



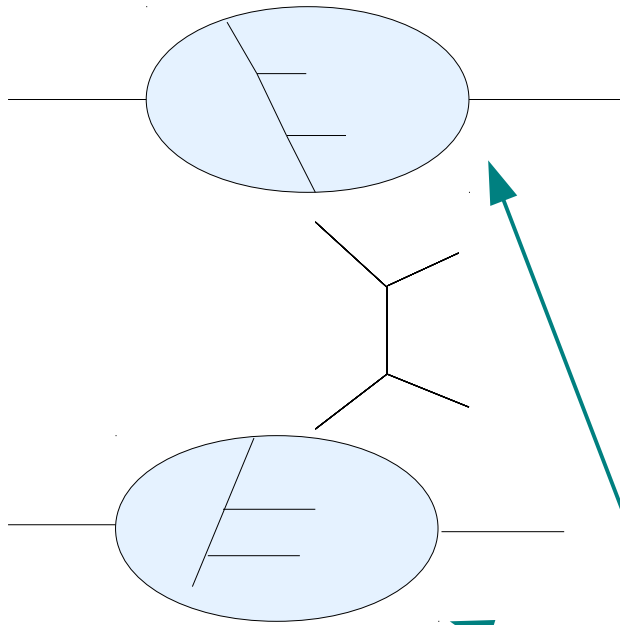
NCN

*Broadening and saturation effects in dijet azimuthal correlations in p-p and p-Pb collisions at  $E= 5.02$  TeV*

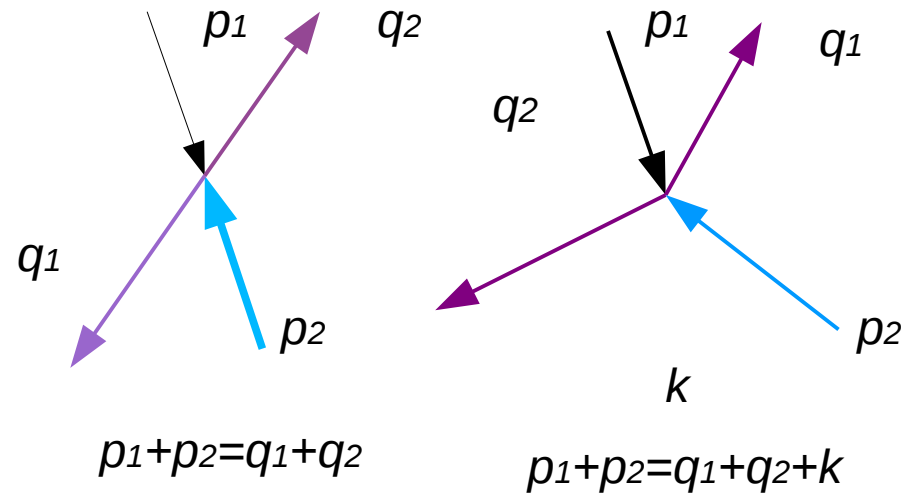
*Krzysztof Kutak*  
*IFJ PAN*

*Based on*  
*Phys.Lett. B795 (2019) 511-515*  
*A. van Hameren, P. Kotko, K. Kutak, S. Sapeta*

# QCD at high energies – high energy factorization



Strongly decreasing  
Longitudinal momentum  
fractions of off-shell partons



$$\frac{d\sigma}{dPS} \propto \mathcal{F}_{a^*}(x_1, k_{\perp 1}) \otimes \hat{\sigma}_{ab \rightarrow cd}(x_1, x_2) \otimes \mathcal{F}_{b^*}(x_2, k_{\perp 2})$$

Ciafaloni, Catani, Hautman '93  
Collins, Ellis '93

New helicity based methods for ME  
Kotko, K.K, van Hameren, '12

# *ITMD: generalization of HEF for forward processes*

*van Hameren, Kotko, Kutak, Sapeta, Petreska '15*

- *accounts for saturation*
- *accounts correctly for gauge structure of the theory*
- *is consistent with Color Glass Condensate in appropriate limit*

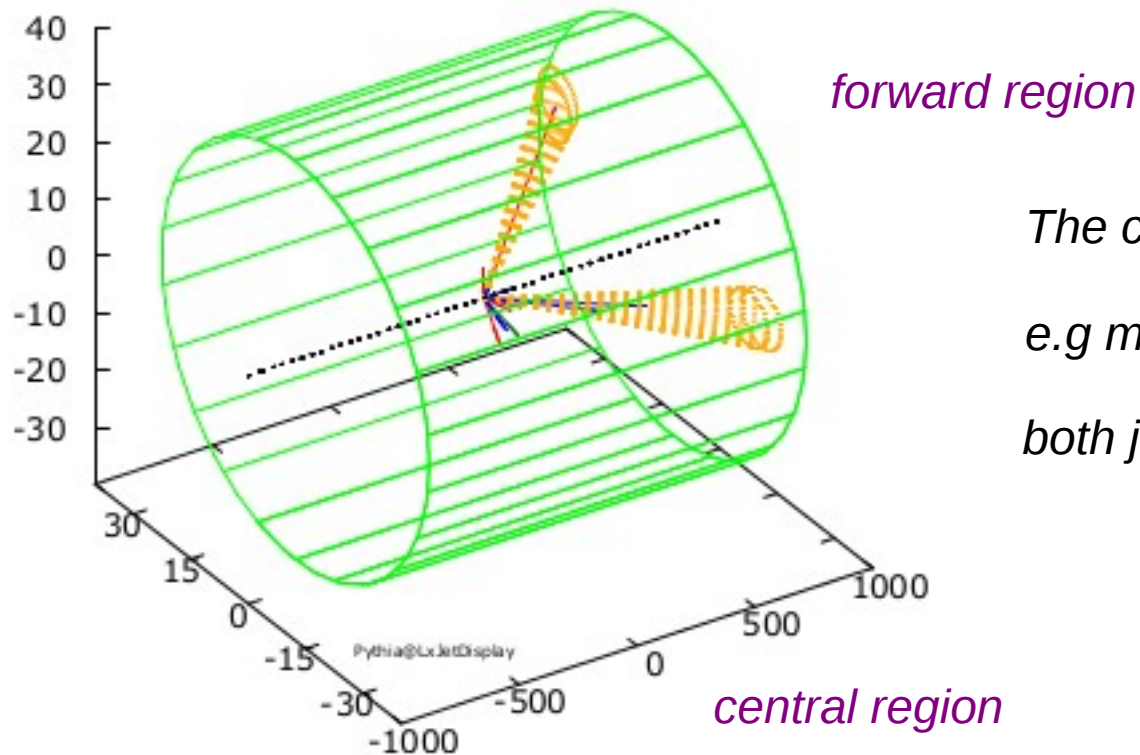
*ITMD = Improved Transversal Momentum Dependent (applied already to dijets (van Hameren, Kotko, Kutak, Sapeta, Petreska'16), particle production (Albacete, Giacalone, Marquet, Matas '18) , UPC processes (Kotko, Sapeta, Staśto, Strikman '17)*

*HEF = High Energy Factorization Catani, Ciafaloni Hautman '93*

See also talk by: *Piotr Kotko*

Poster by: *Andreas van Hameren*

## *Dilute-dense: forward-forward*



*The collisions we consider:*

*e.g minimal  $p_T$  20 GeV*

*both jets are forward  $\rightarrow y > 2.7$*

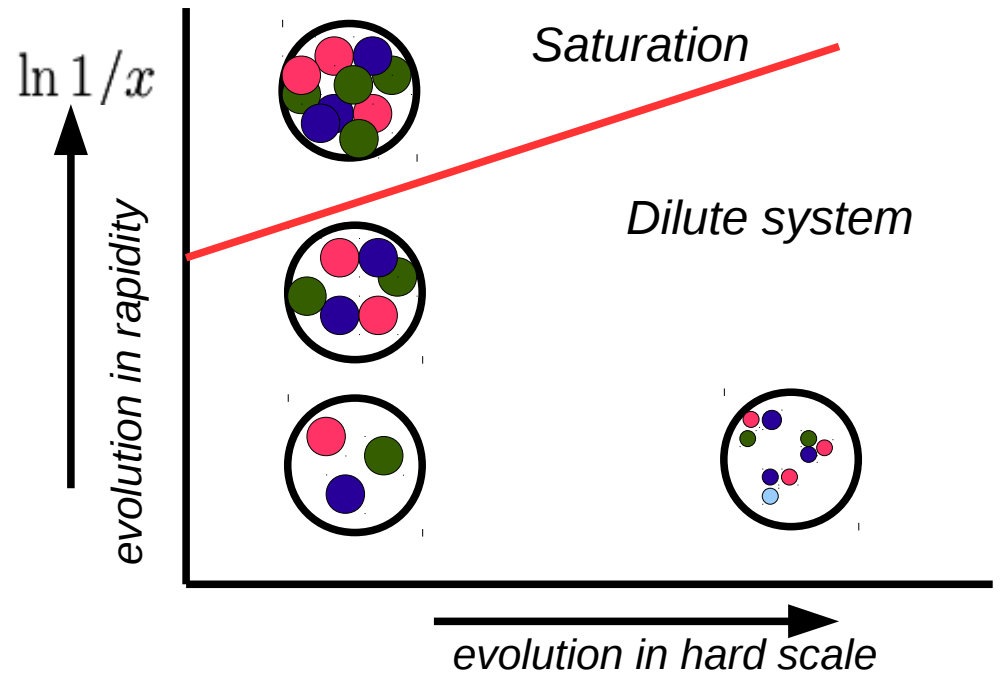
*From: Piotr Kotko  
LxJet*

*There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not*

# Saturation

**Saturation** – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

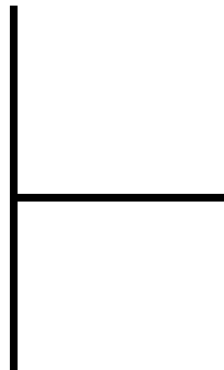
Gribov, Levin, Ryskin '81



On microscopic level it means that gluon apart splitting recombine

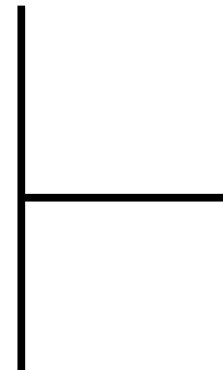
splitting

Linear evolution  
Equation  
BFKL

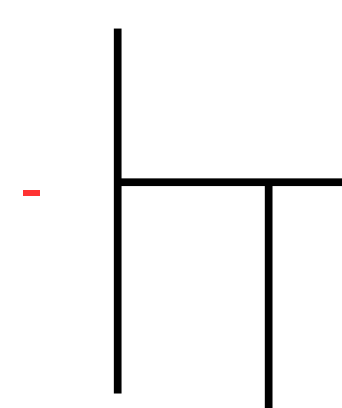


Nonlinear evolution  
equations

splitting



recombination



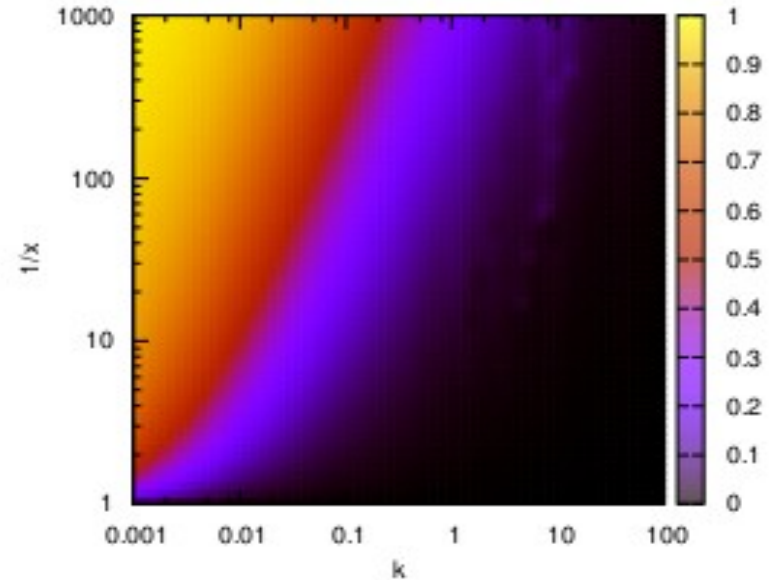
# Saturation

**Saturation** – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

Gribov, Levin, Ryskin '81

Phenomenological status:  
book "QCD at high energy" Kovchegov Levin '12

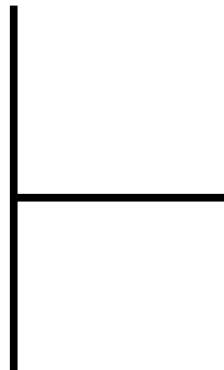
See poster by: M. Hentschinski



On microscopic level it means that  
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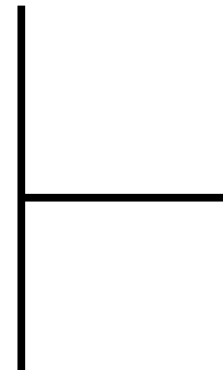
splitting

Linear evolution  
equation  
BFKL

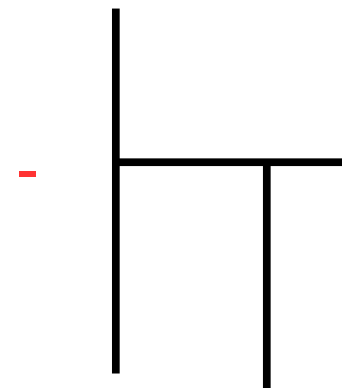


Nonlinear evolution  
equations

splitting



recombination



# The saturation problem: suppressing gluons below $Q_s$

Originally formulated in coordinate space

Balitsky '96, Kovchegov '99

Fit AAMQS '10

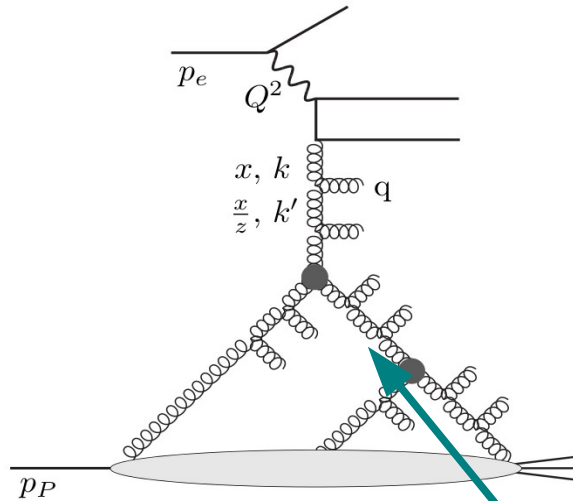
NLO accuracy  
Balitsky, Chirilli '07

and solved  
Lappi, Mantysaari '15

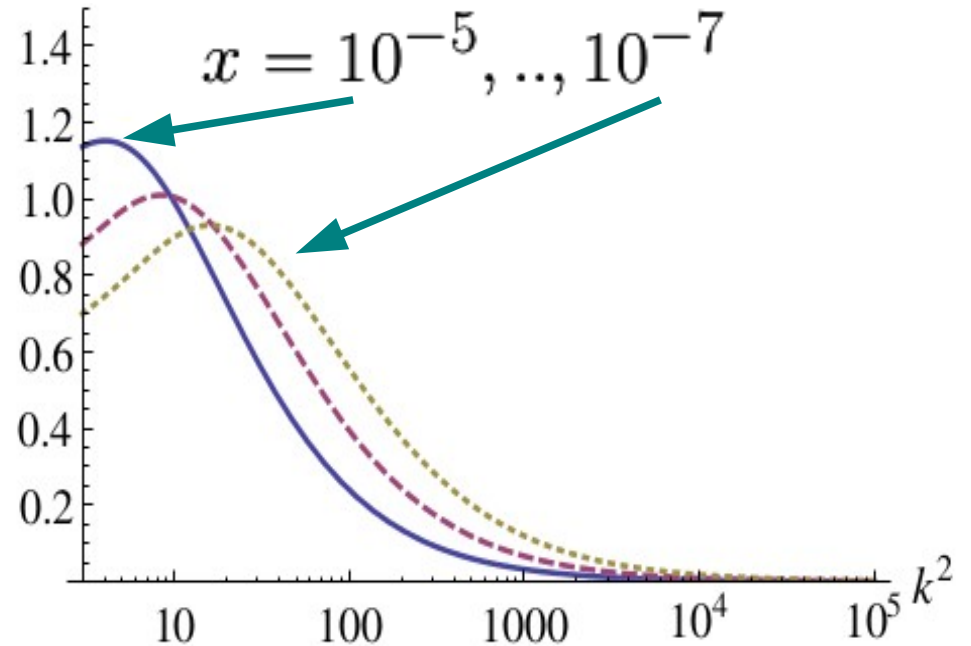
Kinematic corrections  
Iancu et al

Solved  $b$  dependent  
Stasto, Golec-Biernat '02

with kinematic corrections and  $b$   
Cepila, Contrares, Matas '18



Bartels, Wusthoff '95



solution of Balitsky-Kovchegov for gluon density

The BK equation for dipole gluon density

$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F} - \frac{1}{R^2} V \otimes \mathcal{F}^2$$

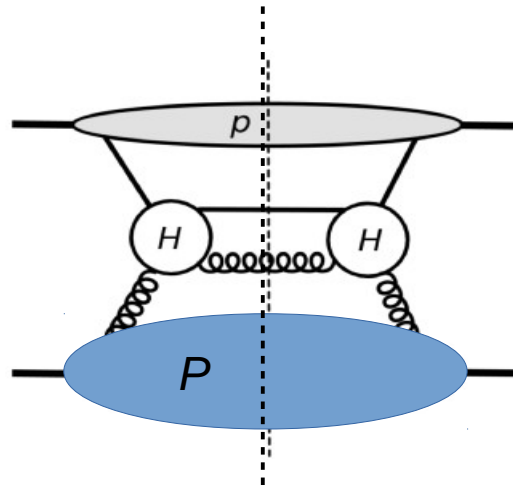
hadron's radius

Kwiecinski, Kutak '02  
Nikolaev, Schafer '06

Fit to  $F_2$  data  
KK. Sapeta '12

## Definition of TMD

The used factorization formula for HEF is strictly valid for large transversal momentum and was obtained in a specific gauge. Ultimately we want to go beyond this

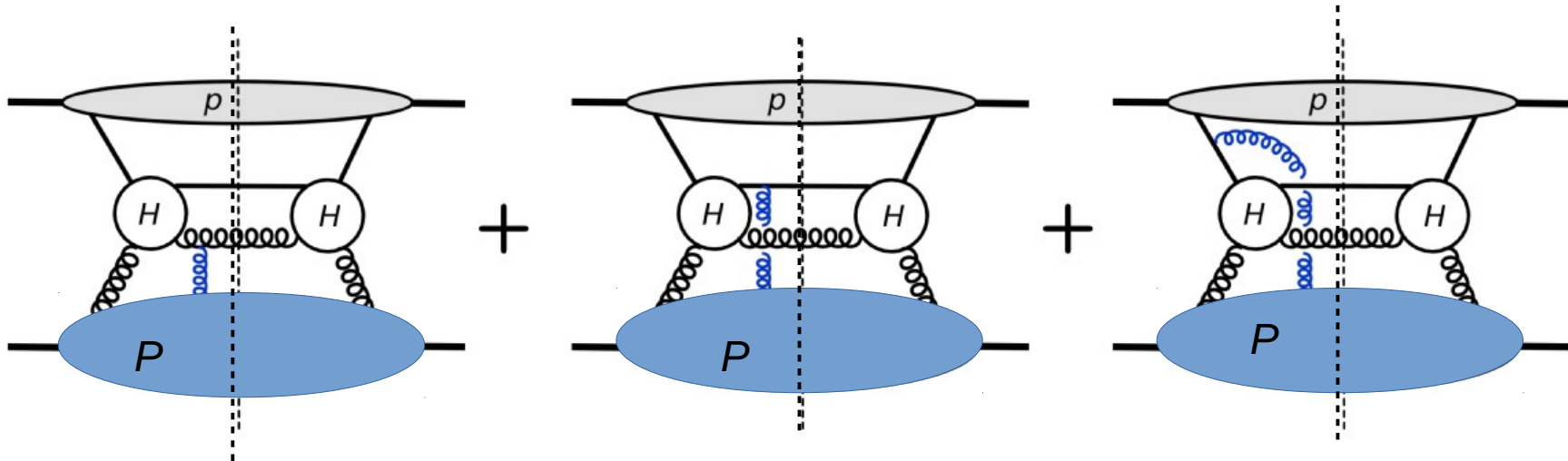


Naive definition

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \hat{F}^{i+}(\xi^+ = 0, \xi^-, \vec{\xi}_T) \right\} | P \rangle$$



# Definition of TMD – gauge links



+ similar diagrams with 2,3,...gluon exchanges.

*Bomhof, Mulders, Pijlman '06*

All this need to be resummed

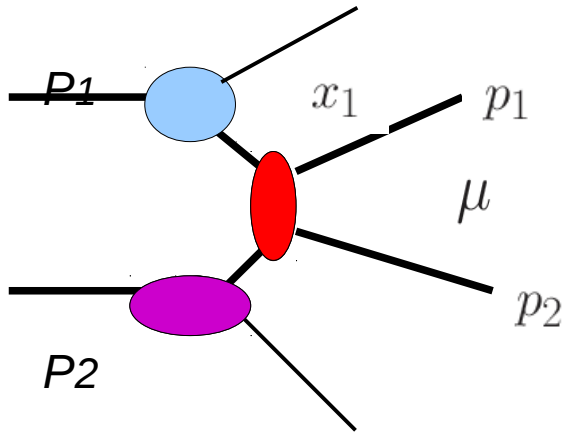
This is achieved via gauge link which renders the gluon density gauge invariant

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \mathcal{U}_{C_1} \hat{F}^{i+}(\xi) \mathcal{U}_{C_2} \right\} | P \rangle$$

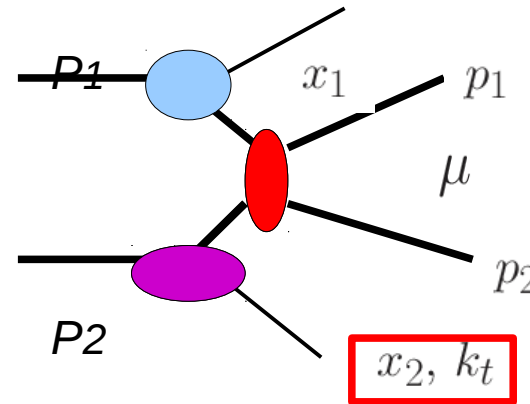
Hard part defines the path of the gauge link

# ITMD for dijets

$$\frac{d\sigma_{\text{SPS}}^{P_1 P_2 \rightarrow \text{dijets} + X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1, \mu^2) |\overline{\mathcal{M}_{ag^* \rightarrow cd}}|^2 \mathcal{F}_{g/P_2}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}$$



can be used for estimates of saturation effects



Generalization but **no possibility to calculate decorrelations** since no  $k_t$  in ME

We found a method to include  $k_t$  in ME and express the factorization formula in terms of gauge invariant sub amplitudes → more direct relation to two fundamental gluon densities: **dipole gluon density** and **Weizacker-Williams gluon density**

Dominguez, Marquet, Xiao, Yuan '11

Conjecture Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren '15  
Proof Altinoluk, Bousarie, Kotko '19 see also Altinoluk, Bousarie '19

gauge invariant amplitudes and TMDs

$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)} \Phi_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}$$

## ITMD formula

Here presented just for gluons

$$d\sigma \sim \vec{A}^\dagger \mathbf{\Phi}_{gg \rightarrow gg} \vec{A}$$

$$\mathbf{\Phi}_{gg \rightarrow gg} = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_2 & \Phi_1 \end{pmatrix}$$

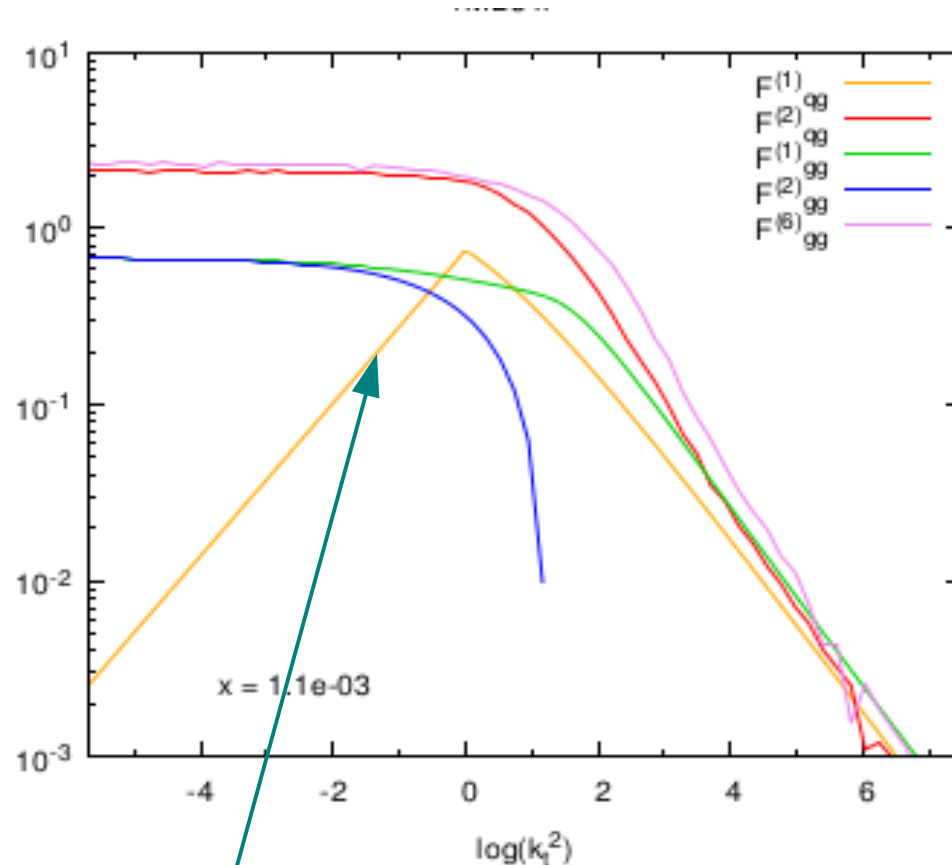
$$\Phi_1 = \frac{1}{2N_c^2} \left( N_c^2 \mathcal{F}_{gg}^{(1)} - 2\mathcal{F}_{gg}^{(3)} + \mathcal{F}_{gg}^{(4)} + \mathcal{F}_{gg}^{(5)} + N_c^2 \mathcal{F}_{gg}^{(6)} \right)$$

$$\Phi_2 = \frac{1}{N_c^2} \left( N_c^2 \mathcal{F}_{gg}^{(2)} - 2\mathcal{F}_{gg}^{(3)} + \mathcal{F}_{gg}^{(4)} + \mathcal{F}_{gg}^{(5)} + N_c^2 \mathcal{F}_{gg}^{(6)} \right)$$

Was extended to 3 and 4 jet final states [KK, Bury, Kotko' 18](#)

# Plots of ITMD gluons

Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren '16



Similar structure obtained  
from solutions of JIMWLK  
Marquet, Petreska, Roiesnel '16

*Standard HEF gluon density*

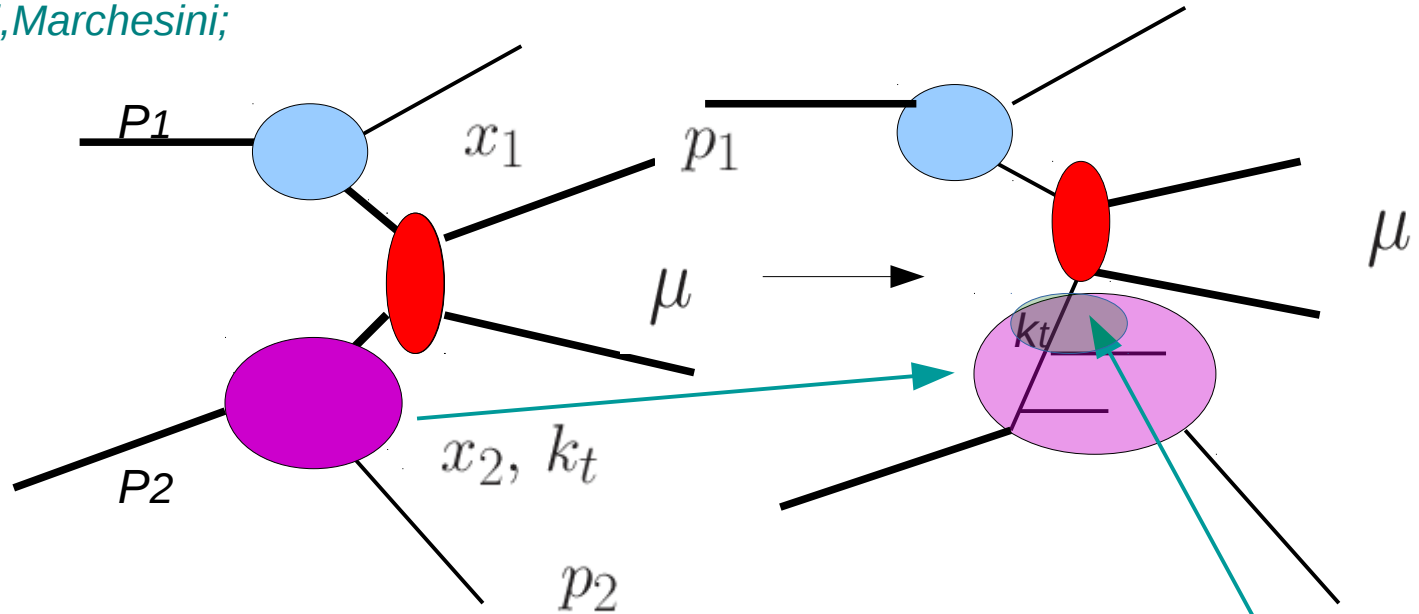
*The other densities are flat at low  $kt \rightarrow$  less saturation*

*Not negligible differences at large  $kt \rightarrow$  differences at small angles*

# Other relevant effects – Sudakov form factor in ISR

The relevance in low  $x$  physics  
 at linear level recognized by:  
 Catani, Ciafaloni, Fiorani, Marchesini;  
 Kimber, Martin, Ryskin;  
 Collins, Jung

Survival probability  
 of the gap without  
 emissions



If hard scale is larger than  $k_t$  the phase space  
 opens for hard scale resummation

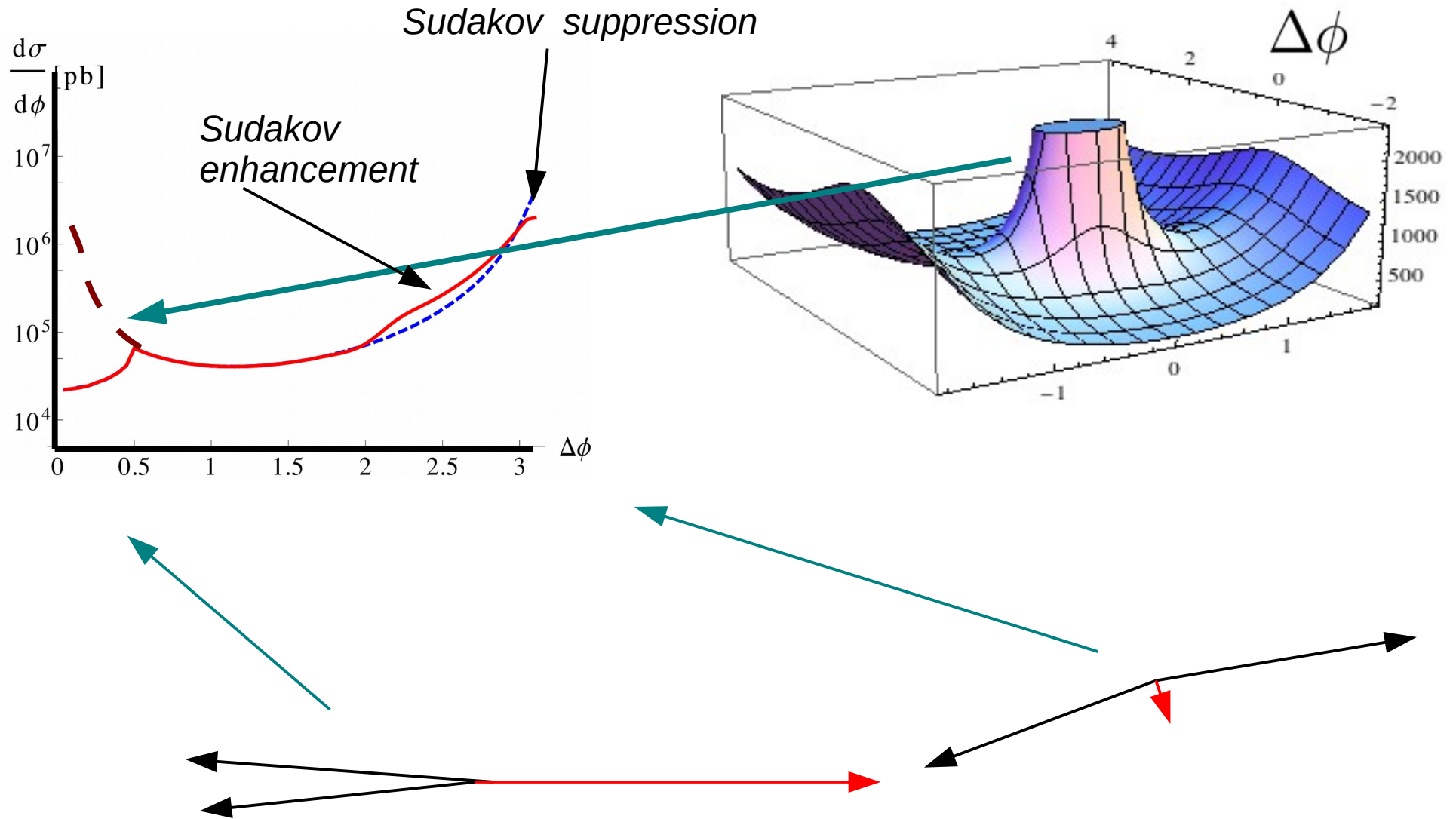
Survival probability of the gap  
 without emissions

Mueller, Xiao, Yuan '12  
 Mueller, Xiao, Yan '13  
 Van Hameren, Kotko, Kutak, Sapeta '14  
 Xiao, Yuan, Zhou '17  
 Zhou '16  
 Kutak '14

# Other relevant effects – Sudakov form factor in ISR

Divergence regularized  
by jet algorithm

K.K '14



# *ITMD with Monte Carlo tools*

*KaTie* (A. van Hameren)

- *complete Monte Carlo program for tree-level calculations*
- *any process within the Standard Model (High Energy Factorization)*
- *any initial-state partons on-shell or off-shell*
- *employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes*
- *automatic phase space optimization*

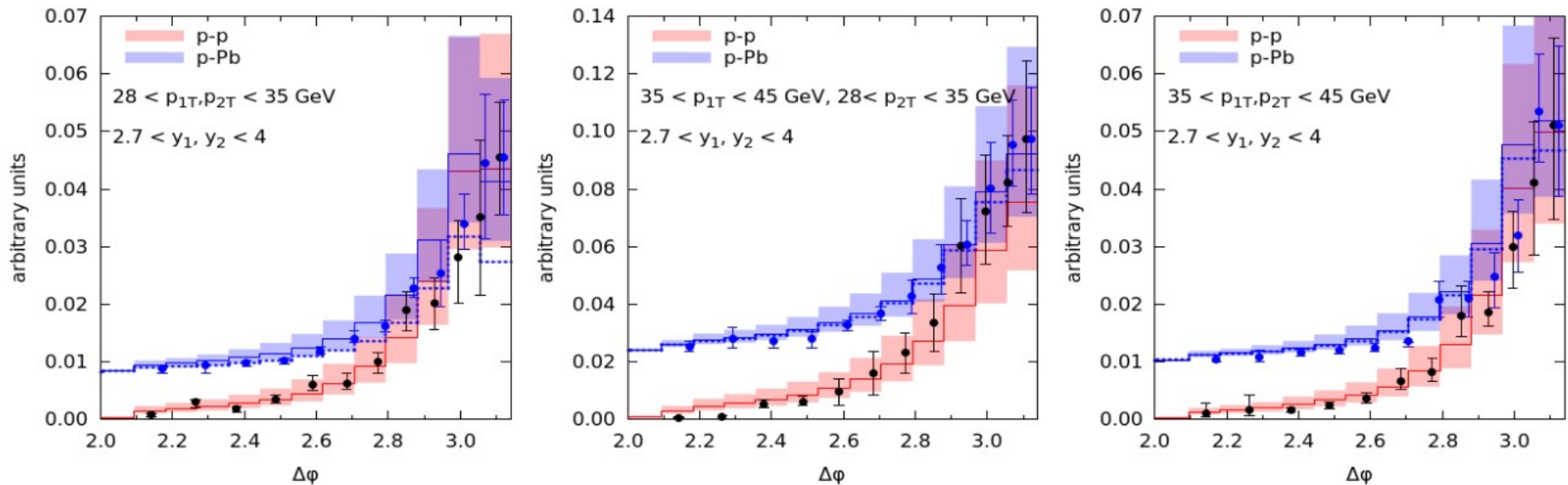
*LxJet* (P. Kotko)

*Specialized Monte Carlo for di-jets and tri-jets for ITMD and HEF processes*

# Signature of saturation in forward-forward dijets

ATLAS 1901.10440

van Hameren, Kotko, Kutak, Sapeta '19



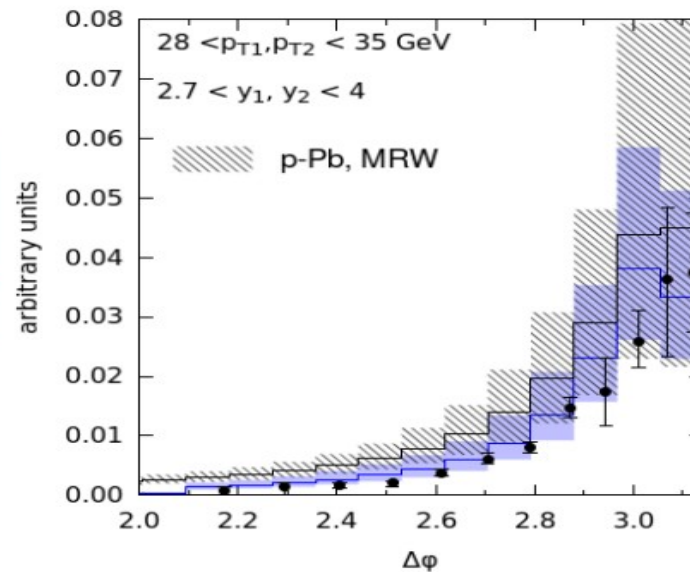
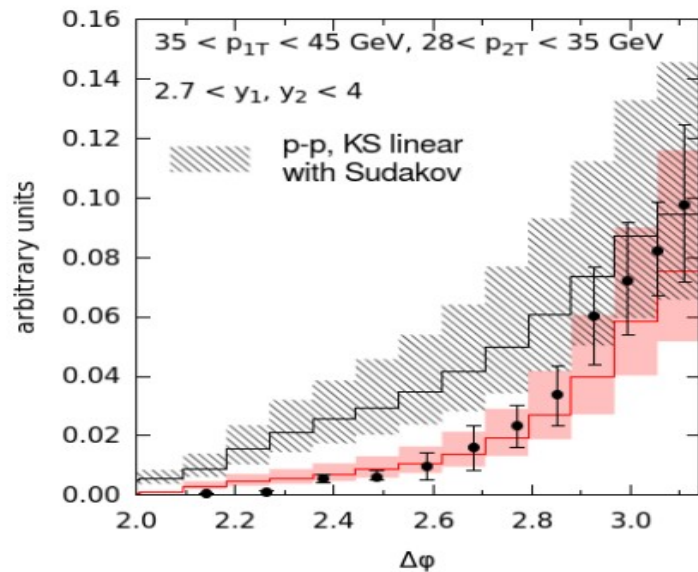
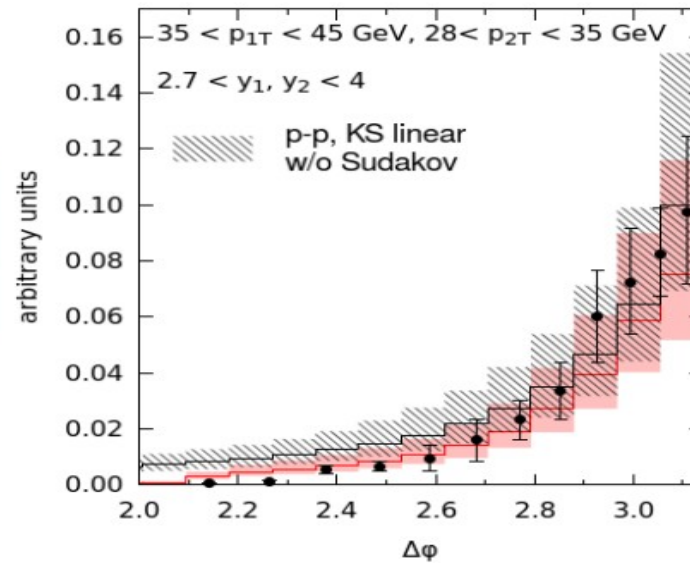
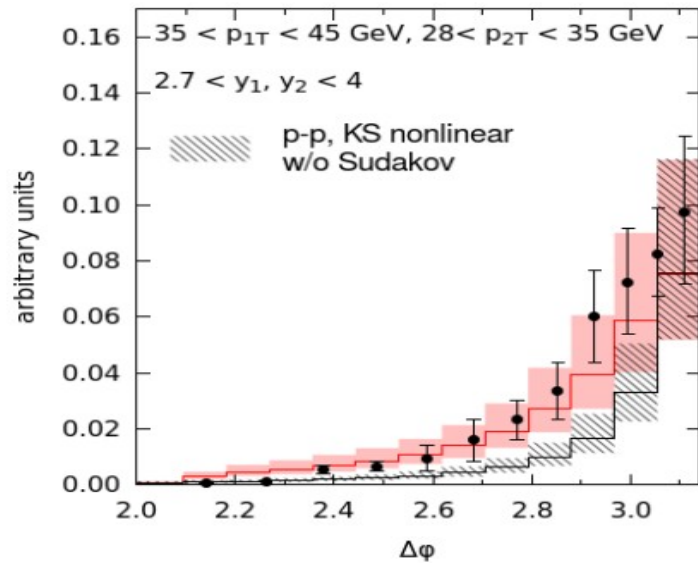
*Data: number of dijets normalized to number of single inclusive jets. We can not calculate that. We can compare shapes.*

*Procedure: fit normalization to p-p data. Use that both for p-p and p-Pb. Shift p-Pb data*

*The procedure allows for visualization of broadening*



# Other possible scenarios



# Summary

*New evidence for saturation*

*Necessity to have both Sudakov resummation and nonlinearities*

*ITMD is not anymore a conjecture – proof from CGC*

# Set of basic TMD's – for any final state jets

KK, Bury, Kotko'  
18

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]}] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \frac{1}{N_c} \left\langle \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger}] \text{Tr} [\hat{F}^{i+}(0) \mathcal{U}^{[\square]}] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]}] \right\rangle,$$

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[-]}] \right\rangle,$$

$$\mathcal{F}_{gg}^{(5)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]}] \right\rangle,$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]}] \right\rangle,$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = 2 \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]}] \right\rangle.$$

# ITMD for 3 jets – basic TMDS

KK, Bury, Kotko'  
18

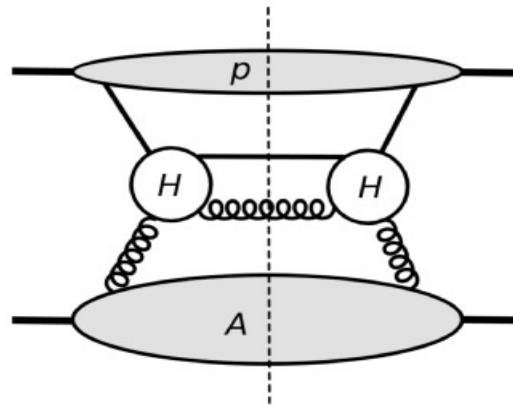
$$\Phi_{gg \rightarrow ggg} = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4^* \\ \Phi_2 & \Phi_1 & \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_3 \\ \Phi_2 & \Phi_3 & \Phi_1 & \Phi_2 & \Phi_4^* & \Phi_3 \\ \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_1 & \Phi_3 & \Phi_2 \\ \Phi_3 & \Phi_2 & \Phi_4^* & \Phi_3 & \Phi_1 & \Phi_2 \\ \Phi_4^* & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}$$

$$\Phi_1 = \frac{1}{4N_c^2} \left( (N_c^2 + 2)\mathcal{F}_{gg}^{(1)} - 4\mathcal{F}_{gg}^{(2)} - 4\mathcal{F}_{gg}^{(3)} + 3N_c^2\mathcal{F}_{gg}^{(6)} + 2\mathcal{F}_{gg}^{(7)} \right),$$

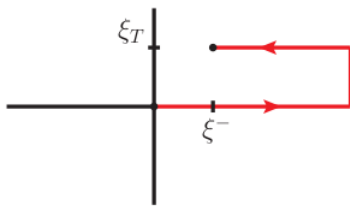
$$\Phi_2 = -\frac{1}{2N_c^2} \left( -2\mathcal{F}_{gg}^{(1)} + 4\mathcal{F}_{gg}^{(2)} + 2\mathcal{F}_{gg}^{(3)} + \mathcal{F}_{gg}^{(4)} + \mathcal{F}_{gg}^{(5)} - 2N_c^2\mathcal{F}_{gg}^{(6)} - 2\mathcal{F}_{gg}^{(7)} \right)$$

⋮

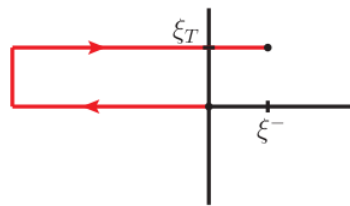
# Backup - Definition of TMD



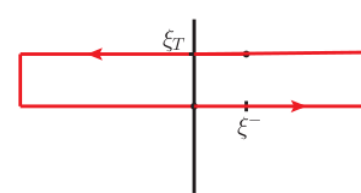
$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \mathcal{U}_{C_1} \hat{F}^{i+}(\xi) \mathcal{U}_{C_2} \right\} | P \rangle$$



$u^{[+]}$



$u^{[-]}$



$u^{[\square]} = u^{[-]\dagger} u^{[+]}$