Perturbations of Slotheon Field Dark Energy and its Evolution

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Kolkata, INDIA

EPS-HEP 2019, GHENT

Plan of Talk

Brief Introduction

Cosmological constant model for Dark Energy

Scalar field Dark Energy model

Evolution of Dark Energy Perturbations for Slotheon Field and Power Spectrum

Cosmological Constant Model

- Cosmological constant was originally introduced by Einstein to explain the static universe.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- After Hubble's discovery in 1929, Einstein discarded his idea.The cosmological constant remarks as an important quantity in explaining the accelerated expansion.

- From the Einstein's equations it is obtained

$$\bar{\rho} = \rho + \frac{\Lambda}{8\pi G} \qquad p = p - \frac{\Lambda}{8\pi G} \longrightarrow \omega_\Lambda = -1$$

- The cosmological constant term yields a negative pressure.

- Problem: Theoretical value of cosmological constant is 10^{121} orders of magnitude larger than the observed value.

Scalar Field Model of Dark Energy

- In order to account for the time varying EOS a scalar field is introduced in the theory. One among those models is the quintessence model.

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} (\nabla\phi)^2 - V(\phi) \right)$$

- Energy momentum tensor of the field is derived by varing the action w.r.t. the metric,

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right)$$

- In the flat FRW backgroung the equation of state for the scalar field dark energy is

$$\omega_\phi = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \,.$$

- The universe accelerates for $\dot{\phi}^2 < V(\phi)$

Evolution of Dark Energy Perturbations for Slotheon Field and Power Spectrum

Slotheon Field Model of Dark Energy

- It is a scalar field model inspired by a model in the theory of extra dimension.
- In the decoupling limit of Dvali Gabadadze Porrati (DGP) model, an extra-dimensional model with one extra dimension, the DGP theory in the Minkowski spacetime is described by a scalar field, obeys Galileon shift symmetry

$$\pi \to \pi + a + b_\mu x^\mu$$

- The Galileon transformation generalised to curved space time, it is shown that the lagrangian density $\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \pi_{;\mu}\pi_{;\nu} + \frac{G^{\mu\nu}}{2M^2}\pi_{;\mu}\pi_{;\nu}$ respects the symmetry. The field is known as Slotheon.
- Adding the Einstein-Hilbert term and a non trivial potential for π, the action is obtained

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(M_{\rm pl}^2 R - \left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\pi_{;\mu}\pi_{;\nu}\right) - V(\pi)\right] + S_m$$

Equations of Motion for Slotheon Field

- Energy momentum tensor for the Slotheon field

$$T^{(\pi)}_{\mu\nu} = \pi_{;\mu}\pi_{;\nu} - \frac{1}{2}g_{\mu\nu}(\nabla\pi)^2 - g_{\mu\nu}V(\pi) + \frac{1}{M^2}\left(\frac{1}{2}\pi_{;\mu}\pi_{;\nu}R - 2\pi_{;\alpha}\pi_{(;\mu}R^{\alpha}_{\nu)}\right.$$
$$\left. + \frac{1}{2}\pi_{;\alpha}\pi^{;\alpha}G_{\mu\nu} - \pi^{;\alpha}\pi^{;\beta}R_{\mu\alpha\nu\beta} - \pi_{;\alpha\mu}\pi^{\alpha}_{;\nu} + \pi_{;\mu\nu}\pi^{\alpha}_{;\alpha} + \frac{1}{2}g_{\mu\nu}\left(\pi_{;\alpha\beta}\pi^{;\alpha\beta} - \left(\pi^{\alpha}_{;\alpha}\right)^2 + 2\pi_{;\alpha}\pi_{;\beta}R^{\alpha\beta}\right)\right).$$

- Friedmann equations for Slotheon field

$$3M_{\rm pl}^2H^2 = \rho_m + \frac{\dot{\pi}^2}{2} + \frac{9H^2\dot{\pi}^2}{2M^2} + V(\pi)$$

$$M_{\rm pl}^2(2\dot{H} + 3H^2) = -\frac{\dot{\pi}^2}{2} + V(\pi) + (2\dot{H} + 3H^2)\frac{\dot{\pi}^2}{2M^2} + \frac{2H\dot{\pi}\ddot{\pi}}{M^2}$$

$$0 = \ddot{\pi} + 3H\dot{\pi} + \frac{3H^2}{M^2}\left(\ddot{\pi} + 3H\dot{\pi} + \frac{2\dot{H}\dot{\pi}}{H}\right) + V_\pi$$

Dimensionless Variables

- The following dimensionless variables will be useful for solving the background equations

$$x = \frac{\dot{\pi}}{\sqrt{6}HM_{\rm pl}} \qquad y = \frac{\sqrt{V(\pi)}}{\sqrt{3}HM_{pl}}$$

$$\epsilon = \frac{H^2}{2M^2} \qquad \lambda = -M_{\rm pl}\frac{V'(\pi)}{V(\pi)}$$

- The following autonomous system of equations are obtained

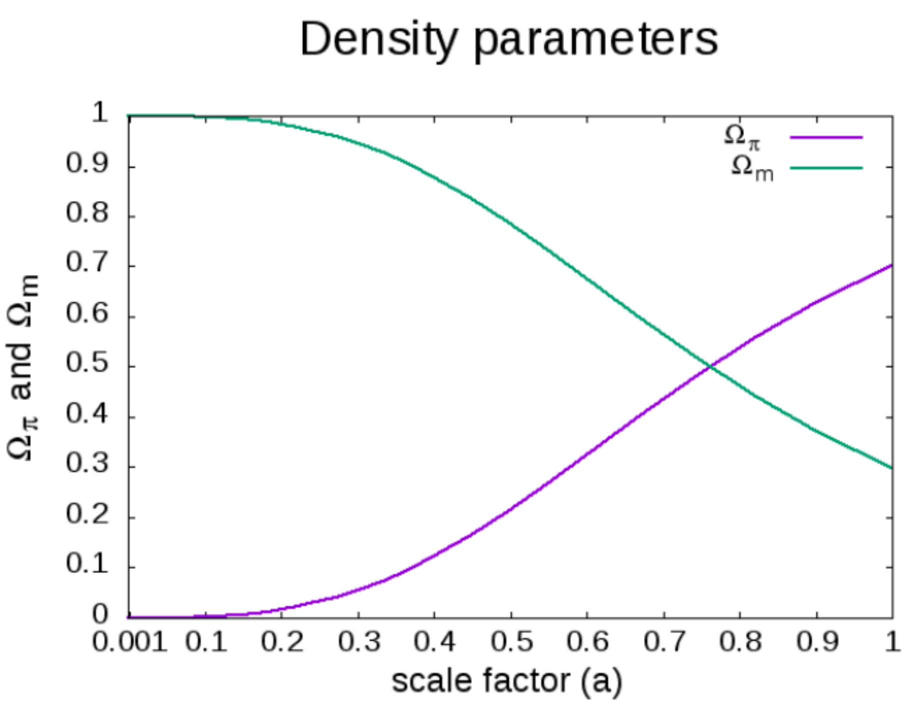
$$\frac{dx}{dN} = x\left(\frac{\ddot{\pi}}{H\dot{\pi}} - \frac{\dot{H}}{H^2}\right) \qquad \frac{d\epsilon}{dN} = 2\epsilon\frac{\dot{H}}{H^2}$$

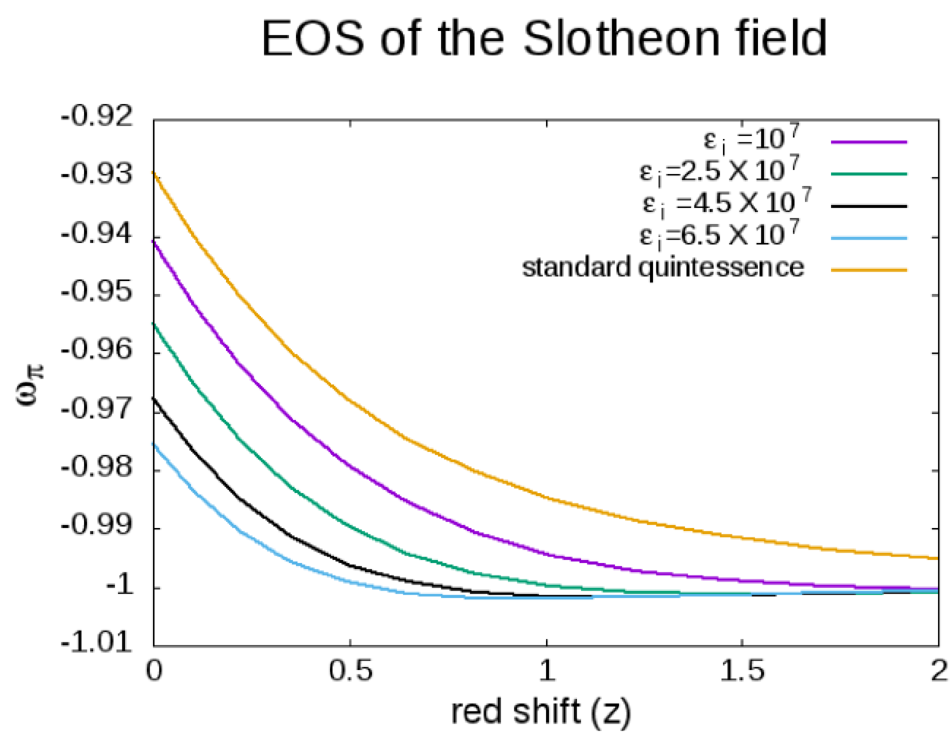
$$\frac{dy}{dN} = -y\left(\sqrt{\frac{3}{2}}\lambda x + \frac{\dot{H}}{H^2}\right) \qquad \frac{d\lambda}{dN} = \sqrt{6}x\lambda^2(1-\Gamma)$$

- Where $\Gamma = \frac{V(\pi)V''(\pi)}{(V'(\pi))^2}$. We use exponential form of potential for calculations.

Evolutions of Density Parameter and Equation of state for Slotheon Field

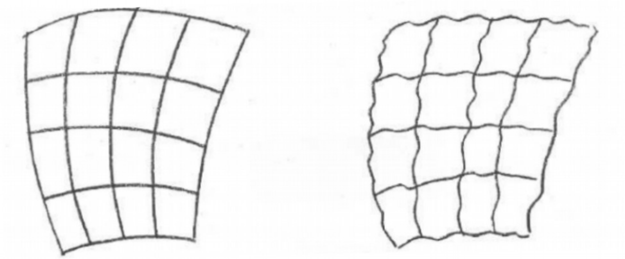
- The density parameter (Ω) and EOS (ω) are expreesed in terms of the dimensionless variables. Thus by solving the set of equations with proper initial conditions the background evolutions of the system are obtained.





Perturbations in General Relativity

- In the perturbation theory of general relativity one consider a spacetimes, the perturbed spacetime, that is close to a simple, symmetric spacetime, the background spacetime, FRW background.



- The metric is $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$
- Einstein tensor and energy momentum tensor

$$G_{\mu\nu} = \bar{G}_{\mu\nu} + \delta G_{\mu\nu} \qquad T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

- Perturbations in energy density, pressure density, four velocity and Slotheon field are defined as

$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho(t, \vec{x}) \qquad p(t, \vec{x}) = \bar{p}(t) + \delta p(t, \vec{x})$$

$$u^\mu = \bar{u}^\mu + v^\mu \qquad \pi(t, \vec{x}) = \bar{\pi}(t) + \delta\pi(t, \vec{x})$$

Perturbations of Slotheon Field

- The scalar perturbed metric is obtained as

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j$$

- Considering energy momentum tensor of ideal fluid and neglecting second and higher order terms of perturbations it is obtained

$$\delta T^0_0 = -\delta\rho \qquad \delta T^i_0 = (\bar{\rho}+\bar{p})v^i \qquad \delta T^i_j = \delta p \delta^i_j$$

- Perturbations are calculated for Slotheon field

$$\delta\rho_\pi = -\frac{1}{a^3M^2}\left[-a^3M^2\delta\pi V_\pi + \dot{\pi}\left(2\dot{a}\nabla^2(\delta\pi) - \delta\dot{\pi}(a^3M^2+9a\dot{a}^2)\right.\right.$$
$$\left.\left.+\dot{\pi}a\left(a^2M^2\Phi - \nabla^2\Phi + 9\dot{a}(a\dot{\Phi}+2\Phi\dot{a})\right)\right)\right]$$

$$(\bar{\rho}_\pi+\bar{p}_\pi)v_i = -\frac{\dot{\pi}}{a^2M^2}\left[-2a\delta\dot{\pi}\dot{a} + \delta\pi(a^2M^2+3\dot{a}^2) + a(a\dot{\Phi}+3\Phi\dot{a})\dot{\pi}\right]|_i$$

$$\delta p_\pi = \frac{1}{a^3M^2}\left[-a^3M^2\delta\pi V_\pi + \dot{\pi}\left(-2a^2\delta\ddot{\pi}\dot{a} + a(-a^2M^2\Phi + a^2\ddot{\Phi} + \nabla^2\Phi - \Phi_{ii})\dot{\pi}\right.\right.$$
$$+\dot{a}(\nabla^2(\delta\pi) - \delta\pi_{ii} + 2a(3a\dot{\Phi}+\Phi\dot{a})\dot{\pi}) + a\delta\dot{\pi}(-\dot{a}^2 + a(aM^2 - 2\ddot{a}))$$
$$\left.\left.+4\Phi\dot{\pi}\ddot{a}a^2\right) + a(\nabla^2\delta\pi - \delta\pi_{ii} - 2a\delta\dot{\pi}\dot{a} + 2a(a\dot{\Phi}+4\Phi\dot{a})\dot{\pi})\ddot{\pi}\right].$$

Perturbed Einstein Equations

- The perturbed Einstein's equation is given by $\delta G^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu}$
- For flat FRW universe, the equations take the forms

$$3H^2\Phi + 3H\dot{\Phi} + \frac{k^2\Phi}{a^2} = -4\pi G \sum_i \delta\rho_i$$

$$k^2(\dot{\Phi} + H\Phi) = 4\pi G a \sum_i (\bar{\rho}_i + \bar{p}_i)\theta_i$$

$$\ddot{\Phi} + 4H\dot{\Phi} + 2\dot{H}\Phi + 3H^2\Phi = 4\pi G \sum_i \delta p_i$$

- The dynamical equation for δπ can be obtained as

$$\frac{1}{M^2}\Bigg[H\dot{\pi}\frac{k^2\Phi}{a} + 18\Phi\dot{a}^3\dot{\pi} - 2\ddot{a}k^2\delta\pi + 2\dot{a}\dot{\pi}k^2\Phi - 3\dot{a}^3\delta\dot{\pi} + H^2 a(-k^2\delta\pi) + \frac{\dot{\pi}k^2\dot{\Phi}}{a} - a^3M^2V_{\pi\pi}\delta\pi +$$
$$2M^2a^3\Phi(V_\pi + 2\ddot{\pi}) + 4a^3M^2\dot{\pi}\dot{\Phi} + \dot{\pi}a^3k^2\dot{\Phi} - a^3M^2\delta\ddot{\pi} + 36a^2\Phi\dot{\pi}H\ddot{a} + 18\Phi a^3H^2\ddot{\pi} - M^2ak^2\delta\pi$$
$$+2\ddot{\pi}ak^2\Phi - 6a^2H\ddot{a}\delta\dot{\pi} + 30a^3H^2\dot{\pi}\dot{\Phi} + 2\dot{\pi}ak^2\dot{\Phi} - 3a^3H^2\delta\ddot{\pi} + 12a^3M^2\Phi H\dot{\pi} + 6\dot{\pi}a^2\ddot{a}\dot{\Phi}$$
$$-a^3H(3M^2\delta\dot{\pi} + 6\ddot{\pi}\dot{\Phi} - \dot{\pi}k^2\Phi - 6\dot{\pi}\ddot{\Phi})\Bigg] = 0$$

Dimensionless Variables for Perturbation Equaions

- To solve the linearised perturbation equations another dimensionless variable is introduced

$$q = \frac{\delta\pi}{\frac{d\pi}{dN}}$$

- Evolutions of the perturbed system are obtained by solving $\frac{dx}{dN}$, $\frac{dy}{dN}$ $\frac{d\lambda}{dN}$, $\frac{d\epsilon}{dN}$, along with following equations

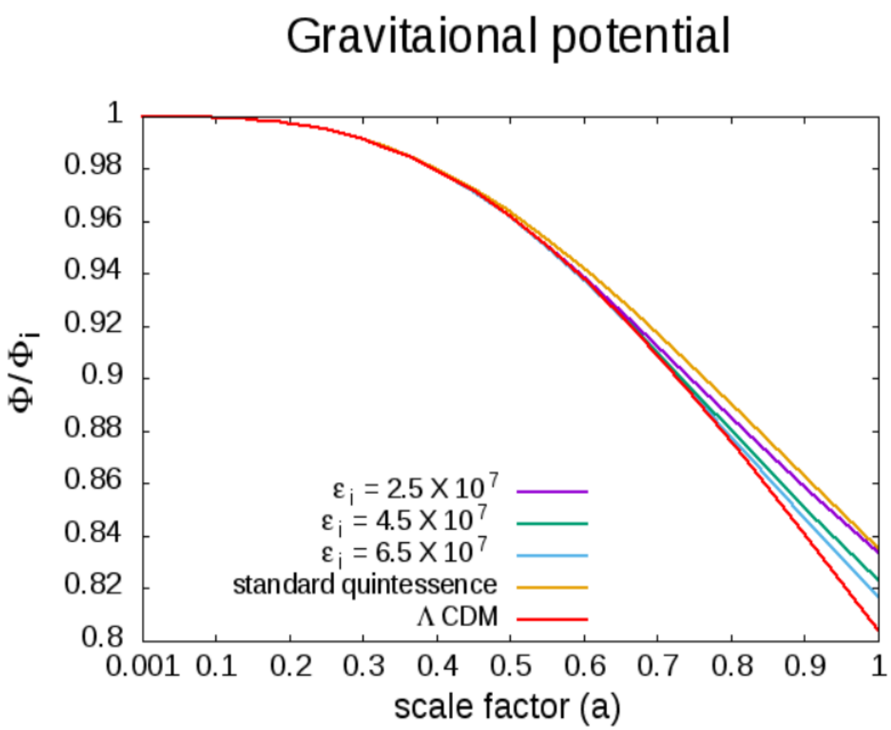
$$\begin{aligned} \frac{dq}{dN} &= q_1 & \frac{dq_1}{dN} &= \delta\pi_f - \frac{q\dot{H}}{xH^2}\frac{dx}{dN} - \frac{2q_1}{x}\frac{dx}{dN} - q_1\frac{\dot{H}}{H^2}\,; \\ \frac{d\Phi}{dN} &= \Phi_1 & \frac{d\Phi_1}{dN} &= \frac{\delta p_f}{4\epsilon} - 4\Phi_1 - 2\frac{\dot{H}}{H^2}\Phi - 3\Phi - \Phi_1\frac{\dot{H}}{H^2} \end{aligned}$$

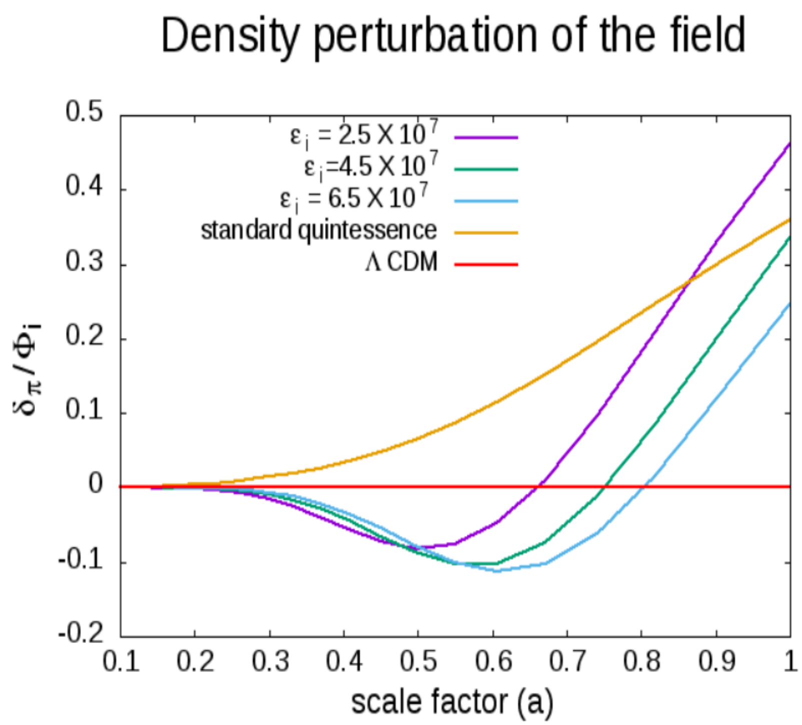
Where, $\delta\pi_f = \delta\ddot{\pi}/H^2\frac{d\pi}{dN}$ $\delta p_f = \delta p_\pi/M^4$

- Fractional density perturbations are calculated

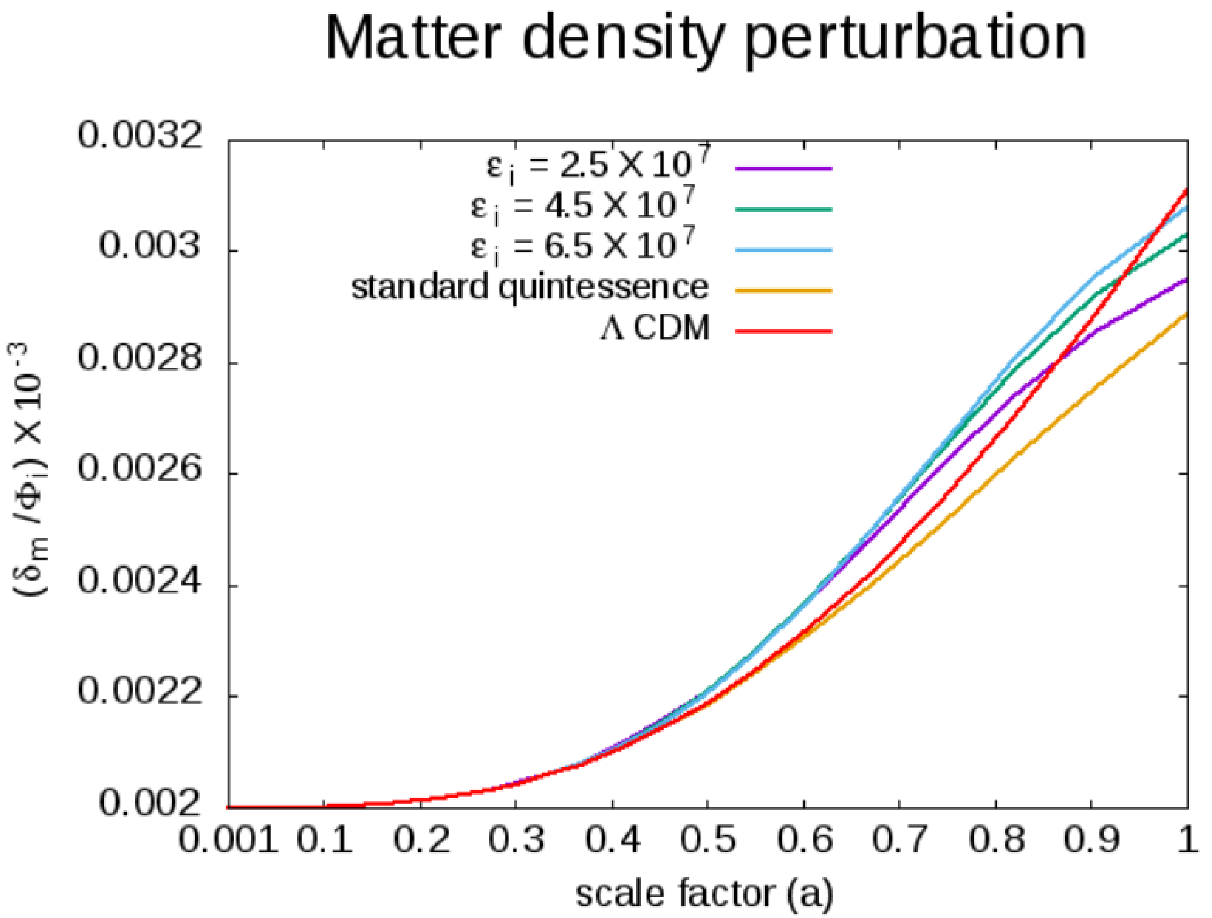
$$\delta = \frac{\delta\rho}{\bar{\rho}}$$

Evolutions of Gravitational Potential and Density Perturbations of the Field





Evolutions of Density Perturbations of Matter



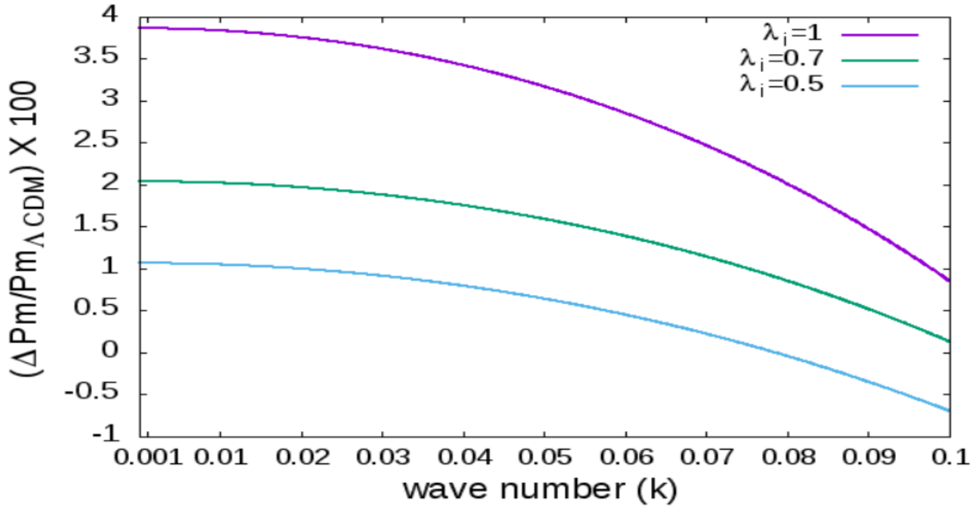
The Effect of the Slotheon Field on Matter Power Spectrum

- Matter power spectrum is defined as the average of the modulus square of the matter density fluctuation

$$Pm = \langle |\delta_m(k,a)|^2 \rangle$$

- We define a percentage suppression X for the Slotheon power spectrum w.r.t. the power spectrum obtained from ΛCDM model as

$$\frac{Pm_{\Lambda\mathrm{CDM}} - Pm_{\mathrm{slotheon}}}{Pm_{\Lambda\mathrm{CDM}}} \times 100 = \frac{\Delta Pm}{Pm_{\Lambda\mathrm{CDM}}} \times 100 = X$$



Summary

- The Dark Energy and late time acceleration of the universe are addressed by considering a type of scalar field, namely Slotheon scalar field.
- The evolution of Dark Energy density for the Slotheon scalar field and the matter density with exponential form of potential are calculated. The epoch of transition is found to be at red shift almost equal to 0.3.
- The evolution of Dark Energy EOS is also calculated.
- The perturbation equations for the Slotheon field and matter are then derived and solved numerically.
- For evolution of gravitational potential and the evolution of matter perturbations, we find that although in the early Universe the Slotheon model results coincide with those of ΛCDM and general quintessence it deviate away in later time.
- The Dark Energy density fluctuations deviate from zero with time as the Dark Energy density grows in the Universe.
- Dark Energy from Slotheon model is more akin to the results of ΛCDM model than that from general quintessence model.
- Matter power spectrum obtained by considering Slotheon field and standard quintessence field are compared.

arXiv:1903.08650

Collaborators

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