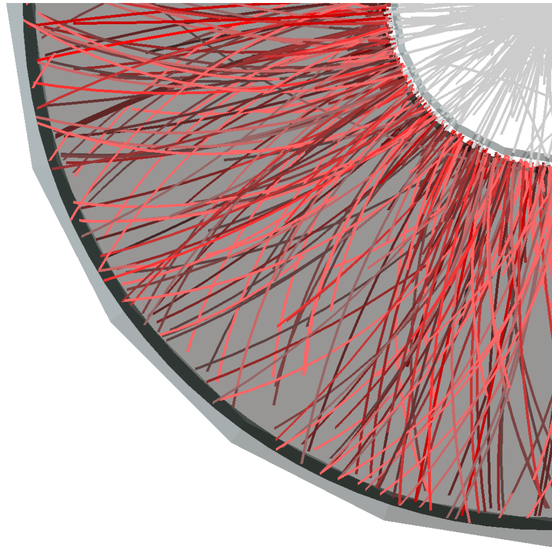
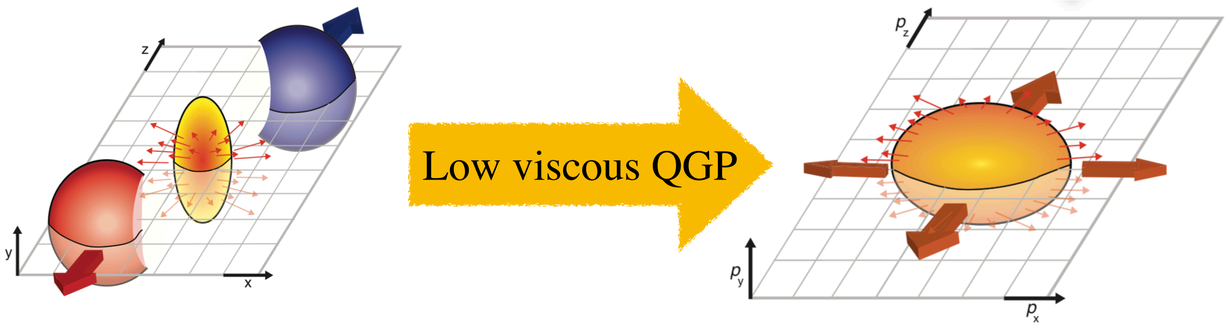


Non-linear flow modes for identified hadrons

Naghmeh Mohammadi
for the ALICE Collaboration

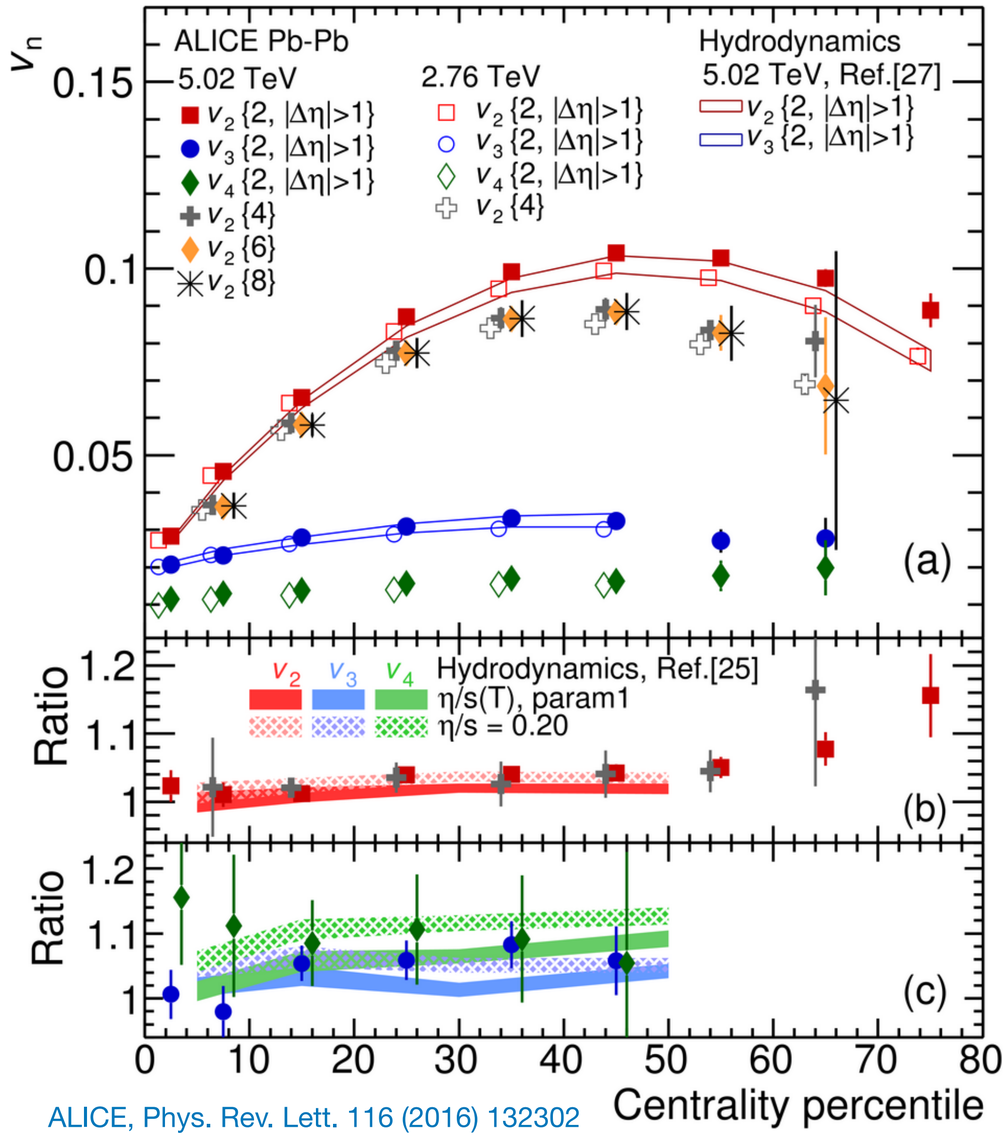
Constraining QGP properties



S. Voloshin, Y. Zhang, Z.Phys.C70:665-672,1996

$$\frac{dN}{d\varphi} \propto \sum_{n=1}^{\infty} v_n(p_T)\cos[n(\varphi - \Psi_n)] \qquad v_n = \langle\cos[n(\varphi - \Psi_n)]\rangle$$

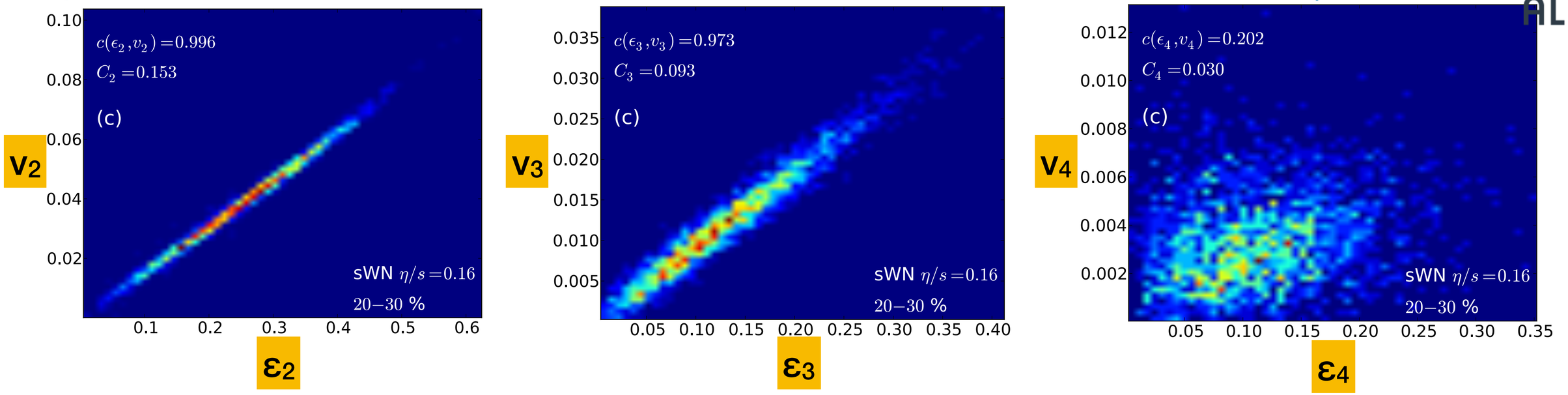
- ✤ Flow harmonics constrain **initial conditions** and **transport properties**:
  - ✤ Shear (η/s) and bulk (ζ/s) viscosity over entropy density ratios, Equation of State (EoS), freeze-out conditions
- ✤ Higher harmonics are **more sensitive**
  - ✤ probe smaller spatial scales
  - ✤ Testing details of hydrodynamical response of QGP



ALICE, Phys. Rev. Lett. 116 (2016) 132302

Linear and non-linear response in higher flow harmonics

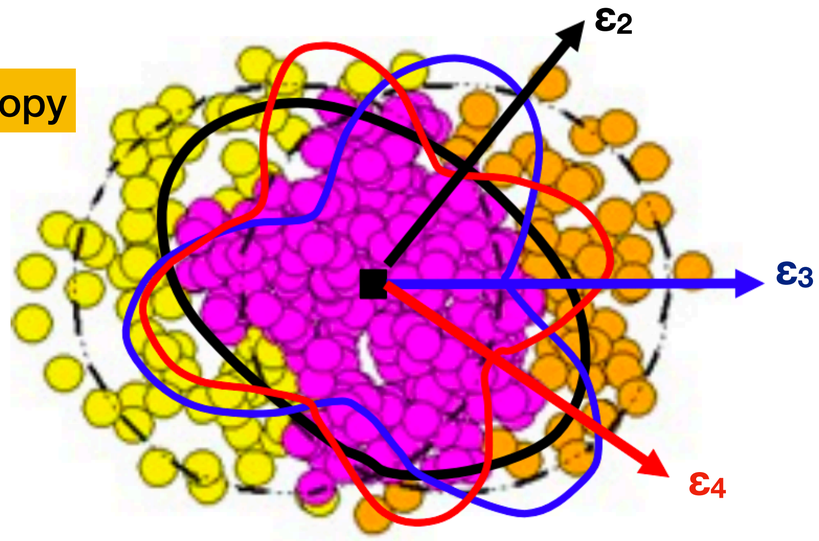
H. Niemi *et al*. Phys. Rev. C 87, 054901



$$V_n = V_n^{\mathrm{L}} + V_n^{\mathrm{NL}} \quad (n > 3)$$

Linear response — Non-linear response

ε_n: initial spatial anistropy

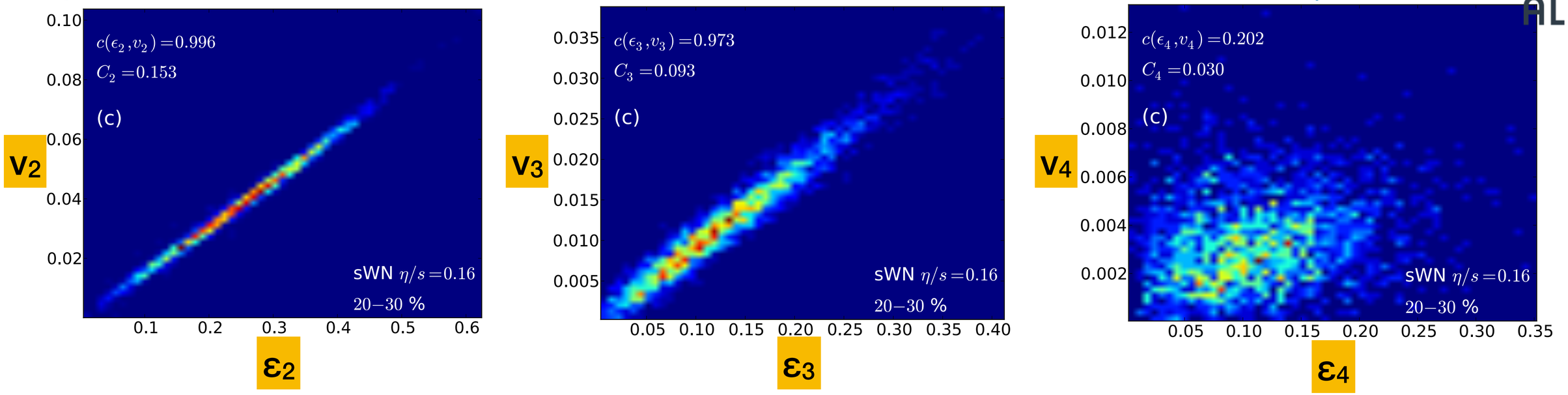


- ✤ V_n^{L} corresponds to the same order anisotropy
- ✤ V_n^{NL} corresponds to the **lower order anisotropies**, e.g. ε_2 and/or ε_3
- ✤ V_n^{NL} and V_n^{L} are **uncorrelated**

Phys.Lett. B773 (2017) 68

Linear and non-linear response in higher flow harmonics

H. Niemi *et al*. Phys. Rev. C 87, 054901



$$V_n = V_n^{\mathrm{L}} + V_n^{\mathrm{NL}} \quad (n > 3)$$

Linear response — Non-linear response

$$V_4 = V_4^{\mathrm{NL}} + V_4^{\mathrm{L}} = \chi_{4,22}(V_2)^2 + V_4^{\mathrm{L}}$$

$$V_5 = V_5^{\mathrm{NL}} + V_5^{\mathrm{L}} = \chi_{5,32} V_3 V_2 + V_5^{\mathrm{L}}$$

$$V_6 = V_6^{\mathrm{NL}} + V_6^{\mathrm{L}} = \chi_{6,222}(V_2)^3 + \chi_{6,33}(V_3)^2 + \chi_{6,24} V_2 V_4^{\mathrm{L}} + V_6^{\mathrm{L}}$$

Phys. Lett. B773 (2017) 68

✤ Magnitude of these non-linear responses in V_n: $v_{n,mk}$

$$v_{4,22} = \frac{\langle v_4 v_2^2 \, cos(4\Psi_4 - 4\Psi_2)\rangle}{\sqrt{\langle v_2^4 \rangle}}$$

$$v_{5,32} = \frac{\langle v_5 v_3 v_2 \, cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2)\rangle}{\sqrt{\langle v_3^2 v_2^2 \rangle}}$$

$$v_{6,33} = \frac{\langle v_6 v_3^2 \, cos(6\Psi_6 - 6\Psi_3)\rangle}{\sqrt{\langle v_3^4 \rangle}}$$

$$v_{6,222} = \frac{\langle v_6 v_2^3 \cos(6\Psi_6 - 6\Psi_2)\rangle}{\sqrt{\langle v_2^6 \rangle}}$$

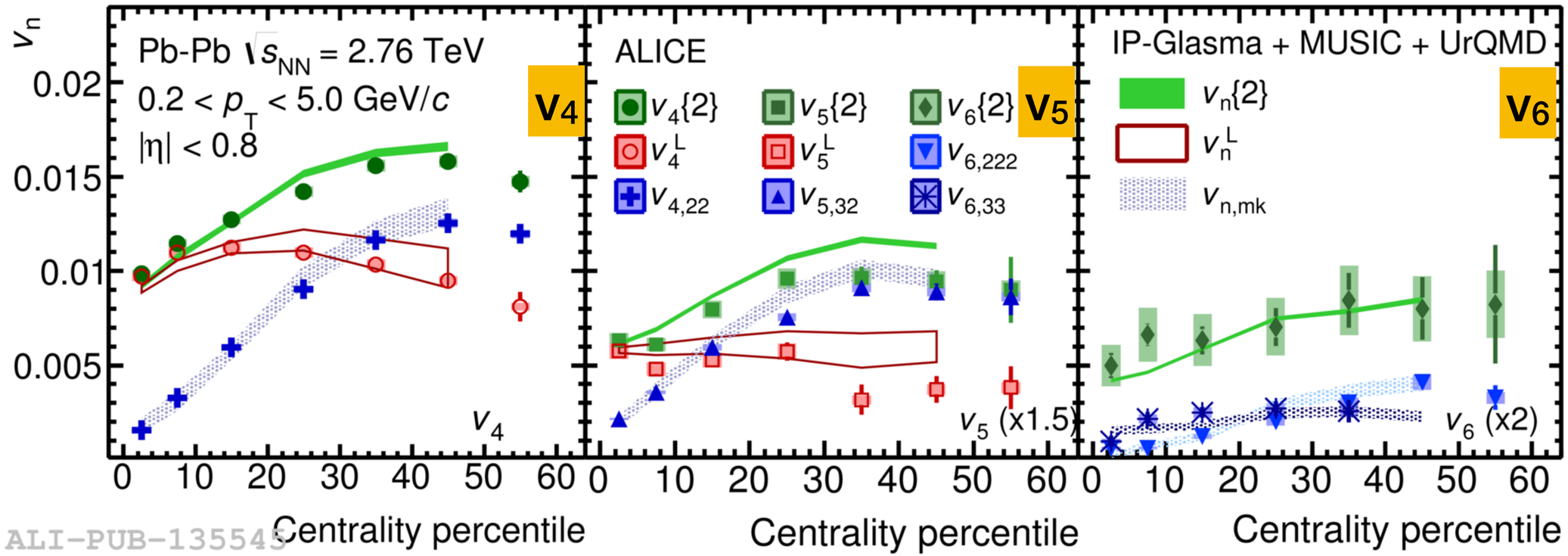
Linear and non-linear response in higher flow harmonics

- p_T-integrated **non-linear flow modes**: $v_{4,22}$, $v_{5,32}$, $v_{6,222}$, $v_{6,33}$
- p_T-integrated **linear flow modes**: v_4^L and v_5^L

$$v_4^{\mathrm{L}} = \sqrt{v_4^2 - v_{4,22}^2} \qquad v_5^{\mathrm{L}} = \sqrt{v_5^2 - v_{5,32}^2}$$

- For charged particles:

ALICE, Phys.Lett. B773 (2017) 68



Linear and non-linear response in higher flow harmonics

- p_T-integrated **non-linear flow modes**: $v_{4,22}$, $v_{5,32}$, $v_{6,222}$, $v_{6,33}$
- p_T-integrated **linear flow modes**: v_4^L and v_5^L

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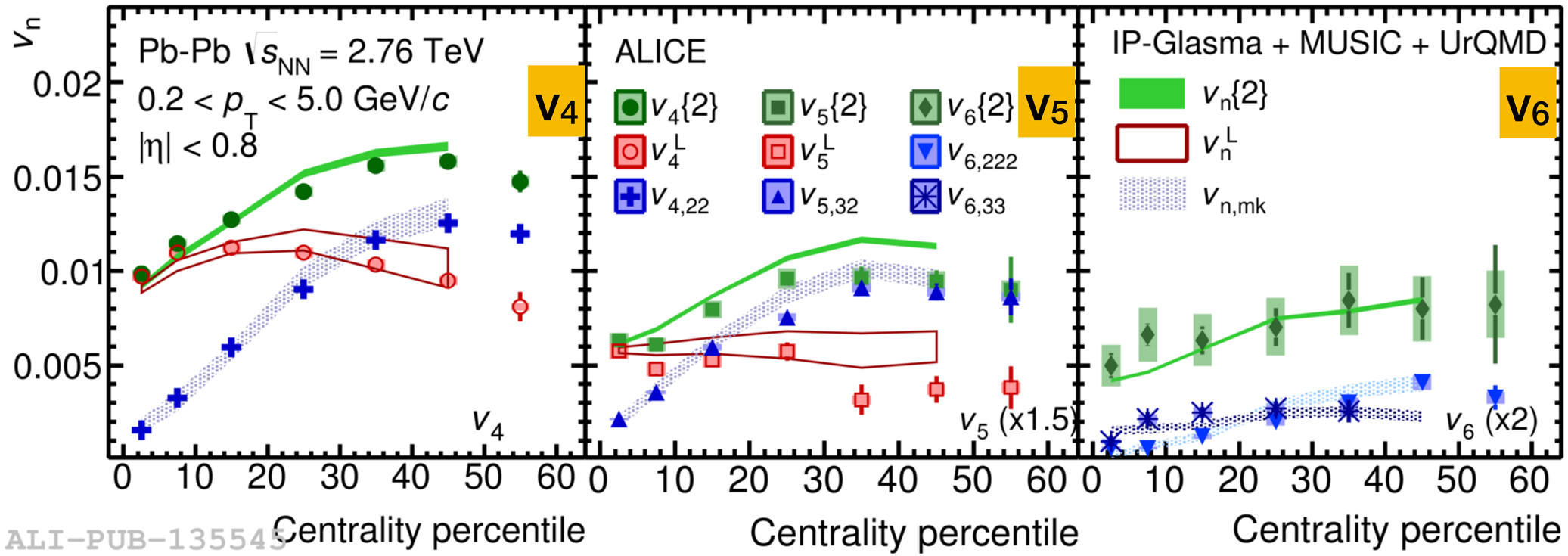
- For charged particles:

ALICE, Phys.Lett. B773 (2017) 68



- p_T-differential Non-linear modes are **more sensitive** to:
  - Initial state fluctuations
  - Transport properties (η/s, ζ/s)

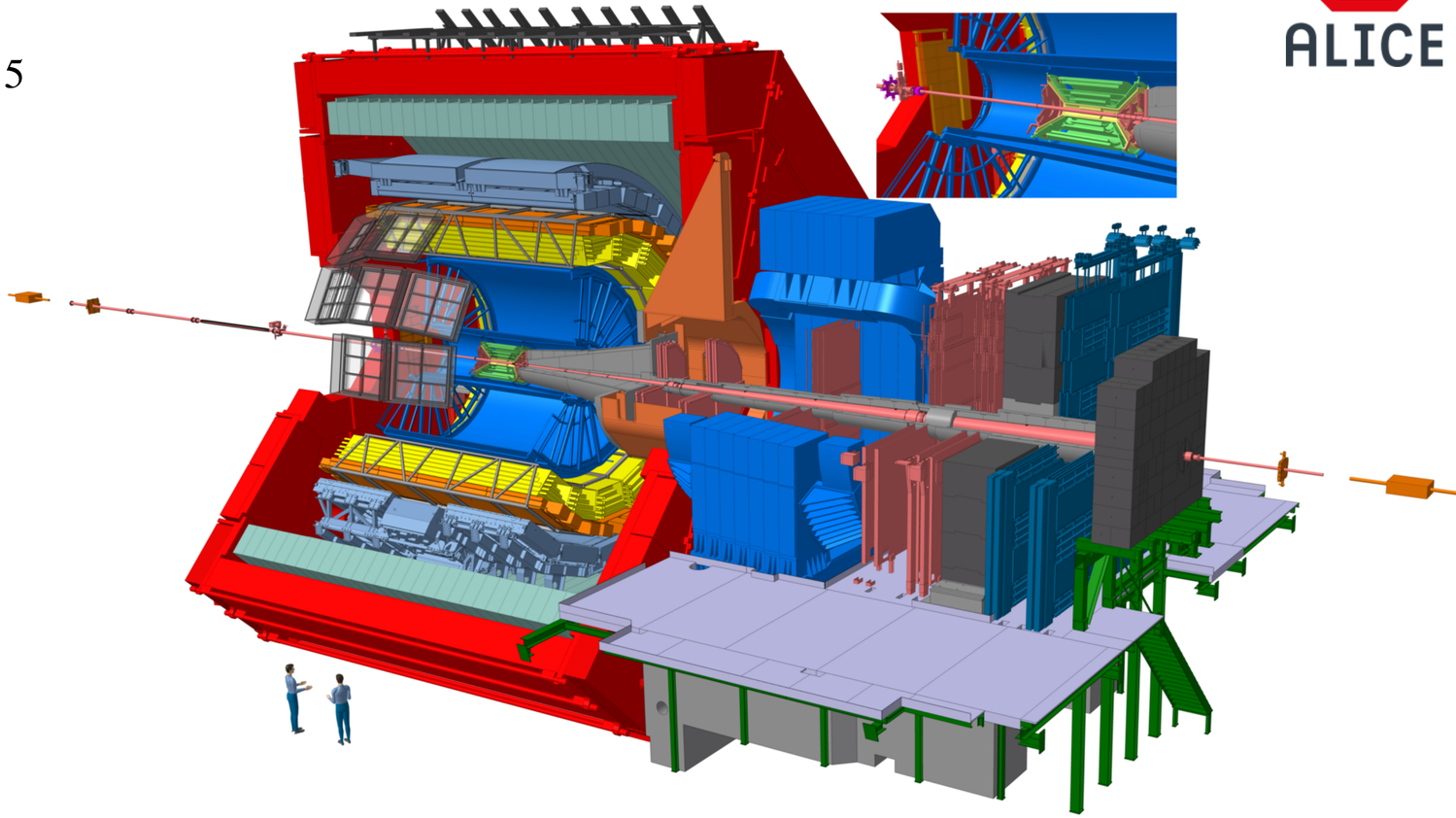
ALICE, Phys.Lett. B773 (2017) 68

- For different particle species, probe in addition:
  - Effects of **hadronisation mechanism**
  - Effects of **hadronic rescattering**

ALICE, JHEP 1609 (2016) 164

Analysis details

- Minimum Bias Pb-Pb data at 5.02 TeV recorded in 2015
  - 45M analysed events
  - 0-5% - 10-20% and 40-50% centrality intervals
- Tracks from TPC acceptance: $|\eta|<0.8$
- 2 non-overlapping sub-events: $|\Delta\eta|>0.0$
- RFPs (Reference particles): charged particles
  - $0.2 < p_T < 5.0$ (GeV/c)
- POIs (Particles of Interest):
  - **$\pi^{\pm}$, $K^{\pm}$ and $p+\bar{p}$**:
    - Particle identification from TPC+TOF



POI	p_T range (GeV/c)	Purity
$\pi^{\pm}$	$0.4 < p_T < 6.0$	>90%
$K^{\pm}$	$0.4 < p_T < 4.0$	>75%
$p+\bar{p}$	$0.4 < p_T < 6.0$	>80%

Analysis details

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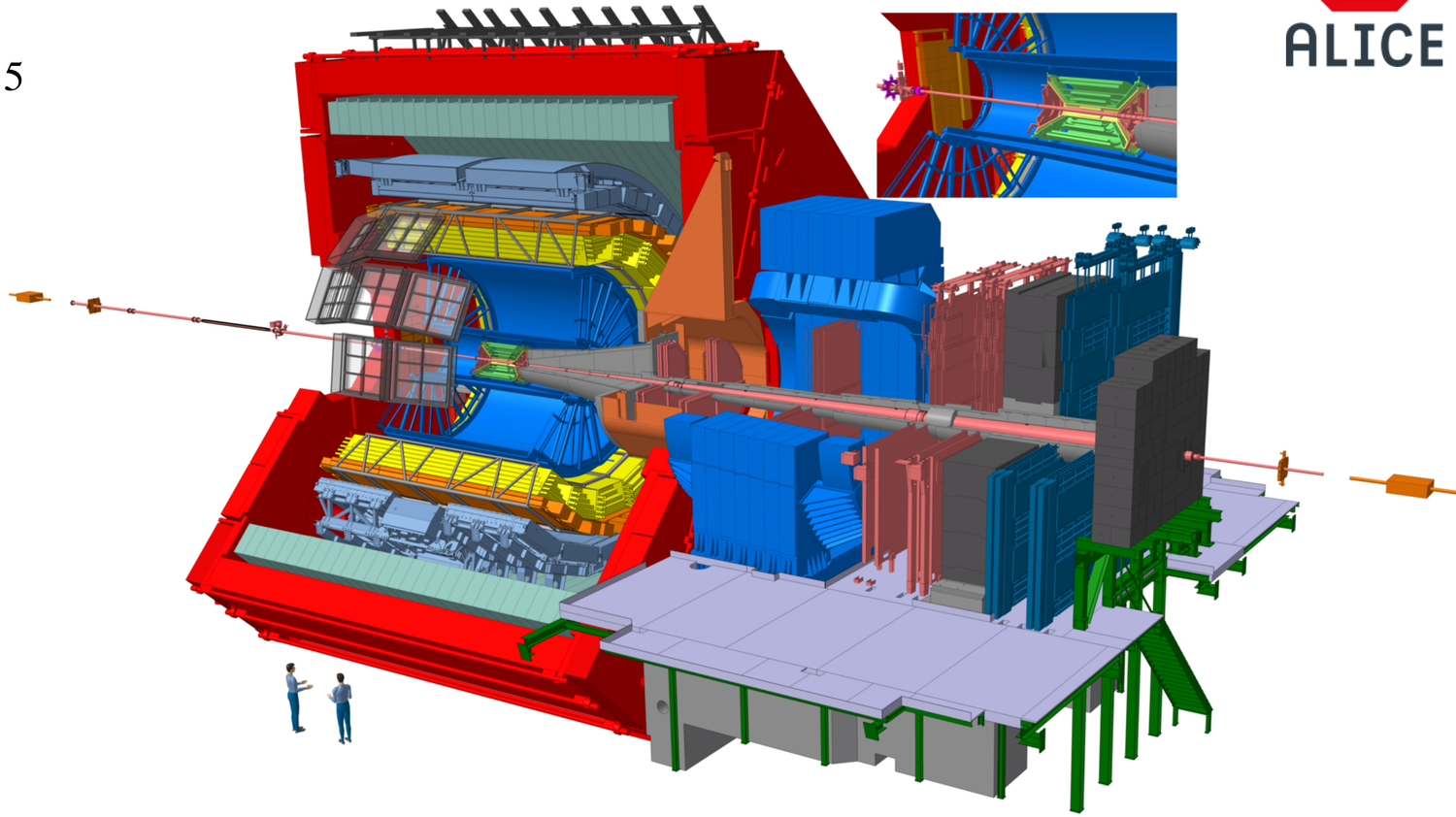
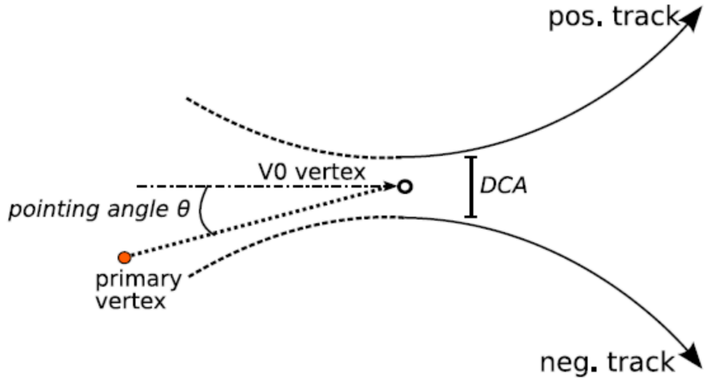
  - $0.2 < p_T < 5.0$ (GeV/c)
- POIs (Particles of Interest):
  - **K^0_S, $\Lambda+\bar{\Lambda}$ and ϕ:**
    - Reconstruction via decay products:
      - Particle Identification: purity > 80%
      - Constraining decay topology



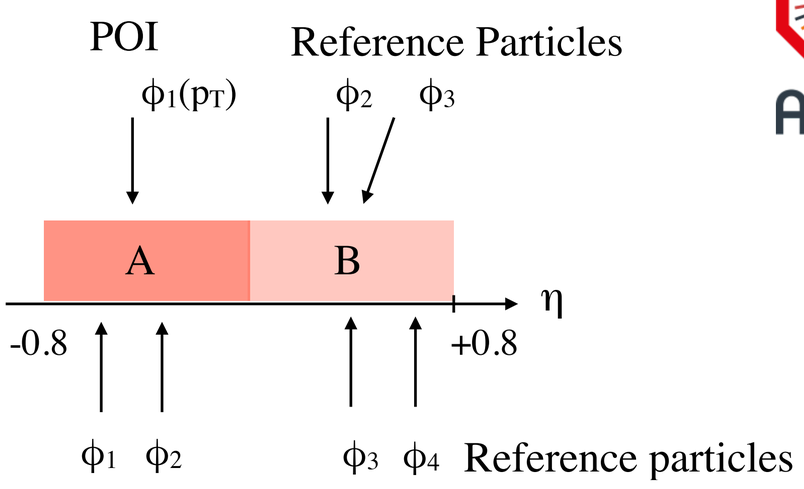


Analysis method

- p_T-differential $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$: ALICE, Phys. Lett. B773 (2017) 68
  - a multi-particle correlation technique
  - 2 non-overlapping sub-events

$$v_{n,mk}(p_T) = \frac{d_{n,mk}(p_T)}{\sqrt{c_{mk,mk}}}$$

for $\pi^{\pm}$, $K^{\pm}$ and $p+\bar{p}$



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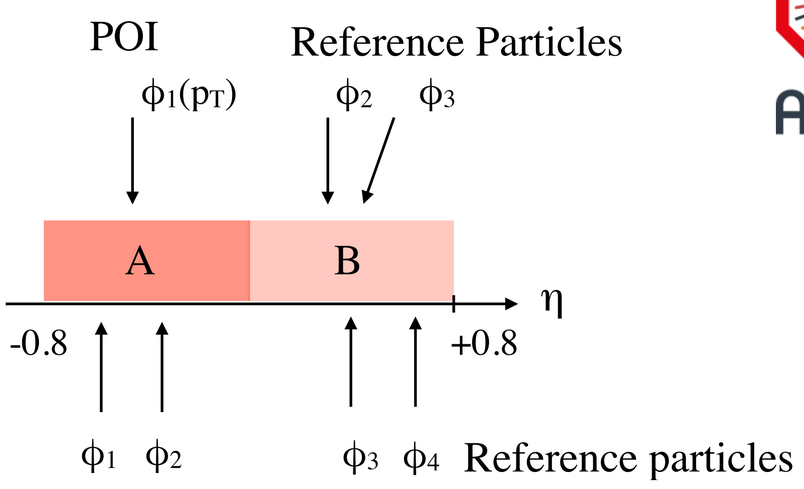


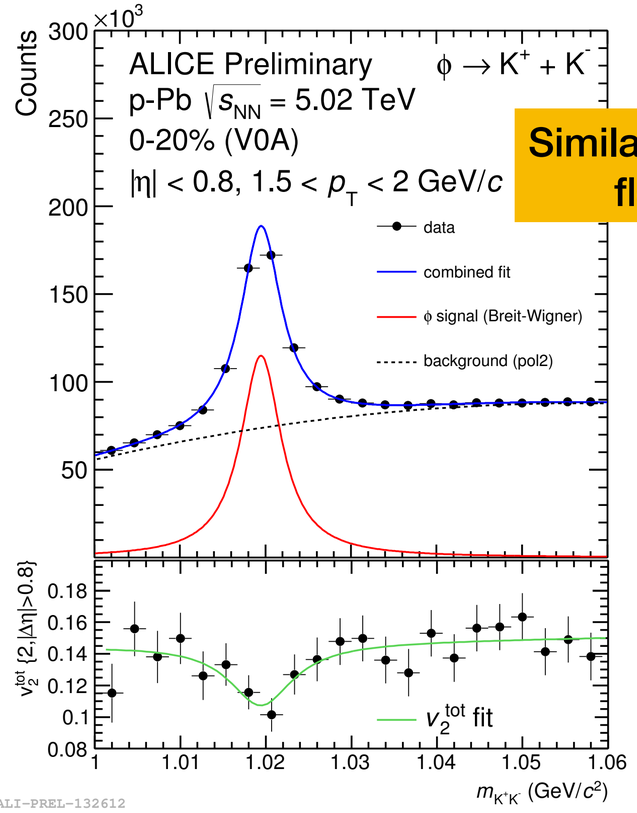
$$v_{n,mk}(p_T) = \frac{d_{n,mk}(p_T)}{\sqrt{c_{mk,mk}}}$$ for $\pi^{\pm}$, $K^{\pm}$ and $p+\bar{p}$

ALICE, JHEP 06 (2015) 190

- For decaying particles: $v_{n,mk}$ is calculated with the m_{inv} method:

$$v_{n,mk}(p_T, m_{inv}) = \frac{d_{n,mk}(p_T, m_{inv})}{\sqrt{c_{mk,mk}}}$$ for K^0_s, $\Lambda+\bar{\Lambda}$ and ϕ

$$d_{n,mk}(m_{inv}) = \frac{N^{sig}}{N^{tot}}(m_{inv})d^{sig}_{n,mk} + \frac{N^{bkg}}{N^{tot}}(m_{inv})d^{bkg}_{n,mk}(m_{inv})$$



Similar for non-linear flow modes

$d_{n,mk}$

Analysis method

- p_T-differential $v_{4,22}$, $v_{5,32}$, $v_{6,33}$ and $v_{6,222}$: ALICE, Phys. Lett. B773 (2017) 68
  - a multi-particle correlation technique
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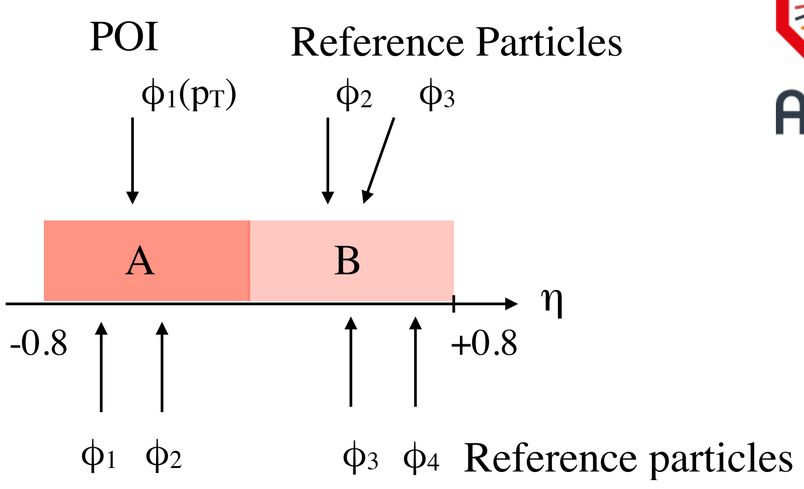

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ALICE, JHEP 06 (2015) 190

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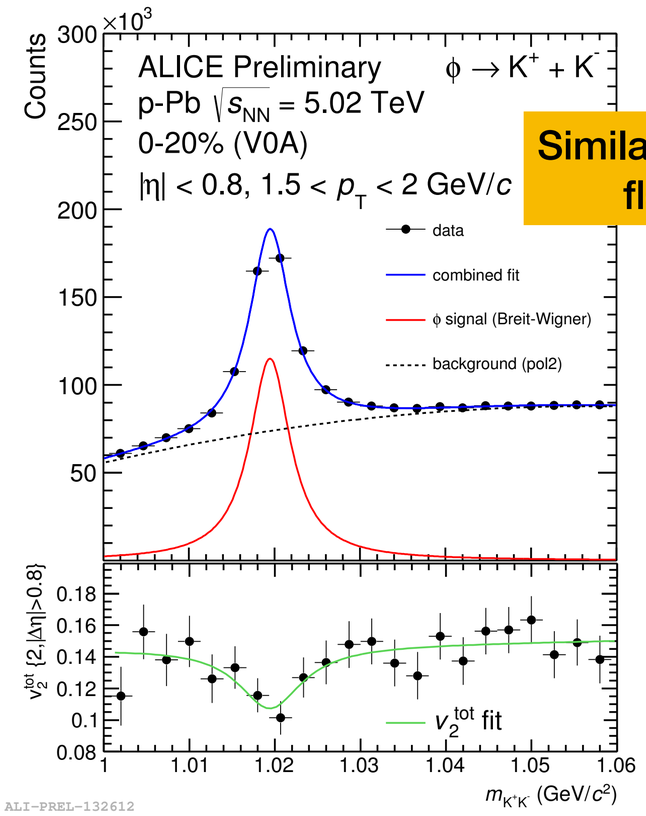
$$d_{n,mk}(m_{inv}) = \frac{N^{sig}}{N^{tot}}(m_{inv}) d^{sig}_{n,mk} + \frac{N^{bkg}}{N^{tot}}(m_{inv}) d^{bkg}_{n,mk}(m_{inv})$$

- Non-flow effects suppressed largely by multi-particle correlations
  - Residual tested with various gaps between the sub-events
    - Included in the systematic uncertainties

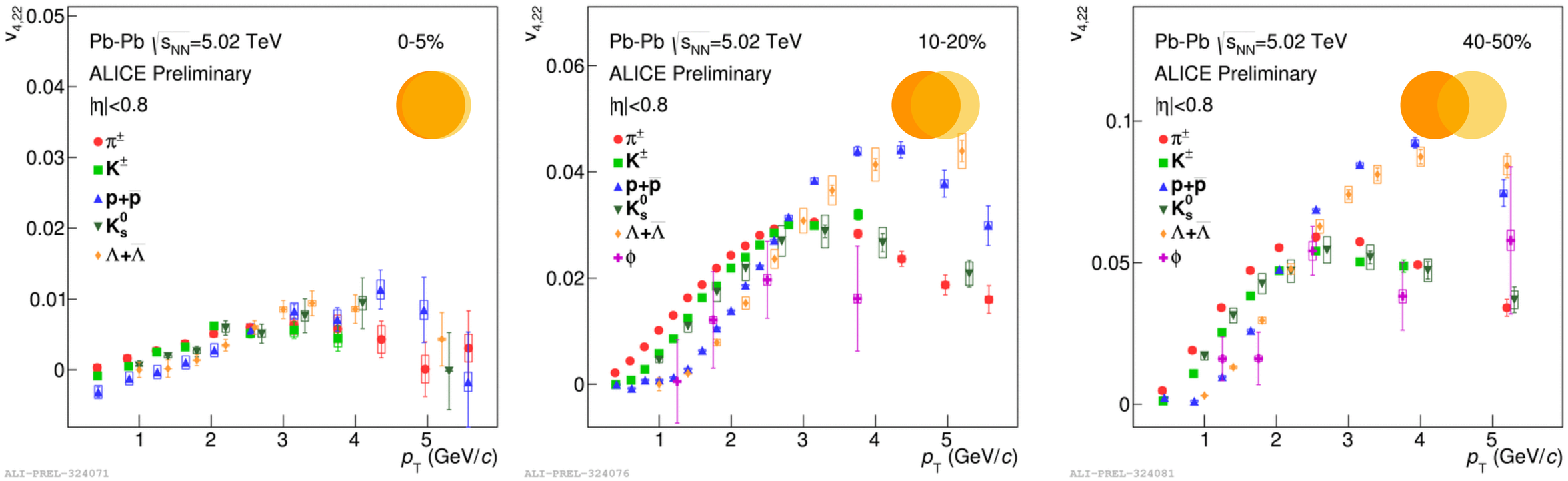


Similar for non-linear flow modes

$d_{n,mk}$

Measurement of $v_{4,22}(p_T)$ for identified particles

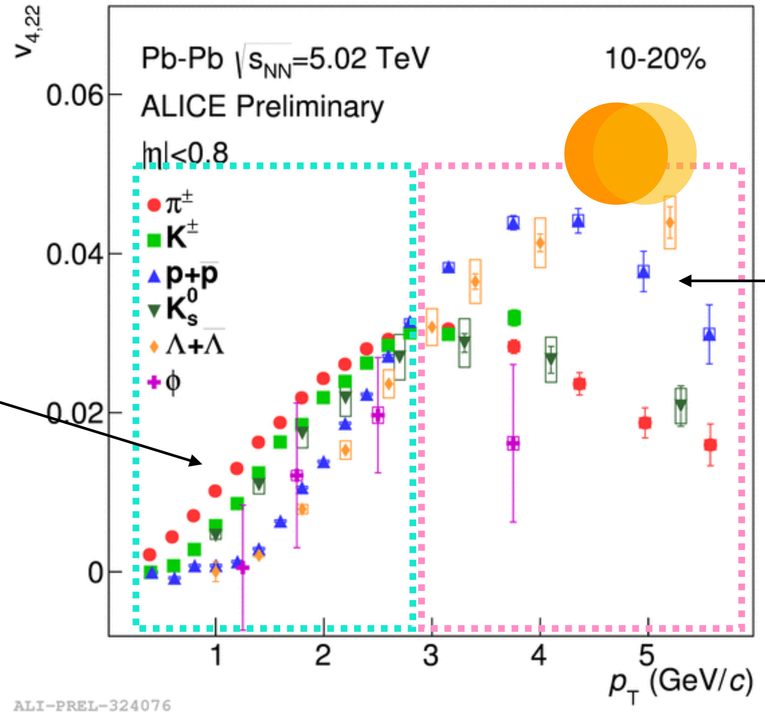
- **Clear centrality dependence**
- Most-central collisions:
  - Small value for all particle species



Measurement of $v_{4,22}(p_T)$ for identified particles

- Clear centrality dependence
- Most-central collisions:
  - Small value for all particle species
- Non-central collisions:
  - **Mass ordering** in the low p_T region (p_T <2.5 GeV/c): Interplay of **radial flow** with non-linear modes
  - **Particle type grouping** in the intermediate p_T region (p_T >2.5 GeV/c): **Quark coalescence(?)** as dominant particle production mechanism

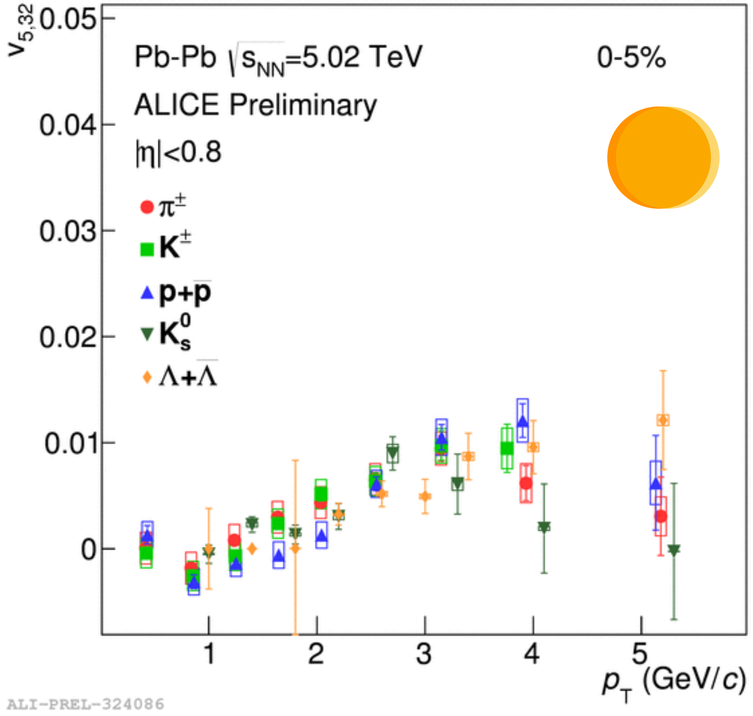
Mass ordering

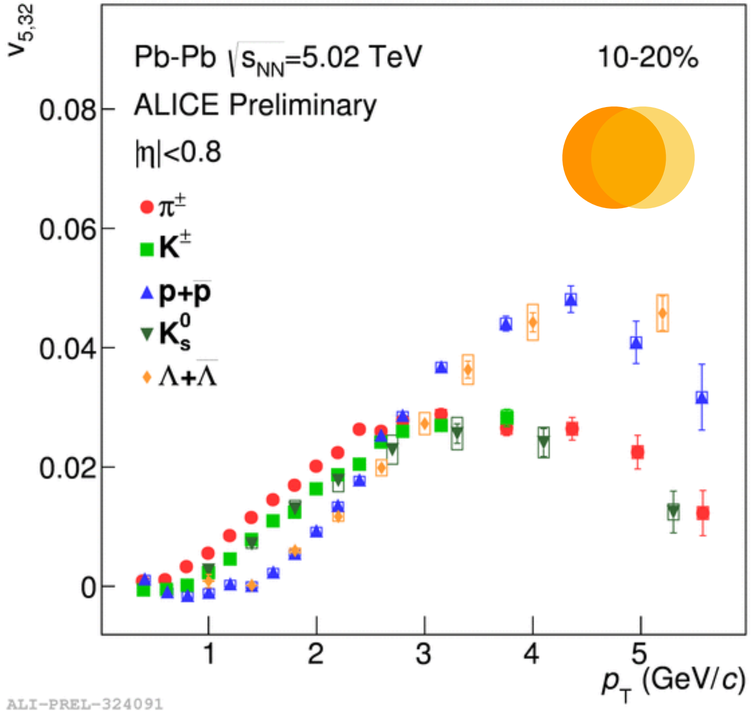


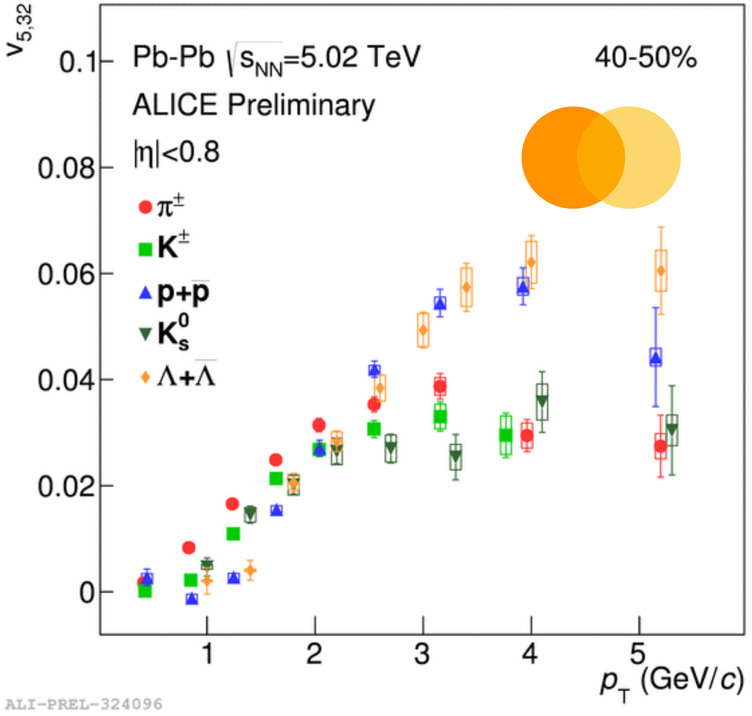
Particle type grouping

Measurement of $v_{5,32}(p_T)$ for identified particles

- Clear centrality dependence
- Most-central collisions:
  - Small value for all particle species
- Non-central collisions:
  - **Mass ordering** in the low p_T region (p_T <2.5 GeV/c): Interplay of **radial flow** with non-linear modes
  - **Particle type grouping** in the intermediate p_T region (p_T >2.5 GeV/c): **Quark coalescence(?)** as dominant particle production mechanism

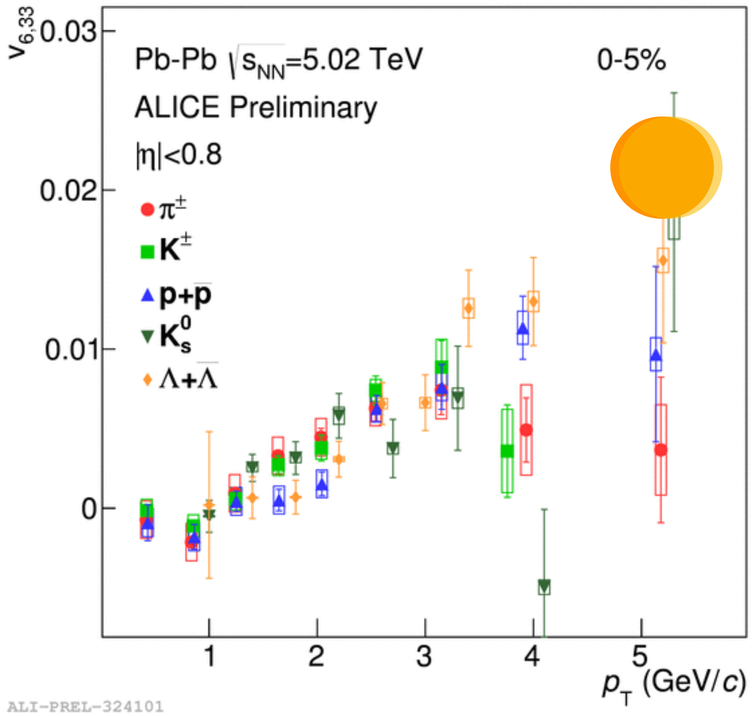


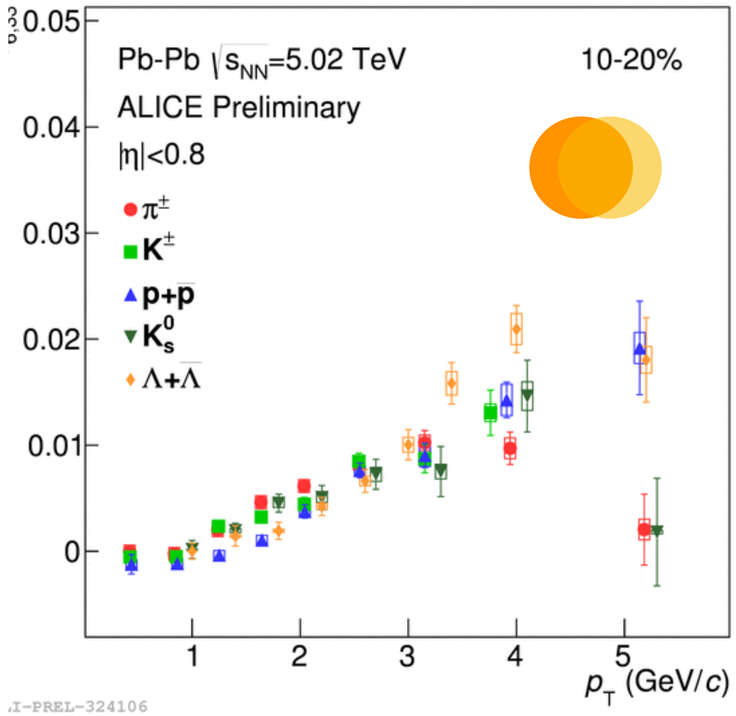


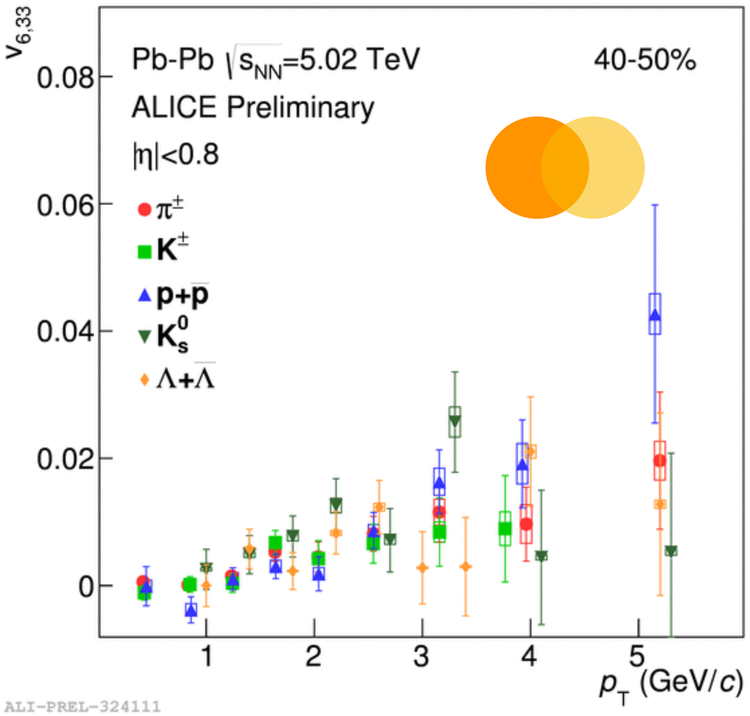


Measurement of $v_{6,33}(p_T)$ for identified particles

- The magnitude **does not exhibit** a strong centrality dependence
- Indication of similar features (**mass ordering** and **particle type grouping**)

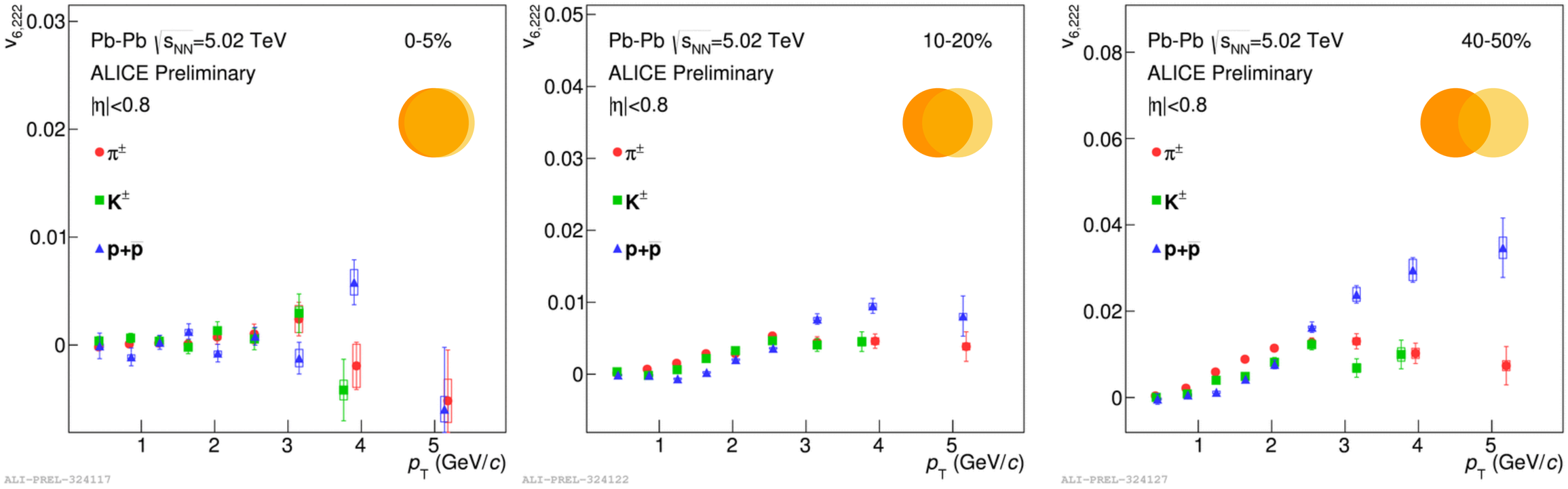






Measurement of $v_{6,222}(p_T)$ for identified particles

- **Clear centrality dependence**
- Most central collisions:
  - Compatible with zero
- Non-central collisions:
  - Same features (**mass ordering** and **particle type grouping**)



Hydrodynamic predictions: $v_n(p_T)$ of identified particles

ALICE, JHEP09(2018)006

Eur.Phys.J. C77 (2017) no.9, 645
Zhao, Wenbin *et al.*

iEBE-VISHNU hybrid model:

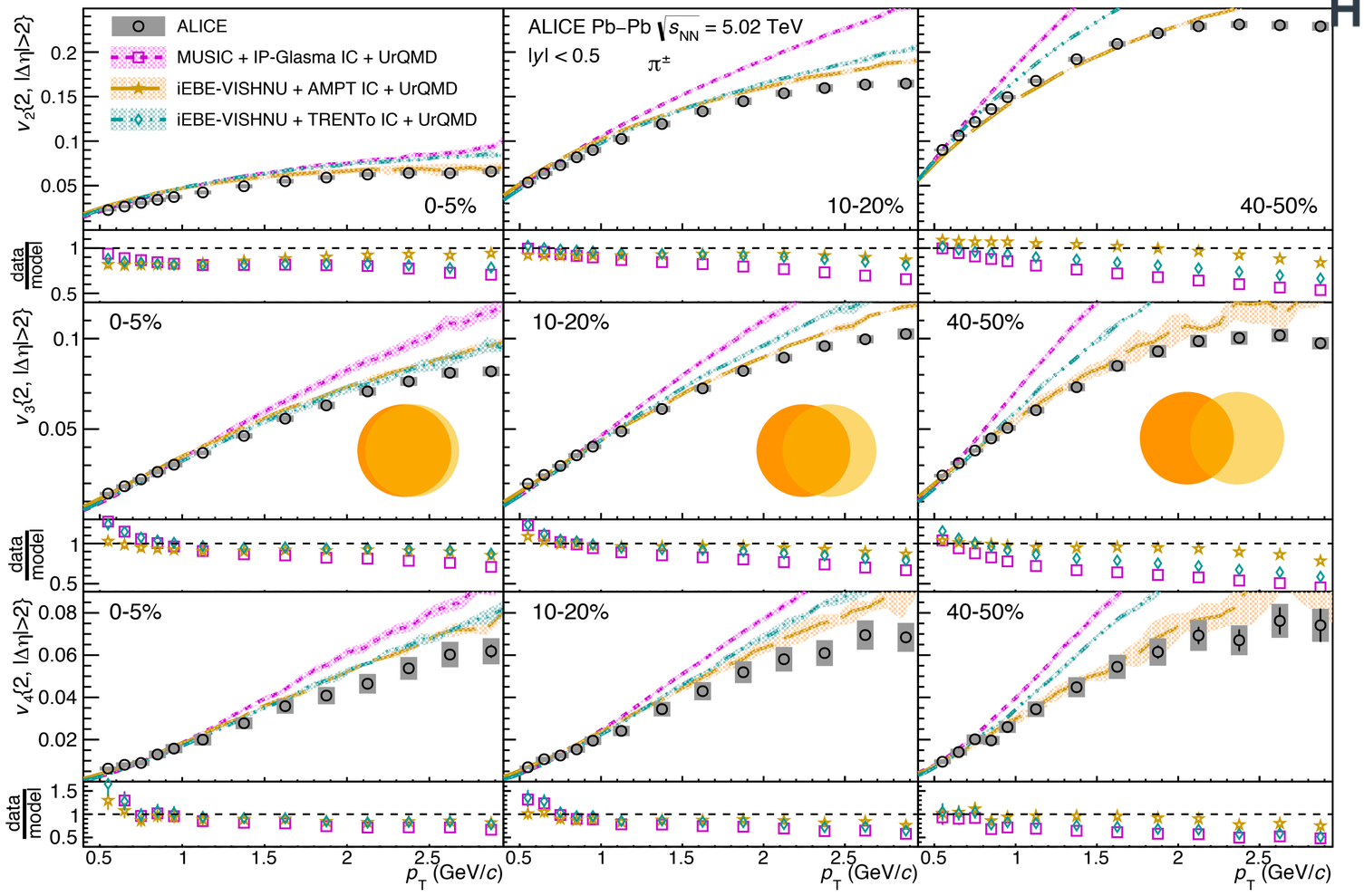
- VISH2+1 coupled to UrQMD
- Two initial conditions: **AMPT, TRENTO**
- T_{switch} = 148 MeV, τ_0 = 0.6 fm/c
- AMPT: **η/s=0.08 and ζ/s=0**
- TRENTo: **η/s(T) and ζ/s(T)**

Phys. Rev. C 94, 024907 (2016)
JE Bernhard *et al.*

v_2

v_3

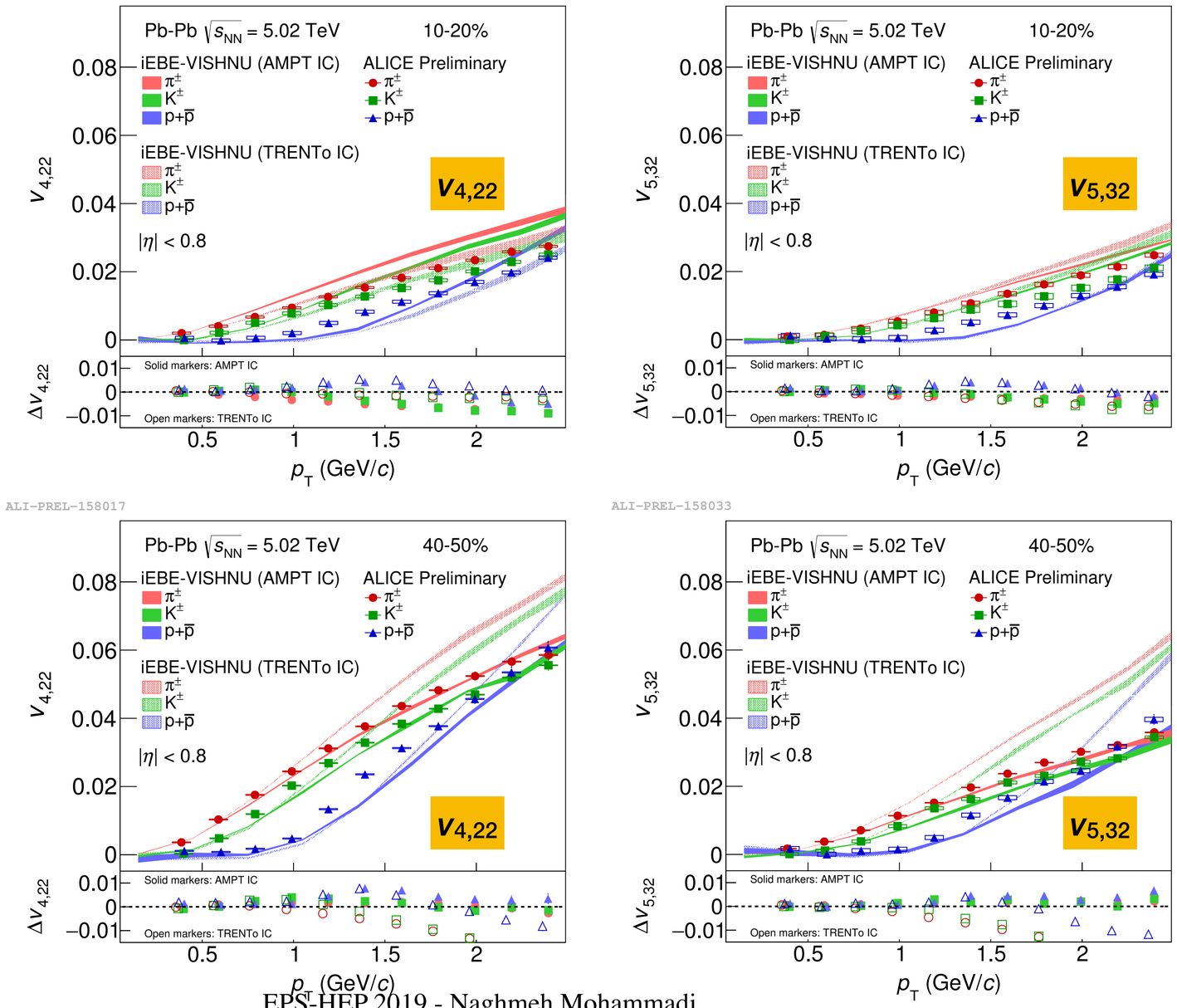
v_4



- AMPT: Better agreement with v_n measurements
- TRENTo: Agreement up to slightly lower transverse momenta depending on the centrality interval

Hydrodynamic predictions: $v_{4,22}(p_T)$ and $v_{5,32}(p_T)$ of identified hadrons

- **Semi-central collisions**: similar performances
  - Also seen in the comparison to v_n measurements
  - TRENTo slightly better in $v_{4,22}$, AMPT better in $v_{5,32}$
- **Mid-peripheral collisions**: AMPT predicts the data better
  - Also seen in the comparisons to v_n measurements
- **Larger separation** between the two calculations compared to anisotropic flow



Hydrodynamic predictions: $v_{6,33}(p_T)$ and $v_{6,222}(p_T)$ of identified hadrons

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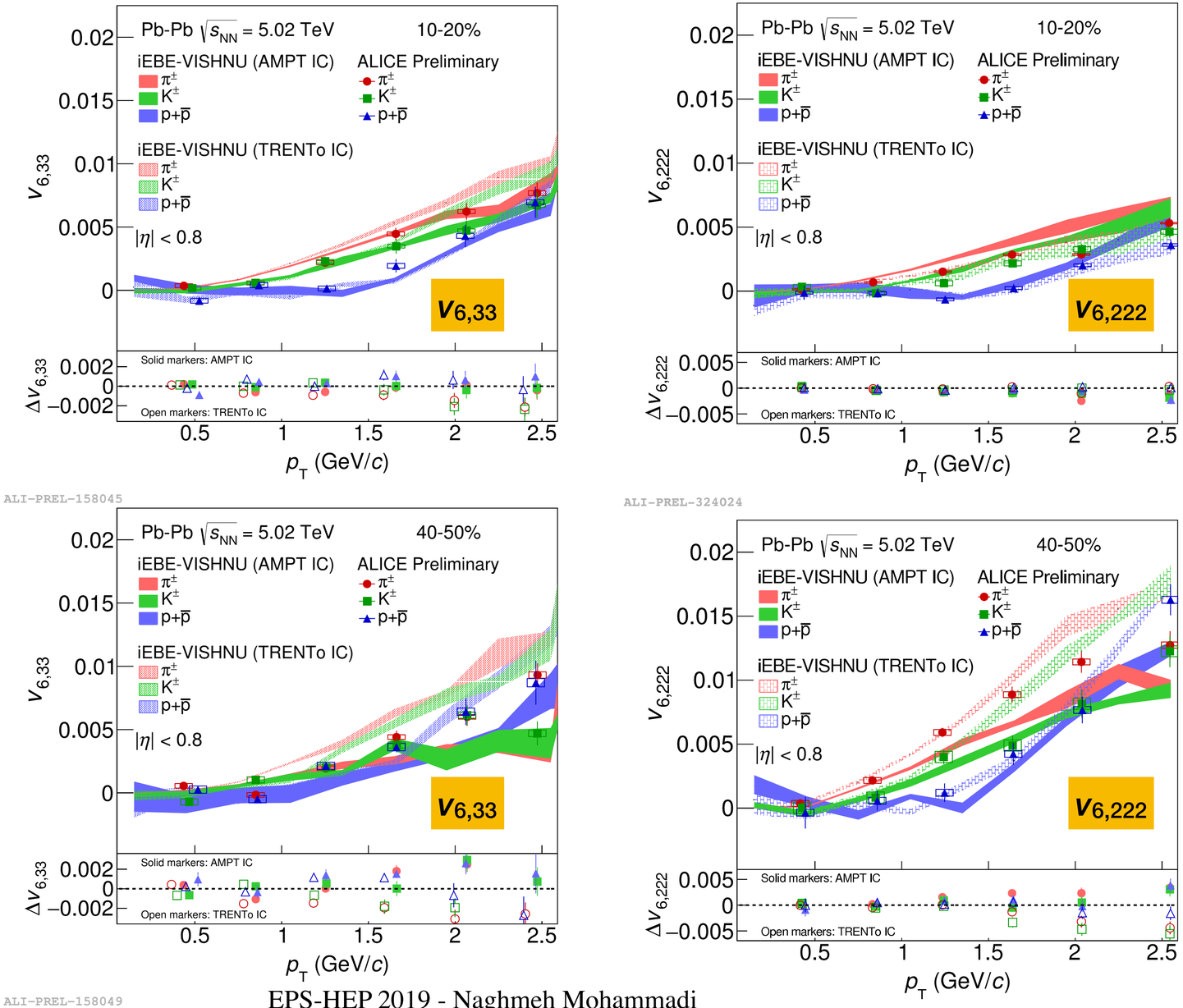
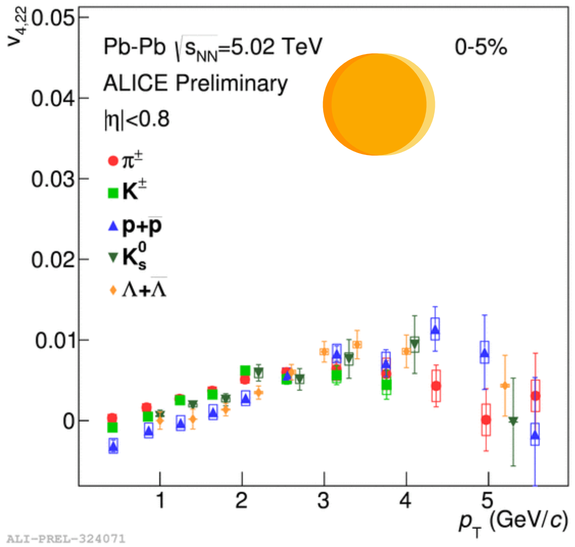
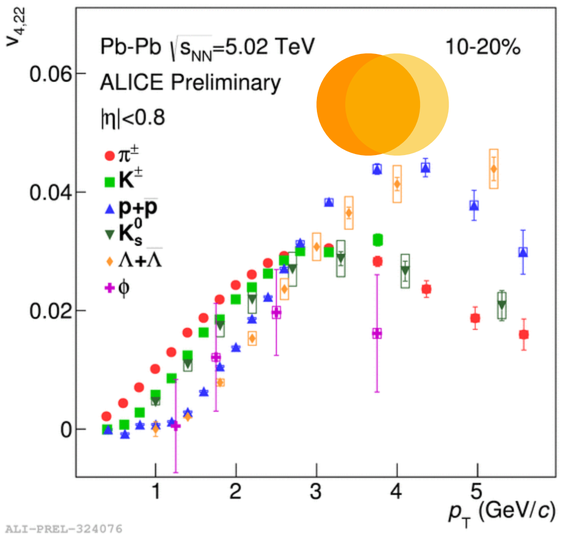
- **Mid-peripheral collisions**: AMPT predicts the data better
  - Also seen in the comparisons to v_n measurements
- **Larger separation** between the two calculations compared to anisotropic flow
- **TRENTo needs to use our measurements for further tuning**



Summary

- **First results** on non-linear flow modes of identified particles: $v_{4,22}$, $v_{5,32}$, $v_{6,33}$, $v_{6,222}$
  - Clear **centrality dependence** for $v_{4,22}$, $v_{5,32}$,$v_{6,222}$ (Less dependence for $v_{6,33}$)





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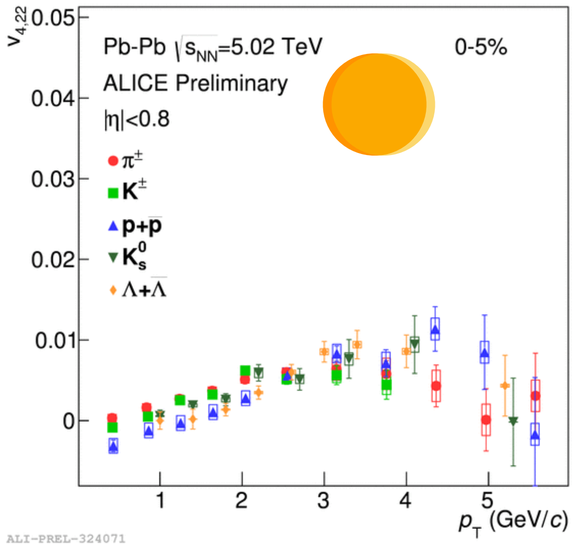
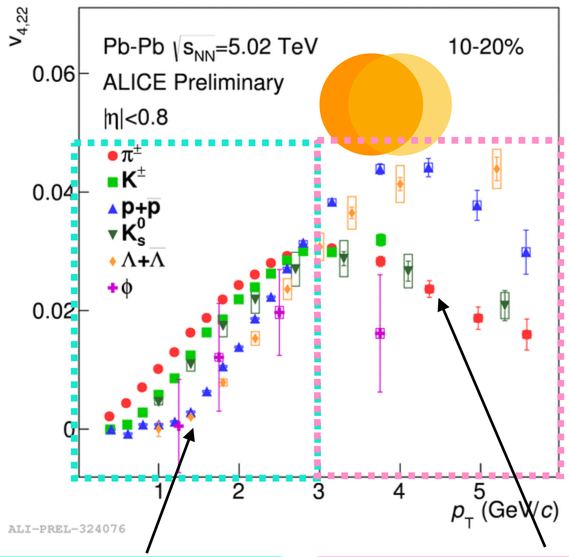
  - Clear **centrality dependence** for $v_{4,22}$, $v_{5,32}$,$v_{6,222}$ (Less dependence for $v_{6,33}$)

- For all flow harmonics at non-central collisions
  - **Mass ordering** in low p_T
  - **Particle type grouping** in the intermediate p_T





Mass ordering

Particle type grouping

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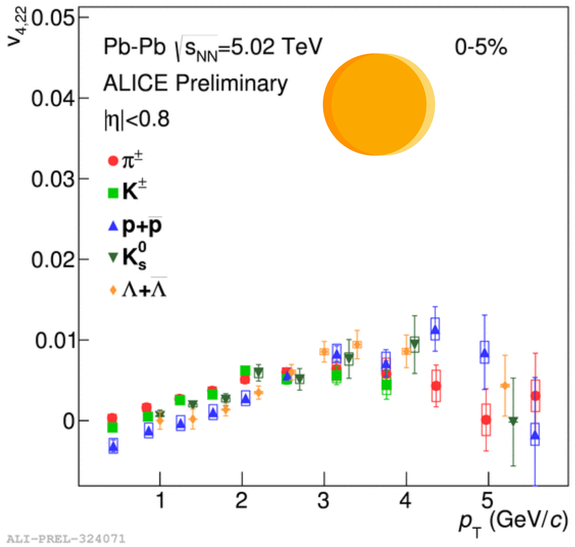
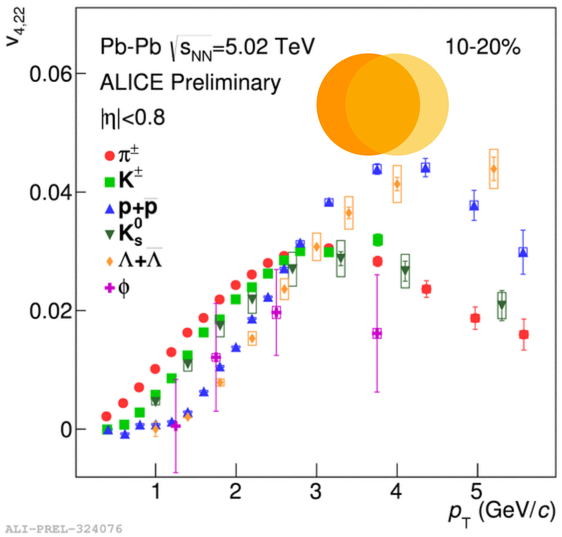
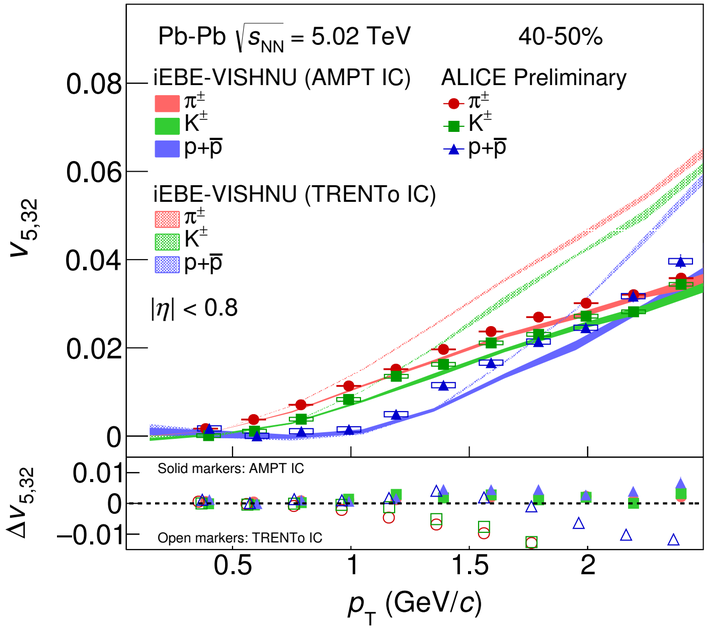
  - Clear **centrality dependence** for $v_{4,22}$, $v_{5,32}$,$v_{6,222}$ (Less dependence for $v_{6,33}$)





- For all flow harmonics at non-central collisions
  - **Mass ordering** in low p_T
  - **Particle type grouping** in the intermediate p_T

- iEBE-VISHNU: **AMPT** and **TRENTo** initial conditions with different sets of parameters
  - AMPT (η/s=0.08 and ζ/s=0) reproduces v_n and $v_{n,mk}$ measurements **slightly better** than TRENTo (η/s(T) and ζ/s(T))
- **Models require a bit more work to describe the details that data reveal**



Thank you

Mathematical background

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$$V_n = V_n^{NL} + V_n^{L} \qquad (n > 3)$$

Assuming V_n^{NL} and V_n^{L} are uncorrelated

$$V_4 = V_4^{NL} + V_4^{L} = \chi_{4,22}(V_2)^2 + V_4^{L}$$

$$\varepsilon_4' e^{i4\Phi_4'} \equiv \varepsilon_4 e^{i4\Phi_4} + \frac{3\langle r^2\rangle^2}{\langle r^4\rangle}\varepsilon_2^2 e^{i4\Phi_2}$$

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$$V_5 = V_5^{NL} + V_5^{L} = \chi_{5,23} V_2 V_3 + V_5^{L}$$

$$V_6 = V_6^{NL} + V_6^{L} = \chi_{6,222}(V_2)^3 + \chi_{6,33}(V_3)^2 + \chi_{6,24} V_2 V_4^{L} + V_6^{L}$$

Analysis method

Using 2 sub-event method with |Δη|>0.0:

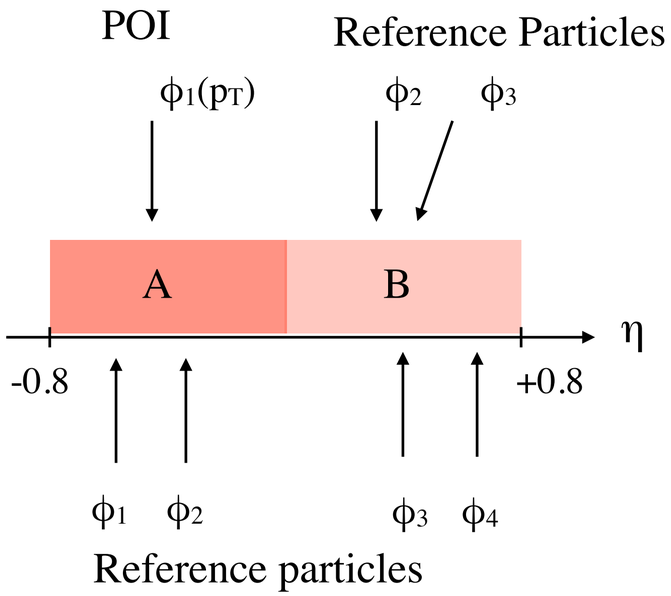
ALICE, Phys. Lett. B773 (2017) 68

$$v^{A}_{4,22}(p_T) = \frac{\langle\langle cos(4\varphi^{A}_{1}(p_{\mathrm{T}}) - 2\varphi^{B}_{2} - 2\varphi^{B}_{3})\rangle\rangle}{\sqrt{\langle\langle cos(2\varphi^{A}_{1} + 2\varphi^{A}_{2} - 2\varphi^{B}_{3} - 2\varphi^{B}_{4})\rangle\rangle}}$$

$$v^{A}_{5,32}(p_T) = \frac{\langle\langle cos(5\phi^{A}_{1}(p_{\mathrm{T}}) - 3\varphi^{B}_{3} - 2\varphi^{B}_{2})\rangle\rangle}{\langle\langle cos(3\varphi^{A}_{1} + 2\varphi^{A}_{2} - 3\varphi^{B}_{3} - 2\varphi^{B}_{4})\rangle\rangle}$$

$$v^{A}_{6,33}(p_T) = \frac{\langle\langle cos(6\varphi^{A}_{1}(p_{\mathrm{T}}) - 3\varphi^{B}_{2} - 3\varphi^{B}_{3})\rangle\rangle}{\langle\langle cos(3\varphi^{A}_{1} + 3\varphi^{A}_{2} - 3\varphi^{B}_{3} - 3\varphi^{B}_{4})\rangle\rangle}$$

$$v^{\mathrm{A}}_{6,222}(p_{\mathrm{T}}) = \frac{\langle\langle \cos(6\varphi^{\mathrm{A}}_{1}(p_{\mathrm{T}}) - 2\varphi^{\mathrm{B}}_{2} - 2\varphi^{\mathrm{B}}_{3} - 2\varphi^{\mathrm{B}}_{4})\rangle\rangle}{\sqrt{\langle\langle \cos(2\varphi^{\mathrm{A}}_{1} + 2\varphi^{\mathrm{A}}_{2} + 2\varphi^{\mathrm{A}}_{3} - 2\varphi^{\mathrm{B}}_{4} - 2\varphi^{\mathrm{B}}_{5} - 2\varphi^{\mathrm{B}}_{6})\rangle\rangle}}.$$



- $v_{n,mk}$ combination of $v_{n,mk}^{A}$ and $v_{n,mk}^{B}$

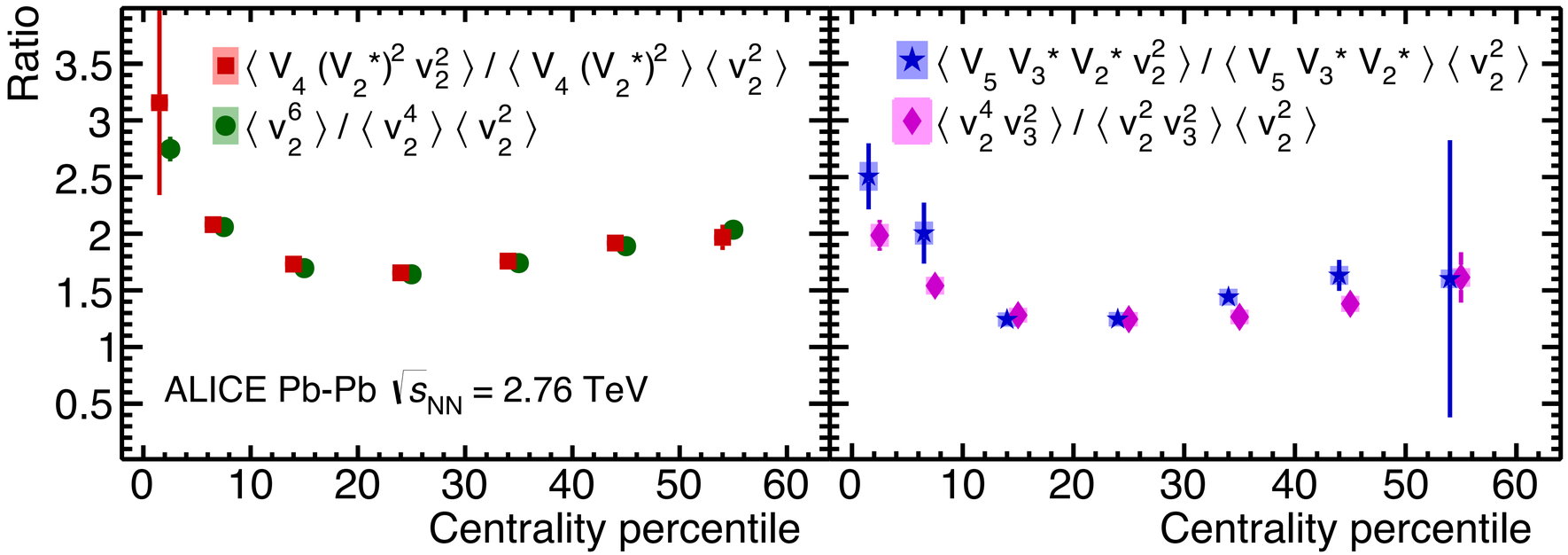
- Non-flow effects suppressed largely by multi-particle correlations in the numerator and denominator
- Residual non-flow tested with various gaps between the sub-events
  - Included in the systematic uncertainties

Are V_n^{NL} and V_n^{L} uncorrelated?

If yes then we should have

$$\frac{\langle V_4 (V_2^*)^2 v_2^2 \rangle}{\langle V_4 (V_2^*)^2 \rangle \langle v_2^2 \rangle} = \frac{\langle v_2^6 \rangle}{\langle v_2^4 \rangle \langle v_2^2 \rangle} \quad \text{eq. 1}$$

$$\frac{\langle V_5 V_3^* V_2^* v_2^2 \rangle}{\langle V_5 V_3^* V_2^* \rangle \langle v_2^2 \rangle} = \frac{\langle v_2^4 v_3^2 \rangle}{\langle v_2^2 v_3^2 \rangle \langle v_2^2 \rangle} \quad \text{eq. 2}$$



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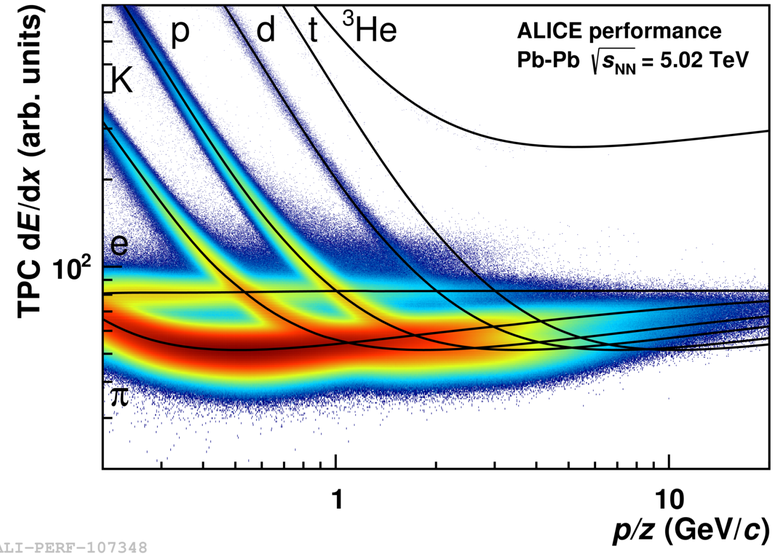
Particle Identification

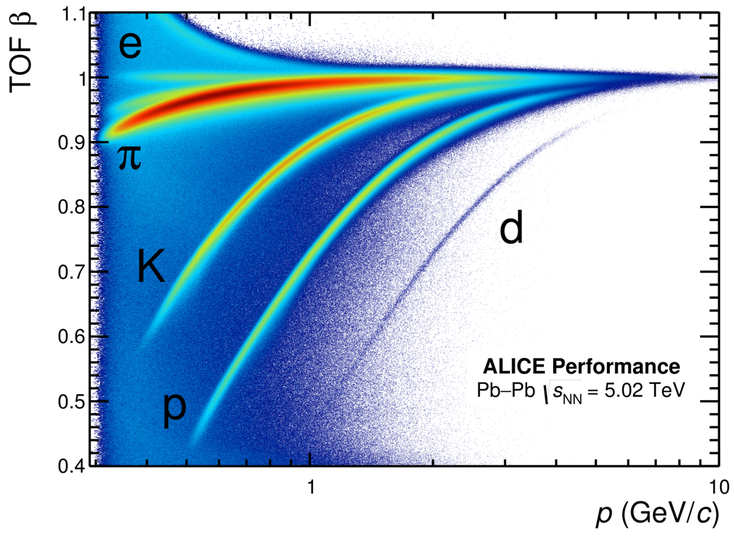
Time Projection Chamber (TPC)

dE/dx : the specific energy loss
Resolution: $\sigma_{dE/dx} \approx 5\%$



Time of Flight (TOF)

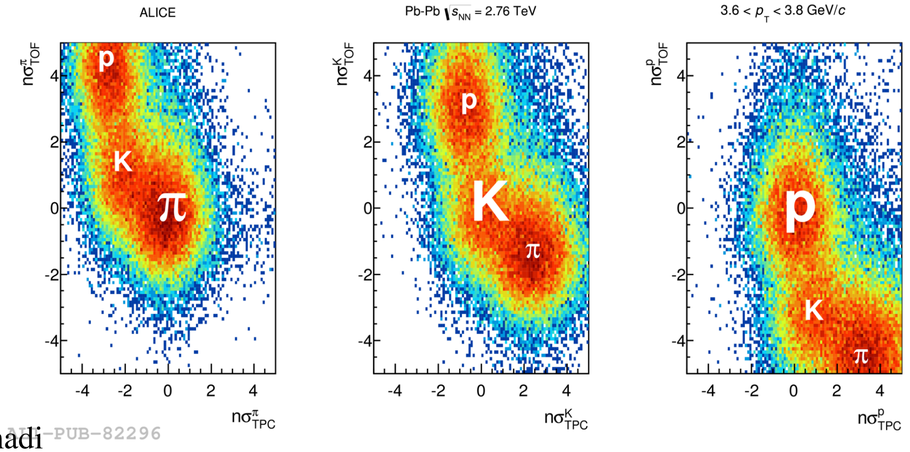
β=Track length/arrival time
Resolution: $\sigma_{TOF} \simeq 86$ ps for Pb-Pb collisions



- ✤ p <0.5 GeV/c TPC (dE/dx signal) (TPCnσ < 3)
- ✤ p>0.5 GeV TPC+TOF combined signals (pT dependent)

Combination of TPC and TOF used for PID

$\pi^{\pm}$: Purity > 90% up to p_T<6 GeV/c
$K^{\pm}$: Purity >75% up to p_T<4 GeV/c
p+p: Purity >80% up to p_T<6 GeV/c



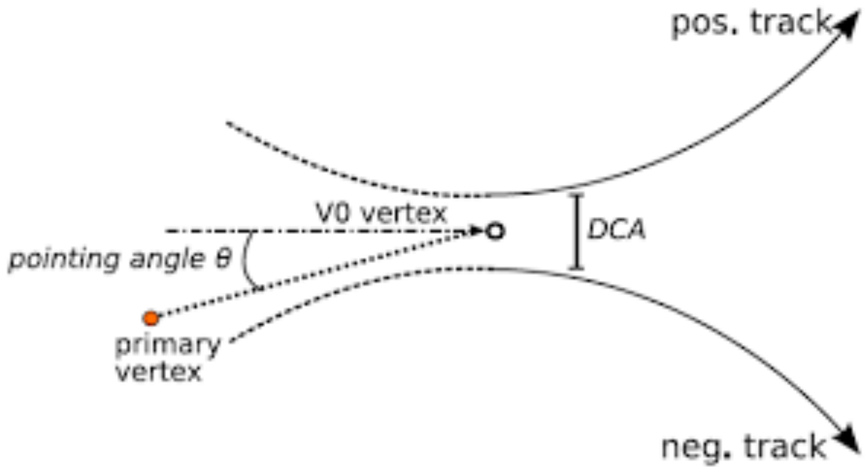
Analysis details

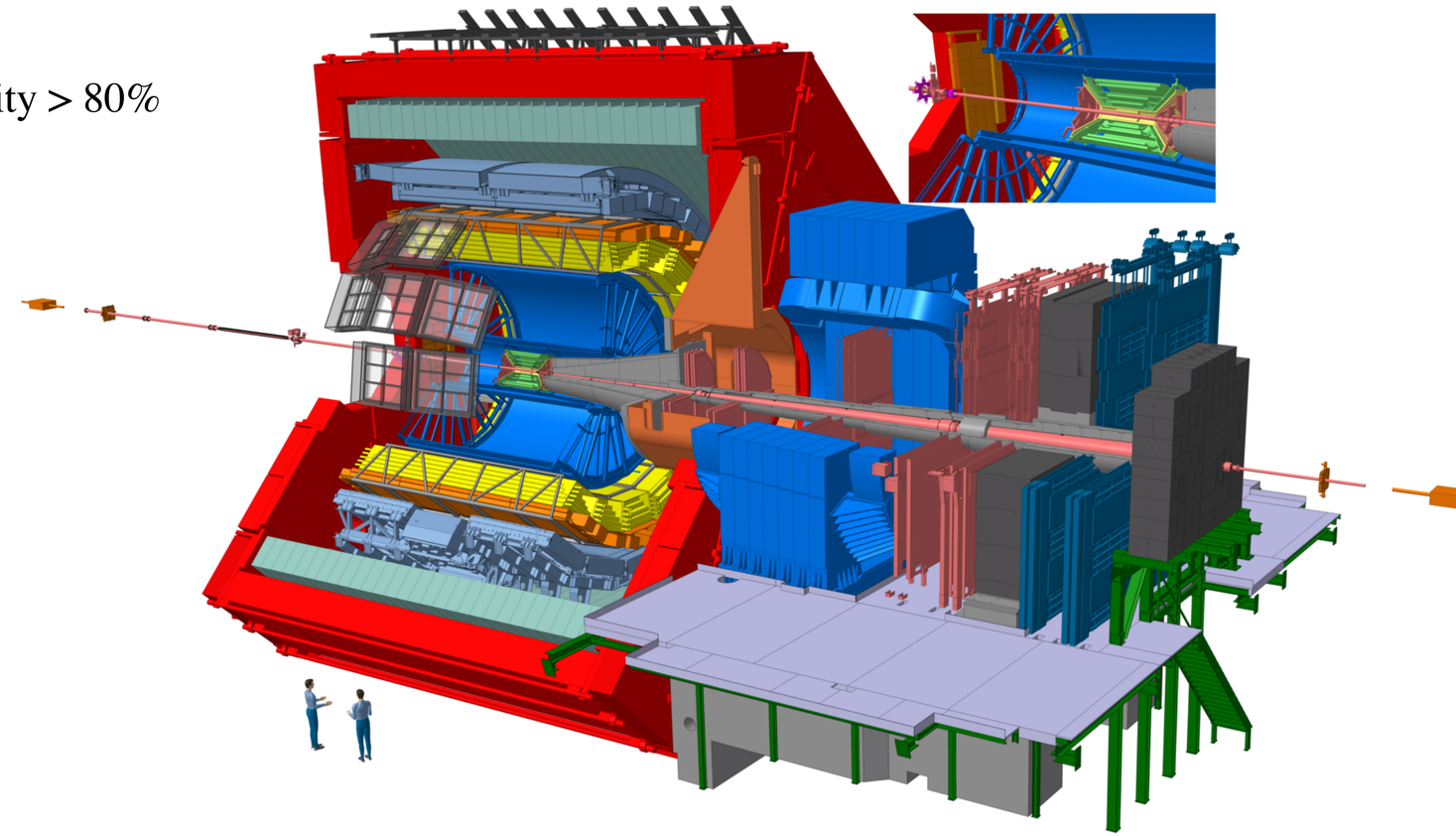
- Reconstruction of **K^0_s, Λ and ϕ**:
  - Via decay products on statistical basis
    - Particle Identification for the decay products: purity > 80%
    - Constraining decay topology

$$K^0_s \rightarrow \pi^+ + \pi^-$$

$$\phi \rightarrow K^+ + K^-$$

$$\Lambda(\bar{\Lambda}) \rightarrow p(\bar{p}) + \pi^-(\pi^+)$$



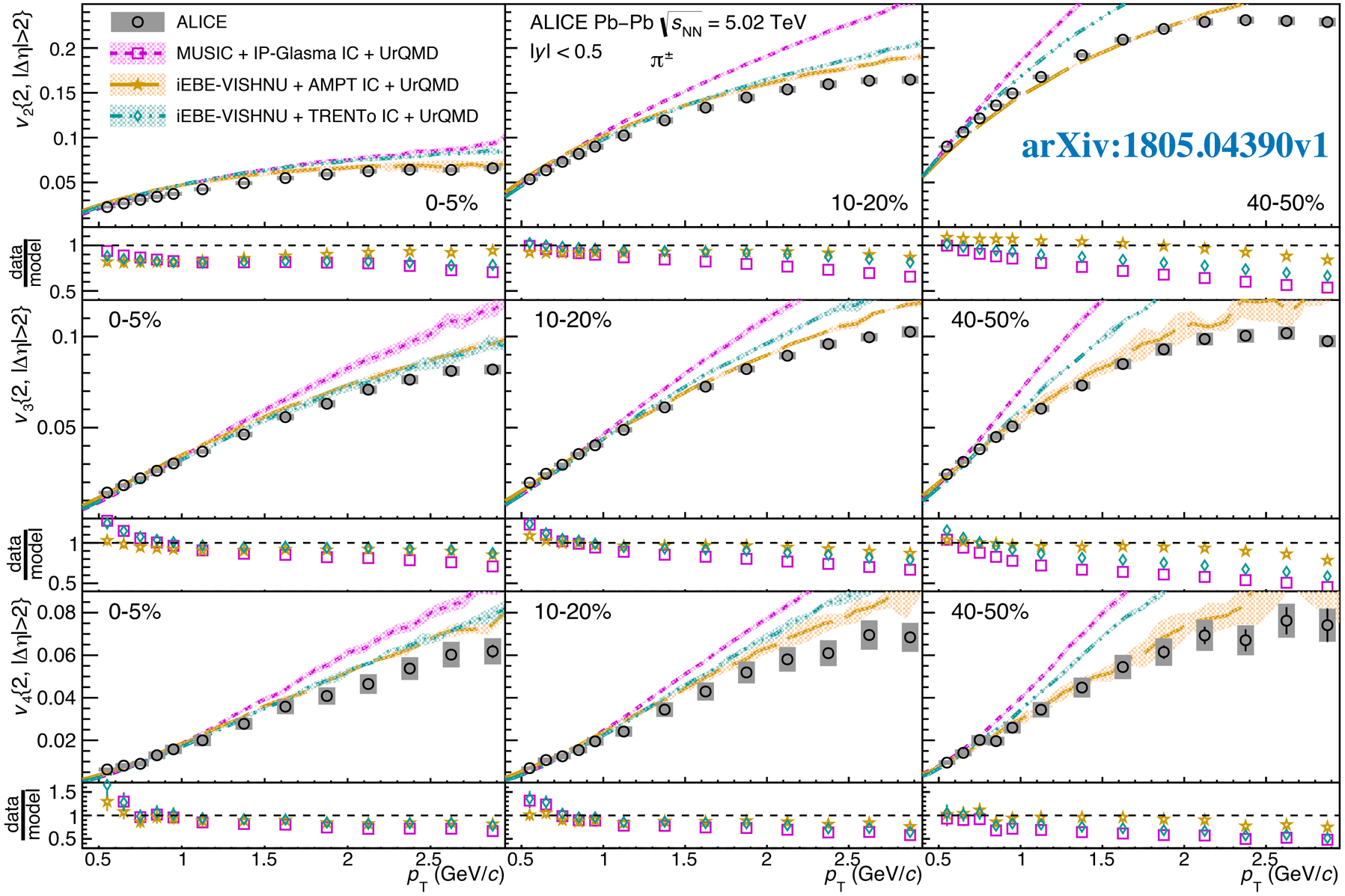


Hydrodynamic predictions:

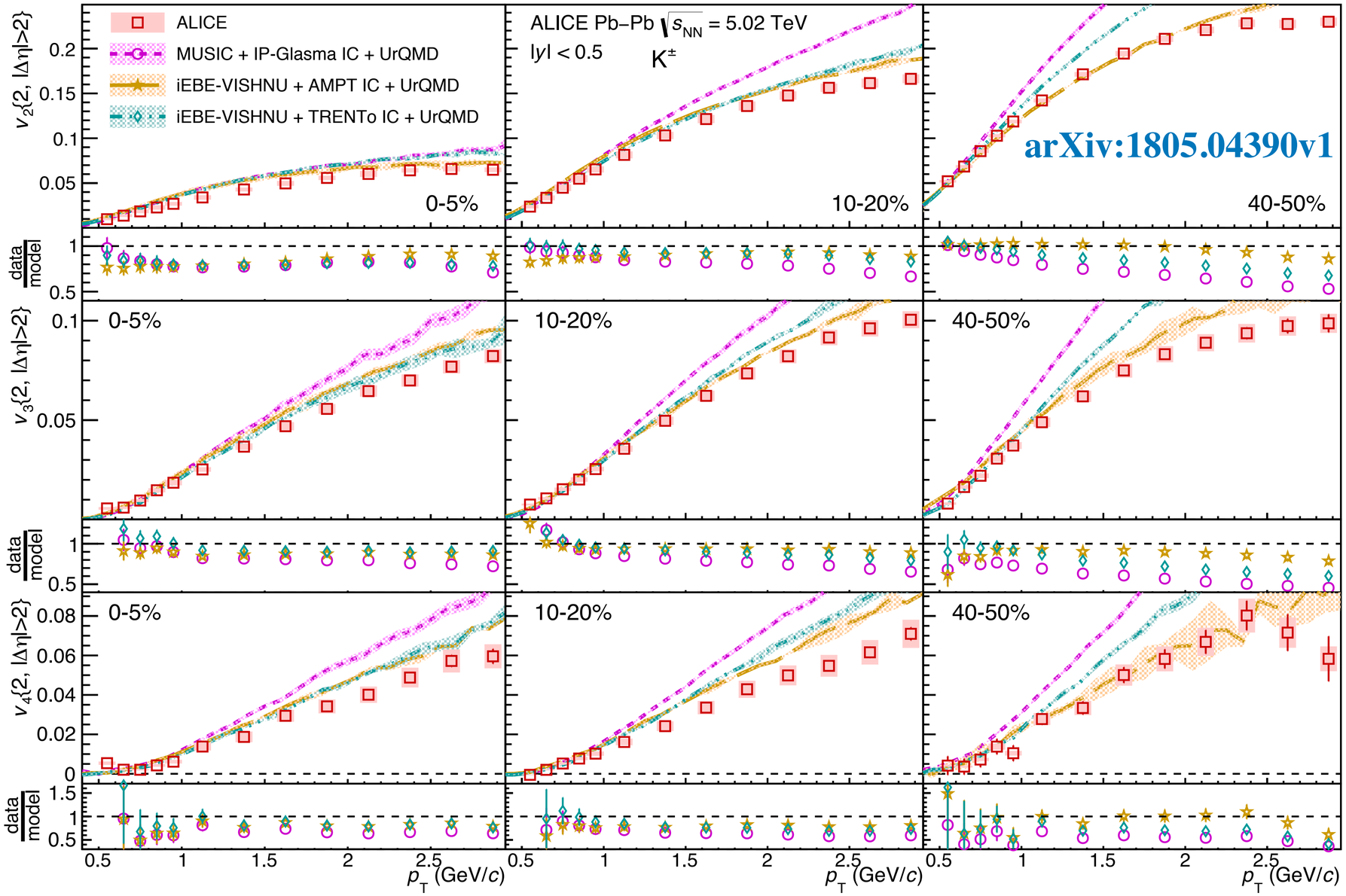
iEBE-VISHNU hybrid model (Eur.Phys.J. C77 (2017) no.9, 645, Zhao, Wenbin *et al.*):

- 2+1 dimensional viscous hydrodynamics (VISH2+1) coupled to hadron cascade model (UrQMD)
- Two initial conditions: AMPT, TRENTO
- Parameters for TRENTo Phys. Rev. C 94, 024907 (2016), JE Bernhard *et al.*
  - Reproduce multiplicity distributions in Pb+Pb, p+Pb, and Au+Au collisions at various collision energies
  - Temperature dependant specific shear viscosity $\eta/s(T)$ and specific bulk viscosity $\zeta/s(T)$
  - Entropy deposition: p= 0
  - T_{switch} = 148 MeV, τ_0 = 0.6 fm/c
- Parameters for AMPT:
  - $\eta/s(=0.08)$ and $\zeta/s(=0)$
  - T_{switch} = 148 MeV, τ_0 = 0.6 fm/c

Hydrodynamic predictions: v_n of pions



Hydrodynamic predictions: v_n of kaons



Hydrodynamic predictions: v_n of protons