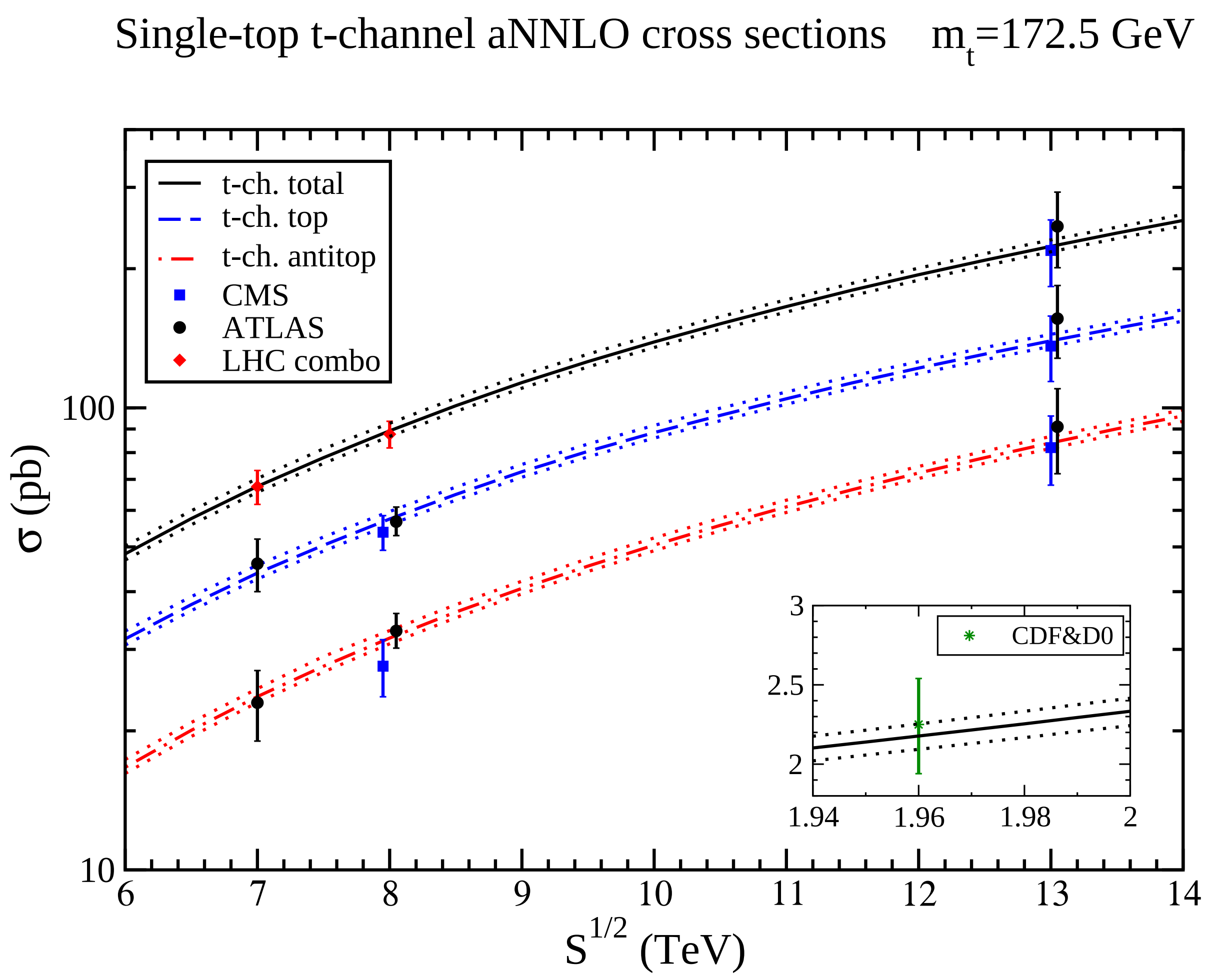
$$\Gamma_{S\,11}^{t\,(2)} = K'^{(2)}\Gamma_{S\,11}^{t\,(1)} + \frac{1}{4}C_F C_A (1-\zeta_3), \qquad \Gamma_{S\,12}^{t\,(2)} = K'^{(2)}\Gamma_{S\,12}^{t\,(1)}$$

$$\Gamma_{S\,21}^{t\,(2)} = K'^{(2)}\Gamma_{S\,21}^{t\,(1)}, \qquad \Gamma_{S\,22}^{t\,(2)} = K'^{(2)}\Gamma_{S\,22}^{t\,(1)} + \frac{1}{4}C_F C_A (1-\zeta_3)$$

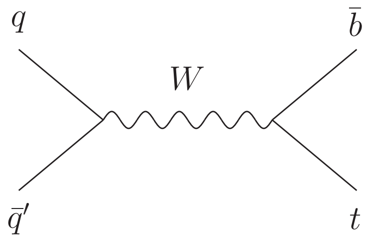
At three loops

$$\Gamma_{S\,11}^{t\,(3)} = K'^{(3)}\Gamma_{S\,11}^{t\,(1)} + \frac{1}{2}K^{(2)} C_A (1-\zeta_3) + C^{(3)}$$

t-channel production at aNNLO with NNLL accuracy



Single-top s-channel production



At one loop

$$\Gamma_{S\,11}^{s\,(1)} = C_F \left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}}\right) - \frac{1}{2}\right], \qquad \Gamma_{S\,12}^{s\,(1)} = \frac{C_F}{2N}\ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right), \qquad \Gamma_{S\,21}^{s\,(1)} = \ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right)$$

$$\Gamma_{S\,22}^{s\,(1)} = C_F \left[\ln\left(\frac{s-m_t^2}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] - \frac{1}{N}\ln\left(\frac{t(t-m_t^2)}{u(u-m_t^2)}\right) + \frac{N}{2}\ln\left(\frac{t(t-m_t^2)}{s(s-m_t^2)}\right)$$

At two loops

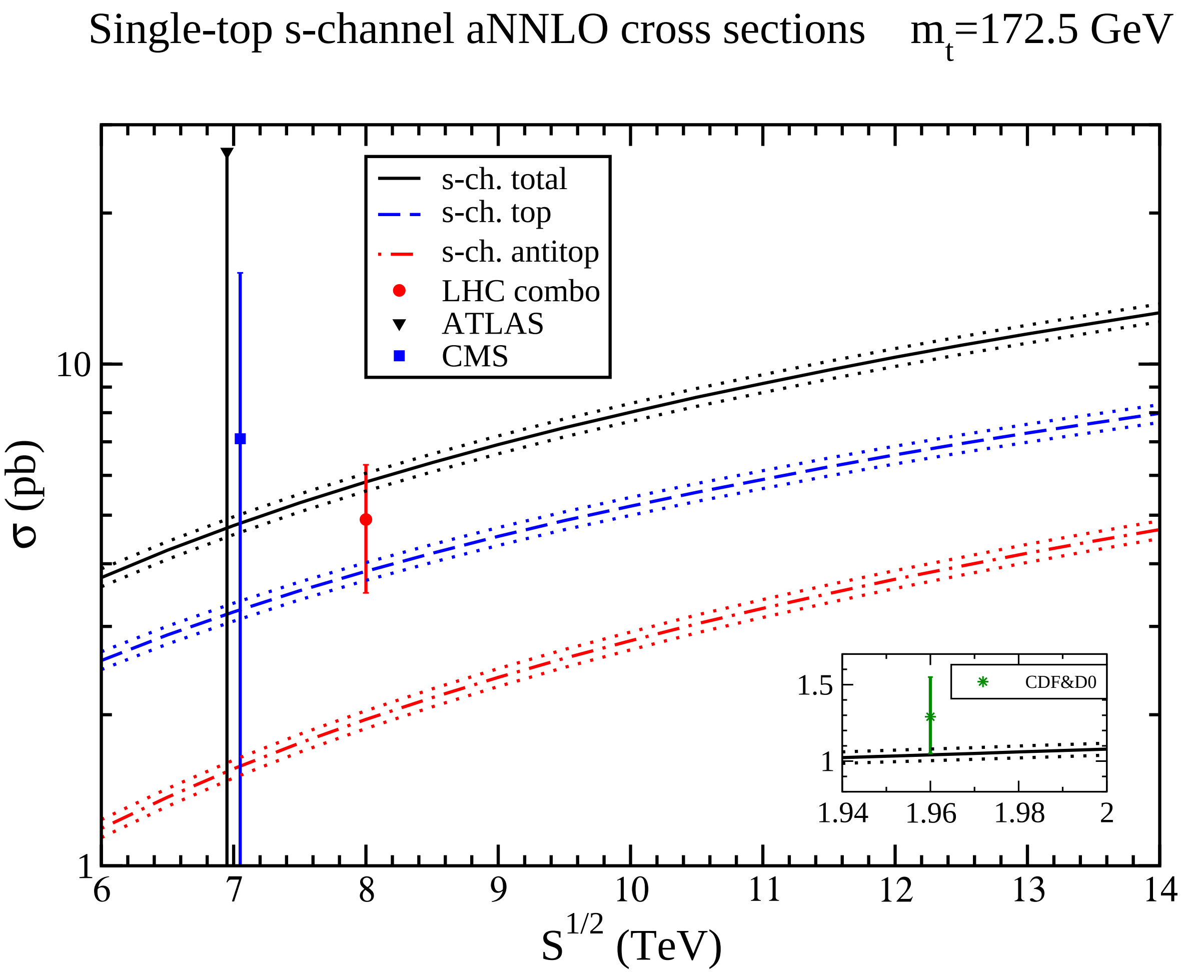
$$\Gamma_{S\,11}^{s\,(2)} = K'^{(2)}\Gamma_{S\,11}^{s\,(1)} + \frac{1}{4}C_F C_A(1-\zeta_3), \qquad \Gamma_{S\,12}^{s\,(2)} = K'^{(2)}\Gamma_{S\,12}^{s\,(1)}$$

$$\Gamma_{S\,21}^{s\,(2)} = K'^{(2)}\Gamma_{S\,21}^{s\,(1)}, \qquad \Gamma_{S\,22}^{s\,(2)} = K'^{(2)}\Gamma_{S\,22}^{s\,(1)} + \frac{1}{4}C_F C_A(1-\zeta_3)$$

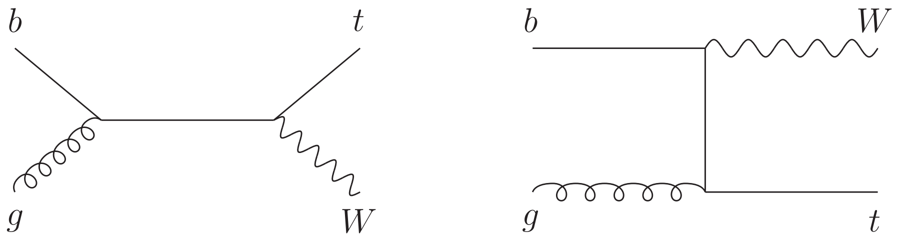
At three loops

$$\Gamma_{11}^{s\,(3)} = K'^{(3)}\Gamma_{11}^{s\,(1)} + \frac{1}{2}K^{(2)}C_A(1-\zeta_3) + C^{(3)}$$

s-channel production at aNNLO with NNLL accuracy



Associated tW production



At one loop

$$\Gamma_S^{tW\,(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{u - m_t^2}{t - m_t^2}\right)$$

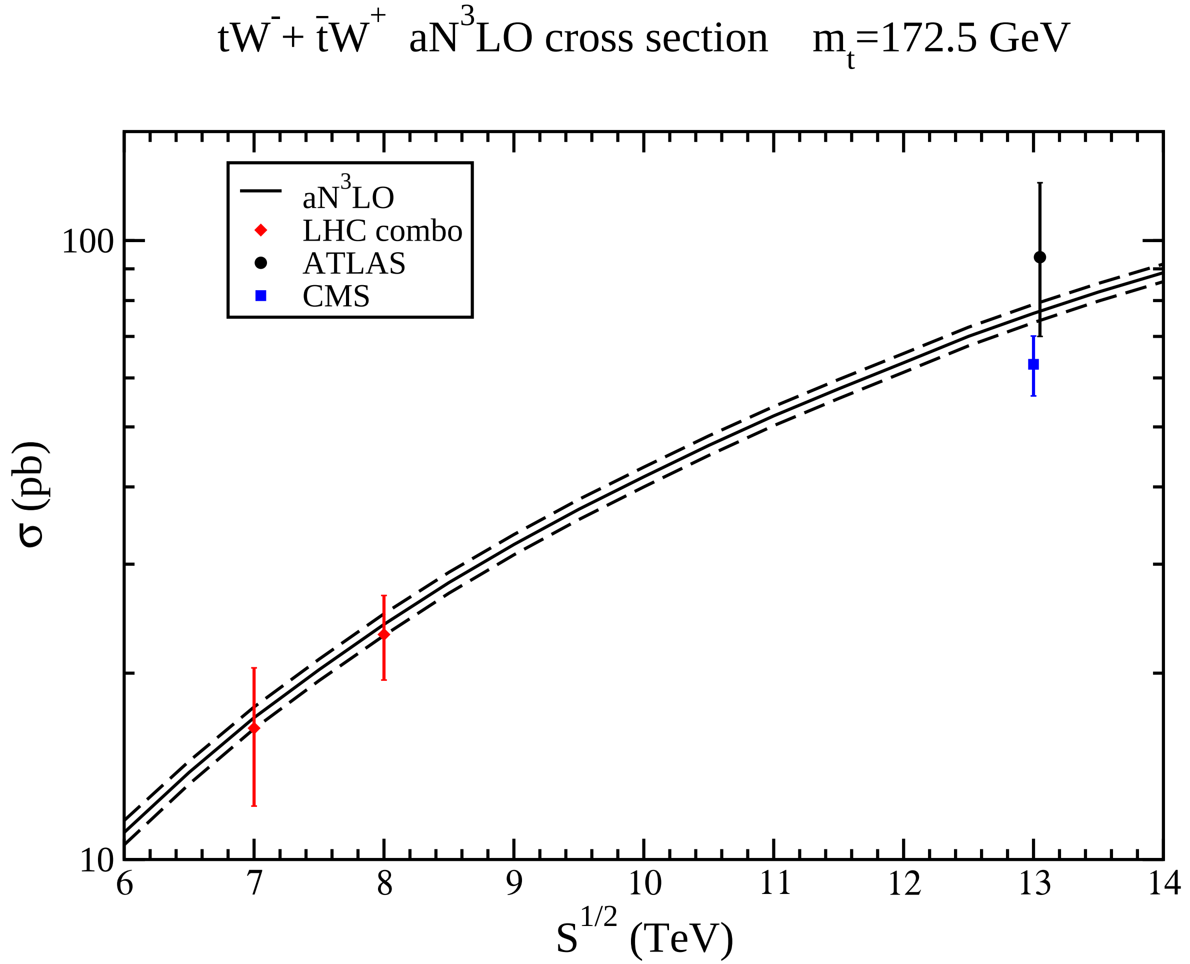
At two loops

$$\Gamma_S^{tW\,(2)} = K'^{(2)}\Gamma_S^{tW\,(1)} + \frac{1}{4}C_F C_A (1 - \zeta_3)$$

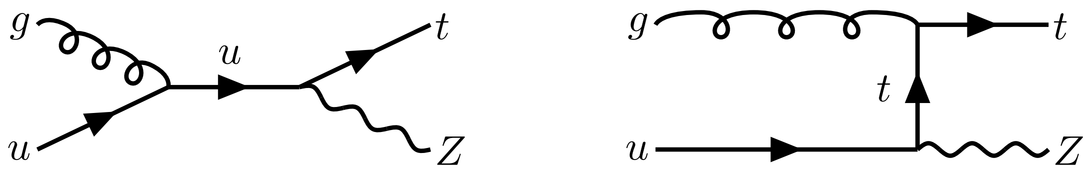
At three loops

$$\Gamma_S^{tW\,(3)} = K'^{(3)}\Gamma_S^{tW\,(1)} + \frac{1}{2}K^{(2)} C_A (1 - \zeta_3) + C^{(3)}$$

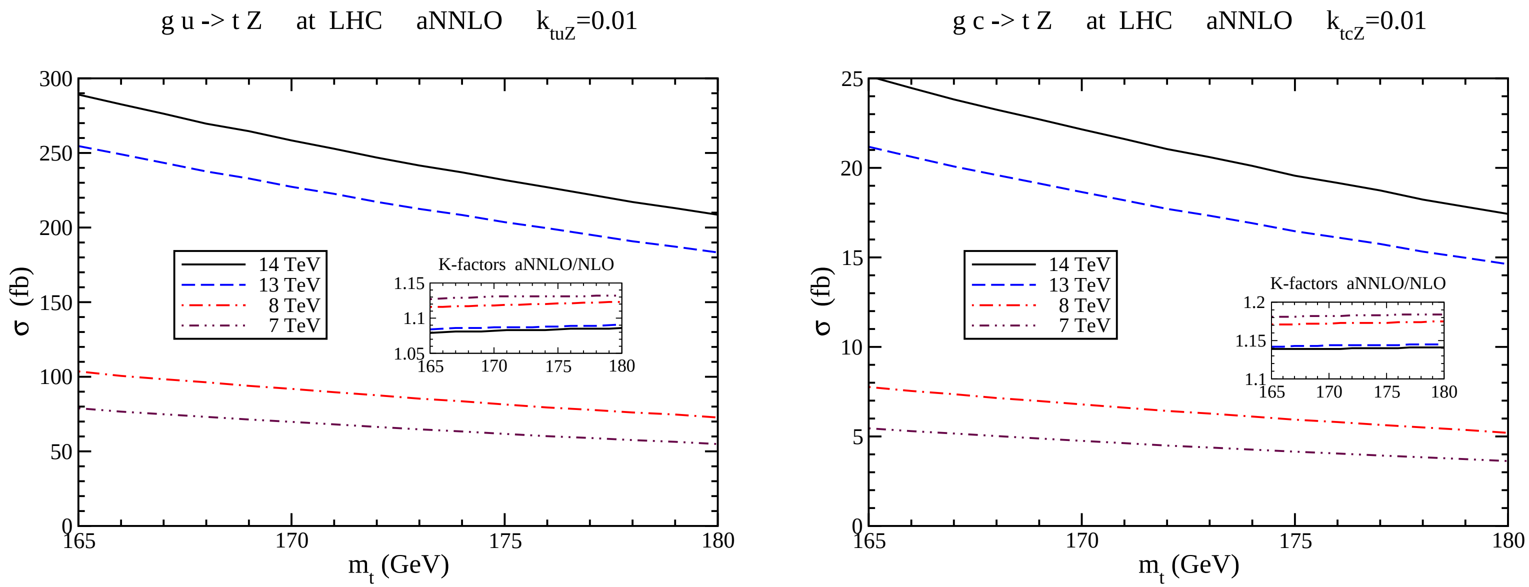
tW production at aN^{3}LO with NNLL accuracy



tZ production via anomalous couplings

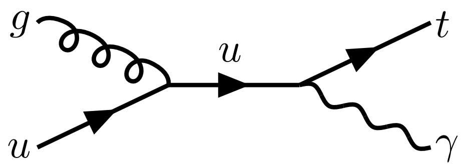


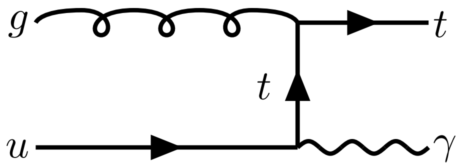
$$\Delta \mathcal{L}^{eff} = \frac{1}{\Lambda}\, \kappa_{tqZ}\, e\, \bar{t}\, (i/2)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)\, q\, F_Z^{\mu\nu} + h.c.$$



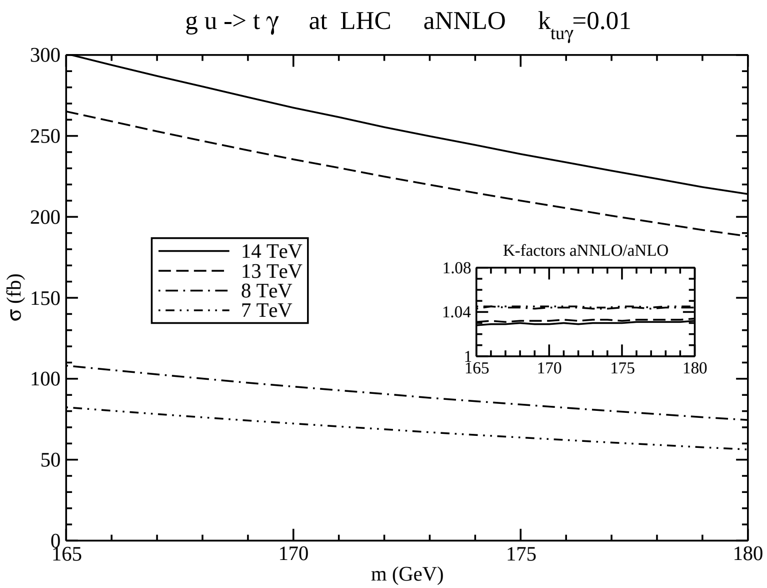
$t\gamma$ production via anomalous couplings

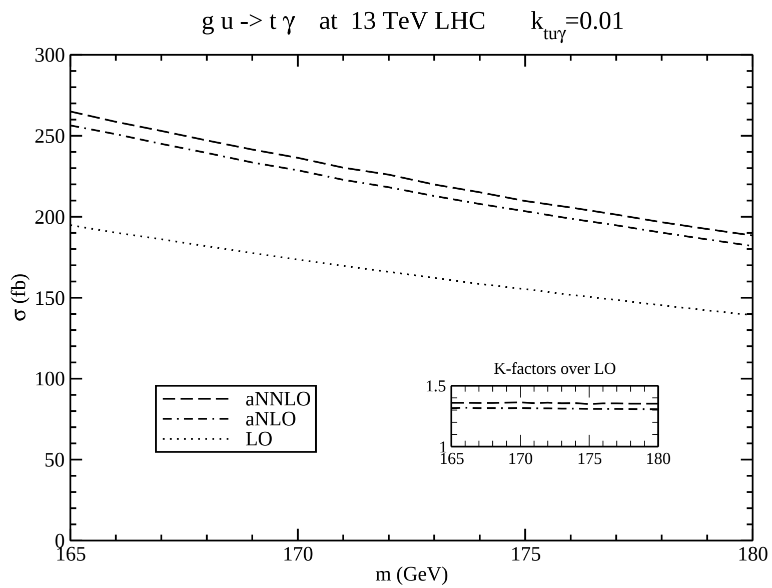
(with Matthew Forslund)





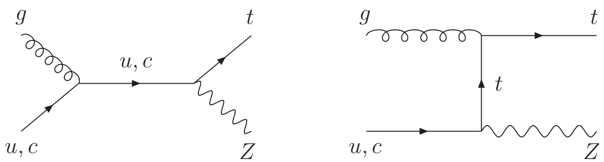
$$\Delta\mathcal{L}^{eff} = \frac{1}{\Lambda}\,\kappa_{tq\gamma}\,e\,\bar{t}\,(i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\,q\,F_\gamma^{\mu\nu} + h.c.$$

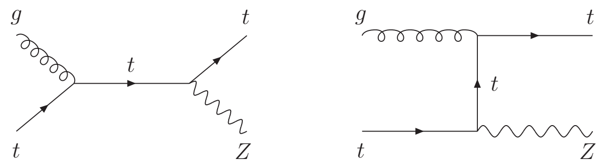




tZ' production

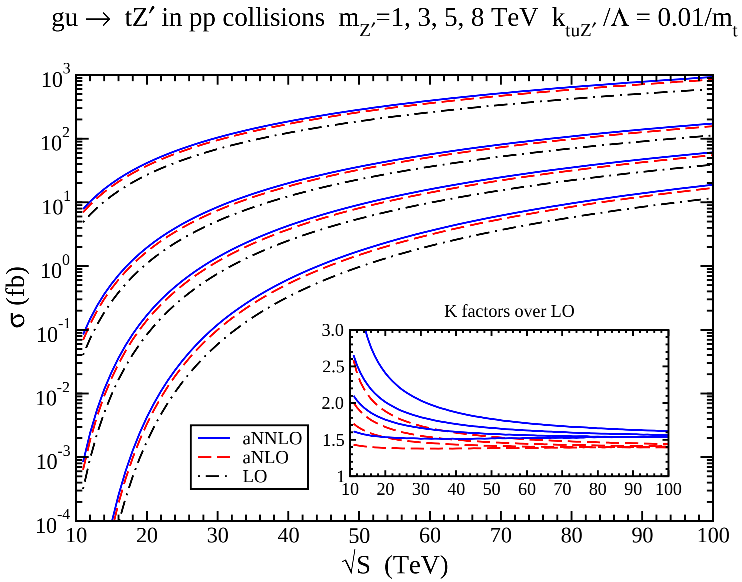
(with Marco Guzzi)

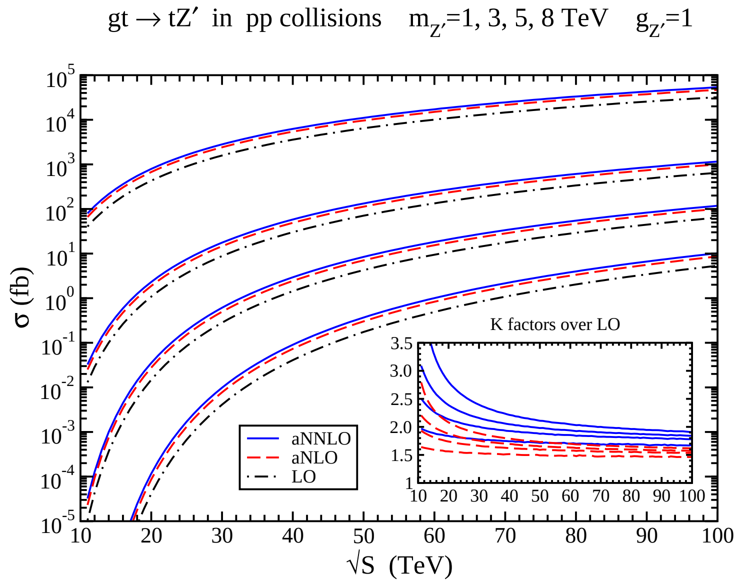




via anomalous couplings

with initial-state top

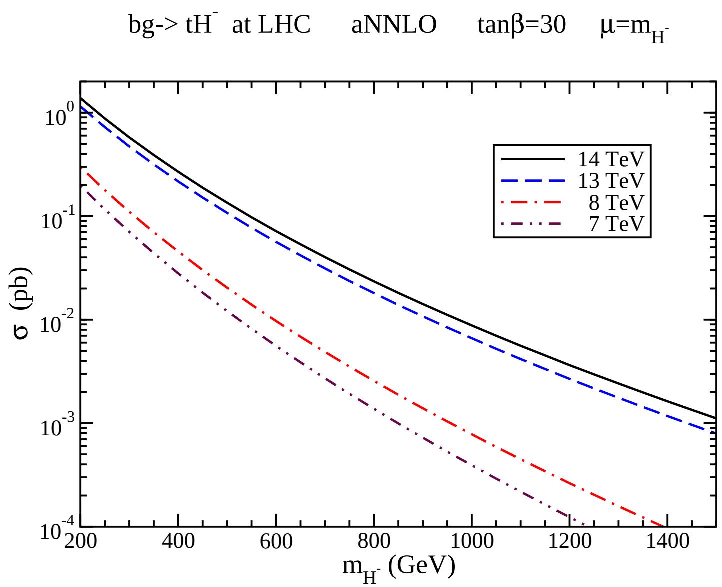


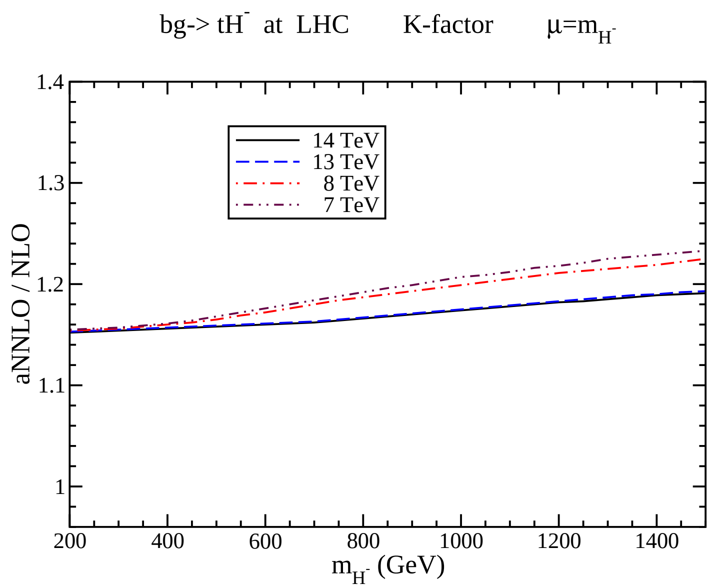


tH^- production

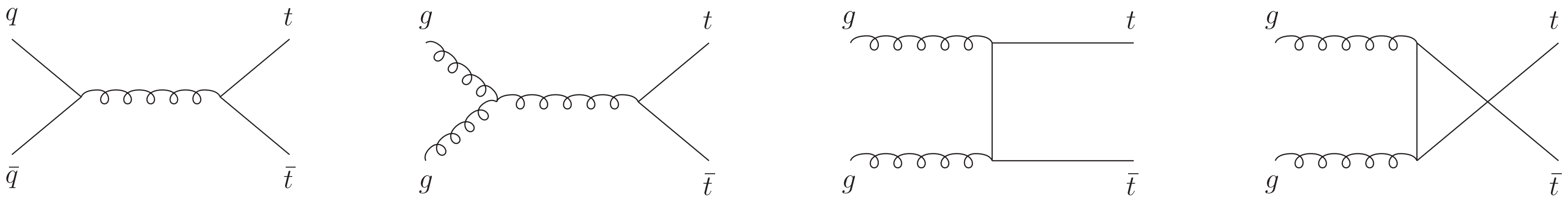
Leading-order cross section for $bg \to tH^- \propto \alpha\alpha_s(m_b^2 \tan^2\beta + m_t^2 \cot^2\beta)$

$\tan\beta = v_2/v_1$ **ratio of vevs of two Higgs doublets**





Top-antitop pair production



At one loop for $q\bar{q} \to t\bar{t}$

$$\Gamma_{11}^{q\bar{q}\,(1)} = \Gamma_{\rm cusp}^{(1)}\,, \quad \Gamma_{12}^{q\bar{q}\,(1)} = \frac{C_F}{C_A}\ln\left(\frac{t_1}{u_1}\right)\,, \quad \Gamma_{21}^{q\bar{q}\,(1)} = 2\ln\left(\frac{t_1}{u_1}\right)$$

$$\Gamma_{22}^{q\bar{q}\,(1)} = \left(1-\frac{C_A}{2C_F}\right)\Gamma_{\rm cusp}^{(1)} + 4C_F\ln\left(\frac{t_1}{u_1}\right) - \frac{C_A}{2}\left[1+\ln\left(\frac{s m_t^2 t_1^2}{u_1^4}\right)\right]$$

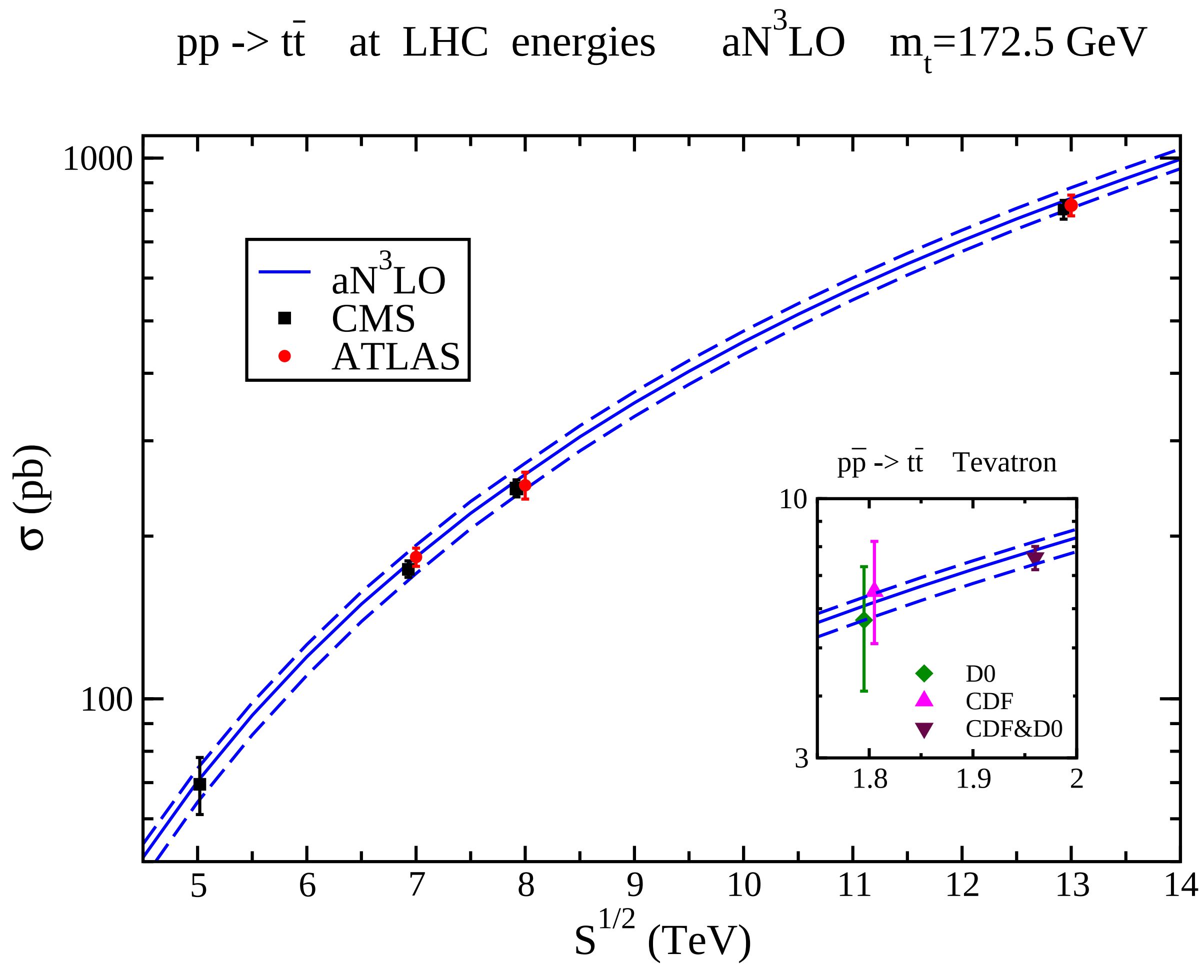
At two loops for $q\bar{q} \to t\bar{t}$

$$\Gamma_{11}^{q\bar{q}\,(2)} = \Gamma_{\rm cusp}^{(2)}\,, \quad \Gamma_{12}^{q\bar{q}\,(2)} = \left(\frac{K}{2}-\frac{C_A}{2}N_{2l}\right)\Gamma_{12}^{q\bar{q}\,(1)}\,, \quad \Gamma_{21}^{q\bar{q}\,(2)} = \left(\frac{K}{2}+\frac{C_A}{2}N_{2l}\right)\Gamma_{21}^{q\bar{q}\,(1)}$$

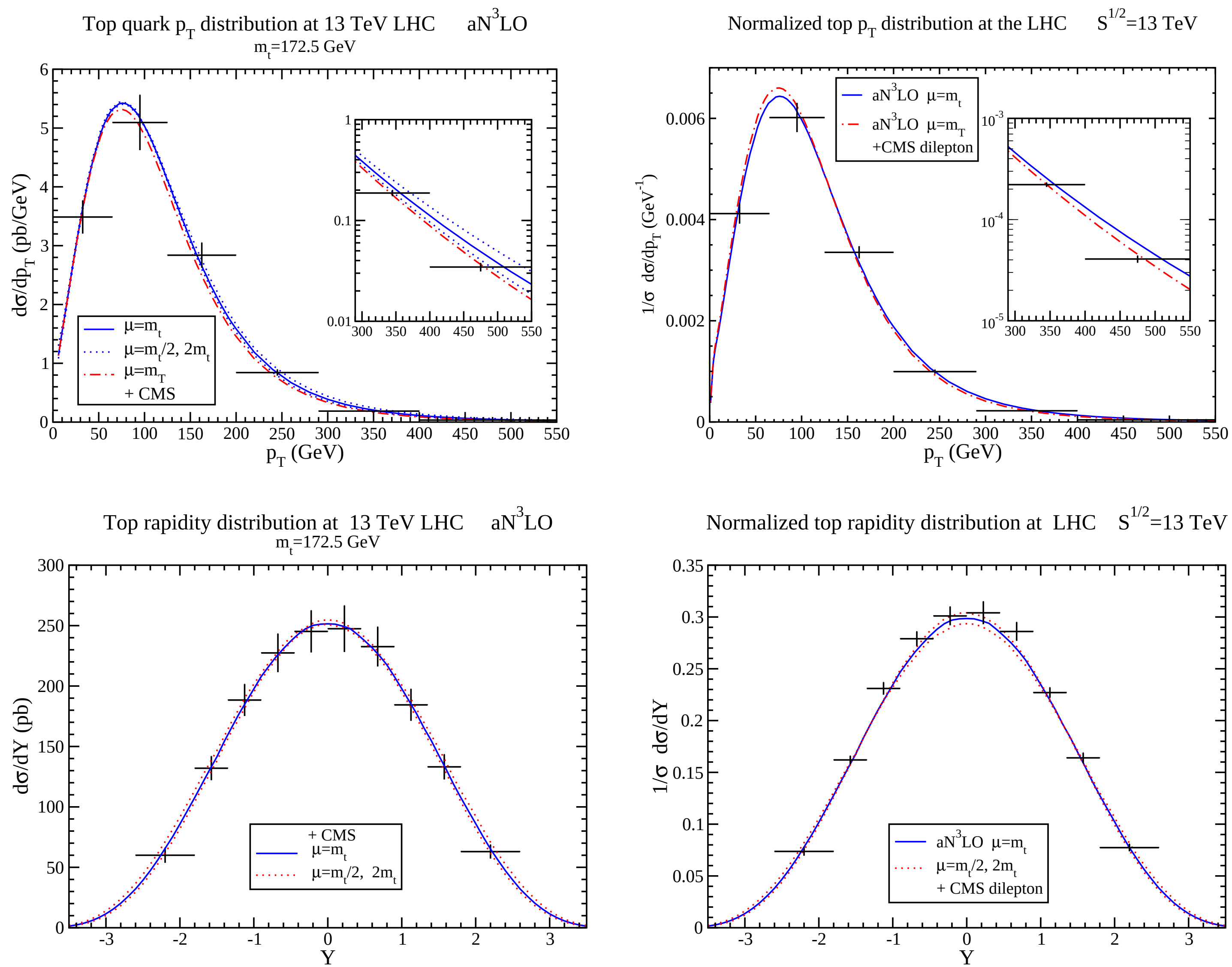
$$\Gamma_{22}^{q\bar{q}\,(2)} = \frac{K}{2}\Gamma_{22}^{q\bar{q}\,(1)} + \left(1-\frac{C_A}{2C_F}\right)\left(\Gamma_{\rm cusp}^{(2)} - \frac{K}{2}\Gamma_{\rm cusp}^{(1)}\right)$$

3×3 matrix for $gg \to t\bar{t}$

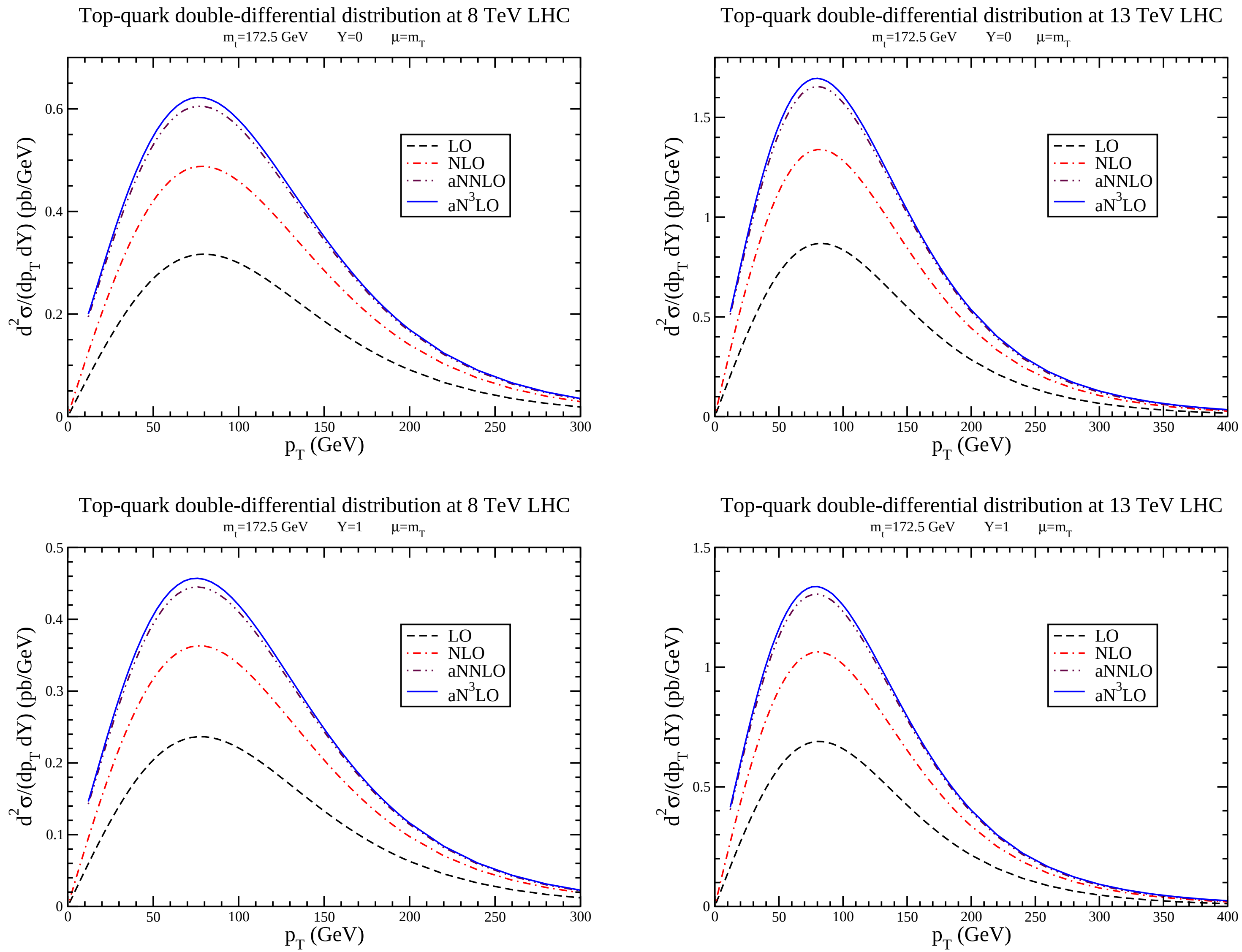
Top-antitop pair production at aN^{3}LO with NNLL accuracy



Top p_T and rapidity distributions in $t\bar{t}$ production



Top double-differential distributions in $t\bar{t}$ production



Summary

- soft-gluon corrections at three loops
- t-channel and s-channel single top at aNNLO
- tW production at aN^{3}LO
- tZ, $t\gamma$, tZ', and tH^- production in new physics models
- $t\bar{t}$ production at aN^{3}LO
- high-order corrections are very significant
- excellent agreement with collider data