Relic neutrinos: clustering and consequences for direct detection

Featuring “Milky Way” & friends

EPS-HEP 2019, Ghent (BE), 10–17/07/2019
1 **Introduction**
- Neutrinos and early Universe
- Relic neutrino capture

2 **Neutrino clustering**
- Theory
- Results from the Milky Way
- Beyond the Milky Way

3 **Direct detection of relic neutrinos**

4 **Conclusions**
History of the universe

BIG BANG

Inflation

Accelerators
- LHC (proton)
- Tevatron
- RHIC (Hvy ion)

Possible dark matter relics

CMB

BBN

Key:
- W, Z bosons
- photon
- quark
- meson
- gluon
- galaxy
- electron
- baryon
- star
- muon
- ion
- tau
- black hole
- atom

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S. Gariazzo
"Relic neutrinos: clustering and consequences for direct detection"
History of the universe

History of the universe

neutrino decoupling

at $T \sim \mathcal{O}(\text{MeV})$
due to insufficient
$\nu e \leftrightarrow \nu e & e^- e^+ \leftrightarrow \nu \bar{\nu}$

$T_\nu \simeq (4/11)^{1/3} T_\gamma$
after $e^- e^+$ annihilation

$T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV}$

$\langle E_\nu \rangle \simeq 3.1 T_{\nu,0} \simeq 5 \times 10^{-4} \text{ eV}$

$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56 \text{ cm}^{-3} \text{ per family}$
∃ at least 2 mass eigenstates with \( m_i \gtrsim 8 \text{ meV} \) \( \left( = \sqrt{\Delta m_{\text{sol}}^2} \right) \) > \( \langle E_\nu \rangle \)

many relic neutrinos are non-relativistic today!

\( T_\nu \sim (4/11)^{1/3} T_\gamma \) after \( e^- e^+ \) annihilation

\( T_{\nu,0} = 1.945 \text{ K} \simeq 1.676 \times 10^{-4} \text{ eV} \)

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Relic neutrinos in cosmology: $N_{\text{eff}}$

Radiation energy density $\rho_r$ in the early Universe:

$$\rho_r = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma = \left[ 1 + 0.2271 N_{\text{eff}} \right] \rho_\gamma$$

$\rho_\gamma$ photon energy density, $7/8$ is for fermions, $(4/11)^{4/3}$ due to photon reheating after neutrino decoupling

- $N_{\text{eff}} \rightarrow$ all the radiation contribution not given by photons
- $N_{\text{eff}} \simeq 1$ correspond to a single family of active neutrino, in equilibrium in the early Universe
- Active neutrinos:
  - $N_{\text{eff}} = 3.046$ [Mangano et al., 2005] (damping factors approximations) $\sim$
  - $N_{\text{eff}} = 3.045$ [de Salas et al., 2016] (full collision terms) due to not instantaneous decoupling for the neutrinos
- $+\,$ Non Standard Interactions: $3.040 < N_{\text{eff}} < 3.059$ [de Salas et al., 2016]

Observations: $N_{\text{eff}} \simeq 3.0 \pm 0.2$ [Planck 2018]

Indirect probe of cosmic neutrino background! $\gg 10\sigma$!
Relic neutrino capture

How to directly detect non-relativistic neutrinos?

Remember that \( \langle E_\nu \rangle \simeq \mathcal{O}(10^{-4}) \) eV today → a process without energy threshold is necessary

[Weinberg, 1962]: neutrino capture in \( \beta \)-decaying nuclei \( \nu + n \rightarrow p + e^- \)

Main background: \( \beta \) decay \( n \rightarrow p + e^- + \bar{\nu}! \)

signal is a peak at \( 2m_\nu \) above \( \beta \)-decay endpoint

only with a lot of material

need a very good energy resolution
PTOLEMY

PonTecorvo Observatory for Light, Early-universe, Massive-neutrino Yield (PTOLEMY)

expected resolution $\Delta \simeq 0.1$ eV?

built mainly for CNB

$M_T = 100$ g of atomic $^3$H

can probe $m_\nu \simeq 1.4 \Delta \simeq 0.1$ eV

$$\Gamma_{\text{CNB}} = \sum_{i=1}^{3} |U_{ei}|^2 [n_i(\nu_{hR}) + n_i(\nu_{hL})] N_T \tilde{\sigma} \sim \mathcal{O}(10) \text{ yr}^{-1}$$

$N_T$ number of $^3$H nuclei in a sample of mass $M_T$ \hspace{1cm} $\tilde{\sigma} \simeq 3.834 \times 10^{-45}$ cm$^2$ \hspace{1cm} $n_i$ number density of neutrino $i$

(without clustering)
PTOLEMY

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~ $\mathcal{O}(10)$ yr$^{-1}$

$N_T$ number of $^3$H nuclei in a sample of mass $M_T$

$\bar{\sigma} \simeq 3.834 \times 10^{-45}$ cm$^2$

$n_i$ number density of neutrino $i$

(can probe $m_\nu \simeq 1.4\Delta \simeq 0.1$ eV)

enhancement from $\nu$ clustering in the galaxy?

enhancement from other effects?

without clustering


[Long et al., JCAP 08 (2014) 038]
[Betts et al., arxiv:1307.4738]
[PTOLEMY LoI, arxiv:1808.01892]
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**ν clustering with N-one-body simulations**

Milky Way (MW) matter attracts neutrinos!

\[
\Gamma_{\text{CNB}} = \sum_{i=1}^{3} \left| U_{ei} \right|^2 f_c(m_i) \left[ n_{i,0}(\nu_{hR}) + n_{i,0}(\nu_{hL}) \right] N_T \bar{\sigma}
\]

\(f_c(m_i) = n_i/n_{i,0}\) clustering factor → How to compute it?

Idea from [Ringwald & Wong, 2004] → **N-one-body** = \(N \times \text{single } \nu \text{ simulations}\)

→ each \(\nu\) evolved from initial conditions at \(z = 3\)
→ spherical symmetry, coordinates \((r, \theta, p_r, l)\)
→ need \(\rho_{\text{matter}}(z) = \rho_{\text{DM}}(z) + \rho_{\text{baryon}}(z)\)

**Assumptions:**

\(\nu\)s are independent

only gravitational interactions

\(\nu\)s do not influence matter evolution \((\rho_\nu \ll \rho_{\text{DM}})\)

how many \(\nu\)s is “\(N\)”?

→ must sample all possible \(r, p_r, l\)
→ must include all possible \(\nu\)s that reach the MW
(fastest ones may come from several (up to \(\mathcal{O}(100)\)) Mpc!)

given \(N\) \(\nu\):

→ weigh each neutrinos
→ reconstruct final density profile with kernel method from [Merritt&Tremblay, 1994]
### Overdensity when $m_{\text{heaviest}} \simeq 60$ meV

<table>
<thead>
<tr>
<th>masses</th>
<th>ordering</th>
<th>matter halo</th>
<th>overdensity $f_c$ ( f_1 \simeq f_2 )</th>
<th>( f_3 )</th>
<th>( \Gamma_{\text{tot}} ) (yr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>any</td>
<td>any</td>
<td>any</td>
<td>no clustering</td>
<td></td>
<td>4.06</td>
</tr>
<tr>
<td>( m_3 = 60 ) meV</td>
<td>NO</td>
<td>NFW (+bar)</td>
<td>( \sim 1 )</td>
<td>1.15 (1.18)</td>
<td>4.07 (4.08)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NFW optimistic</td>
<td></td>
<td>1.21</td>
<td>4.08</td>
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<tr>
<td></td>
<td></td>
<td>EIN (+bar)</td>
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<td>1.09 (1.12)</td>
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<tr>
<td>( m_1 \simeq m_2 = 60 ) meV</td>
<td>IO</td>
<td>NFW (+bar)</td>
<td>1.15 (1.18)</td>
<td>4.66 (4.78)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NFW optimistic</td>
<td>1.21</td>
<td>4.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>EIN (+bar)</td>
<td>1.09 (1.12)</td>
<td>4.42 (4.54)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>EIN optimistic</td>
<td>1.18</td>
<td>4.78</td>
<td></td>
</tr>
</tbody>
</table>

**ordering dependence from** \( \Gamma_{\text{CNB}} = \sum_{i=1}^{3} |U_{ei}|^2 f_i \left[ n_i(\nu_{hR}) + n_i(\nu_{hL}) \right] N_T \bar{\sigma} \)
**Overdensity when** \( m_\nu \approx 150 \) meV

\[ \Rightarrow \text{minimal mass detectable by PTOLEMY if } \Delta \approx 100–150 \text{ meV} \]

### Matter Halo Overdensity \( f_c \)

<table>
<thead>
<tr>
<th>Matter Halo</th>
<th>Overdensity ( f_c )</th>
<th>( \Gamma_{\text{tot}} ) (yr(^{-1}))</th>
</tr>
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<tbody>
<tr>
<td>any</td>
<td>no clustering</td>
<td>4.06</td>
</tr>
<tr>
<td>NFW (+bar)</td>
<td>2.18 (2.44)</td>
<td>8.8 (9.9)</td>
</tr>
<tr>
<td>NFW optimistic</td>
<td>2.88</td>
<td>11.7</td>
</tr>
<tr>
<td>EIN (+bar)</td>
<td>1.68 (1.87)</td>
<td>6.8 (7.6)</td>
</tr>
<tr>
<td>EIN optimistic</td>
<td>2.43</td>
<td>9.9</td>
</tr>
</tbody>
</table>

**No ordering dependence:** \( m_1 \approx m_2 \approx m_3 \) \( \Rightarrow f_1 \approx f_2 \approx f_3 \)

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S. Gariazzo

“Relic neutrinos: clustering and consequences for direct detection”

EPS-HEP 2019, 12/07/2019
Forward-tracking and back-tracking

initial phase space, \( z = 4 \) \( \rightarrow \) homogeneous Fermi-Dirac distribution

final phase space, \( z = 0 \)
Forward-tracking and back-tracking

initial phase space, $z = 4$ $\rightarrow$ homogeneous Fermi-Dirac distribution

compute final position of each particle

final phase space, $z = 0$
Forward-tracking and back-tracking

initial phase space, $z = 4$ → homogeneous Fermi-Dirac distribution

compute final position of each particle

final phase space, $z = 0$
Forward-tracking and back-tracking

initial phase space, $z = 4 \rightarrow$ homogeneous Fermi-Dirac distribution

use positions to find neutrino distribution today

final phase space, $z = 0$
Forward-tracking and back-tracking

initial phase space, $z = 4$ $\rightarrow$ homogeneous Fermi-Dirac distribution

only interested in overdensity at Earth? ★

a lot of time is wasted!

final phase space, $z = 0$
Forward-tracking and back-tracking

initial phase space, $z = 4 \rightarrow$ homogeneous Fermi-Dirac distribution

only interested in overdensity at Earth? ⭐

Smarter way: track backwards only interesting particles!

a lot of time is wasted!

final phase space, $z = 0$
Advantages of tracking back

First advantage is in computational terms: much less points to compute
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Second advantage: no need to use spherical symmetry!

Forward-tracking

Initial conditions need to sample 1D for position + 2D for momentum when using spherical symmetry.

with full grid would require 3+3 dimensions!

Impossible to relax spherical symmetry!

Back-tracking

“Initial” conditions only described by 3D in momentum (position is fixed, apart for checks).

can do the calculation with any astrophysical setup.
Advantages of tracking back

First advantage is in computational terms: much less points to compute

Second advantage: no need to use spherical symmetry!
Preliminary results with back-tracking

In comparison with previous results:

\[ n_\nu/n_\nu^{0} - 1 \]

\[
m_\nu \text{[eV]}
\]

\[
10^{-1}
\]

\[
10^{-2}
\]

\[
10^{-2}
\]

\[
10^{-1}
\]

\[
10^{0}
\]

\[
10^{1}
\]

NFW only
NFW + baryons
Einasto + baryons
NFW + baryons + Virgo
NFWhalo (Ringwald & Wong)
MWnow (Ringwald & Wong)
NFW + baryons (Zhang & Zhang)
NFW only (de Salas et al.)
NFW + baryons (de Salas et al.)

Preliminary!
Preliminary results with back-tracking

In comparison with previous results:

Andromeda is almost negligible

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Warning: NFW is not the same for all the cases!

[de Salas+, 2017] and [Zhang\textsuperscript{2}, 2018] use $\gamma \neq 1$, now we have $\gamma = 1$

[Ringwald&Wong, 2004] uses old parameters
Preliminary results with back-tracking

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many checks are missing: distance of Virgo, Sun position, more on DM, ...
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\[
\frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{(E_e-(E_{\text{end}}+m_i+m_{\text{lightest}}))^2}{2\sigma^2}}
\]

\[
\frac{d\Gamma_\beta}{dE_e} = \frac{\bar{\sigma}}{\pi^2} N_T \sum_{i=1}^{N_\nu} |U_{ei}|^2 H(E_e, m_i)
\]

\[
\frac{d\tilde{\Gamma}_\beta}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} dx \frac{d\Gamma_\beta}{dE_e}(x) \exp \left[ -\frac{(E_e-x)^2}{2\sigma^2} \right]
\]

\(\bar{\sigma}\) cross section, \(N_T\) number of tritium atoms in the source (PTOLEMY: 100 g), \(E_{\text{end}}\) endpoint, \(\sigma = \Delta/\sqrt{8\ln 2}\) standard deviation
\[ \frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{i=1}^{N_\nu} \bar{\sigma} N_T |U_{ei}|^2 n_0 f_c(m_i) \times e^{-\frac{[E_e-(E_{\text{end}}+m_i+m_{\text{lightest}})]^2}{2\sigma^2}} \]

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\(\bar{\sigma}\) cross section, \(N_T\) number of tritium atoms in the source (PTOLEMY: 100 g), \(E_{\text{end}}\) endpoint, \(\sigma = \Delta / \sqrt{8 \ln 2}\) standard deviation

\(m_{\text{lightest}} = 10\) meV
\(\Delta = 10\) meV

\(m_{\text{lightest}} = 10\) meV
\(\Delta = 50\) meV
Detection of the relic neutrinos

using the definition:

\[ N_{\text{th}}^i(\theta) = A_\beta N_\beta^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + A_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + N_b \]

if \( A_{\text{CNB}} > 0 \) at \( N\sigma \), direct detection of CNB accomplished at \( N\sigma \)

**statistical only!**

significance on \( A_{\text{CNB}} > 0 \)

as a function of \( \hat{m}_{\text{lightest}}, \Delta \)
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Conclusions

1. amazing (neutrino) science with direct detection of relic neutrinos (e.g. PTOLEMY) [non-relativistic regime, masses, ordering?, MW structure?, Dirac/Majorana?, ...]

2. But it will be a technological challenge! ($^3$H amount, low background, energy resolution, ...)

3. possible event rate enhancement due to clustering in the Milky Way, and also nearby galaxies/clusters!

4. Clustering cannot increase detection chances, but we could constrain the composition of the astrophysical environment using the event rate!
Conclusions

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but we could constrain the composition of the astrophysical environment using the event rate!

Thank you for the attention!
Milky Way parameterization
Dark matter: profiles today

(\gamma)NFW profile:

\[ \mathcal{N}_{\text{NFW}} \left( \frac{r}{r_s} \right)^{-\gamma} \left( 1 + \frac{r}{r_s} \right)^{-3+\gamma} = \rho_{\text{DM}}(r) = \mathcal{N}_{\text{Ein}} \exp \left\{ -\frac{2}{\alpha} \left( \left( \frac{r}{r_s} \right)^{\alpha} - 1 \right) \right\} \]

\[ \mathcal{N}_{\text{NFW}} = 2^{3-\gamma} \rho_{\text{NFW}}(r_s) \]

Einasto (EIN) profile:

\[ \mathcal{N}_{\text{Ein}} = \rho_{\text{Ein}}(r_s) \]

\[ \mathcal{N}_{\text{Ein}}, r_s, \alpha \]

Best-fit profiles optimistic: close to 2\sigma upper limits fit of data points from [Pato & Iocco, 2015]
DM: Time evolution of the profiles

profile evolution from universe expansion

\[
\rho_{cr}(z) = \frac{3}{8\pi G} H^2(z)
\]
\[
F_{cr}(z) = \Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0}
\]
\[
H^2(z) = H_0^2 F_{cr}(z)
\]
\[
\rho_{cr}(z) = F_{cr}(z) \times \rho_{cr}(z = 0)
\]

\[
M_{vir} = \frac{4\pi}{3} \Delta_{vir}(z) \rho_{cr}(z) a^3 r_vir^3(z)
\]
(constant in time)

virial radius \( r_vir \)

radius of sphere containing \( M_{vir} \), average density \( \Delta_{vir}(z) \times \rho_{cr}(z) \)

but \( \rho_{DM} = \rho_{DM}(r; r_s, N, [\gamma|\alpha]) \)

relation between \( r_s \) and \( r_vir \)?

from N-body [Dutton et al., 2014]

final expression \( \Rightarrow \)

\[
\rho_{DM}(r, z) = N(z) \tilde{\rho}_{DM}(r, r_s(z))
\]

\( r_vir(M_{vir}, z) = \left( \frac{3M_{vir}}{4\pi \rho_{cr,0} \Omega_{m,0}} \right)^{1/3} \left( \frac{\Omega_{m}(z)}{\Delta_{vir}(z)F_{cr}(z)} \right)^{1/3} \)

for EIN,

for NFW.

\[
\Delta_{vir}(z) = \begin{cases} 
200 & \text{for EIN}, \\
18\pi^2 + 82\lambda(z) - 39\lambda(z)^2 & \text{for NFW.}
\end{cases}
\]

\[
\lambda(z) = \Omega_{m}(z) - 1
\]

\( a = 1/(1 + z), \ h = H_0/(100 \text{ Km s}^{-1} \text{ Mpc}^{-1}) \)

- \( h = 0.6727, \ \Omega_{m,0} = 0.3156, \ \Omega_{\Lambda,0} = 0.6844 \) [Planck Collaboration, 2015]
Baryons: the complexity of a structure

Complex problem: how to model baryon content of a galaxy?

e.g. [Pato et al., 2015]:
70 different baryonic models

7 models for the bulge
×
5 for the disc
×
2 for the gas

[Misiriotis et al., 2006]:
5 independent components

warm dust
cold dust
stars
atomic $H$ gas
molecular $H$ gas

our case: [Misiriotis et al., 2006], spherically symmetrized.
Baryons: redshift evolution

baryon evolution with redshift?

$N_{\text{bar}}(z)$ from $M(z)$

mean of 8 simulations ↔ based on Aquarius simulation: $M_{\text{Aq}} \sim M_{\text{MW}}$

from [Marinacci et al., 2013]

results of full N-body simulations

$\overrightarrow{\triangle}$
Milky Way parameterization

PTOLEMY
Events in bin $i$, centered at $E_i$:

\[ N^i_\beta = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\hat{\Gamma}_\beta}{dE_e} dE_e \]

\[ N^i_{\text{CNB}} = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\hat{\Gamma}_{\text{CNB}}}{dE_e} dE_e \]

**Fiducial** number of events:

\[ \hat{N}^i = N^i_\beta(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) + N^i_{\text{CNB}}(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) \]

Add **background** $\hat{N}_b = \hat{\Gamma}_b T$

with $\hat{\Gamma}_b \sim 10^{-5}$ Hz

\[ N^i_t = \hat{N}^i + \hat{N}_b \]

$T$ exposure time – $(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U})$ fiducial endpoint energy, masses, mixing matrix – $\theta = (A_\beta, N_b, \Delta E_{\text{end}}, A_{\text{CNB}}, m_i, U)$
Events in **bin** $i$, centered at $E_i$:

$$N^i_\beta = T \int_{E_i-\Delta/2}^{E_i+\Delta/2} d\tilde{\Gamma}_\beta dE_e$$

$$N^i_{\text{CNB}} = T \int_{E_i-\Delta/2}^{E_i+\Delta/2} d\tilde{\Gamma}_{\text{CNB}} dE_e$$

**Fiducial** number of events: $\hat{N}^i = N^i_\beta(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) + N^i_{\text{CNB}}(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U})$

Add **background** $\hat{N}_b = \hat{\Gamma}_b T$

with $\hat{\Gamma}_b \simeq 10^{-5}$ Hz

$$N^i_t = \hat{N}^i + \hat{N}_b$$

Simulated experimental spectrum:

$$N^i_{\text{exp}}(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) = N^i_t \pm \sqrt{N^i_t}$$

$T$ exposure time – $(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U})$ fiducial endpoint energy, masses, mixing matrix – $\theta = (A_\beta, N_b, \Delta E_{\text{end}}, A_{\text{CNB}}, m_i, U)$
Simulations - I

Events in bin \( i \), centered at \( E_i \):

\[
N_{\beta}^i = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\tilde{\Gamma}_{\beta}}{dE_e} dE_e \\
N_{\text{CNB}}^i = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\tilde{\Gamma}_{\text{CNB}}}{dE_e} dE_e
\]

**Fiducial** number of events:

\[
\hat{N}^i = N_{\beta}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) + N_{\text{CNB}}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U})
\]

**Fiducial** number of background:

\[
\hat{N}_b = \hat{\Gamma}_b T
\]

with \( \hat{\Gamma}_b \simeq 10^{-5} \text{ Hz} \)

Simulated experimental spectrum:

\[
N_{\text{exp}}^i(\hat{E}_{\text{end}}, \hat{m}_i, \hat{U}) = N_t^i \pm \sqrt{N_t^i}
\]

Repeat for theory spectrum, free **amplitudes** and **endpoint position**:

\[
N_{\text{th}}^i(\theta) = A_{\beta} N_{\beta}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + A_{\text{CNB}} N_{\text{CNB}}^i(\hat{E}_{\text{end}} + \Delta E_{\text{end}}, m_i, U) + N_b
\]

\( T \) exposure time – \((\hat{E}_{\text{end}}, \hat{m}_i, \hat{U})\) fiducial endpoint energy, masses, mixing matrix – \( \theta = (A_{\beta}, N_b, \Delta E_{\text{end}}, A_{\text{CNB}}, m_i, U) \)

S. Gariazzo “Relic neutrinos: clustering and consequences for direct detection” EPS-HEP 2019, 12/07/2019 5/10
Simulations - I

Events in bin $i$, centered at $E_i$:

$$N^i_\beta = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\Gamma_\beta}{dE_e} dE_e$$

$$N^i_\text{CNB} = T \int_{E_i - \Delta/2}^{E_i + \Delta/2} \frac{d\Gamma_\text{CNB}}{dE_e} dE_e$$

**Fiducial** number of events: $\hat{N}^i = N^i_\beta(\hat{E}_\text{end}, \hat{m}_i, \hat{U}) + N^i_\text{CNB}(\hat{E}_\text{end}, \hat{m}_i, \hat{U})$

Add **background** $\hat{N}_b = \hat{\Gamma}_b T$

with $\hat{\Gamma}_b \simeq 10^{-5}$ Hz

Simulated experimental spectrum:

$$N^i_\text{exp}(\hat{E}_\text{end}, \hat{m}_i, \hat{U}) = N^i_t \pm \sqrt{N^i_t}$$

Repeat for **theory** spectrum, free **amplitudes** and **endpoint position**:

$$N^i_\text{th}(\theta) = A_\beta N^i_\beta(\hat{E}_\text{end} + \Delta E_\text{end}, m_i, U) + A_\text{CNB} N^i_\text{CNB}(\hat{E}_\text{end} + \Delta E_\text{end}, m_i, U) + N_b$$

Fit

$$\chi^2(\theta) = \sum_i \left( \frac{N^i_\text{exp}(\hat{E}_\text{end}, \hat{m}_i, \hat{U}) - N^i_\text{th}(\theta)}{\sqrt{N^i_t}} \right)^2$$

or $\log \mathcal{L} = -\frac{\chi^2}{2}$

$T$ exposure time – ($\hat{E}_\text{end}, \hat{m}_i, \hat{U}$) fiducial endpoint energy, masses, mixing matrix – $\theta = (A_\beta, N_b, \Delta E_\text{end}, A_\text{CNB}, m_i, U)$
Simulations - II

no random noise?

$m_{\text{lightest}} = 10 \text{ meV}$

$\Delta = 10 \text{ meV}!$

1 year of observation with 100 g of T source
Simulations - II

1 year of observation with 100 g of T source

$m_{\text{lightest}} = 50 \text{ meV}
\Delta = 10 \text{ meV}!$

1 year of observation with 100 g of T source

S. Gariazzo “Relic neutrinos: clustering and consequences for direct detection” EPS-HEP 2019, 12/07/2019 6/10
1 year of observation with 100 g of T source

$m_{\text{lightest}} = 50 \text{ meV}$

$\Delta = 20 \text{ meV}$!
Simulations - II

1 year of observation with 100 g of T source

$m_{\text{lightest}} = 50 \text{ meV}
\Delta = 50 \text{ meV}?$

1 year of observation with 100 g of T source
Simulations - II

no random noise?

\[ m_{\text{lightest}} = 50 \text{ meV} \]
\[ \Delta = 10 \text{ meV?} \]

\[ \Gamma_b = 10^{-5} \text{ Hz} \]
\[ \Gamma_b = 10^{-4} \text{ Hz} \]

1 year of observation with 100 g of T source
Simulations - II

With random noise!

\[ m_{\text{lightest}} = 50 \text{ meV} \]
\[ \Delta = 10 \text{ meV?} \]
\[ \Gamma_b = 10^{-5} \text{ Hz} \]
\[ \Gamma_b = 10^{-4} \text{ Hz} \]

Things are more complicated in this way...low background needed!

1 year of observation with 100 g of T source
Perspectives for the mass determination as a function of $\hat{m}_{\text{lightest}}$, $\Delta$

relative error on $m_{\text{lightest}}$

statistical only!
Perspectives for the mass determination

relative error on $m_{\text{lightest}}$

as a function of $\hat{m}_{\text{lightest}}, \Delta$

wonderful precision in determining the neutrino mass

(well, yes, with 100 g of tritium...)

S. Gariazzo  “Relic neutrinos: clustering and consequences for direct detection”  EPS-HEP 2019, 12/07/2019 7/10
Perspectives for the mass determination as a function of $\hat{m}_{\text{lightest}}$, $\Delta$

relative error on $m_{\text{lightest}}$

statistical only!

wonderful precision in determining the neutrino mass

(mass detection already with 10 mg of tritium!)
Perspectives for the mass determination as a function of $\hat{m}_{\text{lightest}}$, $\Delta$

relative error on $m_{\text{lightest}}$

wonderful precision in determining the neutrino mass

(mass detection already with 10 mg of tritium!)

$\Delta$ has almost no impact

S. Gariazzo

"Relic neutrinos: clustering and consequences for direct detection"
**Neutrino mass ordering**

**Bayesian method:**

Fit fiducial ordering ($\hat{\text{NO}}$ or $\hat{\text{IO}}$) using both **correct** and **wrong** ordering

$\hat{\text{NO}}$/NO vs $\hat{\text{NO}}$/IO

$\hat{\text{IO}}$/NO vs $\hat{\text{IO}}$/IO
Neutrino mass ordering

Bayesian method:

Fit fiducial ordering (\(\hat{\text{NO}}\) or \(\hat{\text{IO}}\)) using both correct and wrong ordering

\[ \hat{\text{NO}}/\text{NO} \text{ vs } \hat{\text{NO}}/\text{IO} \]

\[ \text{worse fit} \]

\[ \hat{\text{IO}}/\text{NO} \text{ vs } \hat{\text{IO}}/\text{IO} \]

\[ \text{better fit} \]
Neutrino mass ordering

Bayesian method:

Fit fiducial ordering (\(\hat{\text{NO}}\) or \(\hat{\text{IO}}\)) using both correct and wrong ordering

\(\hat{\text{NO}}/\text{NO} \) vs \(\hat{\text{NO}}/\text{IO}\)

worse fit

\(\hat{\text{IO}}/\text{NO} \) vs \(\hat{\text{IO}}/\text{IO}\)

good fit

(Bayesian) preference on \(m_{\text{lightest}}\) as a function of \(\hat{m}_{\text{lightest}}, \Delta\)

Statistical only!

\(\text{NO fiducial}\)

\(\text{IO fiducial}\)

always strong significance
Neutrino mass ordering

Bayesian method:

Fit fiducial ordering (\(\hat{\text{NO}}\) or \(\hat{\text{IO}}\)) using both correct and wrong ordering

\(\hat{\text{NO}}/\text{NO} \) vs \(\hat{\text{NO}}/\text{IO}\)

\(\hat{\text{IO}}/\text{NO} \) vs \(\hat{\text{IO}}/\text{IO}\)

worse fit

better fit

(Bayesian) preference on \(m_{\text{lightest}}\)

as a function of \(\hat{m}_{\text{lightest}}, \Delta\)

statistical only!

possible problem: degeneracies between \(m_{\text{lightest}}\) and \(\Delta m^2_{31}\)

always strong significance

\(100 \text{ g yr}\)

\(\text{NO fiducial}\)

\(\text{IO fiducial}\)
### Requirements for PTOLEMY discoveries

**What do we need to discover...**

<table>
<thead>
<tr>
<th></th>
<th>low $\Gamma_b$</th>
<th>extreme $\Delta$</th>
<th>a lot of $^3$H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ldots\nu$ masses?</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$?$</td>
</tr>
<tr>
<td>$\ldots\nu$ mass ordering?</td>
<td>$\times$</td>
<td>$?$</td>
<td>$?$</td>
</tr>
<tr>
<td>$\ldots$ CNB direct detection?</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

✓: strongly required  
? : not so strongly required  
×: loosely required
Dirac and Majorana neutrinos

direct detection through \( \nu_e + ^3 H \rightarrow e^- + ^3 He \)

only neutrinos with correct chirality can be detected!

non-relativistic **Majorana** case: \( \nu \) and \( \bar{\nu} \) cannot be distinguished!

expect more events for the **Majorana** than for Dirac case

---

Dirac normal or inverted ordering differ because lighter \( \nu_1 \) and \( \nu_2 \) in NH are relativistic

almost indistinguishable from **Majorana**