GENUINE, MATTER-INDUCED AND INTERFERENCE COMPONENTS OF CPV, TRV, CPTV ASYMMETRIES FOR NEUTRINO OSCILLATIONS

Based on
J. B., A. Segarra, PRL 121(2018)211802
J. B., A. Segarra, JHEP 1811(2018)063
J. B., A. Segarra, JHEP 1903(2019)103

José Bernabéu
IFIC-Valencia
Historical Problem: Fake CPV in $\nu$ oscillations

Disentangling the Genuine CPV component: Symmetry Guiding

Experimental Signatures for $A(\text{CP}, T) \leftrightarrow A(\text{CP}, \text{CPT})$ Separation

- $\sin\delta \leftrightarrow$ Hierarchy
- Baseline Dependence $\rightarrow$ HKK
- Energy Dependence $\rightarrow$ DUNE

Magic Energy $E_{\text{magic}}(L)$

Fake Interference CP-even $A(T, \text{CPT})$ component

Conclusion
Underground neutrino propagation from the source to the detector

Neutrino Dynamics is affected by the MSW effect of charged-current weak interaction \((V)\) of electron-neutrinos with matter electrons → a new term in the Hamiltonian for Flavour Oscillations to the vacuum term induced by the PMNS mixing matrix \(U\) plus neutrino mass differences. In the flavour basis \([a = 2E.V]\)

\[
H = \frac{1}{2E} \left\{ U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}
\]

\(\nu \rightarrow \bar{\nu} \Rightarrow U \rightarrow U^*, a \rightarrow -a\)

CPV has now two different origins: (1) genuine in the complexity of \(U\) (PMNS), (2) the CP-asymmetry of the Earth, "\(a\)" changing sign from neutrinos to antineutrinos→ Historical problem!

A direct evidence for Symmetry Violation means the measurement, in a single experiment, of a genuine observable odd under the Symmetry.
SEARCH OF EVIDENCE FOR GENUINE CPV ASYMMETRY

- How to search for genuine CPV? Interest: 1) in itself, as symmetries in fundamental physics;
  
  2) Opportunity of Leptogenesis for Matter-Antimatter Asymmetry in the Universe.

- The community seems to have accepted as irreparable this historical problem → use global fits (or single experiment fit in the future) for observables calculated from $H$ to extract the $\delta$ phase of $U$(PMNS) in the vacuum term.

- This methodology is potentially dangerous, not only because it is theory-dependent → you may obtain $\delta \neq 0, \pi$ WITHOUT ANY contribution of genuine CPV terms in your observables.

- $\delta$-dependent terms in the transition probability:

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<td>CPV</td>
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<td>CPC</td>
<td>$\cos \delta$</td>
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For T-symmetric matter $a$ is odd under CP and CPT

- Only the first term is genuine CPV, the first column is CPTI as in vacuum, the entire second column is fake due to CPTV matter, the first row has genuine and fake CPV, the entire second row is CPC.

- Present fits (hints) for $\delta$ demonstrate that the genuine CPV term plays, if any, a minor role! Why? Stay tuned in this talk...
The Guiding Principle for their Disentanglement is their behaviour under the three Discrete Symmetries CP, T, CPT: Whereas both vacuum and matter-induced terms are CPV, the first is CPTI, the second TRI.

Conceptual Basis → Whatever is the theoretical framework, write

\[ H = \frac{1}{2E} \tilde{U} \tilde{M}^2 \tilde{U}^\dagger \]

connecting flavour with definite mass in matter → \((\tilde{U}, \tilde{m}), (\tilde{U}, \tilde{m})\)

The Observables in matter depend on the rephasing-invariant mixings \(\tilde{j}^{ij}_{\alpha\beta} = \tilde{U}_{\alpha i}^\dagger \tilde{U}_{\alpha j}^\dagger \tilde{U}_{\beta i}^\dagger \tilde{U}_{\beta j}\)

and the oscillation phases, \(\tilde{\Delta}_{ij} = \frac{\Delta m_{ij}^2 L}{4E}\) where \(\tilde{j}^{ij}_{\alpha\beta}\) and \(\tilde{\Delta}_{ij}\) are both E-dependent.

From the different behaviour of these ingredients under the discrete T and CPT symmetry transformations, one derives the

**Asymmetry Disentanglement Theorem**

by separating the observable CP asymmetry in any flavour transition \(\alpha \rightarrow \beta\) into L-even (TRI, CPTV) and L-odd (TRV, CPTI) components

\[
A^\text{CPT}_{\alpha\beta} = A^\text{CPT}_{\alpha\beta} + A^T_{\alpha\beta},
\]

T-invariant \(\Rightarrow\) CPT-invariant

FAKE

\[
A^\text{CPT}_{\alpha\beta} = -4 \sum_{j<i} \left[ \text{Re} \tilde{j}^{ij}_{\alpha\beta} \sin^2 \tilde{\Delta}_{ij} - \text{Re} \tilde{j}^{ij}_{\alpha\beta} \sin^2 \tilde{\Delta}_{ij} \right],
\]

GENUINE

\[
A^T_{\alpha\beta} = -2 \sum_{j<i} \left[ \text{Im} \tilde{j}^{ij}_{\alpha\beta} \sin 2\tilde{\Delta}_{ij} - \text{Im} \tilde{j}^{ij}_{\alpha\beta} \sin 2\tilde{\Delta}_{ij} \right].
\]

These Eqs. remain valid for extended neutrino models with more mass eigenstates and a rectangular mixing matrix, not only for the three neutrino model.
DISENTANGLING FOR $\nu_\mu \rightarrow \nu_e$

- **Theory Check** (not for actual experiment) ➔ The separate behaviour of the two components $A(CP, T)$ and $A(CP, CPT)$ on $\sin \delta$ and the matter potential "$a$" at fixed $(L, E)$

$A(CP, T)$ vanishes with $\sin \delta \rightarrow 0$ for all $a$;
$A(CP, CPT)$ vanishes with $a \rightarrow 0$ for all $\sin \delta$.

- They have definite parities under $L$ (model-independent), "$a$", $\sin \delta$ and the hierarchy "$h$"=+- in the neutrino mass ordering:
  - $A(CP,T)$ is CPTI, odd in $L$ and $\sin \delta$ plus even in $a$ and $h$ (!!!),
  - $A(CP,CPT)$ is TRI, even in $L$ and $\sin \delta$ plus odd in $a$ and almost odd in $h$.

This behaviour explicitly seen in **approximations valid for actual** HK, DUNE experiments - energy region between the two MSW resonances - $\Delta m_{21}^2 \ll a \ll \Delta m_{31}^2$

$$A_{\mu e}^{CPT} = 16A \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] (S \sin \Delta_{31} + J_r \cos \delta \Delta_{21} \cos \Delta_{31}) + O(A^3),$$

$$A_{\mu e}^{T} = -16J_r \sin \delta \Delta_{21} \sin^2 \Delta_{31} + O(A^2),$$

where $S \equiv c_{13}^2 s_{13}^2 s_{23}, J_r \equiv c_{12}^2 c_{23} s_{12} s_{13} s_{23}, A \equiv \frac{\alpha}{4\pi} \propto L$ and $\Delta_{ij} \equiv \frac{\Delta m_{21}^2 L}{4\pi} \propto L/E.$
EXPERIMENTAL SIGNATURES FOR $A(\text{CP, T}) \leftrightarrow A(\text{CP, CPT})$ SEPARATION

- **HKK, Two Detectors, Off-Axis Beam:**
  - Kamioka (L = 295 Km), 1st. max = 0.6 GeV, $\theta_{o-a} = 2.7^0$
  - Genuine $A(\text{CP,T})$ may dominate proportional to $\sin\delta$, but $A(\text{CP, CPT})$ – almost $\delta$-independent - to be subtracted if the hierarchy is known.

- **Korea (L = 1000 - 1300 Km), 1st. max = 2.1 - 2.7 GeV, $\theta_{o-a} = 0^0$**
  - The relative contribution of $A$ (CP,CPT) increases with L, now it dominates at large energies and its sign determines the hierarchy.
HKK Korea (L = 1000 - 1300 Km), 2nd. max = 0.70 - 0.92 GeV, \( \theta_{\text{o-a}} = 1.9 - 2.4^0 \) \( \rightarrow \) "MAGIC" ENERGY

\( \rightarrow \) Direct measurement of genuine \( A(\text{CP, T}) \) near the 2nd. Maximum

DUNE, Single Detector \( L = 1300 \) Km, wide-band beam, reconstructed \( E \)

Measure \( A(\text{CP}) \) as function of \( E \)-bins \( \rightarrow \) FIRST-RANK ZERO OF \( A(\text{CP, CPT}) \) at the 2nd. max. 0.92 GeV.

At large energies, measurement of hierarchy from the definite sign of \( A(\text{CP, CPT}) \), independent of \( \sin\delta \).

At the "magic" energy \( E = 0.92 \pm 0.15 \) GeV, measurement of \( A(\text{CP, T}) \) proportional to \( \sin\delta \) \( \rightarrow \) GENUINE.
GENUINE CPV ASYMMETRY

- Understanding the first-rank zero of the Fake Asymmetry

\[ A^T_{\mu e} = -16 J_r \sin \delta \Delta_{21} \sin^2 \Delta_{31} + O(A^2) \]

Max: \( \tan \Delta_{31} = -2 \Delta_{31} \)

\[ A^{\text{CPT}}_{\mu e} = 16 A \left( \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right) \left( S \sin \Delta_{31} + J_r \cos \delta \Delta_{21} \cos \Delta_{31} \right) + O(A^3) \]

\( \delta \)-independent zeros: \( \tan \Delta_{31} = \Delta_{31} \)

\( \delta \)-dependent zeros: \( \tan \Delta_{31} = -0.09 \cos \delta \Delta_{31} \)

\( \Delta_{31} \equiv \Delta m^2_{31} L / 4E \)

\[ E = 0.92 \text{ GeV} \quad \frac{L}{1300 \text{ km}} \quad \frac{\left| \Delta m^2_{31} \right|}{2.5 \times 10^{-3} \text{ eV}^2} \]

\[ L/E = 1420 \text{ km/GeV}, \text{ near 2}^{\text{nd}} \text{ oscillation max.} \]
Is the genuine $A(CP, T)$ component of the CP-Asymmetry coincident with the entire $T$-Asymmetry $A(T)$?

Contrary to the CP-asymmetry, for which the matter effect generates a fake component $A(CP, CPT)$ by itself, matter is $T$-symmetric and it cannot generate a $T$-Asymmetry by itself.

However, in presence of a genuine CPV $\sin \delta - T$-odd -, one can build an **INTERFERENCE TERM** with matter "$a \sin \delta$" which is $T$-odd, **CP-even** and CPT-odd, i.e., a **fake $A(T, CPT)$ component** in the TRV Asymmetry different from the fake $A(CP, CPT)$ component of the CP Asymmetry.

Guided again by the Transformation Properties under the three Discrete Symmetries CP, T, CPT of the rephasing-invariant mixings $\tilde{J}^{ij}_{\alpha \beta} = \tilde{U}_{\alpha i} \tilde{U}_{\beta j}^* \tilde{U}_{\beta i} \tilde{U}_{\alpha j}$ and the phase differences $\tilde{\Delta}_{ij} = \frac{\Delta m^2_{ij} L}{4E}$, we obtain a

**SECOND DISENTANGLEMENT THEOREM**

this time for the TRV-Asymmetry

$$A^T_{\alpha \beta} = A_{\alpha \beta}^{T; CP} + A_{\alpha \beta}^{T; CPT}$$

CPT-invariant $\Rightarrow$ CP-invariant

$$A_{\alpha \beta}^{T; CP} = -2 \sum_{i < j} \left[ \text{Im} \tilde{J}_{\alpha \beta}^{ij} \sin 2\tilde{\Delta}_{ij} - \text{Im} \tilde{J}_{\alpha \beta}^{ij} \sin 2\tilde{\Delta}_{ij} \right],$$

$$A_{\alpha \beta}^{T; CPT} = -2 \sum_{i < j} \left[ \text{Im} \tilde{J}_{\alpha \beta}^{ij} \sin 2\tilde{\Delta}_{ij} + \text{Im} \tilde{J}_{\alpha \beta}^{ij} \sin 2\tilde{\Delta}_{ij} \right].$$

which is **L-odd entirely**
Under the same analytic approximation for energies between the two MSW resonances,

\[ A_{\mu e}^{T;\text{CPT}} = -16 \alpha J_r \sin \delta \Delta_{21} \sin \Delta_{31} \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] + O(\Delta_{31}^3) , \]

\[ A_{\mu e}^{\text{CP};\text{CPT}} = 16 \alpha \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] (S \sin \Delta_{31} + J_r \cos \delta \Delta_{21} \cos \Delta_{31}) + O(\Delta_{a}^3) , \]

where \( S \equiv c_{13}^2 s_{13}^2 s_{23}^2, J_r \equiv c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23}, \Delta_{a} = \frac{\alpha L}{4E} \propto L \) and \( \Delta_{ij} = \frac{\Delta m_{31}^2 L}{4E} \propto L/E . \)

Most characteristic behaviours: **L-even vs. L-odd**, \( \delta \)-even vs. \( \delta \)-odd, both proportional to \( \Delta_{a} \) and with a similar energy dependence, including the **SAME \( \delta \)-independent first-rank zero at the magic energy**. A(T, CPT) is exactly odd in the hierarchy. Whereas A(CP, CPT) contains both \( a, a.\cos\delta \) terms, the new interference A(T, CPT) component contains \( a.\sin\delta \) only \( \leftarrow \) **INTERESTING DISTINCTION**!

The two fake components are odd under the hierarchy, the green \( \delta \)-independent, the red proportional to \( \sin \delta \).
Including the second TRV Asymmetry for antineutrinos $\bar{\mathcal{A}}(T)$, there is a trivial

**Asymmetry Sum Rule for CP, T and CPT:**

$$A(CPT) = A(CP) + \bar{\mathcal{A}}(T)$$

In terms of the components $\rightarrow$ **CPT Disentanglement Lemma** from the two theorems

$$A(CPT, CP) + A(CPT, T) = A(CP, T) + A(CP, CPT) - A(T, CP) + A(T, CPT)$$

Using the basis of the invariant symmetries,

- **CPT-invariant** $\rightarrow$ $A(CP, T) = A(T, CP)$ $\rightarrow$ genuine CPV-Asymmetry
- **T-invariant** $\rightarrow$ $A(CP, CPT) = A(CPT, CP)$ $\rightarrow$ fake CPV by matter
- **CP-invariant** $\rightarrow$ $A(T, CPT) = A(CPT, T)$ $\rightarrow$ fake TRV interference

All Physics contained in these three components which could be **disentangled from the three independent experimental Asymmetries** - in a model independent way - with a single detector

$$A(T), \bar{\mathcal{A}}(T), A(CP) \text{ or } A(T), A(CP), A(CPT)$$

In which Facility? A Neutrino Factory!

Back to Earth $\rightarrow$

- **Separate genuine** $A(CP, T)$ from fake $A(CP, CPT)$: either $L$-dependence (HKK) or $E$-dependence (DUNE).
- **Determine the neutrino mass hierarchy** from the sign of $A(CP, CPT)$
The Physics of Discrete Asymmetries for Neutrino Oscillations in Matter is now completed in terms of 3 independent components in correspondence with invariance under CPT (genuine), T (matter) and CP (interference) → A very rich understandable Physics.

They have definite parities under L-theory independent-, CPV sinδ, matter potential "a", hierarchy "h". In going from neutrino to antineutrino asymmetries, the two CP-odd components change sign, whereas the third (interference) remains invariant. In addition, the T-invariant component is symmetric under exchange of flavour indices, whereas the two T-odd components change sign under α ↔ β.

The two CPT-odd \( A(\text{CP, CPT}) \) and \( A(\text{T, CPT}) \) components are induced by matter in the neutrino propagation and thus they are odd in the matter potential "a". However, there is a very interesting distinction between these two fake components: whereas the first can be induced by CPV matter alone without any fundamental CPV for neutrinos, a positive answer exists to the question on the existence of a matter-induced \( A(\text{T, CPT}) \) component, for even T-symmetric matter.

Are you surprised? The quantum logic is not disjunctive - either vacuum or matter - since interference terms are also relevant: CP-even "a.sinδ".
The two experimental TRV-Asymmetries for neutrinos and antineutrinos, odd in L and $\sin \delta$, could separate the genuine CPV component and the fake interference component, for the same baseline and energy. The first component is even in $a$, the second is odd in $a$.

With muon neutrino and antineutrino sources only, the experimental CPV-Asymmetry can separate the genuine CPV component and the fake matter-induced component by either L-dependence (HKK) or E-dependence (DUNE). The genuine $A(\text{CP, T})$ is blind to the hierarchy $h$ and it is proportional to $\sin \delta$. The fake $A(\text{CP, CPT})$ is almost odd in $h$ and independent of $\delta$.

The two fake components $A(\text{CP, CPT})$ and $A(\text{T, CPT})$ vanish at the same MAGIC ENERGY, independent of the hierarchy, $\delta$ and $a$: only $|\Delta m^2_{31}|$ matters. This magic connection $L/E = 1420 \text{ Km/GeV}$ appears near the second oscillation maximum, where the genuine component $A(\text{CP, T})$ is also maximal.

All in all, the conditions for a direct evidence of genuine CP violation for neutrino oscillations in matter are now met.
THANK YOU VERY MUCH FOR YOUR ATTENTION