Quantum gravity correction to the QCD vacuum density and the cosmological constant

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EPS-HEP 2019, Ghent

QCD vacuum in the IR limit of the theory

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$

$$\simeq -(5 \pm 1) \times 10^9 \ \mathrm{MeV}^4 .$$

large negative
due to gluons!

For a review, see R. Pasechnik, Universe '17

A contribution of a different kind should cancel it,
but existing at the same hadron scale ~ 200 MeV!

Working hypothesis:

there is an extra (non-perturbative) contribution to QCD vacuum energy
which cancel the chromomagnetic one above in the IR-limit of the theory
due to a dynamical "self-tuning" of QCD vacuum (confinement)

A. Addazi, A. Marciano, R. Pasechnik & G. Prokorov, EPJC '19

See my previous talk yesterday for a discussion of a cancellation mechanism

Topological vs collective vacua fluctuations

R. Pasechnik, V. Beylin & G. Vereshkov, JCAP '13

NPT QCD vacuum

Quantum-topological (instanton) fluctuations

instantons/dyons carrying chromomagnetic and chromoelectric charges

$$\varepsilon_{vac(top)} < 0$$

Quantum-wave (hadronic) fluctuations

have quantum numbers of light hadrons

$$m_h \le l^{-1}_{g(min)}$$

$$\varepsilon_{vac(h)} > 0$$

exist at the same typical space-time scales

$$l_{g(min)} < l_g < l_{g(max)},$$

$$l_{g(min)} \simeq (1500\ \mathrm{MeV})^{-1}, \qquad l_{g(max)} \simeq (500\ \mathrm{MeV})^{-1}$$

Can they mutually cancel each other? In principle, YES!

Taking into account ONLY metastable hadrons

$$B = \{N,\ \Lambda,\ \Sigma,\ \Xi\}$$

$$M = \{\pi,\ K,\ \eta,\ \eta'\}$$

$$\varepsilon_{vac(h)} = \frac{1}{32\pi^2}\left(2\sum_B (2J_B+1)m_B^4 \ln\frac{\mu}{m_B} - \sum_M (2J_M+1)m_M^4 \ln\frac{\mu}{m_M}\right)$$

$$\mu \simeq l^{-1}_{g(min)}$$

$$\varepsilon_{vac(top)} + \varepsilon_{vac(h)} = 0 \quad \text{for} \quad \mu = 1.22\ \ \mathrm{GeV}$$

!!!

Zeldovich-Sakharov scenario

Ya. Zeldovich (1967): $\Lambda \sim G m^6$

Gravitational constant

Characteristic mass scale of elementary particles

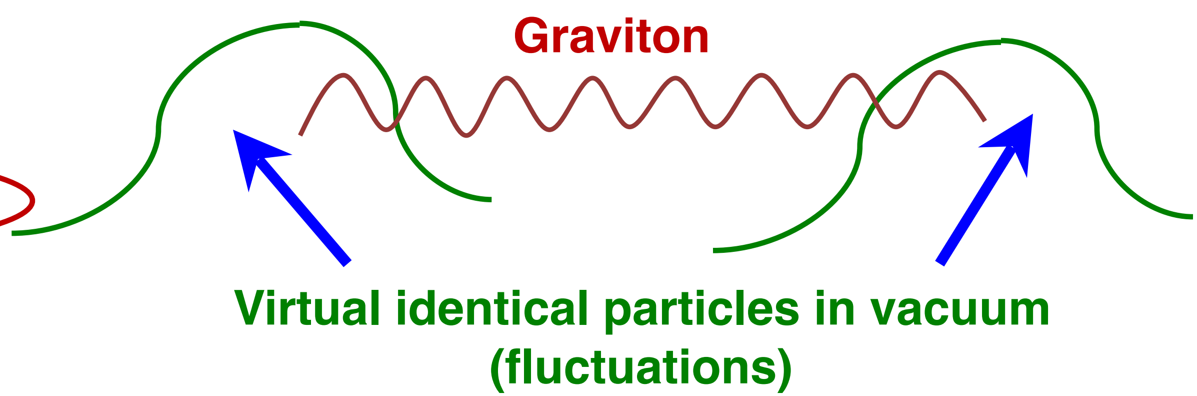
A. Sakharov (1967):

extra terms describing an effect of graviton exchanges between *identical particles (bosons occupying the same quantum state) should appear in the right hand side of Einstein equations (averaged over quantum ensemble)*

…first quantum gravity correction to "bare" vacuum!

Basic idea:

Gravity-induced Λ-term



Can QCD vacuum substructure be responsible for the Λ-term generation?

Does it give both the right value and the right sign?

Quasiclassical (semiquantum) gravity

Zeldovich-Sakharov scenario can be realized in the following way:

Action $$S=\int L d^4x, \qquad L=-\frac{1}{2\varkappa}\sqrt{-\hat{g}}\,\hat{g}^{ik}\hat{R}_{ik}+L\left(\hat{g}^{ik},\chi_A\right)$$

Metric operator $\hat{g}^{ik}$

Macroscopic geometry (c-number part) g^{ik}

Quantum graviton field Φ_i^k

$$\langle 0|\Phi_i^k|0\rangle=0$$

Independent variations over classical and quantum fields:

$$\delta\int L d^4x=-\frac{1}{2}\int d^4x\left(\sqrt{-g}\,\delta g^{ik}\hat{G}_{ik}\right)_{\Phi_i^k=\mathrm{const}}$$
$$=-\frac{1}{2}\int d^4x\left(\sqrt{-g}\,\delta\Phi^{ik}\hat{G}_{ik}\right)_{g^{ik}=\mathrm{const}}$$

Heisenberg state vector containing info about initial states of all fields exists!

Averaging over initial states

same operator eqns:

$$\hat{G}_i^k=\frac{1}{2}\left(\delta_l^k\delta_i^m+g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{E}_m^l=0,$$
$$\hat{E}_m^l=\frac{1}{\varkappa}\left(\hat{g}^{lp}\hat{R}_{pm}-\frac{1}{2}\delta_m^l\hat{g}^{pq}\hat{R}_{pq}\right)-\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

e.o.m. for macroscopic geometry

$$\langle 0|\hat{G}_i^k|0\rangle=0$$

e.o.m. for graviton field

$$\hat{G}_i^k-\langle 0|\hat{G}_i^k|0\rangle=0$$

Metric fluctuations

R. Pasechnik, V. Beylin & G. Vereshkov, JCAP '13

Independent variations fix the exponential parameterization:

$$\sqrt{-\hat{g}}\hat{g}^{ik} = \sqrt{-g}g^{il}(\exp\psi)_l^k = \sqrt{-g}g^{il}\left(\delta_l^k + \psi_l^k + \tfrac{1}{2}\psi_l^m\psi_m^k + \ldots\right) \qquad \psi_i^k = \Phi_i^k - \frac{1}{2}\delta_i^k\Phi$$

up to quadratic terms in graviton field we get:

$$\hat{G}_i^k = \frac{1}{2\varkappa}\left(\psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k\psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k\psi_l^m R_m^l\right) + \frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) - \hat{T}_i^k$$

where $$\hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

with energy-momentum tensor of gravitons

$$\hat{T}_{i(G)}^k = \frac{1}{4\varkappa}\left(\psi_{m;i}^l\psi_l^{m;k} - \frac{1}{2}\psi_{;i}\psi^{;k} - \psi_{i;m}^l\psi_l^{m;k} - \psi_l^{k;m}\psi_{m;i}^l\right) - \frac{1}{8\varkappa}\delta_i^k\left(\psi_{m;n}^l\psi_l^{m;n} - \frac{1}{2}\psi_{;n}\psi^{;n} - 2\psi_{n;m}^l\psi_l^{m;n}\right)$$

$$-\frac{1}{4\varkappa}\left(2\psi_n^l\psi_i^{k;n} - \psi_n^k\psi_i^{l;n} - \psi_i^n\psi^{kl}{}_{;n} + \psi_i^{n;k}\psi_n^l + \psi_{n;i}^k\psi^{nl} + \delta_i^k\left(\psi_m^n\psi_n^l\right)^{;m}\right)_{;l}$$

$$-\frac{1}{4\varkappa}\left(\psi_i^m\psi_n^l R_l^k + \psi_n^k\psi_l^n R_i^l - \delta_i^k\psi_l^n\psi_n^m R_m^l\right) + O(\psi^3).$$

e.o.m. for macroscopic geometry $$\frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) = \langle 0|\hat{T}_i^k|0\rangle$$

e.o.m. for graviton field $$\psi_{i;l}^{k;l} - \psi_{i;l}^{l;k} - \psi_{l;i}^{k;l} + \delta_i^k\psi_{l;m}^{m;l} + \psi_i^l R_l^k + \psi_l^k R_i^l - \delta_i^k R_l^m\psi_m^l = 2\varkappa\left(\hat{T}_i^k - \langle 0|\hat{T}_i^k|0\rangle\right)$$

QCD effective energy-momentum tensor

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To one-loop approximation: $\beta[g_s^2(J)] = -\frac{b g_s^2(J)}{16\pi^2}, \qquad \frac{g_s^2(J)}{4\pi} \equiv \alpha_s(J) = \frac{8\pi}{b\ln(J/\lambda^4)}$

An account for quarks changes b-factor $\quad b = b(3) = 9$

Phenomenology provides with:

correlation length of fluctuations!

$$\langle 0|:\bar{s}s:|0\rangle \simeq \langle 0|:\bar{u}u:|0\rangle = \langle 0|:\bar{d}d:|0\rangle = -\langle 0|:\frac{\alpha_s}{\pi}F^a_{ik}F^{ik}_a:|0\rangle L_g = -(225 \pm 25\ \mathrm{MeV})^3$$

the same for quantum-wave fluctuations! $\quad L_g \simeq (1500 \pm 300\ \mathrm{MeV})^{-1}$

summing gluon and quark contributions:

$$\hat{T}^k_{i(QCD)} = \frac{b_{eff}}{32\pi^2}\left(-\mathcal{F}^a_{il}\mathcal{F}^{kl}_a + \frac{1}{4}\delta^k_i \mathcal{F}^a_{ml}\mathcal{F}^{ml}_a\right)\ln\frac{eJ}{\lambda^4} - \delta^k_i \frac{b_{eff}}{128\pi^2}\mathcal{F}^a_{ml}\mathcal{F}^{ml}_a$$

$$b_{eff} = b(3) + 8L_g(m_u + m_d + m_s) \simeq 9.6$$

instead of 11 for pure gluodynamics!

to a good approximation:

$$\ln\frac{eJ}{\lambda^4} = \ln\frac{e\langle 0|J|0\rangle}{\lambda^4} + \frac{J - \langle 0|J|0\rangle}{\langle 0|J|0\rangle} + \dots \qquad \ln\frac{e\langle 0|J|0\rangle}{\lambda^4} = 4\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

Averaged conformal anomaly!

coming back to original fields:

$$\hat{T}^k_{i(QCD)} = \frac{b_{eff}\alpha_s}{2\pi}\left(-F^a_{il}F^{kl}_a + \frac{1}{4}\delta^k_i F^a_{ml}F^{ml}_a\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}} - \delta^k_i \frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}F^a_{ml}F^{ml}_a|0\rangle$$

Λ-term calculation

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We start from the Einstein equations for macroscopic geometry:

$$\frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) = \langle 0|\hat{T}_i^k|0\rangle \qquad \hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

Trace: $R + 4\varkappa\Lambda = 0$

$$\Lambda = -\frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{il}\hat{g}^{km}\hat{F}_{ik}^a\hat{F}_{lm}^a|0\rangle + \frac{1}{4}\langle 0|\hat{T}_{(G)}|0\rangle$$

Stress tensor in Riemann space is found from YM eqs:

$$\left(\delta^{ab}\frac{\partial}{\partial x^k} - g_s f^{abc}\hat{A}_k^c\right)\sqrt{-\hat{g}}\hat{g}^{il}\hat{g}^{km}\hat{F}_{lm}^b = 0$$

$$\hat{F}_{ik}^a = F_{ik}^a + \underbrace{\frac{1}{2}\psi F_{ik}^a - \psi_i^l F_{lk}^a - \psi_k^l F_{il}^a}_{} + O(\alpha_s G)$$

induce interactions of YM field with metric fluctuations

Equation for gravitons turns into:

$$\psi_{i,l}^{k,l} - \psi_{i,l}^{l,k} - \psi_{l,i}^{k,l} + \delta_i^k\psi_{l,m}^{m,l} = \frac{\varkappa b_{eff}\alpha_s}{\pi}\left(-F_{il}^a F_a^{kl} + \frac{1}{4}\delta_i^k F_{ml}^a F_a^{ml}\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

After exact cancellation of unperturbed part of EMT tensor we get:

$$\Lambda = -\frac{b_{eff}}{16}\ln\frac{L_g^{-1}}{e\Lambda_{QCD}}\langle 0|\frac{\alpha_s}{\pi}F_{il}^a F_a^{kl}\left(\psi_k^i - \frac{1}{4}\delta_k^i\psi\right)|0\rangle$$

linear in graviton field!

Λ-term calculation

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Fock gauge: $\psi^{k}_{i;\,k} = 0$

Metric fluctuations are induced by QCD vacuum fluctuations!

Exact solution of graviton equation:

$$\psi^k_i(x) = \varkappa b_{eff} \ln \frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4x' \underbrace{\mathcal{G}(x-x')}_{} \times \left(\frac{\alpha_s}{\pi} F^a_{il}(x') F^{kl}_a(x') - \delta^k_i \frac{\alpha_s}{4\pi} F^a_{ml}(x') F^{ml}_a(x') \right)$$

Green function: $\mathcal{G}_{;\,l}^{;\,l} = -\delta(x-x')$

After explicit calculation of averages, we get

$$\Lambda = -\pi G \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik} F^{ik}_a : |0\rangle^2 \times \left(\frac{b_{eff}}{8}\right)^2 \ln \frac{L_g^{-1}}{e\Lambda_{QCD}} \ln \frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4y \mathcal{G}(y) D^2(y) =$$
$$= (1 \pm 0.5) \times 10^{-29} \Delta \text{ MeV}^4.$$

where

$$\langle 0| \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x') |0\rangle = \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(0) F^{ik}_a(0) : |0\rangle D(x-x'),$$

$$\Delta = -\frac{1}{L_g^2} \int d^4y \mathcal{G}(y) D^2(y)$$

$$D(x-x') = D_{top}(x-x') - D_h(x-x'), \qquad D(0) = 0.$$

In terms of known NPT QCD parameters

$$1/L_{top} \sim 1/L_h \sim 1/L_g,$$

$$|1/L_{top} - 1/L_h| \sim m_u + m_d + m_s$$

$$\Delta = k \cdot \frac{(m_u + m_d + m_s)^2 L_g^2}{(2\pi)^4} \sim 3 \cdot 10^{-6}$$

!!!

must be established in a dynamical theory of NPT QCD vacuum!

⇩

It is expected to be generated by chiral symmetry breaking

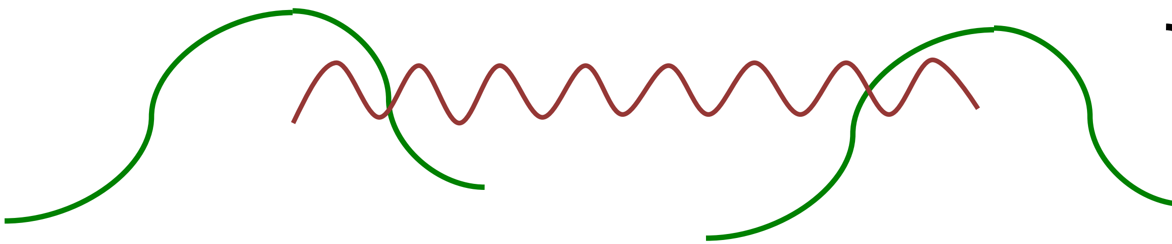
Outlook

"zeroth" order in QG

QCD vacuum

Dynamically cancel exactly in the IR, leading to emergent QCD confinement
(no fine tuning!)

"first" order in QG



virtual (strong) NPT fluctuations of quark and gluon fields dynamically induce metric fluctuations (gravitons)!

cancel NOT exactly!
(due the chiral SB in QCD)

Observable Λ-term!

$$\varepsilon_\Lambda \sim G\Lambda^6_{\rm QCD}$$

$$\Lambda = \frac{m_\pi^6}{(2\pi)^4 M_{Pl}^2} \simeq 2.98 \times 10^{-35}\ {\rm MeV}^4 \qquad m_\pi \simeq 138\ {\rm MeV}$$

$$\Lambda_{\rm exp} = (3.0 \pm 0.7) \times 10^{-35}\ {\rm MeV}^4$$

Only NPT QCD vacuum fluctuations coupled to Gravity at lowest (hadron) scales of Particle Physics gives rise to Λ-term if UV terms are canceled

Summary

Physically motivated conditions for the Zeldovich-Sakharov scenario are formulated.

- ➢ Perturbative part of the Physical Vacuum should be compensated at every known energy scale separately

- ➢ QCD vacuum is a special case: has strongest non-perturbative component (responsible e.g. for color confinement) at the lowest Particle Physics energy scale (~200 MeV)

- ➢ Large negative NPT (chromomagnetic) QCD vacuum component is exactly canceled by the chromoelectric component yielding zero net gluon field density at distances beyond the confinement length-scale

- ➢ A small quantum gravity correction to NPT QCD vacuum fluctuations (graviton exchanges in the vacuum) induces an uncompensated Λ-term. Our estimate based upon phenomenological parameters reproduces the observable value within a factor of few. Uncertainties are due to unknown NPT dynamics of the QCD vacuum in real time.

- ➢ The estimated gravity correction to the QCD ground state is a conventional physics prediction which must be taken into account in any model of (more exotic) Dark Energy

Backup slides

Macroscopic cancellation of QCD vacua

Classical EYM fields equations **with vacuum polarisation:**

$$\frac{6}{\varkappa}\frac{a''}{a^3} = \varepsilon - 3p + 4\bar{\Lambda} + T^{\mu,YM}_{\mu}\,, \qquad T^{\mu,YM}_{\mu} = \frac{33}{16\pi^2}\frac{1}{a^4}\left(A'^2 - A^4\right)$$

$$\frac{\partial}{\partial\eta}\left(A' \ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{QCD})^4}\right) + 2A^3 \ln\frac{6e|A'^2 - A^4|}{a^4(\xi\Lambda_{QCD})^4} = 0.$$

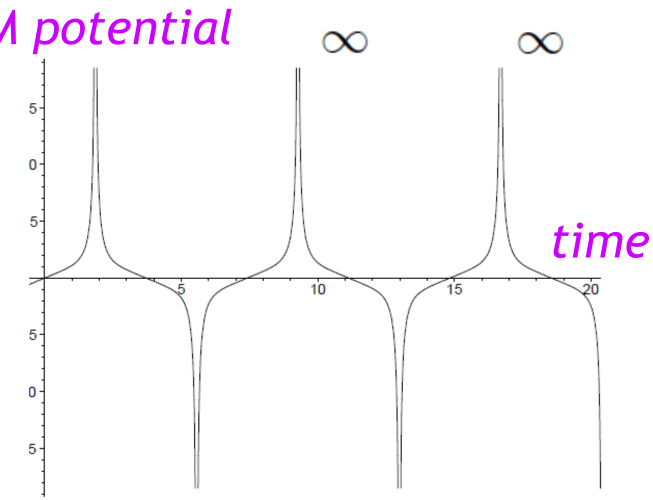
$$\bar{\Lambda} = \Lambda_{inst} + \Lambda_{cosm}$$

instanton QCD vacuum contribution — **observable cosmol. constant**

Exact first integral with POSITIVE constant energy exists!

*Simple! System with **minimal energy**!*

$$\frac{6e(A'^2 - A^4)}{a^4(\xi\Lambda_{QCD})^4} = 1$$



$$T^{0,\,tot}_{0} = T^{0,\,mat}_{0} + \bar{\Lambda} + \frac{33}{64\pi^2}\frac{(\xi\Lambda_{QCD})^4}{6e} \qquad \Lambda_{QCD} \simeq 280\,\mathrm{MeV}$$

$$\frac{33}{64\pi^2}\frac{(\xi\Lambda_{QCD})^4}{6e} + \Lambda_{inst} = 0 \qquad \Lambda_{inst} \simeq -265^4\,\mathrm{MeV}^4 \qquad \xi \simeq 4$$

$$\frac{3}{\varkappa}\frac{a'^2}{a^4} = \varepsilon + \Lambda_{cosm},$$

$$A'^2 - A^4 = a^4\frac{(\xi\Lambda_{QCD})^4}{6e}$$

1) dynamical cancellation of the quantum-topological QCD contribution

2) decoupling of microscopic evolution of YMC and cosmological evolution

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