New Einstein-Hilbert type action for space-time and matter
- Nonlinear-supersymmetric general relativity theory -

Kazunari Shima
Saitama Institute of Technology

OUTLINE

1. **Motivation**
2. **Nonlinear-supersymmetric general relativity theory(NLSUSYGR)**
3. **Evolutioon of NLSUSYGR and the vacuum structure**
4. **Low energy particle physics and cosmology of NLSUSYGR**
5. **Rarita-Schwinger NLSUSYGR**
6. **Summary**

1. Motivation

@ **The success of Two SMs, i.e. GR and GWS model.**

However, many unsolved fundamental problems in SMs: e.g.,

- **Unification of two SMs.**
- **Space-time dimension _four_,**
- **Three generations of quarks and leptons,**
- **Tiny Neutrino mass M_ν, proton stability in GUT**
- **Dark Matter, Dark Enegy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda$(cosmological term)?**

$\Longrightarrow$ **SUGRA!?,**

- **Origin of SUSY breaking ?, $\cdots$ etc.**

@ **GR describes geometry of space-time.**

However, unpleasant differences between GR and SUGRA:

- **GR $\Longleftrightarrow$ Geometry of Riemann space-time(Physical:[x^μ], GL(4,R))**
- **SUGRA $\Longleftrightarrow$ Geometry of superspace (Mathematical:[x^μ, θ_α], sPoicaré)**

$\Longrightarrow$ **New SUSY paradigm on specific physical space-time!.**

@ As for the three-generations structure:

Among all SO(N) sP,   **[$N = 8$ by M. Gell-Mann]**

SM with just 3 generations emerges in one irreducible rep. of only SO(10) sP.

• **10 supercharges** $Q^I, (I = 1, 2, \cdots .10)$ **are assigned as follows:**

$\underline{\mathbf{10}}_{SO(10)} = \underline{\mathbf{5}}_{SU(5)} + \underline{\mathbf{5}}^{*}{}_{SU(5)}$

$\underline{\mathbf{5}}_{SU(5)} = [\ \{Q_a(a = 1,2,3); \underline{\mathbf{3}}^{*c}, \underline{\mathbf{1}}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3})\}, \ \{Q_m(m = 4,5); \underline{\mathbf{1}}^{c}, \underline{\mathbf{2}}^{ew}, (-e, 0)\}\ ].$

$\Leftrightarrow$ **Supercharge** $\underline{\mathbf{5}}_{SU(5)}$ **has the same quantum numbers as $\underline{\mathbf{5}}$ of SU(5) GUT.**

• **Massless helicity states of gravity multiplet of SO(10) sP with CPT conjugation are specified by the helicity** $h = (2 - \frac{n}{2})$ **and the dimension** $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!}$:

$$|h> = Q^n Q^{n-1} \cdots Q^2 Q^1 |2>, \quad Q^n \ (n = a, m, a^*, m^*)\text{: supercharge}$$

$|h|$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
			$\underline{\mathbf{1}}_{[0]}$	$\underline{\mathbf{10}}_{[1]}$	$\underline{\mathbf{45}}_{[2]}$	$\underline{\mathbf{120}}_{[3]}$	$\underline{\mathbf{210}}_{[4]}$
$\underline{d}_{[n]}$	$\underline{\mathbf{1}}_{[10]}$	$\underline{\mathbf{10}}_{[9]}$	$\underline{\mathbf{45}}_{[8]}$	$\underline{\mathbf{120}}_{[7]}$	$\underline{\mathbf{210}}_{[6]}$	$\underline{\mathbf{252}}_{[5]}$	$\underline{\mathbf{210}}_{[4]}$

© Dirac particle survivors after *tentative* superHiggs-like mechanism

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
1	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ (E)
3	5/3 2/3 $-1/3$ $-4/3$	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$
6	4/3 1/3 $-2/3$	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
8	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

© SM Higgs-doublet state survives in $h = 0$ state.

- $N > 10$ case is unrealistic/ugly and **excluded.**

@ How to construct N=10 SUSY with gravity
beyond No-Go theorem in S-matrix ?

- To circumvent the No-Go theorem degeneracy of space-time is considered.

$$\Downarrow$$

We show in this talk:

- N=10 SUSY with gravity is obtained by the geometric description of General Relativity principle on specific unstable *physical* (Riemann) space-time whose tangent space possesses NLSUSY structure.

A quick review of NLSUSY:

- **Take (flat) space-time specified by x^a for SO(1,3) and ψ_α for SL(2,C).**
- **Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar\psi\gamma^a d\psi - d\bar\psi\gamma^a\psi)$,**

κ **is an arbitrary constant with the dimension l^{+2}.**

- $\delta\omega^a = 0$ **under** $\delta x^a = \frac{i\kappa^2}{2}(\bar\zeta\gamma^a\psi - \bar\psi\gamma^a\zeta)$ **and** $\delta\psi = \zeta$

with a global spinor parameter ζ.

- **An invariant acction($\sim$ invariant volume) is obtained:**

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4 x L_{VA}$,

L_{VA} **is N=1 Volkov-Akulov model of NLSUSY given by**

$L_{\text{VA}} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 - t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b), \quad t^a{}_b = -i\kappa^2(\bar\psi\gamma^a\partial_b\psi - \bar\psi\gamma^a\partial_b\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar\zeta\gamma^a\psi - \bar\zeta\gamma^a\psi)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_{\text{P}}$.

- ψ **is Nambu-Goldstone(NG) fermion for $\frac{superPoincare}{Poincare}$.**

- ψ **is quantized canonically in compatible with SUSY algebra.**

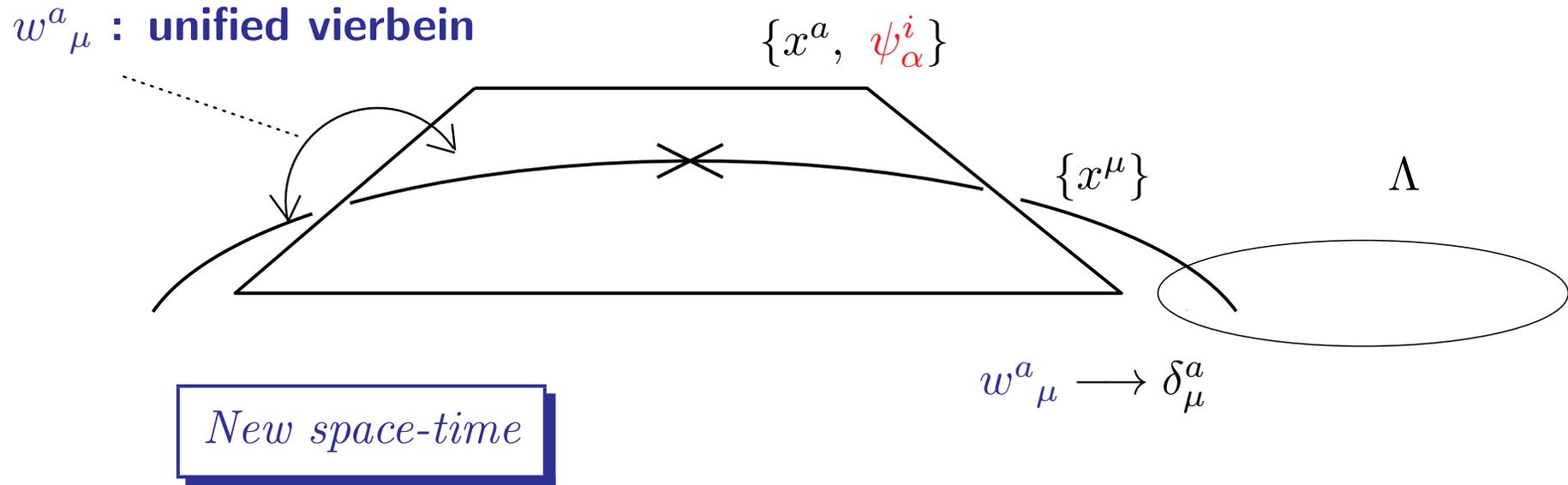# 2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

2.1. New Space-time as Ultimate Shape of Nature

We consider **new (unstable)** *physical* **space-time** inspired by **nonlinear(NL) SUSY:**

The tangent space of new space-time is specified by **Grassmann coordinates** ψ_α for **SL(2,C)** besides **the ordinary Minkowski coordinates** x^a **for SO(1,3),**

i.e.,
the coordinate ψ_α of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the **NLSUSY NG fermion** (called *superon* hereafter) are attached at every curved space-time point besides x^a.

- **New (empy) unstable space-time:**

$w^a{}_\mu$: **unified vierbein**

$\{x^a, \ \psi^i_\alpha\}$

$\{x^\mu\}$

Λ

New space-time

$w^a{}_\mu \longrightarrow \delta^a_\mu$

(Non-compact groups **SO(1,3)** and **SL(2,C)** for space-time symmetry are analogous to compact groups **SO(3)** and **SU(2)** for gauge symmetry of 't Hooft-Polyakov monopole, though **SL(2,C) is realized nonlinearly**.)

- **We will see that** $SO(1,3) \cong SL(2,C)$ **is crucial for NLSUSYGR scenario.**

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

**The geometrical arguments of Einstein general relativity(GR)
can be extended to new (unstable) space-time.**

- **Unified vierbein $w^a{}_\mu(x)$(*ulvierbein*) of new space-time:**

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi) \equiv w^a{}_b e^b{}_\mu,$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a \equiv (w^a{}_b e^b{}_\mu)^{-1},$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar\psi^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar\psi^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

(Note that Grassmann d.o.f. induces <u>the imaginary part of $w^a{}_\mu(x)$</u>.)

- N-**extended NLSUSYGR action of Eienstein-Hilbert(EH)-type
for new space-time.** $\implies$

N-**extended NLSUSYGR action:**

$$L_{\mathrm{NLSUSYGR}}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w) + \Lambda\}, \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \tag{2}$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I\gamma^a\partial_\mu\psi^I - \partial_\mu\bar{\psi}^I\gamma^a\psi^I), (I = 1,2,..,N) \tag{3}$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: **the unified vierbein of new space-time(***ulvierbein***)**
- $e^a{}_\mu(x)$: **the ordinary vierbein for the local SO(1,3) d.o.f.of GR,**
- $t^a{}_\mu(\psi(x))$: **the mimic vierbein for the local SL(2,C) d.o.f. composed of the stress-energy-momentum of NG fermion** $\psi(x)^I$**(called** *superons***),**
- $\Omega(w)$: **Ricci scalar curvature of new space-time computed in terms of** $w^a{}_\mu$,
- $s_{\mu\nu} \equiv w^a{}_\mu\eta_{ab}w^b{}_\nu,\ \ s^{\mu\nu}(x) \equiv w^\mu{}_a(x)\eta^{ab}w^\nu{}_a(x)$: **metric tensors of new space-time.**
- G : **the Newton gravitational constant.**
- $\Lambda > 0$: **cosmological constant required by the correct NLSUSY structure of tangent space.**

- **NLSUSYGR scenario** fixes the **arbitrary constatnt** κ^2 **to**

$$\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G}\right)^{-1},$$

with the dimension $(length)^4 \sim (enegy)^{-4}$.

- $\Lambda > 0$ **in the action** L_{NLSUSYGR} **allows**
negative dark energy density interpretation of $\frac{\Lambda}{G}$
in the Einstein equation. $\rightarrow$ **Sec.4.**

- **No-go theorem** for $\underline{N > 8}$ **SUSY** has been circumvented
by the global NLSUSY, i.e. by the vacuum(flat space) degeneracy.

- **Note that** $SO(1, D-1) \cong SL(d, C)$, **i.e.** $\frac{D(D-1)}{2} = 2(d^2 - 1)$ **holds**
for only $\underline{D = 4, d = 2}$.

NLSUSYGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSUSYGR(N-extended action)

- **Space-time symmetries** $(\sim sP)$**:**

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \tag{4}$$

- **Internal symmetries for N-extended NLSUSY GR (N-superons ψ^I $(I = 1, 2, ..N)$):**

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \tag{5}$$

For example:

- **<u>Invariance under the new NLSUSY transformation</u>**;

$$\delta_\zeta \psi^I = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}. \quad (6)$$

induce GL(4,R) transformations on $w^a{}_\mu$ **and the unified metric** $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \quad (7)$$

where ζ **is a constant spinor parameter,** $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ **and** $\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I$**.**

- **Commutators of two new NLSUSY transformations (6) on** ψ^I **and** $e^a{}_\mu$ **close to GL(4,R),**

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu\partial_\mu\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \quad (8)$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2 - \xi_1^\rho\xi_2^\sigma e_a{}^\mu\partial_{[\rho}e^a{}_{\sigma]}$**.**

$q.e.d.$

- **New NLSUSY (6) is the square-root of GL(4,R)**;

$$[\delta_1, \delta_2] = \delta_{\mathrm{GL(4,R)}}, \quad i.e. \quad \delta \sim \sqrt{\delta_{\mathrm{GL(4,R)}}}.$$

c.f. SUGRA (LSUSY)

$$[\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

- **The ordinary local GL(4,R) invariance is manifest by the construction.**

- **Invariance under new local Lorentz transformation**;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I\gamma_5\gamma_d\psi^I(\partial_\mu\epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., the new form on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ **is given by** $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$.
$q.e.d.$

3.   Evolution of NLSUSYGR and vacuum structure:

3.1. Big Collapse of new space-time toward true vacuum

@ $\Lambda > 0$ allows $L_{\rm NLSUSYGR}(w)$ breaks down spontaneously(Big Collapse) to ordinary Riemann space-time(graviton) and NG fermion(superon): $L_{\rm SGM}(e, \psi)$.

$$L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e, \psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w(\psi^I)|\Lambda + \tilde{T}(e, \psi^I)\}. \qquad (12)$$

- $R(e)$: the Ricci scalar curvature of ordinary Riemann space-time
- Λ : the cosmological constant
- $|w(\psi^I)| = \det w^a{}_b = \det \{\delta^a{}_b + t^a{}_b(\psi^I)\}$: NLSUSY action for superon
- $\tilde{T}(e, \psi^I)$: the gravitational interaction of superon

@ Big Collapse may induce the rapid spacial expansion of space-time.

@ $L_{\rm SGM}(e, \psi^I)$ evolves toward the true vacuum by constituting gravitational composite (massless) eigenstates of broken LSUSY SO(N) sP, which is the ignition of the Big Bang SMs scenario. $\Longrightarrow$
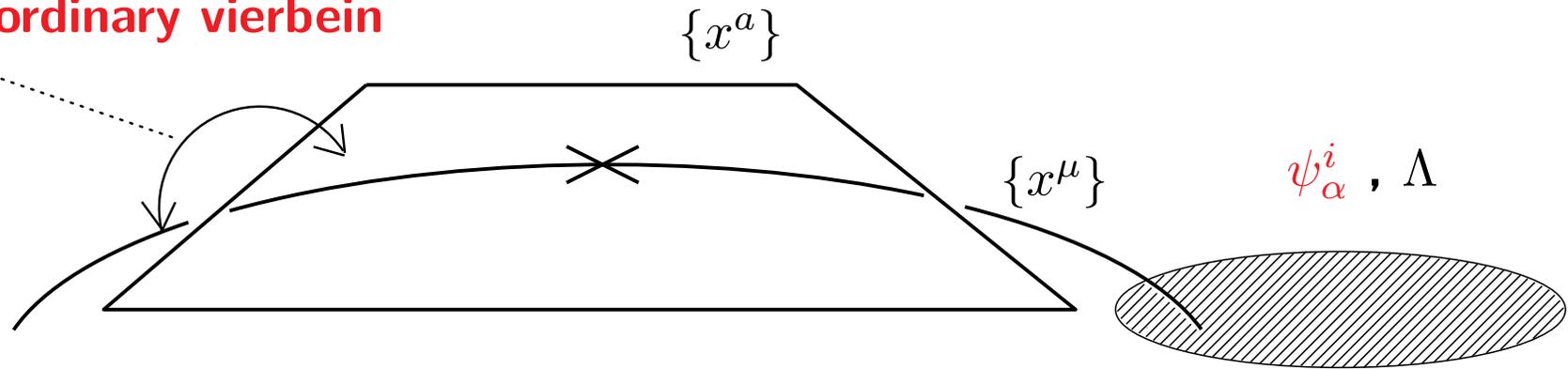
$w^a{}_\mu$: **unified vierbein**

$\{x^a, \; \psi^i_\alpha\}$

$\{x^\mu\}$

Λ

$w^a{}_\mu \longrightarrow \delta^a_\mu$
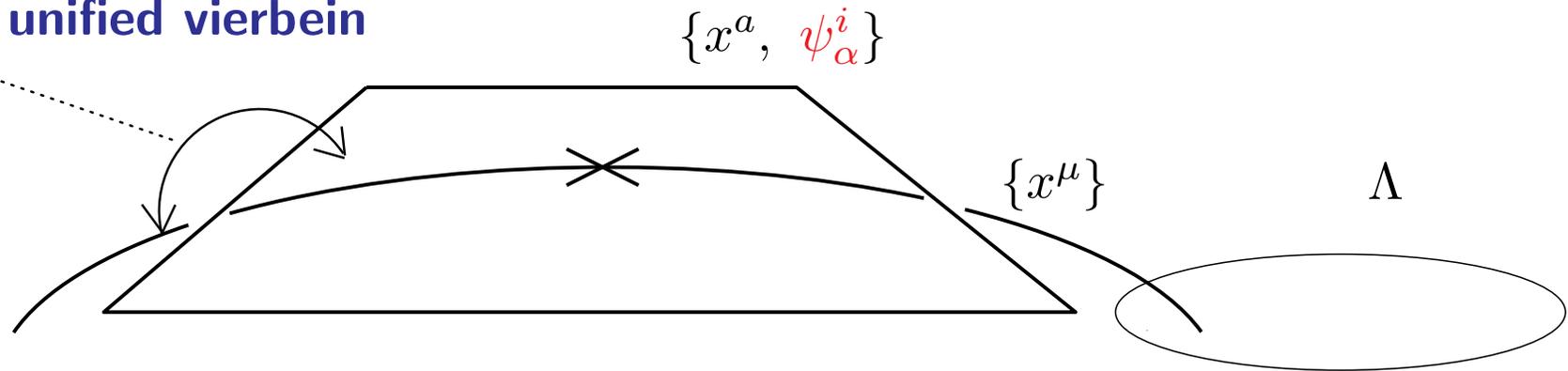
New space-time

$\Downarrow$**Big Collapse**

$e^a{}_\mu$: **ordinary vierbein**

$\{x^a\}$

$\{x^\mu\}$

ψ^i_α , Λ

$e^a{}_\mu \longrightarrow \delta^a_\mu$

Riemann space-time and matter ψ

Ignition of the Big Bang

- **The Noether's theorem gives the conserved supercurrent:**

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}} e_a{}^\mu \gamma^a \psi^I + \cdots. \qquad (13)$$

- **The supercurrent couples the graviton and the superon(NG fermion) to the vacuum**

$$< e_b{}^\nu \psi_\beta{}^J | S_\alpha{}^{I\mu} | 0 > = i\sqrt{\frac{c^4\Lambda}{8\pi G}} \delta^{\mu\nu} \delta^{IJ} (\gamma_b)_{\alpha\beta} \qquad (14)$$

with the strength $g_{sv} = \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}.$

3.2. Linearization of NLSUSY and vacuum of $L_{\mathrm{SGM}}(e, \psi^I)$

The graviton is the universal attractive force and creates all possible gravitational composites of superons, which are helicity eigenstates of LSUSY sP algebra of asymptotic space-time symmetry

• **We study the vacuum structure of $L_{\mathrm{SGM}}(e, \psi^I)$ by linearizing NLSUSY into LSUSY based on NLSUSY and sP algebra**

and obtain the linearized broken LSUSY theory for composites of superon and $e^a{}_\mu$:

$$L_{\mathrm{LSUSY}}(e^a{}_\mu, \psi_\nu(e, \psi), v^a(e, \psi), \lambda(e, \psi), \phi(e, \psi), M(e, \psi), N(e, \psi), \cdots)$$

which satisfies:

$$\text{NL/L SUSY relation } (\textit{equivalence}):$$
$$L_{\mathrm{NLSUSYGR}}(w) = L_{\mathrm{SGM}}(e, \psi) = L_{\mathrm{LSUSY}}(e^a{}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \cdots).$$

3.3. NL/L SUSY relation (equivalence)

We demonstrate **NL/L SUSY relation for $N=2$ SUSY** in *flat space*:

$$L_{\mathrm{NLSUSYGR}}(w) = L_{\mathrm{SGM}}(e, \psi) \rightarrow \ L_{\mathrm{NLSUSY}}(\psi) = L_{\mathrm{LSUSY}}(v^a(\psi), \phi(\psi), \cdots)$$

($N = 2$ **SGM** reduces to $N = 2$ **NLSUSY of Λ term** of **NLSUSYGR**)

@ N=2, d=2 NLSUSY model:

$$L_{NLSUSY} = -\frac{1}{2\kappa^2}|w_{NLSUSY}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \quad (15)$$

$|w_{NLSUSY}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b), \ \ t^a{}_b = -i\kappa^2(\bar{\psi}_j\gamma^a\partial_b\psi^j - \bar{\psi}_j\gamma^a\partial_b\psi^j), \ (j=1,2),$

which is invariant under **N=2 NLSUSY transformation,**

$\delta_\zeta\psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar{\zeta}_k\gamma^a\psi^k - \bar{\zeta}_k\gamma^a\psi^k)\partial_a\psi^j, (j=1,2).$

N=2, d=2 LSUSY Theory (SUSY QED):

- **Helicity states of N=2 vector supermultiplet:**

$$\begin{pmatrix} +1 \\ +\frac{1}{2}, \ +\frac{1}{2} \\ 0 \end{pmatrix} + [\mathrm{CPT conjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell minimal vector supermultiplet:
$(v^a, \lambda^i, A, \phi, D; \ \text{i=1,2})$ **in** WZ $gauge$. (A **and** ϕ **are two singlets,** 0^+ **and** 0^-, **scalar fields.**)

- **Helicity states of N=2 scalar supermultiplet:**

$$\begin{pmatrix} +\frac{1}{2} \\ 0, \ 0 \\ -\frac{1}{2} \end{pmatrix} + [\mathrm{CPT conjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets:
$(\chi, \ B^i, \ \nu, \ F^i; \ i = 1, 2)$, B^i **and** F^i **are complex.**

- **The most genaral $N = 2, d = 2$ SUSYQED action ($m = 0$ case) :**

$$L_{\text{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf},\tag{16}$$

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i\slashed{\partial}\lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi 0} = \frac{i}{2}\bar{\chi}\slashed{\partial}\chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu}\slashed{\partial}\nu + \frac{1}{2}|F^i|^2,$$

$$L_e = e\left\{ iv_a\bar{\chi}\gamma^a\nu - \epsilon^{ij}v^a B^i\partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5\nu \right.$$

$$\left. + B^i(\bar{\lambda}^i\chi - \epsilon^{ij}\bar{\lambda}^j\nu) - \frac{1}{2}|B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2}e^2(v_a{}^2 - A^2 - \phi^2)|B^i|^2,$$

$$L_{Vf} = f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}\tag{17}$$

- **Note that**
  $J = 0$ **states in the vector multiplet** for $N \geq 2$ **SUSY** induce **Yukawa coupling.**

$L_{\mathrm{N=2SUSYQED}}$ **is invariant under** $N = 2$ **LSUSY transformation:**

- **For the minimal vector off-shell supermultiplet:**

$$\delta_\zeta v^a = -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j,$$

$$\delta_\zeta \lambda^i = (D - i\slashed{\partial}A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\slashed{\partial}\phi\zeta^j,$$

$$\delta_\zeta A = \bar{\zeta}^i\lambda^i,$$

$$\delta_\zeta \phi = -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j,$$

$$\delta_\zeta D = -i\bar{\zeta}^i\slashed{\partial}\lambda^i. \tag{18}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{19}$$

where $\zeta^i, i = 1, 2$ **are constant spinors and** $\delta_g(\theta)$ **is the** $U(1)$ **gauge transformation for only** v^a **with** $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2,$ $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\ v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi).$

- **For the two scalar off-shell supermultiplets:**

$$\delta_\zeta \chi = (F^i - i\slashed{\partial}B^i)\zeta^i - e\epsilon^{ij}V^iB^j,$$

$$\delta_\zeta B^i = \bar{\zeta}^i\chi - \epsilon^{ij}\bar{\zeta}^j\nu,$$

$$\delta_\zeta \nu = \epsilon^{ij}(F^i + i\slashed{\partial}B^i)\zeta^j + eV^iB^i,$$

$$\delta_\zeta F^i = -i\bar{\zeta}^i\slashed{\partial}\chi - i\epsilon^{ij}\bar{\zeta}^j\slashed{\partial}\nu$$

$$-e\{\epsilon^{ij}\bar{V}^j\chi - \bar{V}^i\nu + (\bar{\zeta}^i\lambda^j + \bar{\zeta}^j\lambda^i)B^j - \bar{\zeta}^j\lambda^jB^i\},$$

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\chi = \Xi^a\partial_a\chi - e\theta\nu,$$

$$[\delta_{\zeta_1},\delta_{\zeta_2}]B^i = \Xi^a\partial_aB^i - e\epsilon^{ij}\theta B^j,$$

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\nu = \Xi^a\partial_a\nu + e\theta\chi,$$

$$[\delta_{\zeta_1},\delta_{\zeta_2}]F^i = \Xi^a\partial_aF^i + e\epsilon^{ij}\theta F^j, \tag{20}$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ **and the U(1) gauge parameter** θ.

<u>$N = 2$ **NL/L SUSY relation(***equivalence***)**:</u>

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\mathrm{N=2NLSUSY}} + [\text{surface terms}], \quad (21)$$

is realized by
(I) commutator-based (heuristic aspect) linearization
or
(II) superfield-based (systematic aspect) linearization.
Z.B.
(I) Commutator-based linearization:

@The product of Lorentz tensors composed of ψ^i **and** $|w|$ **play a basic role.**

$$Q^n \sim (\psi + \bar{\psi}\gamma \cdot \partial\psi\psi + \cdots)^n \sim \psi^n,$$

• **For such Lorentz tensors (of current) of** ψ^i **miltiplied by** $|w|$,

$$b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n \left((\psi^i)^{2(n-1)} |w| \right) = \kappa^{2n-3}\bar{\psi}^i\gamma_A\psi^j\bar{\psi}^k\gamma_B\psi^l\cdots\bar{\psi}^m\gamma_C\psi^n|w|, \quad (22)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\cdots n}{}_C{}^p \left((\psi^i)^{2n-1} |w| \right) = \kappa^{2(n-1)}\psi^i\bar{\psi}^j\gamma_A\psi^k\bar{\psi}^l\gamma_B\psi^m\cdots\bar{\psi}^n\gamma_C\psi^p|w|, \quad (23)$$

the variations under the NLSUSY transformations become

$$\delta_\zeta b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n = \kappa^{2(n-1)} \left[\left\{ \left(\bar{\zeta}^i \gamma_A \psi^j + \bar{\psi}^i \gamma_A \zeta^j \right) \bar{\psi}^k \gamma_B \psi^l \cdots \bar{\psi}^m \gamma_C \psi^n + \cdots \right\} |w| \right.$$

$$\left. + \kappa \partial_a \left(\xi^a \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \cdots \bar{\psi}^m \gamma_C \psi^n |w| \right) \right], \tag{24}$$

$$\delta_\zeta f^{ij}{}_A{}^{kl}{}_B{}^{ml\cdots n}{}_C{}^p = \kappa^{2n-1} \left[\left\{ \zeta^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p \right. \right.$$

$$\left. + \psi^i \left(\bar{\zeta}^j \gamma_A \psi^k + \bar{\psi}^j \gamma_A \zeta^k \right) \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p + \cdots \right\} |w|$$

$$\left. + \kappa \partial_a \left(\xi^a \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \cdots \bar{\psi}^n \gamma_C \psi^p |w| \right) \right], \tag{25}$$

where $\xi^a = i\kappa \bar{\zeta}^i \gamma^a \psi^i$.

- **These show that the tables of products of Lorentz tensors (of currents) of ψ^i multiplied by $|w|$ give a finite representation of NLSUSY algebra.**

- **The self-contained part of Lorentz-tensor table corresponds to smaller LSUSY multiplet.**
- **They satisfy the basic commutator under NLSUSYGR tr.**

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v),$$

where $\delta_P(v)$ is a translation with a parameter $v^a = 2i(\bar{\zeta}^i_{1L} \gamma^a \zeta^i_{2L} - \bar{\zeta}^i_{1R} \gamma^a \zeta^i_{2R})$

- **These results show that**
the commutator-based linearization **closes on the all possible Lorentz tensors composed of** ψ^i and gives a **finite dimensional representation of sP algebra**.

- **Assign each composite Lorentz tensor to the component field of the LSUSY supermultiplet including the auxiliary field (susy compositenenss)**,
which **reproduces the familiar LSUSY transformation among the supermultiplet under the NLSUSY transformations of constituents** ψ^i.

- **Substituting SUSY compositeness into** $L_{\mathrm{N=2LSUSYQED}}$,
we obtain $L_{\mathrm{N=2NLSUSY}}$, **i .e. the NL/L SUSY relation(equivalence)**.

- **SUSY compositeness** for the vector off-shell **minimal** supermultiplet:

$$v^a = -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|,$$

$$\lambda^i = \xi\psi^i|w|,$$

$$A = \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|,$$

$$\phi = -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|,$$

$$D = \frac{\xi}{\kappa}|w|, \tag{26}$$

where ξ is a **VEV** factor of the auxiliary field D.

- **Note that ψ^i is the low energy leading term of the supercharge Q^i.**

- **SUSY compositeness** for scalar off-shell **minimal** supermultiplets:

$$\chi = \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right]$$

$$B^i = -\kappa \left(\frac{1}{2}\xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right],$$

$$F^i = \frac{1}{\kappa}\xi^i \left\{ |w| + \frac{1}{8}\kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|)$$

$$-\frac{1}{4}e\kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|. \tag{27}$$

- **The quartic fermion self-interaction term in** F^i **realizes the local** $U(1)$ **gauge symmetry of LSUSY.**

- ξ^i **is the VEV factor of the auxiliary field** F^i.

- **SUSY compositeness** produces **under NLSUSY transformation**

a **new off-shell commutator algebra which closes on only a translation**:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{28}$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i) \tag{29}$$

- **Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.**

• **Substituting SUSY copositeness into** $L_{\mathrm{N=2LSUSYQED}}$,
we find NL/L SUSY relation for the <u>**minimal**</u> **supermultiplet**:

$$L_{\mathrm{N=2LSUSYQED}} = f(\xi, \xi^i) L_{\mathrm{N=2NLSUSY}} + [\text{suface terms}], \tag{30}$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \tag{31}$$

$\Rightarrow$ **LSUSY may be regarded as composite eigenstates of space-time symmetries.**

• **NL/L SUSY relation bridges naturally**
the cosmology and the low energy particle physics in NLSUSYGR. ($\Rightarrow$ Sec. 4).

• **The direct linearization of highly nonlinear SGM action (12)**
in curved space remains to be carried out.
$\rightarrow$

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ of SGM and materializes the true vacuum of SGM
(as gravitational composites of NG fermion)

SM can be a low energy effective theory of SGM/NLSUSYGR.

(II) Linearization of NLSUSY by the superfield formulation($d = 2$)

- **General superfields** are given for the $N = 2$ **vector supermultiplet** by

$$\mathcal{V}(x, \theta^i) = C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2}\bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2}\bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i \gamma_5 \theta^j \phi(x)$$

$$- \frac{i}{4}\epsilon^{ij}\bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8}\bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \tag{32}$$

and for the $N = 2$ **scalar supermultiplet** by

$$\Phi^i(x, \theta^i) = B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij}\bar{\theta}^j \nu(x) - \frac{1}{2}\bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i\bar{\theta}^i \slashed{\partial} B^j(x)\theta^j$$

$$+ \frac{i}{2}\bar{\theta}^j \theta^j (\bar{\theta}^i \slashed{\partial}\chi(x) - \epsilon^{ik}\bar{\theta}^k \slashed{\partial}\nu(x)) + \frac{1}{8}\bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \tag{33}$$

- **Extend the superspace to the following superspace** (x'^a, θ') **with** $-\kappa\psi(x)$**,**

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{34}$$

and denote the extended superfields on (x'^a, θ'^i) **and their** θ**-expansions as**

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{35}$$

- **Extended global SUSY transformations** $\tilde{\delta} = \delta^L(x.\theta) + \delta^{NL}(\psi)$ **on** (x'^a, θ'^i) **give:**

$$\tilde{\delta}\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \tilde{\delta}\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{36}$$

- **Therefore, the conditions(SUSY invariant constraints):**

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\text{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\text{constant}), \tag{37}$$

make extended superfields invariant under the extended SUSY transformations. which provide SUSY compositeness.

- **Putting constants in the most general case as follows:**

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad (38)$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (39)$$

- **We obtain straightforwardly SUSY compositeness $\varphi_{\mathcal{V}}^I = \varphi_{\mathcal{V}}^I(\psi)$ for the vector supermultiplet**

$$
\begin{aligned}
C &= \xi_c + \kappa\bar{\psi}^i\xi_\Lambda^i + \frac{1}{2}\kappa^2(\xi_M^{ij}\bar{\psi}^i\psi^j - \xi_M^{ii}\bar{\psi}^j\psi^j) + \frac{1}{4}\xi_\phi\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - \frac{i}{4}\xi_v^a\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j \\
&\quad - \frac{1}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\xi_\lambda^j - \frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\Lambda^i &= \xi_\Lambda^i + \kappa(\xi_M^{ij}\psi^j - \xi_M^{jj}\psi^i) + \frac{1}{2}\xi_\phi\kappa\epsilon^{ij}\gamma_5\psi^j - \frac{i}{2}\xi_v^a\kappa\epsilon^{ij}\gamma_a\psi^j \\
&\quad - \frac{1}{2}\xi_\lambda^i\kappa^2\bar{\psi}^j\psi^j + \frac{1}{2}\kappa^2(\psi^j\bar{\psi}^i\xi_\lambda^j - \gamma_5\psi^j\bar{\psi}^i\gamma_5\xi_\lambda^j - \gamma_a\psi^j\bar{\psi}^i\gamma^a\xi_\lambda^j)
\end{aligned}
$$

$$-\frac{1}{2}\xi\kappa^2\psi^i\bar{\psi}^j\psi^j - i\kappa\slashed{\partial}C(\psi)\psi^i,$$

$$M^{ij} = \xi_M^{ij} + \kappa\bar{\psi}^{(i}\xi_\lambda^{j)} + \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j + i\kappa\epsilon^{(i|k|}\epsilon^{j)l}\bar{\psi}^k\slashed{\partial}\Lambda^l(\psi) - \frac{1}{2}\kappa^2\epsilon^{ik}\epsilon^{jl}\bar{\psi}^k\psi^l\partial^2 C(\psi),$$

$$\phi = \xi_\phi - \kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\xi_\lambda^j - \frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\slashed{\partial}\Lambda^j(\psi) + \frac{1}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2 C(\psi),$$

$$v^a = \xi_v^a - i\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\xi_\lambda^j - \frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j - \kappa\epsilon^{ij}\bar{\psi}^i\slashed{\partial}\gamma^a\Lambda^j(\psi) + \frac{i}{2}\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j\partial^2 C(\psi)$$

$$-i\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma^b\psi^j\partial^a\partial_b C(\psi),$$

$$\lambda^i = \xi_\lambda^i + \xi\psi^i - i\kappa\slashed{\partial}M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa\epsilon^{ab}\epsilon^{ij}\gamma_a\psi^j\partial_b\phi(\psi)$$

$$-\frac{1}{2}\kappa\epsilon^{ij}\left\{\psi^j\partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab}\gamma_5\psi^j F_{ab}(\psi)\right\}$$

$$-\frac{1}{2}\kappa^2\{\partial^2\Lambda^i(\psi)\bar{\psi}^j\psi^j - \partial^2\Lambda^j(\psi)\bar{\psi}^i\psi^j - \gamma_5\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma_5\psi^j$$

$$-\gamma_a\partial^2\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j + 2\slashed{\partial}\partial_a\Lambda^j(\psi)\bar{\psi}^i\gamma^a\psi^j\} - \frac{i}{2}\kappa^3\slashed{\partial}\partial^2 C(\psi)\psi^i\bar{\psi}^j\psi^j,$$

$$D = \frac{\xi}{\kappa} - i\kappa\bar{\psi}^i\slashed{\partial}\lambda^i(\psi)$$

$$+ \frac{1}{2}\kappa^2\left\{\bar{\psi}^i\psi^j\partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\partial^2\phi(\psi)\right.$$

$$+ \frac{i}{2}\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j\partial_a\partial_b v^b(\psi)\bigg\}$$

$$- \frac{i}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\slashed{\partial}\partial^2\Lambda^j(\psi) + \frac{1}{8}\kappa^4\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j\partial^4 C(\psi), \tag{40}$$

and SUSY compositeness for the scalar multiplet $\varphi^I_\Phi = \varphi^I_\Phi(\psi)$**:**

$$B^i = \xi^i_B + \kappa(\bar{\psi}^i\xi_\chi - \epsilon^{ij}\bar{\psi}^j\xi_\nu) - \frac{1}{2}\kappa^2\{\bar{\psi}^j\psi^j F^i(\psi) - 2\bar{\psi}^i\psi^j F^j(\psi) + 2i\bar{\psi}^i\slashed{\partial}B^j(\psi)\psi^j\}$$

$$- i\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\slashed{\partial}\chi(\psi) - \epsilon^{ik}\bar{\psi}^k\slashed{\partial}\nu(\psi)\} + \frac{3}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 B^i(\psi),$$

$$\chi = \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\slashed{\partial}B^i(\psi)\psi^i\}$$

$$- \frac{i}{2}\kappa^2[\slashed{\partial}\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\slashed{\partial}\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}]$$

$$+\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\slashed{\partial}F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j,$$

$$\nu = \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\slashed{\partial}B^i(\psi)\psi^j\}$$

$$-\frac{i}{2}\kappa^2[\slashed{\partial}\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\slashed{\partial}\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}]$$

$$+\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\slashed{\partial}F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j,$$

$$F^i = \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\slashed{\partial}\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\slashed{\partial}\nu(\psi)\}$$

$$-\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\slashed{\partial}F^j(\psi)\psi^j$$

$$+\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi). \qquad (41)$$

- **Choosing the following Lorentz invariant and SUSY invariant constraints** of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (42)$$

give **previous SUSY compositeness for the minimal supermultiplet**.

Therefore,

- **under SUSY invariant constraints**,

the $N = 2$ **NLSUSY** action $S_{N=2\mathrm{NLSUSY}}$ **is related to** $N = 2$ **SUSY QED action:**

$$L_{N=2\mathrm{SUSYQED}} \equiv L_{\mathcal{V}\mathrm{free}} + L_{\mathcal{V}f} + L_{\mathrm{gauge}} = f(\xi, \xi^i) S_{N=2\mathrm{NLSUSY}} \qquad (43)$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

- **NL/L SUSY relation bridges the cosmology and the low energy particle physics in NLSUSYGR scenario** $\Longrightarrow$ **Sec. 4.**

@ SGM scenario predicts the magnitude of the bare gauge coupling constant.

For more general SUSY invariant constraints(**vev of** 0^+ **auxiliary field**):

$$\underline{\tilde{C} = \xi_c}, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (44)$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \ e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \quad (45)$$

where e is the **bare** gauge coupling constant.

• **This mechanism is natural and favorable for SGM scenario as a theory of everything.**

**Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.**

3.3. $N=3$ NL/L SUSY relation and SUSY Yang-Mills theory

- **Physical helicity states of $N=3$ LSUSY vector supermultiplet:**

$$\left[\underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right) \right] + [\text{CPT conjugate}], \qquad (46)$$

where $\underline{n}(\lambda)$ means the dimension $\underline{n}$ and the helicity λ, are accomodated in $N=3$ off-shell vector supermultiplet($d=2$):

- $N=3$ **superYang-Mills(SUSYYM)** minimal off-shell **gauge multiplet,**

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), \chi_\alpha{}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I=1,2,\cdots,dim.G) \qquad (47)$$

Each component field belongs to the adjoint representation of the YM gauge group G:
$[T^I, T^J] = i f^{IJK} T^K$ **and denoted as** $\varphi^i = \varphi^{iI} T^I$**, etc..**

- $N = 3$ **(pure) SUSYYM action:**

$$S_{\text{SYM}} = \int d^2x \, \text{tr} \left\{ -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{D} \lambda^i + \frac{1}{2}(D_a A^i)^2 + \frac{i}{2}\bar{\chi}\not{D}\chi + \frac{1}{2}(D_a\phi)^2 + \frac{1}{2}(D^i)^2 \right.$$

$$-ig\{\epsilon^{ijk}A^i\bar{\lambda}^j\lambda^k - [A^i,\bar{\lambda}^i]\chi + \phi(\bar{\lambda}^i\gamma_5\lambda^i + \bar{\chi}\gamma_5\chi)\}$$

$$\left. +\frac{1}{4}g^2([A^i,A^j]^2 + 2[A^i,\phi]^2) \right\}, \tag{48}$$

where g **is the gauge coupling constant,** D_a **and** F_{ab} **are the covariant derivative and the YM gauge field strength defined as**

$$D_a\varphi = \partial_a\varphi - ig[v_a,\varphi],$$

$$F_{ab} = \partial_a v_b - \partial_b v_a - ig[v_a,v_b]. \tag{49}$$

- **SUSYYM action is invariant under $N = 3$ LSUSY transformations:**

$$\delta_\zeta v^a = i\bar{\zeta}^i\gamma^a\lambda^i,$$

$$\delta_\zeta \lambda^i = \epsilon^{ijk}(D^j - i\slashed{D}A^j)\zeta^k + \frac{1}{2}\epsilon^{ab}F_{ab}\gamma_5\zeta^i - i\gamma_5\slashed{D}\phi\zeta^i$$

$$+ig([A^i, A^j]\zeta^j + \epsilon^{ijk}[A^j, \phi]\gamma_5\zeta^k),$$

$$\delta_\zeta A^i = \epsilon^{ijk}\bar{\zeta}^j\lambda^k - \bar{\zeta}^i\chi,$$

$$\delta_\zeta \chi = (D^i + i\slashed{D}A^i)\zeta^i + ig(\epsilon^{ijk}A^iA^j\zeta^k - [A^i, \phi]\gamma_5\zeta^i),$$

$$\delta_\zeta \phi = \bar{\zeta}^i\gamma_5\lambda^i,$$

$$\delta_\zeta D^i = -i\epsilon^{ijk}\bar{\zeta}^j\slashed{D}\lambda^k - i\bar{\zeta}^i\slashed{D}\chi + ig(\bar{\zeta}^i[\lambda^j, A^j] + \bar{\zeta}^j[\lambda^i, A^j] - \bar{\zeta}^j[\lambda^j, A^i]$$

$$-\epsilon^{ijk}\bar{\zeta}^j[\chi, A^k] + \epsilon^{ijk}\bar{\zeta}^j\gamma_5[\lambda^k, \phi] + \bar{\zeta}^i\gamma_5[\chi, \phi]), \tag{50}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{51}$$

where $\delta_G(\theta)$ means $\delta_G(\theta)\varphi = ig[\theta, \varphi]$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^iv_a - \epsilon^{ijk}\bar{\zeta}_1^i\zeta_2^jA^k - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- **SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet**

$$v^{aI} = -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\slashed{\partial}\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5),$$

$$\lambda^{iI} = \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\slashed{\partial}\psi^l)$$

$$+\frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4),$$

$$A^{iI} = \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\slashed{\partial}\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5),$$

$$\chi^I = \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\slashed{\partial}\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4),$$

$$\phi^I = -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\slashed{\partial}\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5),$$

$$D^{iI} = \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\slashed{\partial}\psi^k)\}$$

$$-\frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \tag{52}$$

- **Arbitrary real constants ξ^{iI} of auxirially fields D^{iI} bridge $N = 3$ SUSY and the YM gauge group G.**

- **Substituting (52) into the SYM action (48), we can show the NL/L SUSY relation for $N = 3$ SUSY:**

$$L_{\mathrm{SUSYYM}}(\psi) = -(\xi^{iI})^2 L_{\mathrm{NLSUSY}} + [\text{surface terms}].$$

(53)

3.4. Linearization of $L_{SGM}(e, \psi)$ in curved space-time

@ Commutator-based linearization:

• **Show**

<div align="center">

NL/L SUSY relation (*equivalence*):

$L_{\mathrm{NLSUSYGR}}(w) = L_{\mathrm{SGM}}(e, \psi) = L_{\mathrm{LSUSY}}(e^a{}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \cdots).$

</div>

• **NLSUSY transformations**

$$\delta_\zeta \psi^i = \tfrac{1}{\kappa} \zeta^i + \xi^\mu \partial_\mu \psi^i, \ \ \delta_\zeta e^a{}_\mu = 2i\kappa \bar{\zeta}^i \gamma^\nu \psi^i \partial_{[\mu} e^a{}_{\nu]}, \ \ \xi^\mu = i\kappa \bar{\psi}^j \gamma^\mu \zeta^j.$$

• **Based commutator algebra, LSUSY as well,**

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{GL(4,R)}(\Xi^\mu), \ \ \Xi^\mu = 2(i\bar{\zeta}^i_1 \gamma^\mu \zeta^i_2 - \xi^\nu{}_1 \xi^\kappa{}_2 e^\mu{}_a \partial_{[\mu} e^a{}_{\nu]})$$

• **In curved space-time, Lorentz- and Riemann-tensor functionals with γ-matrices**

$$F^{IA}_M = F^{IA}_M(\psi^i, e^a_\mu, \partial\psi^i, \partial_{[\nu} e^a{}_{\kappa]})$$

satisfies the commutator algebra, favourable to the SUGRA-like LSUSY structure.

4. Low energy particle physics and Cosmology of NLSUSYGR

4.1. Low Energy Particle Physics of NLSUSYGR :

@ As we have seen that
$N = 2$ **SGM is essentially N=2 NLSUSY action in tangent(flat)) space-time,**
we focus on N=2 NLSUSY action for extracting physical implications of SGM.

● The low energy theorem for NLSUSY gives
the following superon(massless NG fermion)-vacuum coupling

$$< \psi^j{}_\alpha(x)|J^{k\mu}{}_\beta|0> = i\sqrt{\frac{c^4\Lambda}{8\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk} + \cdots, \tag{54}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^k + \cdots$ **is the conserved supercurrent.**

$\sqrt{\frac{c^4\Lambda}{8\pi G}} = \frac{1}{\sqrt{2}\kappa}$ **is the coupling constant ($\equiv g_{sv}$) of superon with the vacuum.**

- **How to extract the vacuum configuration of SGM.**

In Riemann-flat space-time, **NL/L SUSY relation(equivalence)** **gives:**

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \tag{55}$$

- **We study vacuum structures of $N=2$ LSUSY QED action**
  in stead of $N=2$ SGM.

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of
$L_{N=2\mathrm{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2\right\}D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \tag{56}$$

- **Substituting the solution of the equation of motion for the auxiliary field D we obtain**

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)|B^i|^2 \geq 0. \quad (57)$$

- **Two different types of vacua $V = 0$ exist in (A, ϕ, B^i)-space:**

$$(I) \quad A = \phi = 0, \quad |\tilde{B}^i|^2 = -k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (58)$$

and

$$(II) \quad |\tilde{B}^i|^2 = 0, \quad A^2 - \phi^2 = k^2. \quad (59)$$

- **Expansions of $A, \phi, \tilde{B}^i$ around vacuum values give**

low energy particles $\hat{A}, \hat{\phi}, \hat{B}^i$ in the true vacuum.

- **For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$, $e\xi < 0$,**

$$
\begin{aligned}
L_{N=2\text{SUSYQED}} = {}& \frac{1}{2}\{(\partial_a \rho)^2 - 2(-ef)k^2 \rho^2\} \\
& + \frac{1}{2}\{(\partial_a \hat{A})^2 + (\partial_a \hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \slashed{\partial} \lambda^i + \frac{i}{2}\bar{\chi} \slashed{\partial} \chi + \frac{i}{2}\bar{\nu} \slashed{\partial} \nu + \sqrt{-2ef}(\bar{\lambda}^1 \chi - \bar{\lambda}^2 \nu) + \cdots,
\end{aligned}
$$

(60)

and following mass spectra

$$m_\rho^2 = m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa},$$

$$m_{\lambda^i} = m_\chi = m_\nu = 0. \tag{61}$$

- **The vacuum breaks both SUSY and the local $U(1)(O(2))$ spontaneously and Higgs-Kibble mechanism works.**

- **All bosons have the same mass and all fermions remain massless.**

- **λ^i are NG fermions.**

- **For the type (II) vacuum with $SO(1,1)$ symmetry for (A, ϕ),   e.g. $f\xi > 0$,**

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = \ & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2 k^2 \hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2 k^2 (|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \slashed{\partial} \lambda^i - 2fk\bar{\lambda}^i \lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\slashed{\partial}\chi + \bar{\nu}\slashed{\partial}\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots .
\end{aligned}
\tag{62}
$$

and following mass spectra:

$$m_{\hat{A}}^2 = m_{\lambda^i}^2 = 4f^2 k^2 = \frac{4\xi f}{\kappa},$$

$$m_{\hat{B}^1}^2 = m_{\hat{B}^2}^2 = m_\chi^2 = m_\nu^2 = e^2 k^2 = \frac{\xi e^2}{\kappa f},$$

$$m_{v_a} = m_{\hat{\phi}} = 0, \tag{63}$$

which produces <u>mass hierarchy</u> by the factor $\frac{e}{f}$ independent of κ. $\left(\kappa^{-2} = \frac{c^4 \Lambda}{16\pi G}\right)$

**● The vacuum <u>breaks both SUSY and $SO(1,1)$ for (A,ϕ)</u>
and <u>restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$</u>, spontaneously,**

**which <u>produces</u> NG-Boson $\hat{\phi}$ and <u>massless photon v_a</u>
and <u>gives soft masses $< A >$ to λ^i</u>.**

• **We have shown explicitly that**

N=2 LSUSY QED, i.e. the matter sector(Λ term) of $N = 2$ SGM (in flat-space), possesses a true vacuum type (II).

• **The resulting model describes qualitatively**

lepton-Higgs-U(1) sector analogue of SM:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\hat{B}^1 + i\hat{B}^2$),
one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are composites of superons.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the NLSUSY cosmological term of SGM
as composites of NG fermion in the true vacuum.

4. Cosmological implications of SGM scenario

The variation of SGM action $L_{\mathrm{SGM}}(e, \psi)$ with respect to $e^a{}_\mu$ yields Einstein equation equipping with matter and cosmological term:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\}. \tag{64}$$

where $\tilde{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

• Note that the cosmological term $-\frac{c^4\Lambda}{8\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density ρ_D.

- **Big collapse** may induce **3 dimensional expansion** of space-time **by Pauli principle**:

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \Phi_{\mu\nu}(e,\psi)\}dx^\mu dx^\nu.$$
$$\{\psi(x),\bar{\psi}(y)\} = 0 \;\Rightarrow\; \{\psi(x),\bar{\psi}(y)\} = \delta^{(3)}(\mathbf{x}-\mathbf{y})$$

- **Big Collapse produces composite (massless) eigenstates of SO(N) sP angebra due to the universal gravitational force**,

which is the ignition of the Big Bang(BB) SM scenario.

- **As shown in the toy model, the vacuum of the composite SGM scenario may explain naturally observed mysterious (numerical) relations:**

$$dark\ energy\ density\ \rho_D \sim O(\kappa^{-2}) \sim m_\nu{}^4 \sim (10^{-12}GeV)^4 \sim g_{sv}{}^2,$$

provided $\lambda_D{}^0$ is identified with neutrino and $f\xi \sim O(1)$.

5. Rarita-Schwinger NLSUSYGR

- **New SUSY algebra containing spinor-vector generators Q_α^μ:**

$$\{Q_\alpha^\mu, Q_\beta^\nu\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta}, \tag{65}$$

$$[Q_\alpha^\mu, P^\nu] = 0, \tag{66}$$

$$[Q_\alpha^\mu, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho} Q^\mu)_\alpha + i\eta^{\lambda\mu} Q_\alpha^\rho - i\eta^{\rho\mu} Q_\alpha^\lambda, \tag{67}$$

where Q_α^μ are vector-spinor generators satisfying Majorana condition $Q_\alpha^\mu = C_{\alpha\beta} \overline{Q}_\alpha^\mu$.
- **Consider the following global (3/2 super)translations:**

$$\psi_\alpha^a \longrightarrow \psi_\alpha^a + \zeta_\alpha^a. \tag{68}$$

$$x_a \longrightarrow x_a + i\kappa\varepsilon_{abcd} \overline{\psi}^b \gamma^c \gamma_5 \zeta^d, \tag{69}$$

where ζ_α^a is a constant Majorana tensor-spinor parameter.
- **The invariant differential forms become:**

$$\omega_a = dx_a + i\kappa\varepsilon_{abcd} \overline{\psi}^b \gamma^c \gamma_5 d\psi^d. \tag{70}$$

- **Invariant action of nonlinear representation of vector-spinor SUSY:**

$$S = \frac{1}{\kappa} \int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa} \int \det w_{ab} d^4 x, \tag{71}$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa \varepsilon_{acde} \bar{\psi}^c \gamma^d \gamma_5 \partial_b \psi^e, \tag{72}$$

- **By similar geometrical arguments to SGM**
we obtain **vector-spinor NLSUSY GR:**

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G} |w| \{ \Omega(w^a{}_\mu) + \Lambda \}, \tag{73}$$

$$|w| = det w^a{}_\mu = det(e^a{}_\mu + t^a{}_\mu), \tag{74}$$

- **Unified vierbeins are:**

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa \varepsilon^{abcd} \bar{\psi}_b \gamma_c \gamma_5 \partial_\mu \psi_d, \tag{75}$$

- $L_{vsNLSUSYGR}$ **possesses similar symmetry properties as SGM.**

6. Summary

NLSUSYGR(SGM) for unity of nature:

• **Ultimate entity; Unstable** $d = 4$ **space-time:**$[x^a, \psi_\alpha{}^N; x^\mu]$ **described by**
$[L_{\mathrm{NLSUSYGR}}(w^a{}_\mu)]$: **NLSUSYGR on New space-time with** $\Lambda > 0$

$\Longrightarrow$ **Big Collapse (BC)**

• **The creation of Riemann space-time [graviton** $e^a{}_\mu$**] and**
massless fermionic matter [superon $\psi_\alpha{}^{\mathbf{N}}$**]**
$[L_{\mathrm{SGM}} = L_{\mathrm{EH}}(e) - \Lambda + T(\psi.e)]$: **Einstein GR with** $\Lambda > 0$ **and** N **superon**

• **The universal attractive force graviton dictates the phase transition**
by forming gravitational composite LSUSY supermultiplet
corresponding to (massless) eigenstates of space-time symmetry SO(10) sP.

$\Longrightarrow$ **Ignition of Big Bang Scenario** $\Longrightarrow$ **(MS)SM: superon-quintet composit view**

• **In flat space-time, broken** N**-LSUSY theory emerges from the** N**-NLSUSY**
cosmological term of $L_{\mathrm{SGM}}(e, \psi)$ **[NL/L SUSY relation].** $\longleftrightarrow$ **BCS vs GL**

The cosmological constant is the origin of everything!

Predictions and Speculations(*preliminary*)

@SO(10) sP algebra with $\underline{10} = \underline{5}_{\mathrm{SU(5)}} + \underline{5}^*_{\mathrm{SU(5)}}$**:**
superon-quintet model(SQM) of matter

- **Two new** 1^{C} **states besides SM particles:**

  double-charge spin 1/2 fermion $E^{2\pm}$**:** $\epsilon^{abc} Q_a Q_b Q_c \epsilon^{mn} Q^*{}_m Q^*{}_n$
  One neutral massive vector boson S**:** $\delta^{ab} Q_a Q^*{}_b$

- **Proton decay diagrams of SU(5) GUT in composite SQM view**
  are forbidden by superon selection rule. $\Rightarrow$ **stable proton**

@Field theory via Linearization:

- **NLSUSYGR(SGM) scenario predicts 4 dimensional space-time.**
- **The bare gauge coupling constant is determined.**
- N-**L/NL SUSY relation** $\Longleftrightarrow$ **superon-quintet hypothesis**

cosmological term $\leftrightarrow$ **dark energy density** $\leftrightarrow$ **SUSY Br.** $\rightarrow m_\nu$

Many Open Questions ! e.g.,

• **Direct linearization of SGM action,**
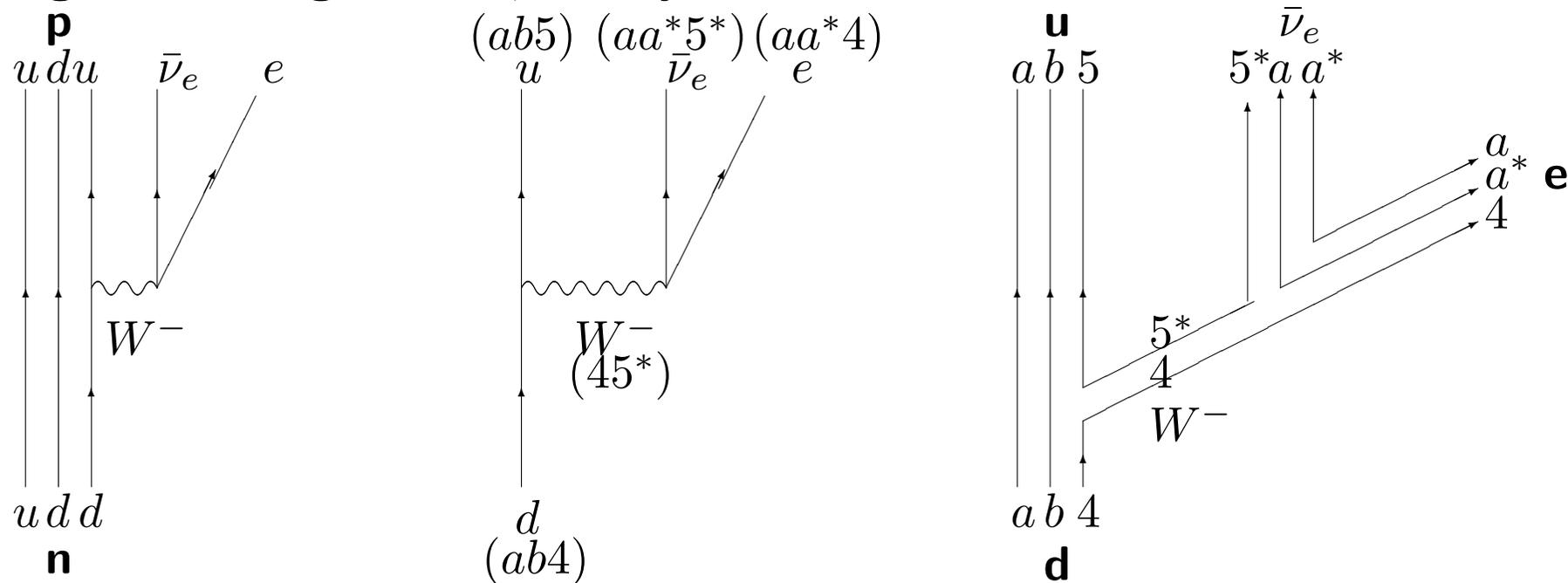i.e. **NL/L SUSY relation in curved space-time, LSUSY of high spin states .**

• **Revisit SM and GUT from** $N = 10$ **SQM composite diagram viewpoints:**

(e, ν_e): $\delta^{ab} Q_a Q^*_b Q_m$, (μ, ν_μ): $\delta^{ab} Q_a Q^*_b \epsilon^{lm} Q_l Q_m Q^*_n$, (τ, ν_τ): $\epsilon^{abc} Q_b Q_c \epsilon^{ade} Q^*_d Q^*_e Q_m$
(u, d): $\epsilon^{abc} Q_b Q_c Q_m$, (c, s): $\epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c Q^*_n$, (t, b): $\epsilon^{abc} Q_a Q_b Q_c Q^*_d Q_m$,
$HiggsBoson$: $\delta^{ab} Q_a Q^*_b Q_l Q^*_m$, $GaugeBoson$: $Q_a Q^*_b$, $\cdots$.

e.g. SQM diagram for β-decay.

- SQM diagram interpretation of dangerous Feynman diagrams of SM and GUT (e.g., proton decay, FCNC, $\cdots$).

- **Physical consequences of spin $\frac{3}{2}$ NLSUSYGR.**

[Ref.] K. Shima, Invited talk at *Conference on Cosmology, Gravitational Waves and Particles*, 6-10, January, 2017, NTU, Singapore. Proceeding of *CCGWP*, ed. Harald Fritzsch, (World Scientific, Singapore, 2017), 301.
K. Shima Invited talk at *14th Rencontres du Vietnam on Windows on the Universe, 25th Anniversary*, 8/5-11, 2018, Quy Nhon, Vietnam. Proceedings of Windows on the Universe, ed. Jacques Dumarchez. at press.