Positivity Bounds on Vector Boson Scattering

Shuang-Yong Zhou (University of Science and Technology of China)

EPS-HEP2019 Conference, Ghent, 13 July 2019

Cen Zhang & **SYZ**, arXiv:1808.00010
Bi Qi, Cen Zhang & **SYZ**, arXiv:1902.08977

Effective field theories (EFTs)

- EFTs are widely used in physics

  BSM physics, GR, inflation, dark energy,…

- Separation of physics at different scales

  write down all local operators consistent
  with symmetries suppressed by cut-off scale

$$\mathscr{L} = \sum_i \Lambda^4 f_i \, \mathcal{O}_i \left(\frac{\text{boson}}{\Lambda}, \frac{\text{fermion}}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right)$$

Are all EFTs allowed?

Answer: No!

Not every set of Wilson coefficients are allowed!

$$e^{\frac{i}{\hbar}S_W[\text{light}]} = \int D[\text{heavy}]\; e^{\frac{i}{\hbar}S_{\text{UV}}[\text{light,heavy}]}$$

UV completion satisfies: Lorentz invariance, unitarity, locality, causality, analyticity, …

$\Downarrow$

$\boxed{\text{Positivity bounds on Wilson coefficients}}$

Simple example: P(X)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \cdots$$

Positivity bound → $\lambda > 0$

Theories with $\lambda < 0$ do not have an analytic UV completion

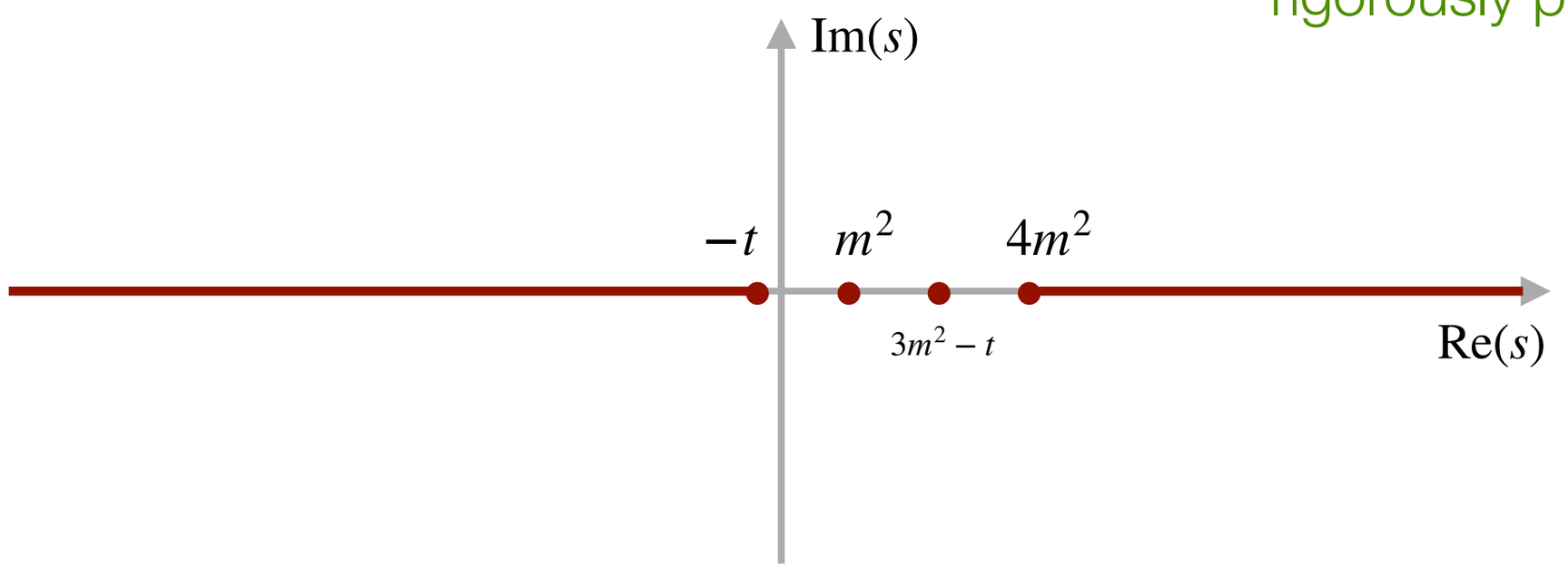
How does it work?

Analyticity and unitarity

Complex amplitude $A(s,t)$

2 to 2 scattering

rigorously proven in 60'



Optical theorem: $\mathrm{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\sigma(s) > 0$

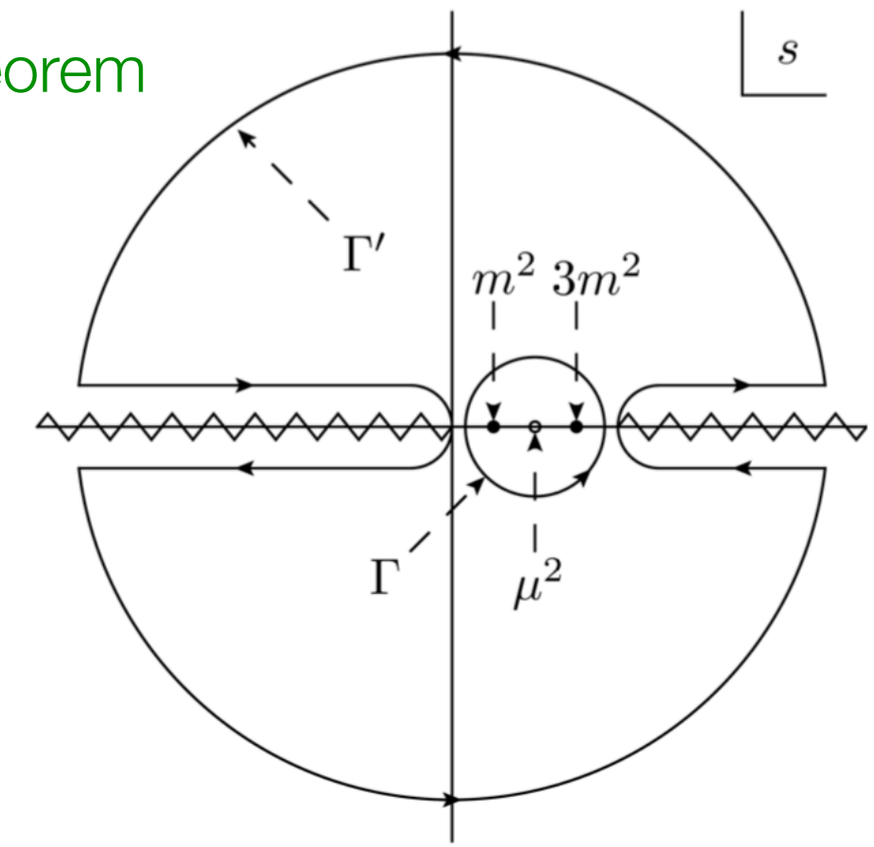
Frossart bound: as $s \to \infty, |A(s,0)| < Cs\ln^2 s$

Froissart, 1961

Dispersion relation

$$f := \frac{1}{2\pi i}\oint_{\Gamma} \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$

Cauchy's theorem



Frossart bound: as $s \to \infty, |A(s,0)| < Cs\ln^2 s$

$$f = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{+\infty}\right)\mathrm{d}s \frac{\mathrm{Disc}\,A(s,0)}{(s-\mu^2)^3}$$

Crossing and $\mathrm{Disc}\,A(s,0) = 2i\,\mathrm{Im}A(s,0)$

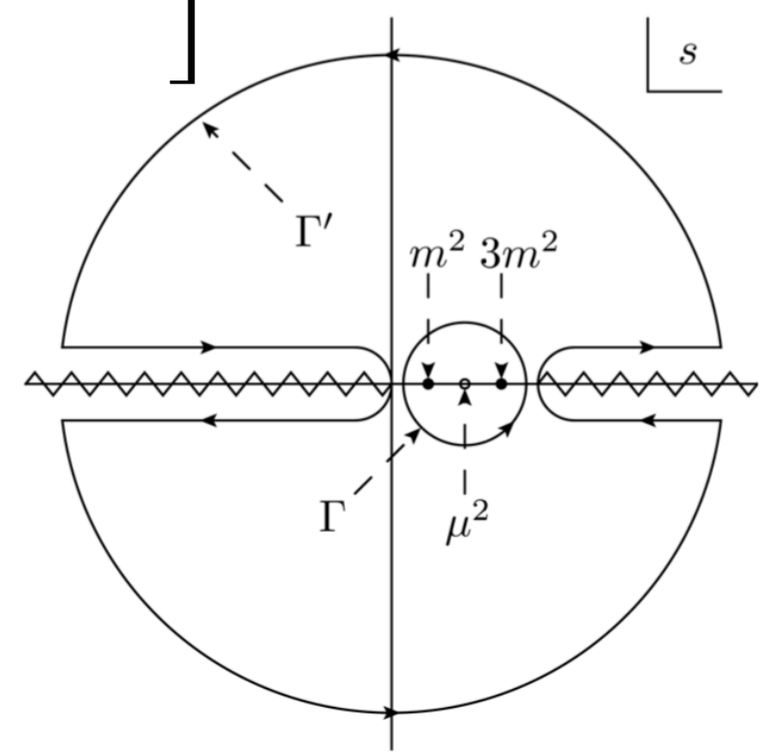
$$f = \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}s\left[\frac{\mathrm{Im}A(s,0)}{(s-\mu^2)^3} + \frac{\mathrm{Im}A^{(u)}(s,0)}{(s+\mu^2-4m^2)^3}\right]$$

Positivity bound

Optical theorem: $\mathrm{Im}[A(s,0)] = \sqrt{s(s-4m^2)}\sigma(s) > 0$

$$f = \frac{1}{\pi}\int_{4m^2}^{\infty} \mathrm{d}s \left[\frac{\sqrt{s(s-4m^2)}}{(s-\mu^2)^3}\sigma(s) + \frac{\sqrt{s(s-4m^2)}}{(s+\mu^2-4m^2)^3}\sigma^{(u)}(s) \right]$$

For $s > 4m^2$, $0 < \mu^2 < 4m^2$



Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

$$f > 0$$

$$f = \sum_{\Gamma} \mathrm{Res}\left[\frac{A(s,0)}{(s-\mu^2)^3}\right]$$

Calculable within low energy EFT!

Application to the SMEFT

Standard Model Effective Field Theory

- Write down all possible operators with
  - SM particle contents and global symmetries
  - SM gauge group structure $SU(3)_c \times SU(2)_L \times U(1)$

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_j \frac{f_j^{(6)} O_j^{(6)}}{\Lambda^2} + \sum_i \frac{f_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Vector boson scattering (VBS)

$$V_1 + V_2 \to V_3 + V_4, \quad V_i \in \{Z, W^+, W^-, \gamma\}$$

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]$$
$$O_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,2} = \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,3} = \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,4} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi\right] \times \hat{B}^{\beta\nu}$$
$$O_{M,5} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi\right] \times \hat{B}^{\beta\mu}$$
$$O_{M,7} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu} D^\nu\Phi\right]$$

$$O_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right]$$
$$O_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right]$$
$$O_{T,2} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right]$$
$$O_{T,5} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,6} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,7} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},$$

Leading order bounds

If dim-6 ops alone, all positivity bounds are violated

$$\mathcal{O}\left(\frac{1}{\Lambda^4}\right): \qquad \sum_i (-C_i)\left(\sum_j D_j f_j^{(6)}\right)^2 \geq 0, \quad C_i > 0$$

Positivity bounds require the existence of higher-D ops!

Then, add dim-8 ops

(dim-8 part) + (dim-6 part) > 0

$\mathcal{O}\left(\frac{1}{\Lambda^4}\right)$:

weak but simpler bounds

(dim-8 part) > 0 $\rightarrow$ $\sum_i E_i f_i^{(8)} \geq 0$

Cen Zhang & **SYZ**, arXiv:1808.00010

Leading order bounds for dim-8

$$M_{S,ij}F_{S,j} > 0$$
$$M_{M,ij}F_{M,j} > 0$$
$$M_{T,ij}F_{T,j} > 0$$

+ a few nonlinear bounds

$$F_{S,i} = (F_{S,0}, F_{S,1}, F_{S,2})^T$$
$$F_{M,i} = (F_{M,0}, F_{M,1}, F_{M,2}, F_{M,3}, F_{M,4}, F_{M,5}, F_{M,7})^T$$
$$F_{T,i} = (F_{T,0}, F_{T,1}, F_{T,2}, F_{T,5}, F_{T,6}, F_{T,7}, F_{T,8}, F_{T,9})^T$$

$$M_S = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

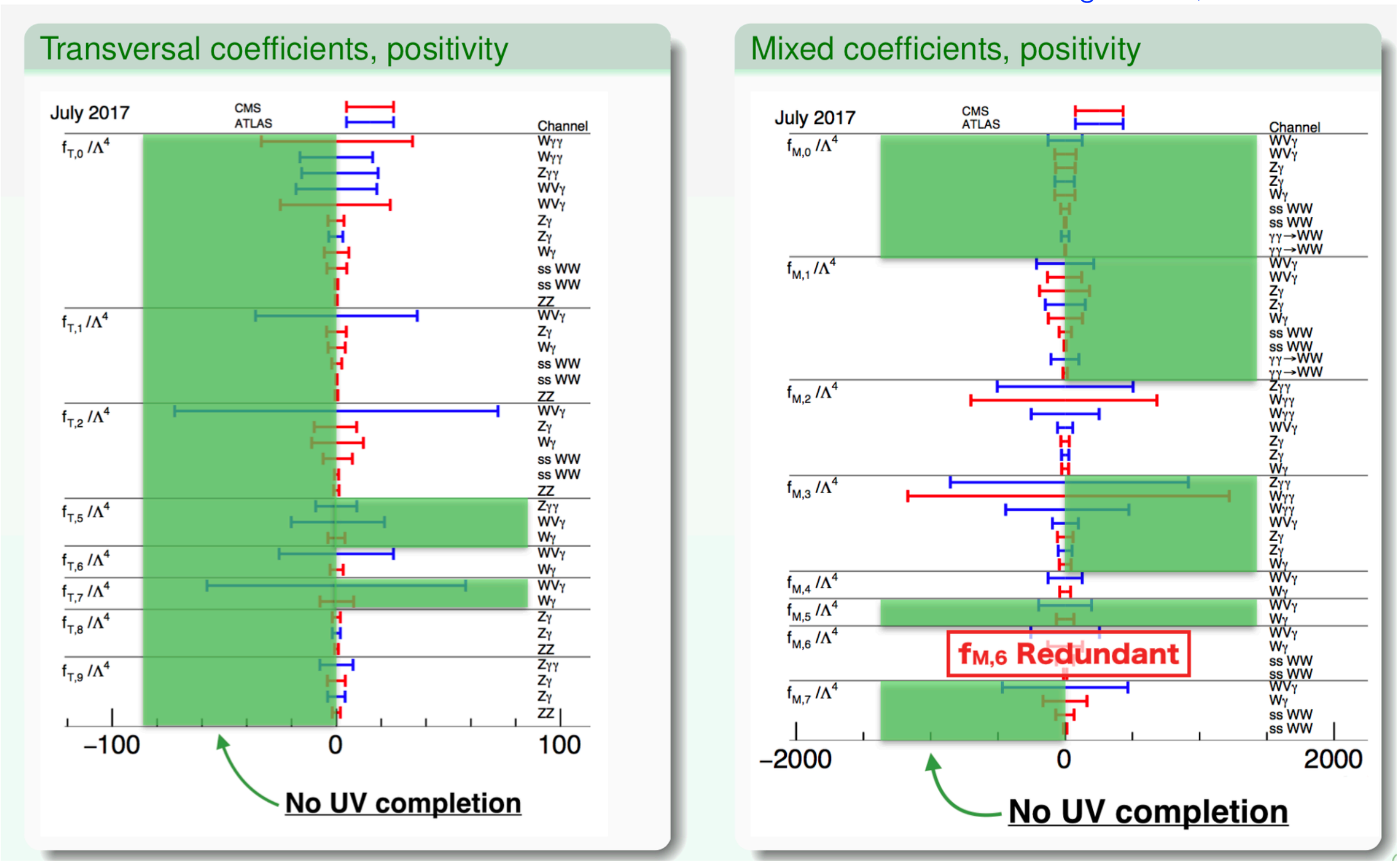
$$M_M = \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 c_W^2 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -s_W^2 c_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix}$$

$$M_T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\ 8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\ 0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\ 0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\ 0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 8c_W^4 & 0 & 0 & \left(c_W^2 - s_W^2\right)^2 & 0 & 2s_W^4 \\ 32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4\left(c_W^2 - s_W^2\right)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\ 8 & 8 & 4 & 4 & 4 & 2 & 2 & 1 \end{pmatrix}$$

Bi, Zhang & **SYZ**, arXiv:1902.08977

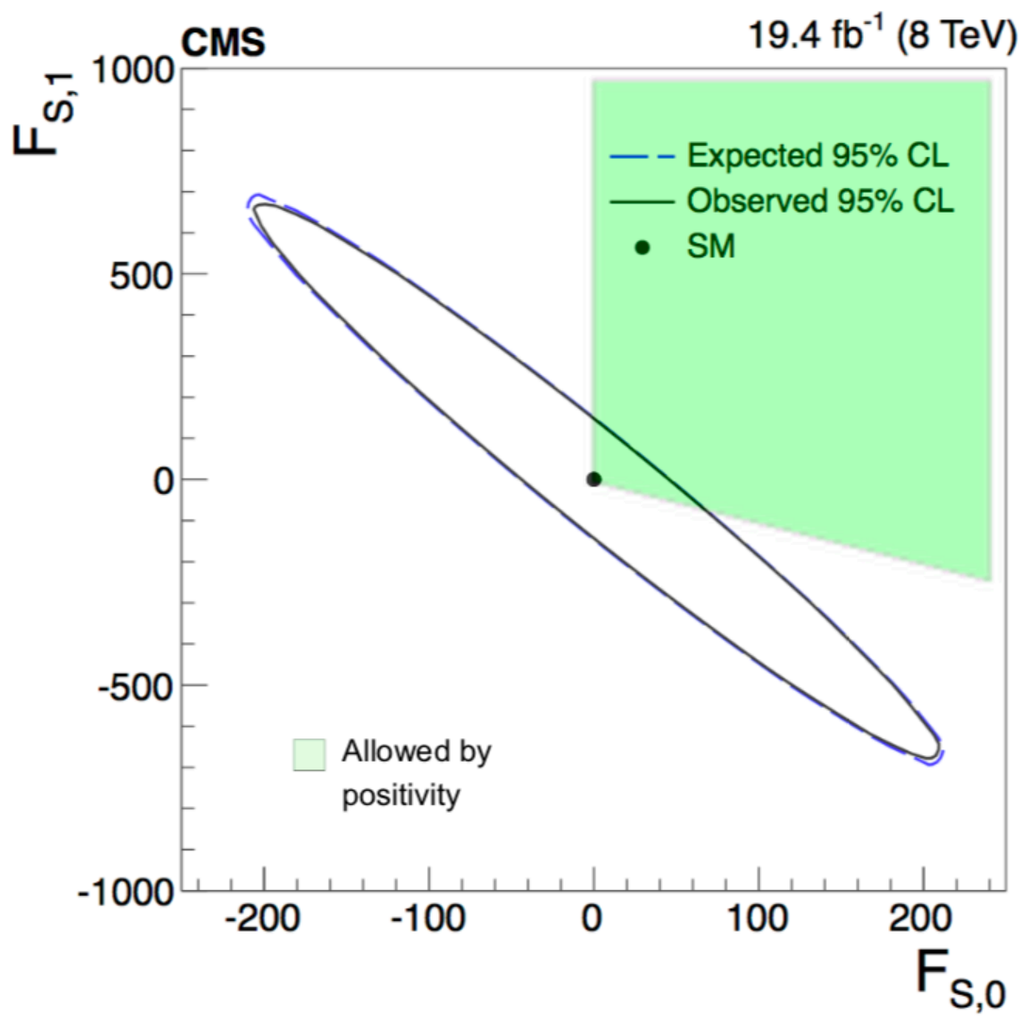
1D bounds on aQGCs

Cen Zhang & **SYZ**, arXiv:1808.00010



2D bounds: an example

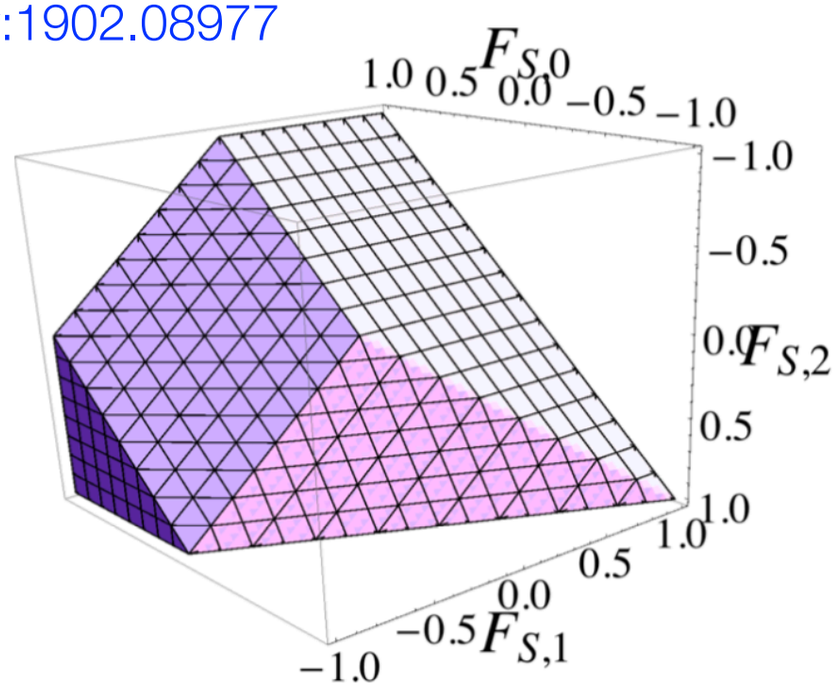
The case of O_{S0} and O_{S1}

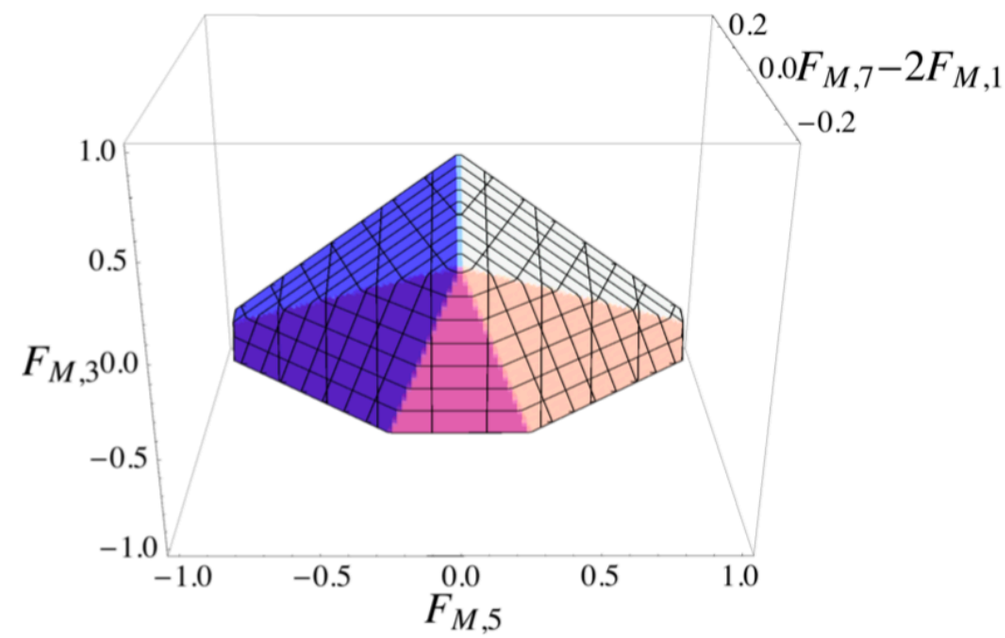


Morphology of higher-D bounds

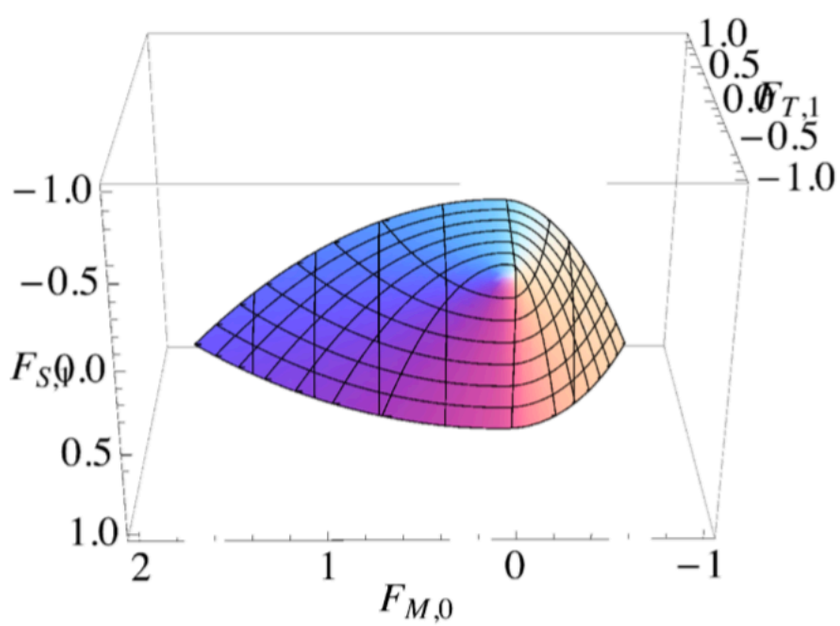
Bi, Zhang & **SYZ**, arXiv:1902.08977

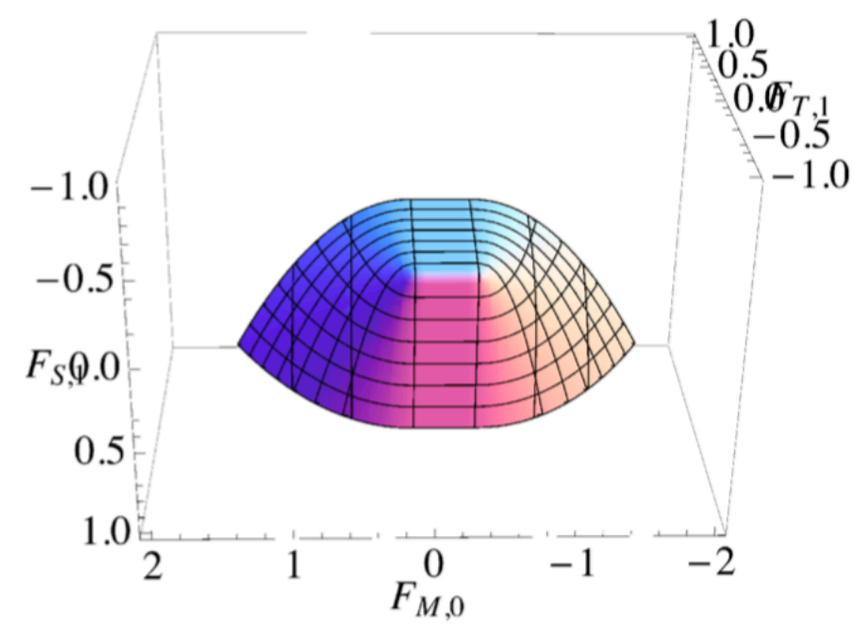
Pyramids





Cones

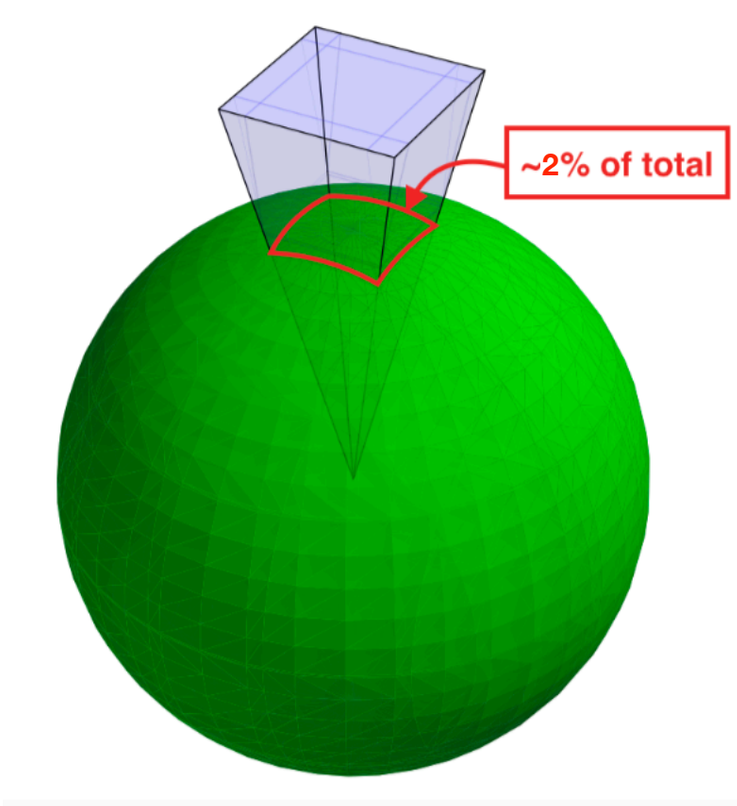




General bounds on aQGCs

All 18 parameters:

Randomly take points on18D parameter space and count how many of them satisfy all the positivity bounds.



Only ~2% of the total parameter space admit an analytic UV completion!

Summary

Not all EFTs have a UV completion!

Positivity bounds: new constraints on coupling constants

Most of the VBS parameter space in SMEFT
do not admit a UV completion.