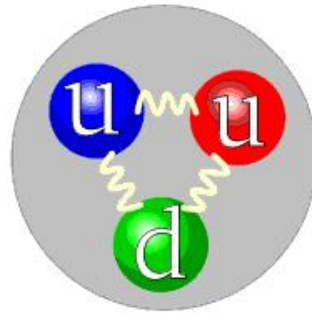


QGP production from the quantum ground-state of QCD: Hamiltonian picture

beautiful math or “New Physics” of QCD?



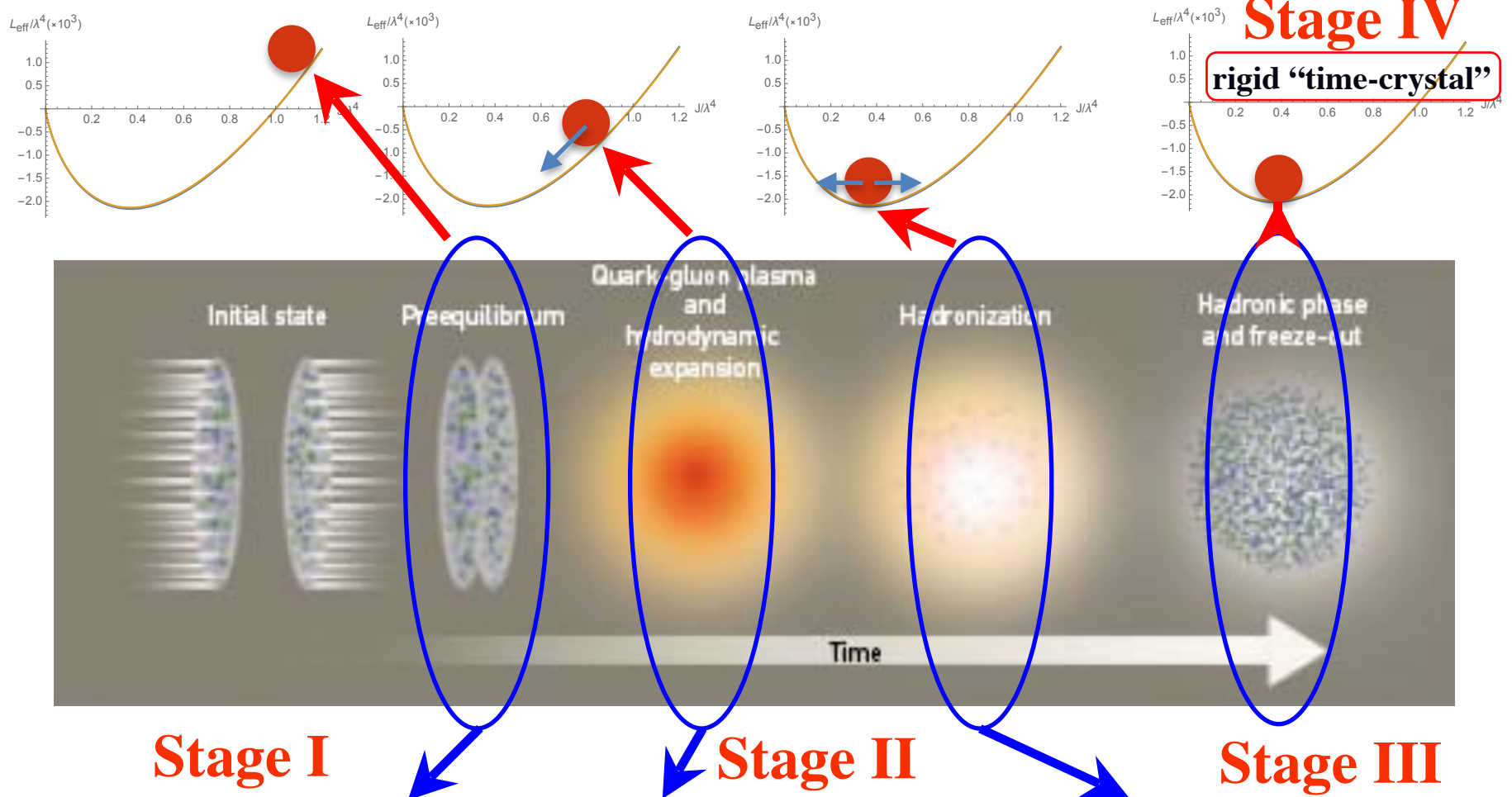
Roman Pasechnik

*“The greatest adventure my generation will ever have
- the confined field theory of QCD”*

Bo Andersson

EPS-HEP 2019, Ghent

Outlook: stages of the “micro Big Bang”

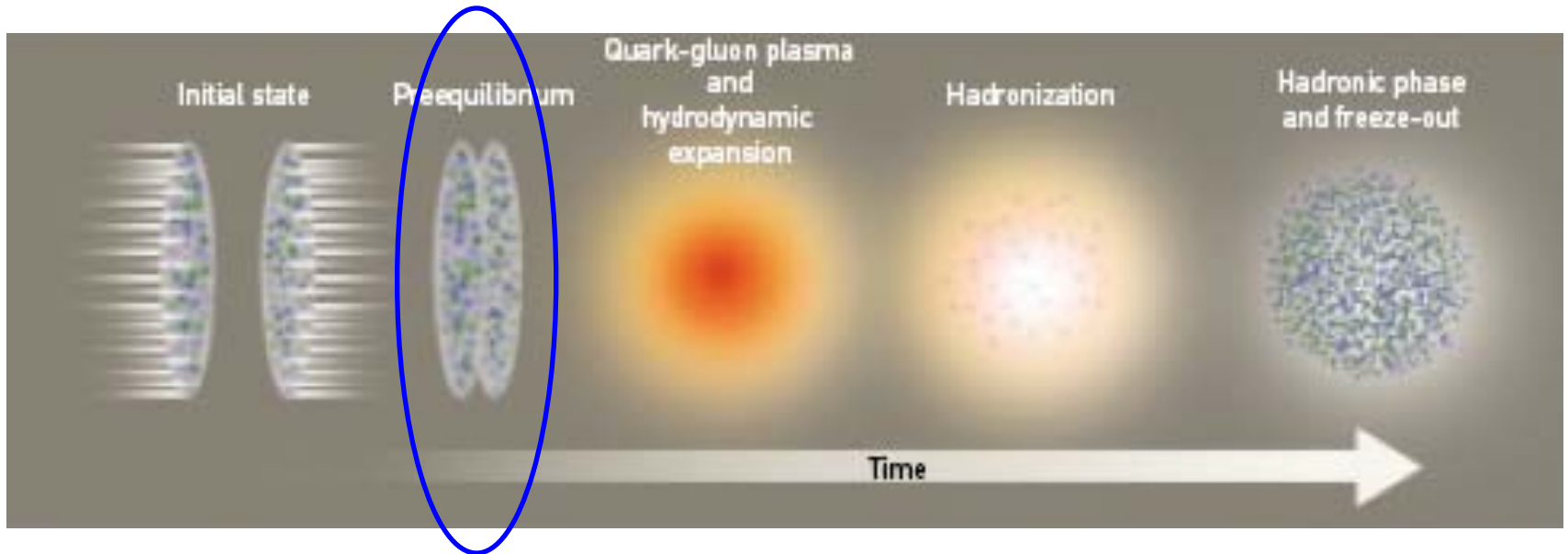


- (almost) classical homogeneous CE gluon condensate evolution
- small inhomogeneities
- perturbative regime (short distances)

- energy “swap” from condensate to the fluctuations (quasiclassical pic.)
- large inhomogeneities (plasma modes)
- parametric resonance effect (particle production mechanism)

- quantum ground state formation (CE mostly + initiation of CM)
- large distances/essentially quantum dynamics
- domain-wall formation

Stage I



Homogeneous gluon condensate: semi-classics

R. Pasechnik, G. Prokhorov & G. Vereshkov JHEP '14

Corrections are small

for $g_{\text{YM}} \ll 1$

(short distances!)

Classical YM Lagrangian:

$$\mathcal{L}_{\text{cl}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_{\text{YM}} f^{abc} A_\mu^b A_\nu^c$$

Basis for canonical (Hamiltonian) quantisation of “condensate+waves” system:

temporal (Hamilton)
gauge

$$A_0^a = 0$$

$$e_i^a A_k^a \equiv A_{ik}$$

$$e_i^a e_k^a = \delta_{ik}$$

$$e_i^a e_i^b = \delta_{ab}$$

due to local $\text{SU}(2) \sim \text{SO}(3)$ isomorphism

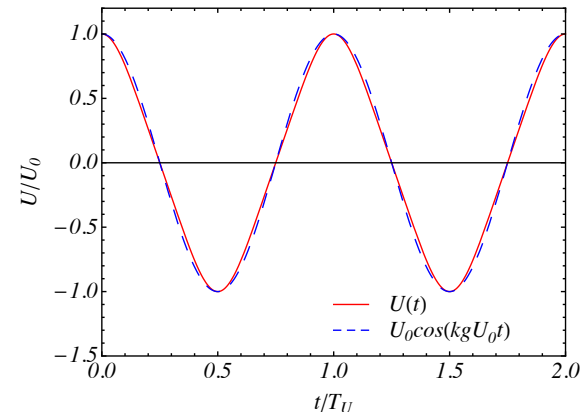
$$A_{ik}(t, \vec{x}) = \delta_{ik} U(t) + \tilde{A}_{ik}(t, \vec{x})$$

$$U(t) \equiv \frac{1}{3} \delta_{ik} \langle A_{ik}(t, \vec{x}) \rangle_{\vec{x}}, \quad \langle A_{ik}(t, \vec{x}) \rangle_{\vec{x}} = \frac{\int_{\Omega} d^3x A_{ik}(t, \vec{x})}{\int_{\Omega} d^3x}$$

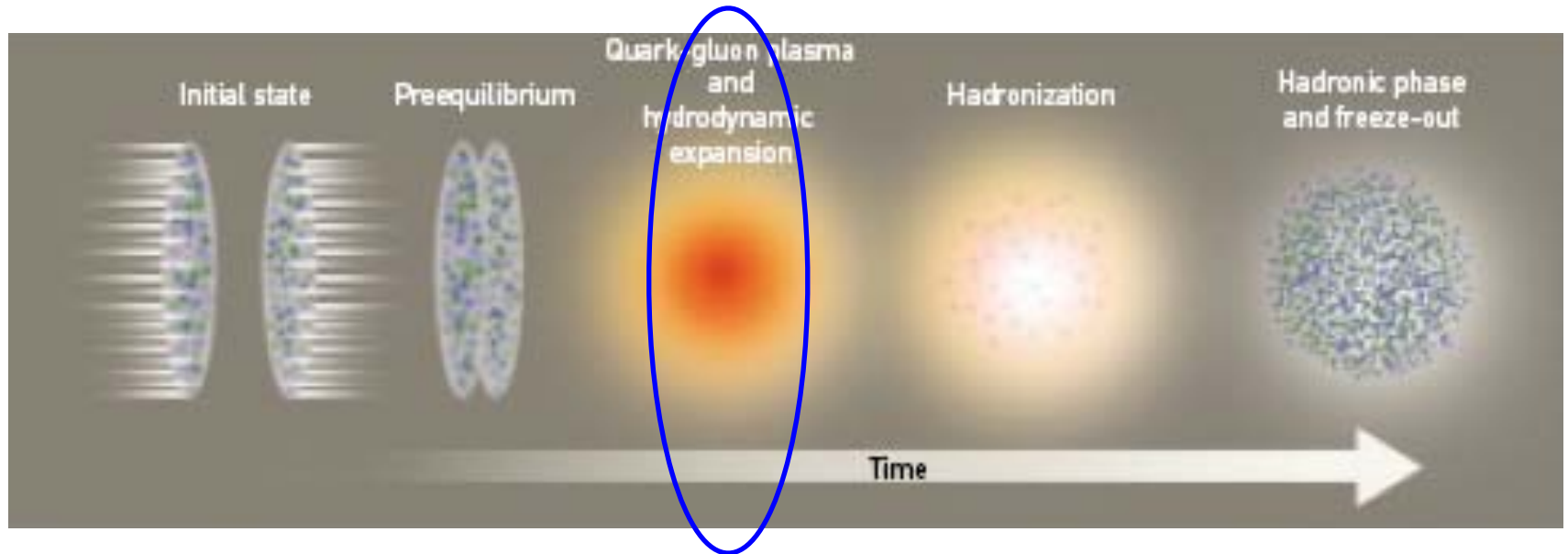
Zeroth-order in waves = “pre-equilibrium state”?

$$\mathcal{H}_{\text{YM}} \simeq \mathcal{H}_{\text{YMC}} = \frac{3}{2} \left[(\partial_0 U)^2 + g^2 U^4 \right], \quad \partial_0 \partial_0 U + 2g^2 U^3 = 0$$

$$t = - \int_{U_0}^U \frac{dU}{\sqrt{g^2 U_0^4 - g^2 U^4}}, \quad U(0) = U_0, \quad U'(0) = 0$$



Stage II



Condensate+waves semi-classical system

R. Pasechnik, G. Prokhorov & G. Vereshkov JHEP '14

“condensate+waves” system evolution:

$$\begin{aligned}
 & -\delta_{lk}(\partial_0\partial_0 U + 2g^2 U^3) + (-\partial_0\partial_0 \tilde{A}_{lk} + \partial_i\partial_i \tilde{A}_{lk} - \partial_i\partial_k \tilde{A}_{li} - g_{elmk}\partial_i \tilde{A}_{mi} U - 2g_{elip}\partial_i \tilde{A}_{pk} U \\
 & - g_{elmi}\partial_k \tilde{A}_{mi} U + g^2 \tilde{A}_{kl} U^2 - g^2 \tilde{A}_{lk} U^2 - 2g^2 \delta_{lk} \tilde{A}_{ii} U^2) + (-g_{elmp}\partial_i \tilde{A}_{mi} \tilde{A}_{pk} \\
 & - 2g_{elmp} \tilde{A}_{mi} \partial_i \tilde{A}_{pk} - g_{elmp}\partial_k \tilde{A}_{mi} \tilde{A}_{pi} + g^2 \tilde{A}_{li} \tilde{A}_{ik} U + g^2 \tilde{A}_{li} \tilde{A}_{ki} U + g^2 \tilde{A}_{ik} \tilde{A}_{il} U \\
 & - 2g^2 \tilde{A}_{ii} \tilde{A}_{lk} U - g^2 \delta_{lk} \tilde{A}_{pi} \tilde{A}_{pi} U) + g^2 (\tilde{A}_{li} \tilde{A}_{pk} \tilde{A}_{pi} - \tilde{A}_{pi} \tilde{A}_{pi} \tilde{A}_{lk}) = 0
 \end{aligned}$$

tensor basis decomposition

$$\chi_l^{\vec{p}} = s_l^\sigma \eta_\sigma^{\vec{p}} + n_l \lambda^{\vec{p}}$$

$$\tilde{A}_{ik} = \psi_{ik} + e_{ikl} \chi_l$$

$$\psi_{ik}^{\vec{p}} = \psi_\lambda^{\vec{p}} Q_{ik}^\lambda + \varphi_\sigma^{\vec{p}} (n_i s_k^\sigma + n_k s_i^\sigma) + (\delta_{ik} - n_i n_k) \Phi^{\vec{p}} + n_i n_k \Lambda^{\vec{p}}$$

Full Hamiltonian

$$\begin{aligned}
 \mathcal{H}_{\text{YM}}^{\text{waves}} = & \frac{1}{2} \left\{ \partial_0 \psi_\lambda \partial_0 \psi_\lambda^\dagger + \partial_0 \phi_\sigma \partial_0 \phi_\sigma^\dagger + \partial_0 \Phi \partial_0 \Phi^\dagger + \frac{1}{2} \partial_0 \Lambda \partial_0 \Lambda^\dagger + \partial_0 \eta_\sigma \partial_0 \eta_\sigma^\dagger \right. \\
 & + \partial_0 \lambda \partial_0 \lambda^\dagger + p^2 \psi_\lambda \psi_\lambda^\dagger + \frac{p^2}{2} \phi_\sigma \phi_\sigma^\dagger + p^2 \Phi \Phi^\dagger + \frac{p^2}{2} \eta_\sigma \eta_\sigma^\dagger + p^2 \lambda \lambda^\dagger \\
 & - \frac{p^2}{2} e^{\gamma\sigma} (\eta_\sigma \phi_\gamma^\dagger + \phi_\gamma \eta_\sigma^\dagger) + igp U e^{\sigma\gamma} \eta_\sigma \eta_\gamma^\dagger - igp U Q^{\lambda\gamma} \psi_\lambda \psi_\gamma^\dagger \\
 & - igp U e^{\sigma\gamma} \phi_\sigma \phi_\gamma^\dagger - igp U (2\Phi \lambda^\dagger - 2\lambda \Phi^\dagger + \Lambda \lambda^\dagger - \lambda \Lambda^\dagger) \\
 & \left. + 2g^2 U^2 \eta_\sigma \eta_\sigma^\dagger + 2g^2 U^2 \lambda \lambda^\dagger + g^2 U^2 (4\Phi \Phi^\dagger + 2\Phi \Lambda^\dagger + 2\Lambda \Phi^\dagger + \Lambda \Lambda^\dagger) \right\}
 \end{aligned}$$

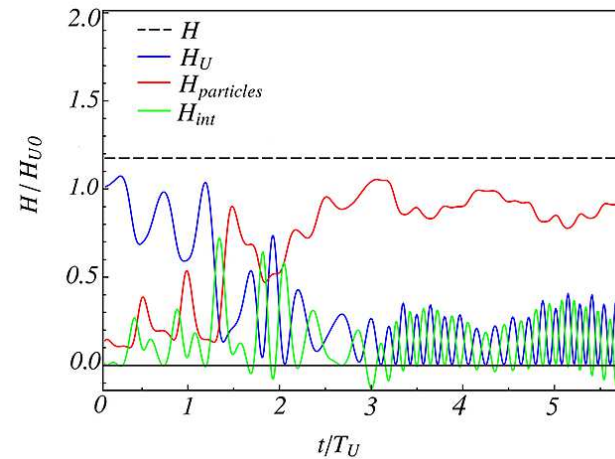
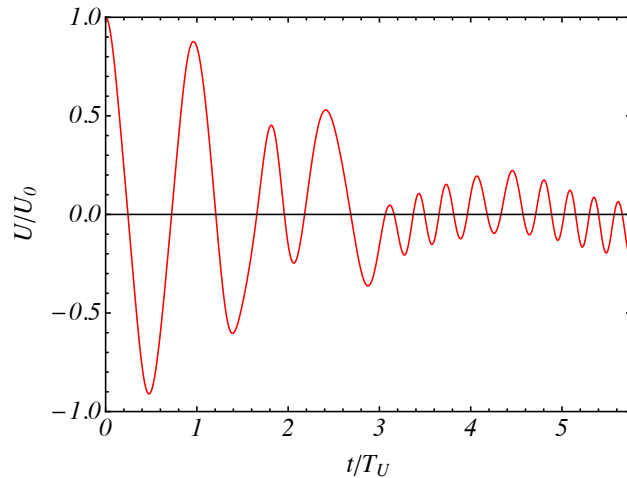
$$\mathcal{H}_{\text{YM}} = \mathcal{H}_{\text{YMC}} + \sum_{\vec{p}} \mathcal{H}_{\text{YM}}^{\text{waves}}$$

Longitudinally polarised (plasma) mode becomes physical due to interactions with the homogeneous condensate!

Decay of the homogeneous condensate

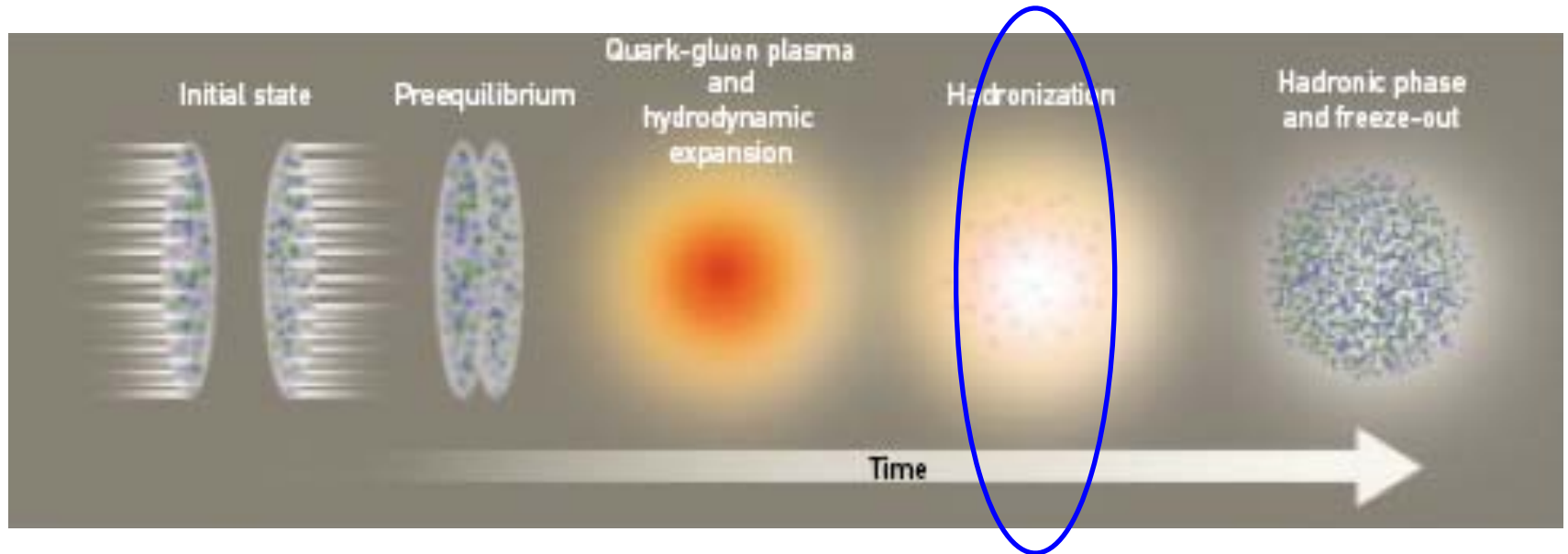
R. Pasechnik, G. Prokhorov & G. Vereshkov JHEP '14

$$\begin{aligned}
 \mathcal{H}_U &= \frac{3}{2} (\partial_0 U \partial_0 U + g^2 U^4), \\
 \mathcal{H}_{\text{particles}} &= \frac{1}{2} \sum_{\vec{p}} \left(\partial_0 \psi_\lambda \partial_0 \psi_\lambda^\dagger + \partial_0 \phi_\sigma \partial_0 \phi_\sigma^\dagger + \partial_0 \Phi \partial_0 \Phi^\dagger + \frac{1}{2} \partial_0 \Lambda \partial_0 \Lambda^\dagger + \partial_0 \eta_\sigma \partial_0 \eta_\sigma^\dagger \right. \\
 &\quad + \partial_0 \lambda \partial_0 \lambda^\dagger + p^2 \psi_\lambda \psi_\lambda^\dagger + \frac{p^2}{2} \phi_\sigma \phi_\sigma^\dagger + p^2 \Phi \Phi^\dagger + \frac{p^2}{2} \eta_\sigma \eta_\sigma^\dagger + p^2 \lambda \lambda^\dagger \\
 &\quad \left. - \frac{p^2}{2} e^{\gamma\sigma} (\eta_\sigma \phi_\gamma^\dagger + \phi_\gamma \eta_\sigma^\dagger) \right), \\
 \mathcal{H}_{\text{int}} &= \frac{1}{2} \sum_{\vec{p}} \left[igp U e^{\sigma\gamma} \eta_\sigma \eta_\gamma^\dagger - igp U Q^{\lambda\gamma} \psi_\lambda \psi_\gamma^\dagger \right. \\
 &\quad - igp U e^{\sigma\gamma} \phi_\sigma \phi_\gamma^\dagger - igp U (2\Phi \lambda^\dagger - 2\lambda \Phi^\dagger + \Lambda \lambda^\dagger - \lambda \Lambda^\dagger) \\
 &\quad \left. + 2g^2 U^2 \eta_\sigma \eta_\sigma^\dagger + 2g^2 U^2 \lambda \lambda^\dagger + g^2 U^2 (4\Phi \Phi^\dagger + 2\Phi \Lambda^\dagger + 2\Lambda \Phi^\dagger + \Lambda \Lambda^\dagger) \right].
 \end{aligned}$$



Ultra-relativistic gluon plasma production!

Stage III



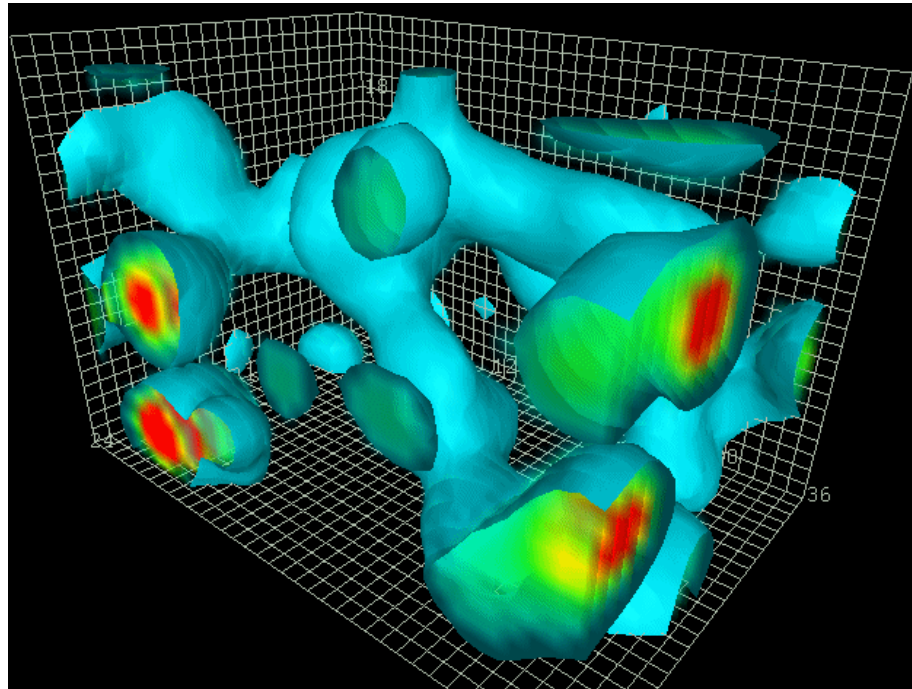
Long distances: chromo-magnetic condensate

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32} \langle 0 | : \frac{\alpha_s}{\pi} F_{ik}^a(x) F_a^{ik}(x) : | 0 \rangle + \frac{1}{4} (\langle 0 | : m_u \bar{u}u : | 0 \rangle + \langle 0 | : m_d \bar{d}d : | 0 \rangle + \langle 0 | : m_s \bar{s}s : | 0 \rangle)$$
$$\simeq -(5 \pm 1) \times 10^9 \text{ MeV}^4.$$

CM condensate:

$$\epsilon_{vac} \sim 10^{-2} \text{ GeV}^4$$



Ground-state at long distances:

$$\Lambda_{\text{cosm}} \sim 10^{-47} \text{ GeV}^4$$

Vacuum in QCD has incredibly wrong energy scale... or

We must be missing something very important!?

Effective YM action

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

**At least, for SU(2) gauge symmetry,
the all-loop and one-loop effective Lagrangians
are practically indistinguishable (by FRG approach)**

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

$$\mathcal{L}_{\text{cl}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \quad \begin{aligned} \mathcal{A}_\mu^a &\equiv g_{\text{YM}} A_\mu^a \\ \mathcal{F}_{\mu\nu}^a &\equiv g_{\text{YM}} F_{\mu\nu}^a \end{aligned}$$



Effective YM Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \quad \mathcal{J} = -\mathcal{F}_{\mu\nu}^a \mathcal{F}_a^{\mu\nu}$$



The energy-momentum tensor:

$$T_\mu^\nu = \frac{1}{\bar{g}^2} \left[\frac{\beta(\bar{g}^2)}{2} - 1 \right] \left(\mathcal{F}_{\mu\lambda}^a \mathcal{F}_a^{\nu\lambda} + \frac{1}{4} \delta_\mu^\nu \mathcal{J} \right) - \delta_\mu^\nu \frac{\beta(\bar{g}^2)}{8\bar{g}^2} \mathcal{J}$$

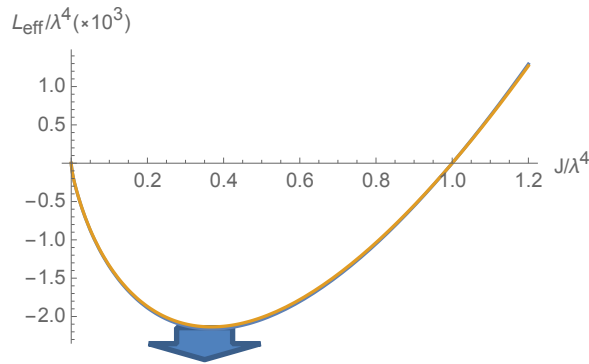
Equations of motion:

$$\begin{aligned} \vec{\mathcal{D}}_\nu^{ab} \left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2} \left(1 - \frac{\beta(\bar{g}^2)}{2} \right) \right] &= 0, \\ \vec{\mathcal{D}}_\nu^{ab} &\equiv \left(\delta^{ab} \vec{\partial}_\nu - f^{abc} \mathcal{A}_\nu^c \right), \end{aligned}$$

trace anomaly:

$$T_\mu^\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2} \mathcal{J}$$

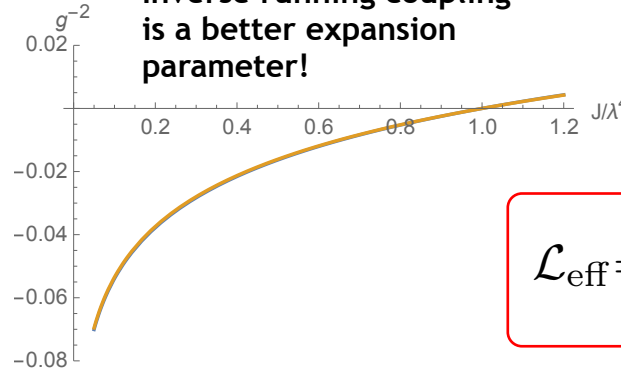
Effective Lagrangian:



chromoelectric (CE) condensate $\mathcal{J}^* > 0$
(Savvidy vacuum)

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

**Inverse running coupling
is a better expansion
parameter!**



NOTE: the RG equation

$$\frac{d \ln |\bar{g}^2|}{d \ln |\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

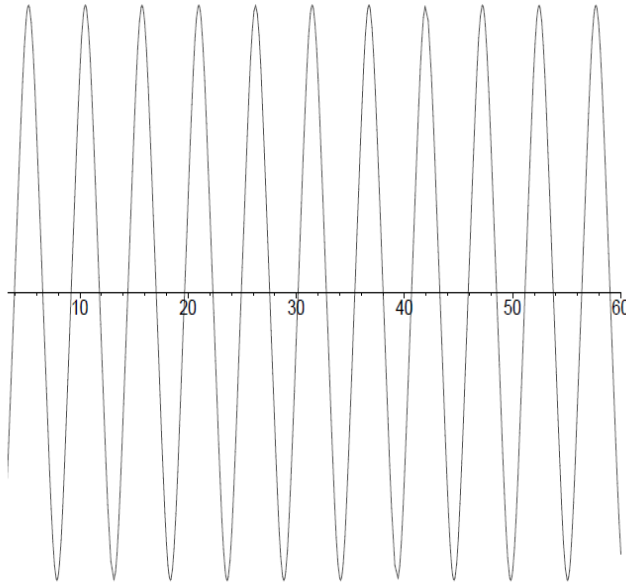
**appears to be
invariant under**

$$\begin{aligned} \mathcal{J} &\longleftrightarrow -\mathcal{J} \\ \bar{g}^2 &= \bar{g}^2(|\mathcal{J}|) \end{aligned}$$

Chromo-electric condensate solution

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

Classical YM condensate



“Radiation” medium

$$\epsilon_{\text{YM}} \propto 1/a^4$$

Unstable solution!

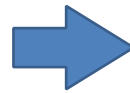
$$Q \equiv \frac{32}{11} \pi^2 e (\xi \Lambda_{\text{QCD}})^{-4} T_{\mu}^{\mu} [U]$$

$$= 6e \left[(U')^2 - \frac{1}{4} U^4 \right] a^{-4} (\xi \Lambda_{\text{QCD}})^{-4}$$

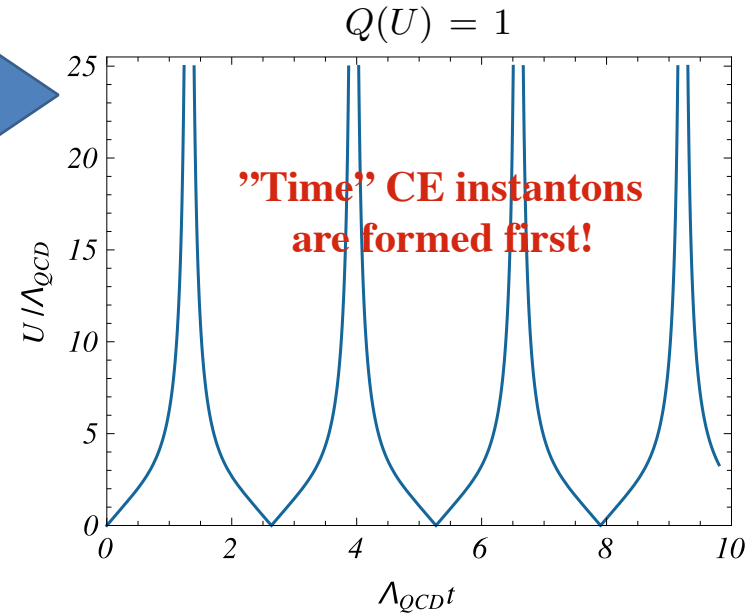
Exact partial solution:

$$|Q| = 1$$

Quantum corrections



Savvidy (CE) vacuum



QCD vacuum:

**a ferromagnetic undergoing
spontaneous magnetisation
(Pagels&Tomboulis)**

Asymptotic tracker solution!

$$\epsilon_{\text{CE}} \rightarrow +\text{const} \quad t \rightarrow \infty$$

Stable solution!

- In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

“Mirror” symmetry of the ground state

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC ‘19

In a vicinity of the ground state, the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{\mathcal{J}}{4\bar{g}^2} \quad \mathcal{J} \simeq \mathcal{J}^*$$

is invariant under

$$\mathbb{Z}_2: \quad \mathcal{J}^* \longleftrightarrow -\mathcal{J}^*, \quad \bar{g}^2(\mathcal{J}^*) \longleftrightarrow -\bar{g}^2(\mathcal{J}^*), \quad \beta(\bar{g}_*^2) \longleftrightarrow -\beta(\bar{g}_*^2)$$

For pure gluodynamics at **one-loop**: $\beta_{(1)} = -\frac{bN}{48\pi^2} \bar{g}_{(1)}^2 \quad b = 11$

$$\alpha_s = \frac{\bar{g}^2}{4\pi} \quad \alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \alpha_s(\mu_0^2) \ln(\mu^2/\mu_0^2)} \quad \mu^2 \equiv \sqrt{|\mathcal{J}|}$$

Choosing the ground state value of the condensate $\mu_0^2 \equiv \sqrt{|\mathcal{J}^*|}$ as the physical scale

we observe that **the mirror symmetry**, indeed, holds provided

$$\mathcal{J} \simeq \mathcal{J}^* \quad \alpha_s(\mu_0^2) \longleftrightarrow -\alpha_s(\mu_0^2)$$

i.e. in the ground state only!

Heterogenic quantum YM ground state: two-scale vacuum

The running coupling at one-loop

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1 + \frac{bN}{96\pi^2} \bar{g}_1^2(\mu_0^4) \ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN \ln(|\mathcal{J}|/\lambda_{\pm}^4)}$$

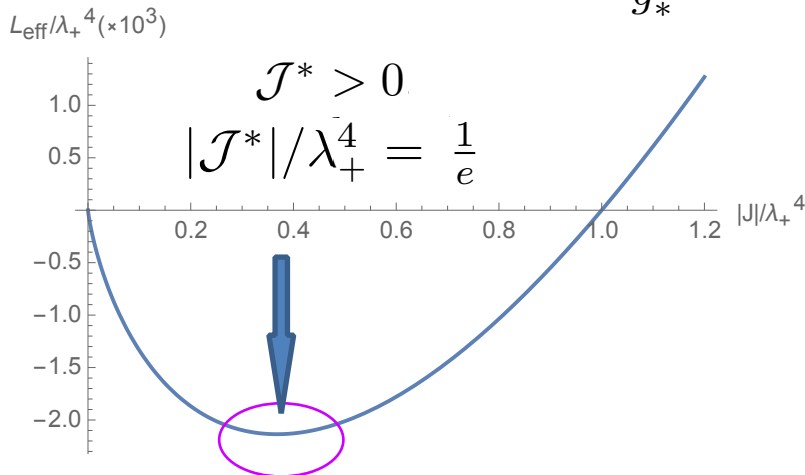
$$\mathcal{L}_{\text{eff}}^{(1)} = \frac{bN}{384\pi^2} \mathcal{J} \ln\left(\frac{|\mathcal{J}|}{\lambda_{\pm}^4}\right) \quad \text{with two energy scales}$$

$$\lambda_{\pm}^4 \equiv |\mathcal{J}^*| \exp\left[\mp \frac{96\pi^2}{bN |\bar{g}_1^2(\mathcal{J}^*)|}\right] \quad |\mathcal{J}^*| = \lambda_+^2 \lambda_-^2$$

CE vacuum: $\beta(\bar{g}_*^2) = 2$

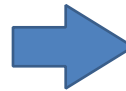
e.o.m. is automatically satisfied!

Trace anomaly: $T_{\mu, \text{CE}}^{\mu} = -\frac{1}{\bar{g}_*^2} \mathcal{J}^*$



Cosmological CE attractor

Mirror symmetry



One-loop:

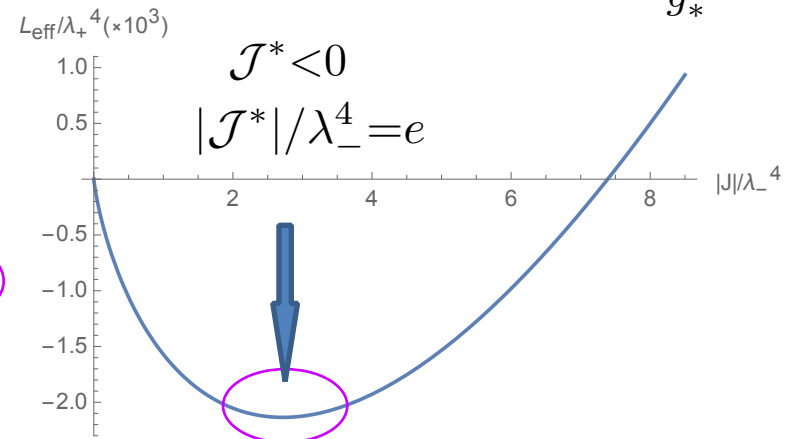
$$\lambda_+^2 / \lambda_-^2 = e$$

CM vacuum: $\beta(\bar{g}_*^2) = -2$

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

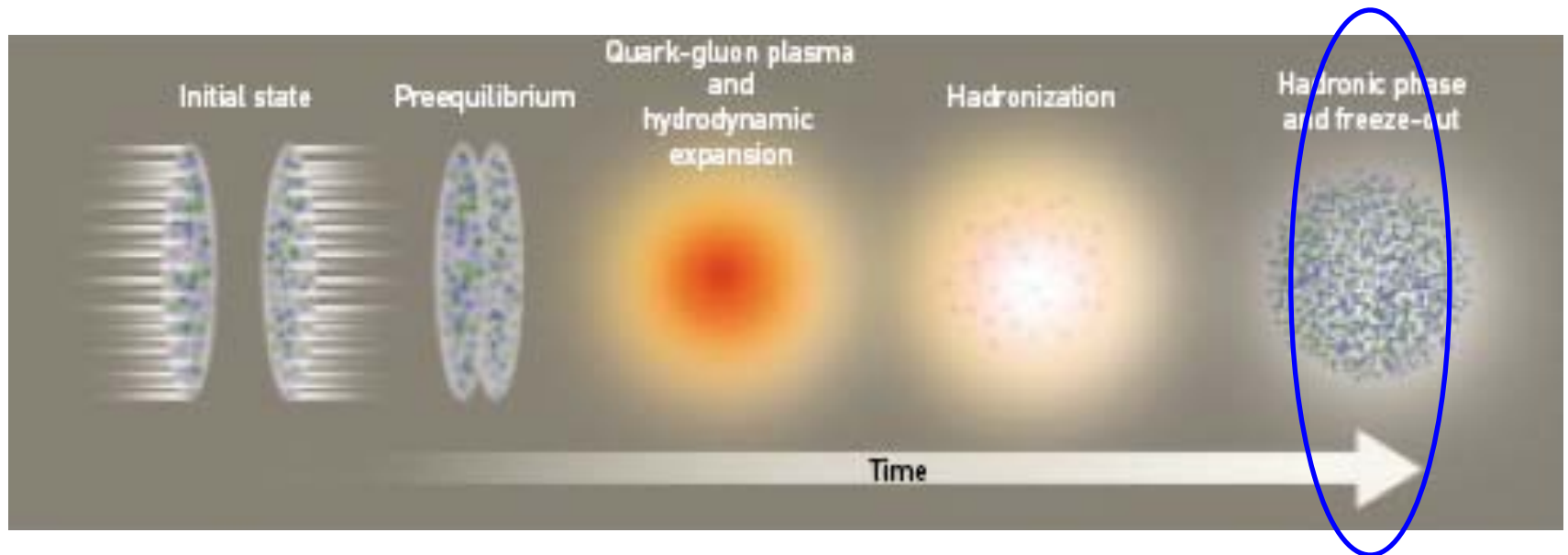
$$\vec{D}_{\nu}^{ab} \left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2} \right] = 0, \quad \bar{g}^2 \simeq \bar{g}_*^2$$

Trace anomaly: $T_{\mu, \text{CM}}^{\mu} = +\frac{1}{\bar{g}_*^2} \mathcal{J}^*$



Cosmological CM attractor

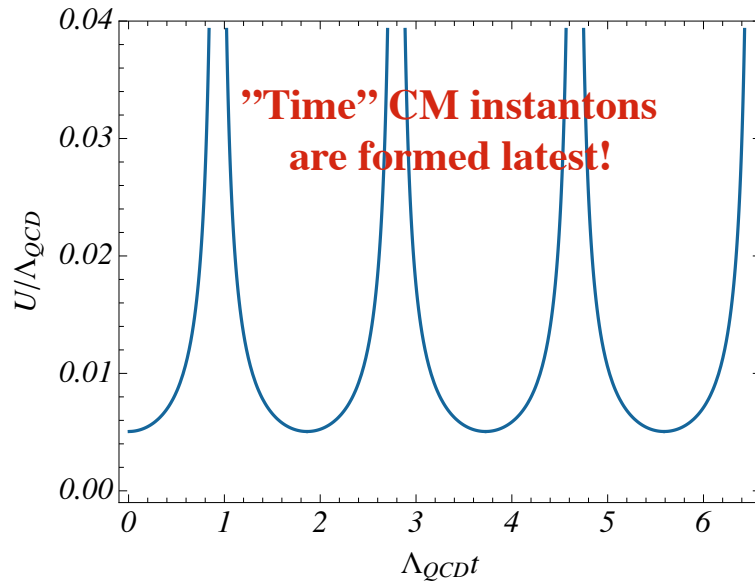
Post-confinement: stage IV



- Both CE and CM reach their attractors
- CM/CE domains “crystallisation”
- rigid “time-crystal” (no fluctuations)
- CC is formed

Macroscopic evolution and vacua cancellation

$$Q(U) = -1$$



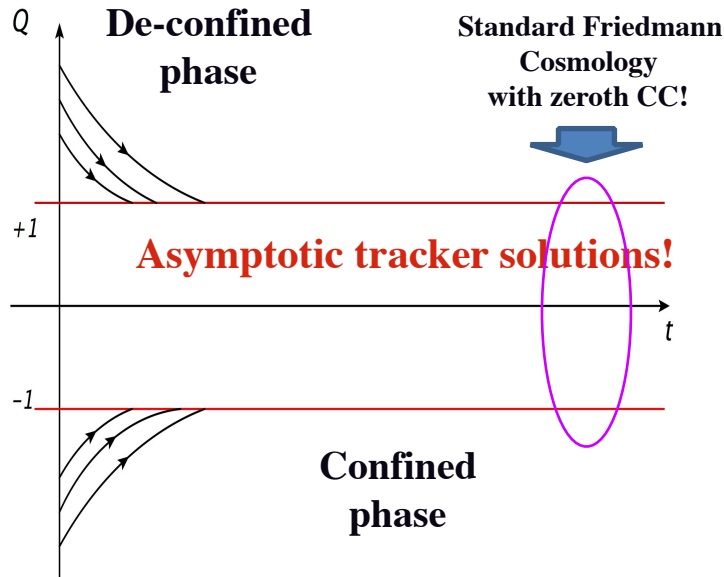
$$\epsilon_{\text{vac}} \equiv \frac{1}{4} \langle T_{\mu}^{\mu} \rangle_{\text{vac}} = \mp \mathcal{L}_{\text{eff}}(\mathcal{J}^*)$$



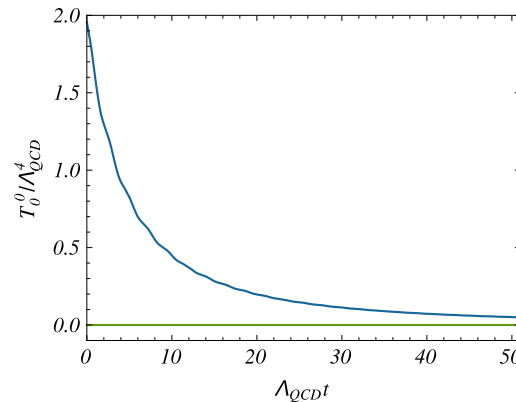
$$\epsilon_{\text{vac}}^{\text{CE}}|_{\mathcal{J}^* > 0} + \epsilon_{\text{vac}}^{\text{CM}}|_{\mathcal{J}^* < 0} \equiv 0$$



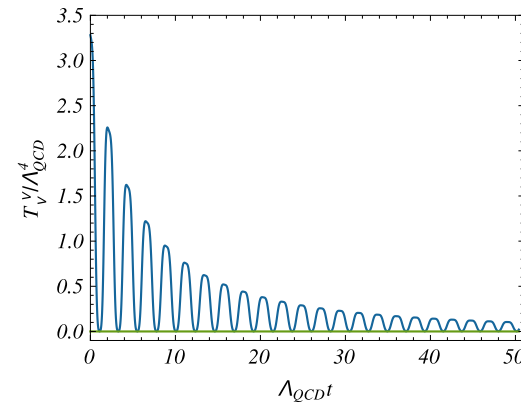
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace



System with very unusual dynamical properties!

Summary

- **Hamiltonian (real-time) picture in temporal gauge** offers a novel look at physical evolution of “micro-Big Bang” events in heavy-ion collisions that is not reachable in Lattice QCD (Maiani-Testa theorem PLB ‘90)
- **Semi-classical dynamics of the homogeneous gluon condensate** (with small inhomogeneities) represent **the initial (pre-equilibrium) state** in a typical such event
- Real-time evolution of such pre-equilibrium state unavoidably leads to a decay of the gluon condensate and **resonant-like production of inhomogeneous quantum-wave fluctuations**. This provides a consistent dynamical mechanism of QGP production from the ground state in QCD
- While semi-classical picture above is approximately valid at small times/distances, at large separation scales, quantum corrections to the ground state become crucial. The **vacuum polarisation dramatically modifies the e.o.m** for the homogeneous gluon condensate, leading to the ground-state solutions of a new type (obeying the vacuum e.o.s)

For more information, please, come to my next talk today:

Quantum Yang-Mills vacua in expanding Universe: Do we live in a time crystal?



11 Jul 2019, 14:53

22m

ICC - Baeckeland 2 (Ghent)

Parallel talk

Quantum Field and Str...

Quantum Field and String...