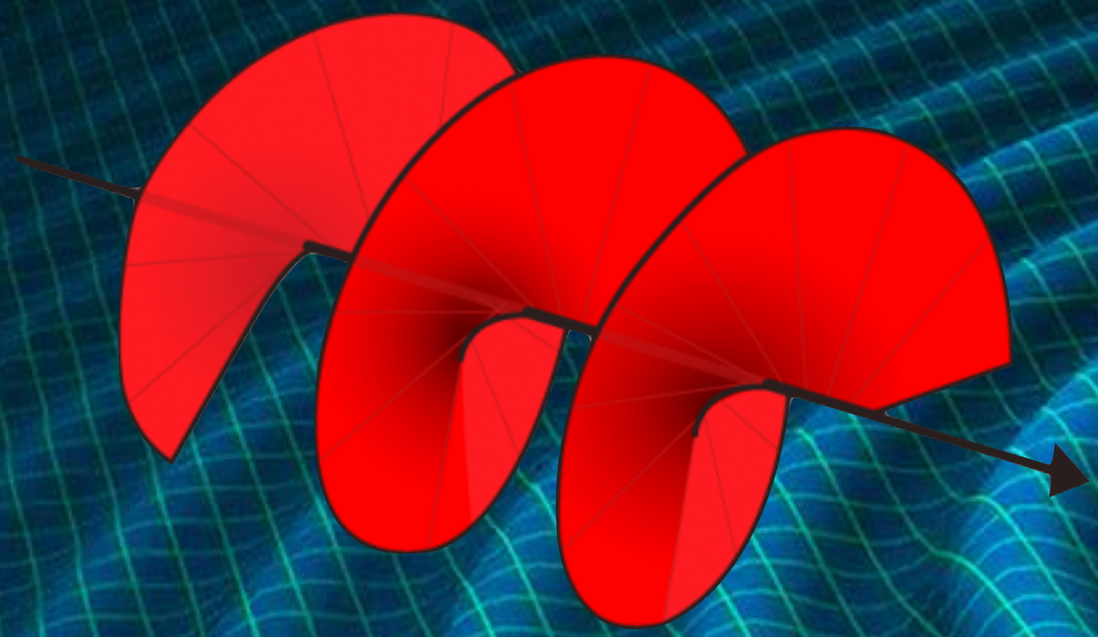

EPS - HEP 2019 Ghent

Detecting Gravitational Waves by Twisted Light - Dipole Interaction of Photons and Gravitational Waves

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School of Physics and Astronomy,

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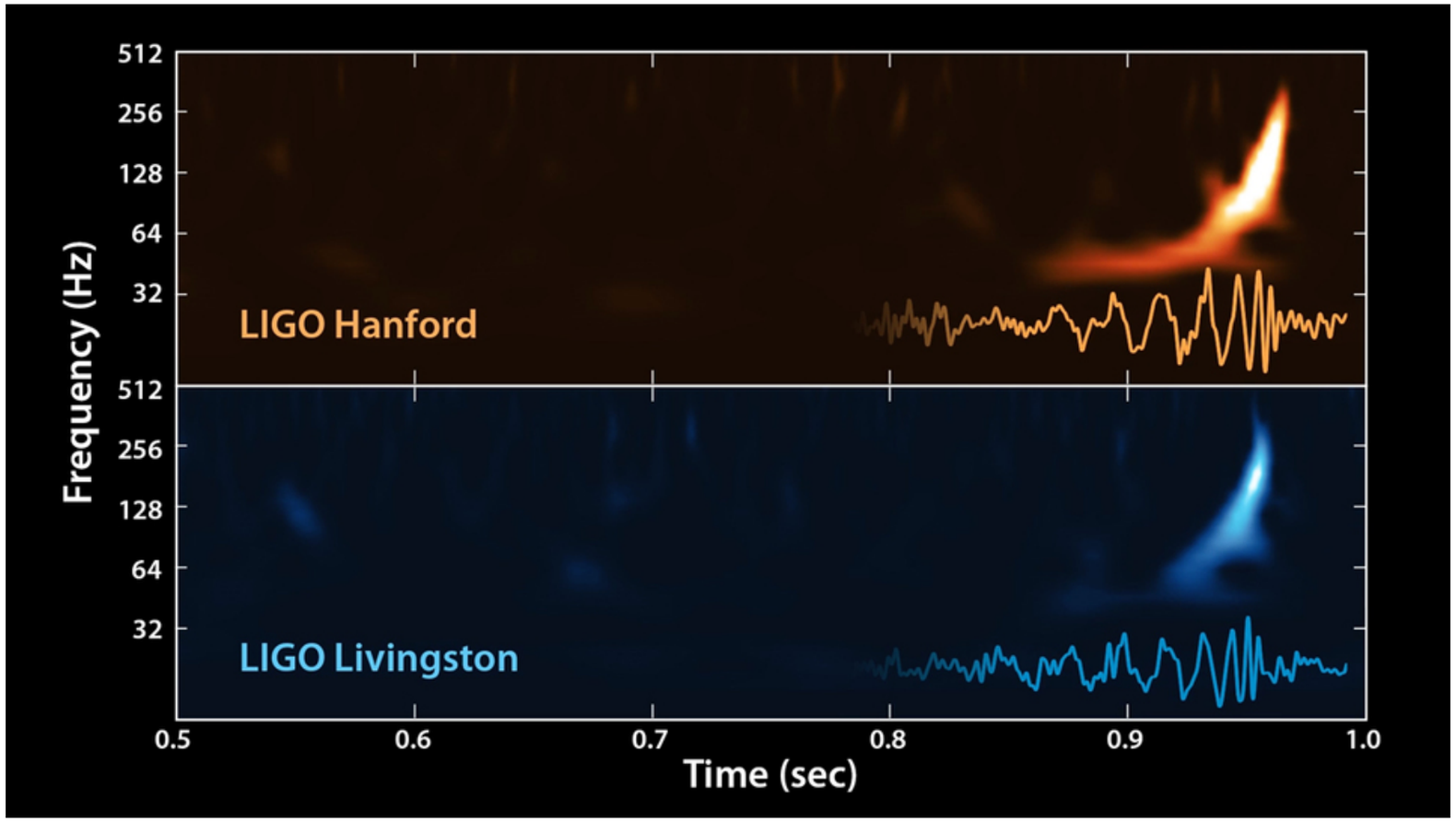
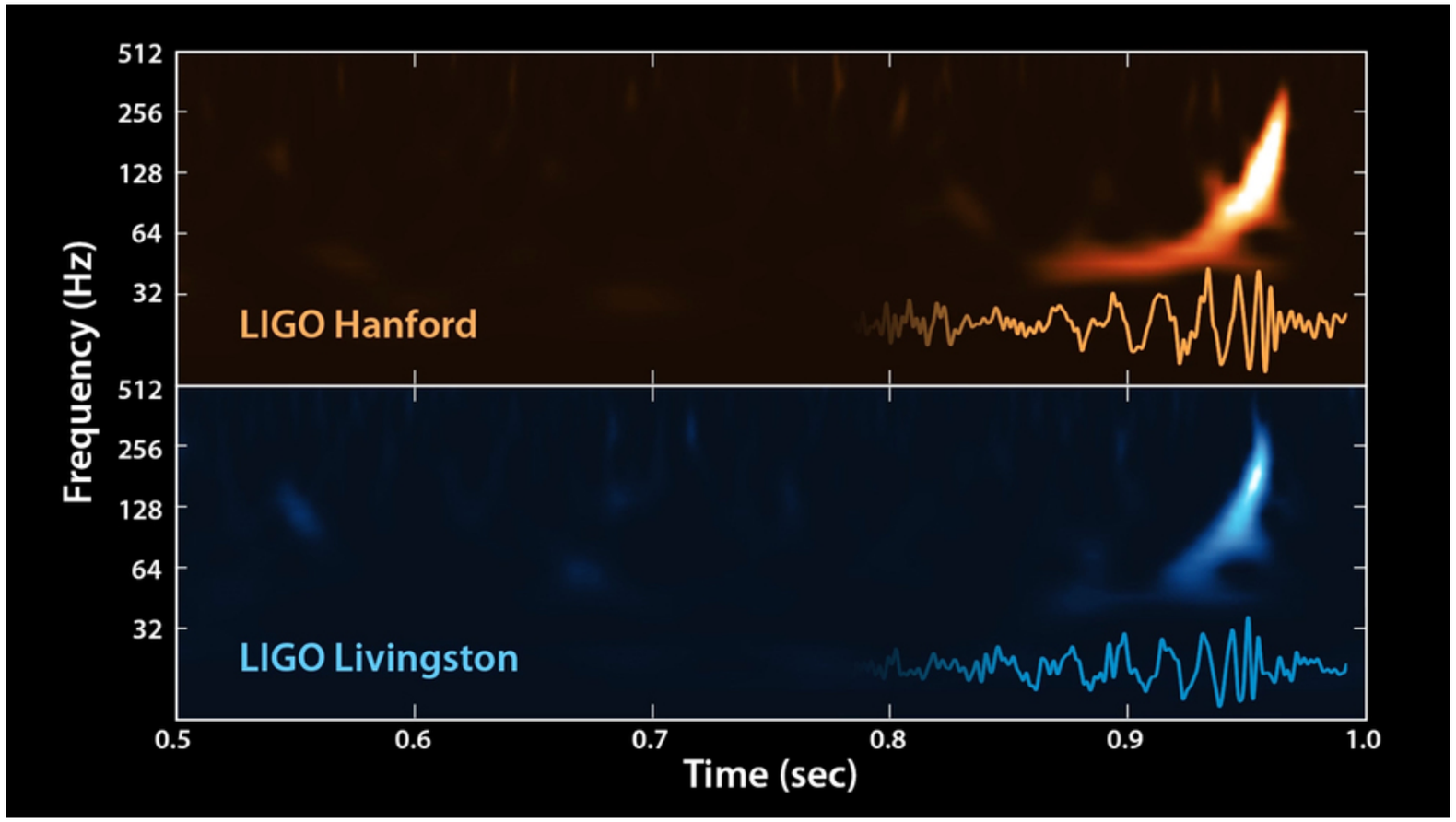
Motivations

- **The Detection of Gravitational Wave : the Advanced LIGO Interferometer**



Motivations

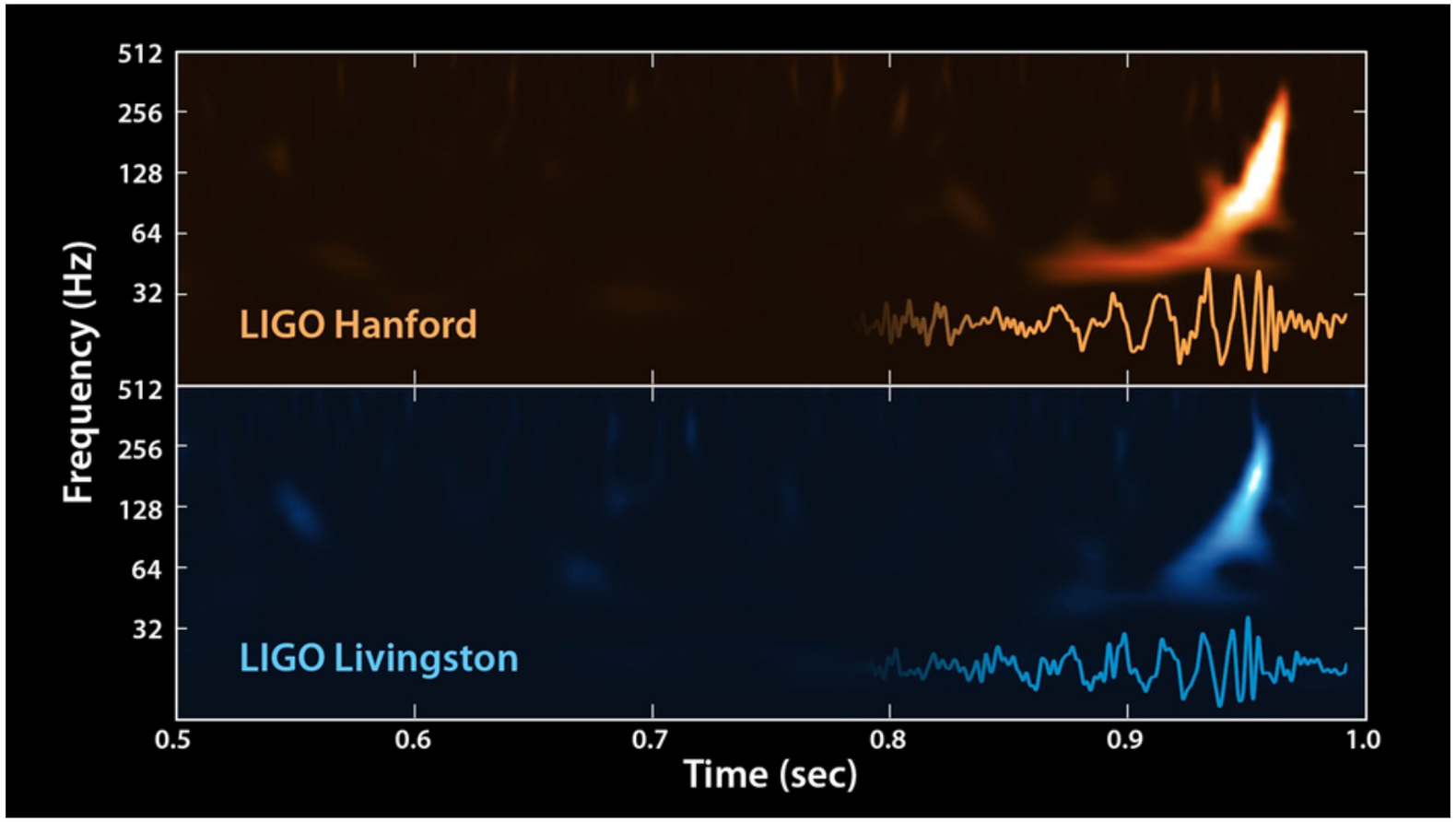
- **The Detection of Gravitational Wave : the Advanced LIGO Interferometer**



- **Transformation Optics: Optical Activity from Helicity-Rotation Interaction**

Motivations

- The Detection of Gravitational Wave : the Advanced LIGO Interferometer



- Transformation Optics: Optical Activity from Helicity-Rotation Interaction
- The Next-Generation of Laser-Interferometer GW Detectors

Maxwell - Schrodinger Equation

- **Vector Wavefunction**

$$|\Psi\rangle = \mathbf{D} + i\mathbf{B}$$

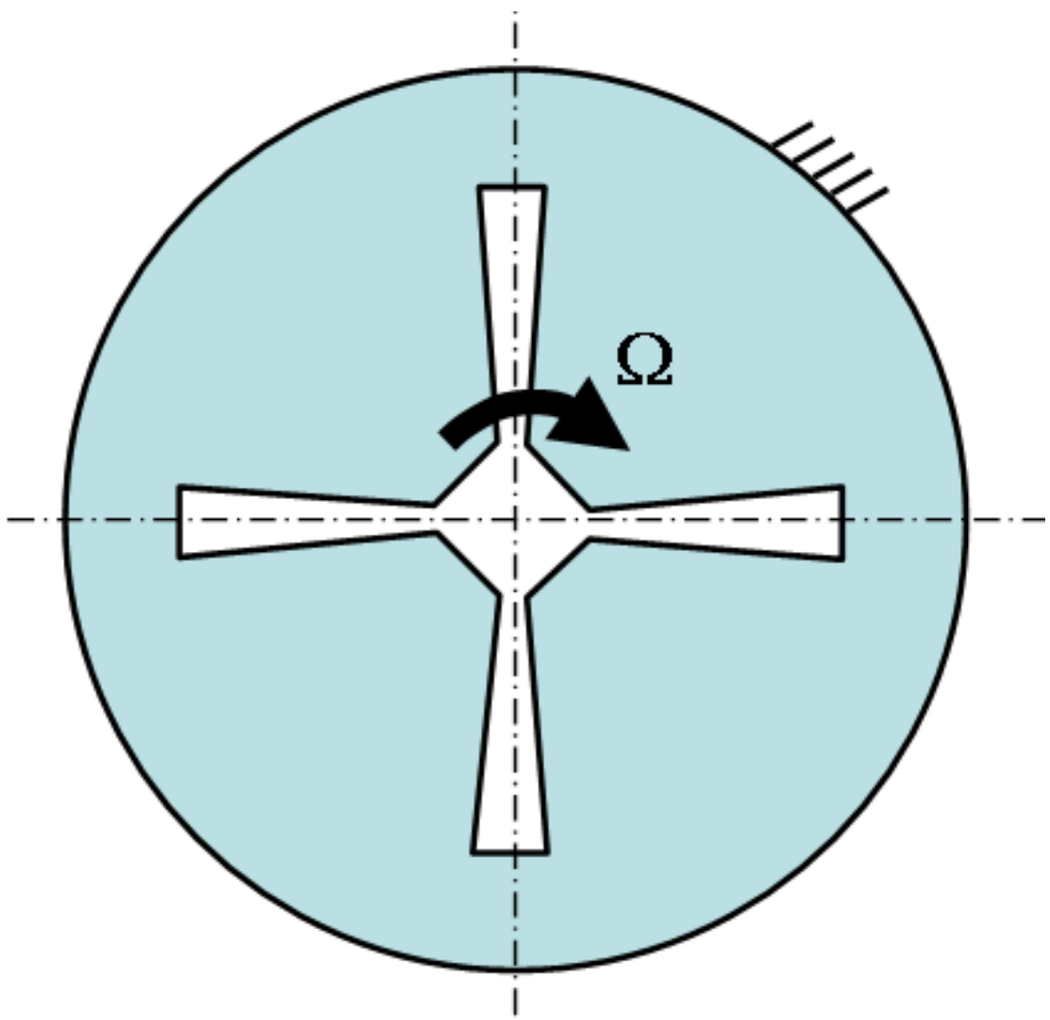
- **Schrodinger Equation**

$$i\frac{\partial}{\partial t}|\Psi\rangle = \nabla \times (|\Psi\rangle/\sqrt{g^{00}} + i\mathbf{g} \times |\Psi\rangle)$$

- **Transverse Condition**

$$\nabla \cdot |\Psi\rangle = 0$$

Electromagnetism in a Rotating Frame of Reference



- **Schrodinger Equation**

$$i\frac{\partial}{\partial t}|\Psi\rangle = (\mathbf{k}\cdot\mathbf{s} + \Omega\cdot(\mathbf{L}+\mathbf{s}))|\Psi\rangle$$

$$\mathbf{k}\cdot|\Psi\rangle = 0$$

$$\mathbf{k} = -i\nabla \qquad \mathbf{L} = \mathbf{x}\times\mathbf{k}$$

- **Time Evolution**

$$|\Psi(t)\rangle = e^{it\Omega\cdot\mathbf{J}}|\Psi_i\rangle$$

Light Propagating in Gravitational Waves

- **Maxwell Equation in a linear Medium**

$$i\frac{\partial}{\partial t}\mathbf{\Psi}^h = \nabla \times (\mathbf{U}\mathbf{\Psi}^h) \qquad \mathbf{U} = \begin{pmatrix} \mathbf{I}+\mathbf{Q} & \mathbf{q} \\ \mathbf{q}^\dagger & 1+q_0 \end{pmatrix}$$

- **Multipole Components**

$$q_0 = h_{33}$$

$$\mathbf{q} = \{Q_{+1}, Q_{-1}\}^T, \quad Q_{\pm 1} = \frac{1}{\sqrt{2}}(h_{13} \mp i h_{23})$$

$$Q_{11} = Q_{22} = \frac{1}{2}(h_{11} + h_{22})$$

$$Q_{12} = \frac{1}{2}(h_{11} - h_{22}) - i h_{12}, \quad Q_{21} = Q_{12}^*$$

Paraxial Equation for Twisted Light

Positive Helicity State: $|\mathbf{\Psi}\rangle_{\perp}^{h} = \Psi_0^{+} e^{-i\omega t + ikz} \mathbf{e}_{+}$

$$i\frac{\partial \Psi_0^{+}}{\partial z} = -\frac{1}{k'}\nabla_{+}\nabla_{-}\Psi_0^{+} - (\mathbf{q}\cdot\frac{1}{i}\nabla_{\perp})\Psi_0^{+}$$

Paraxial Equation for Twisted Light

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Paraxial Equation in the Flat Space

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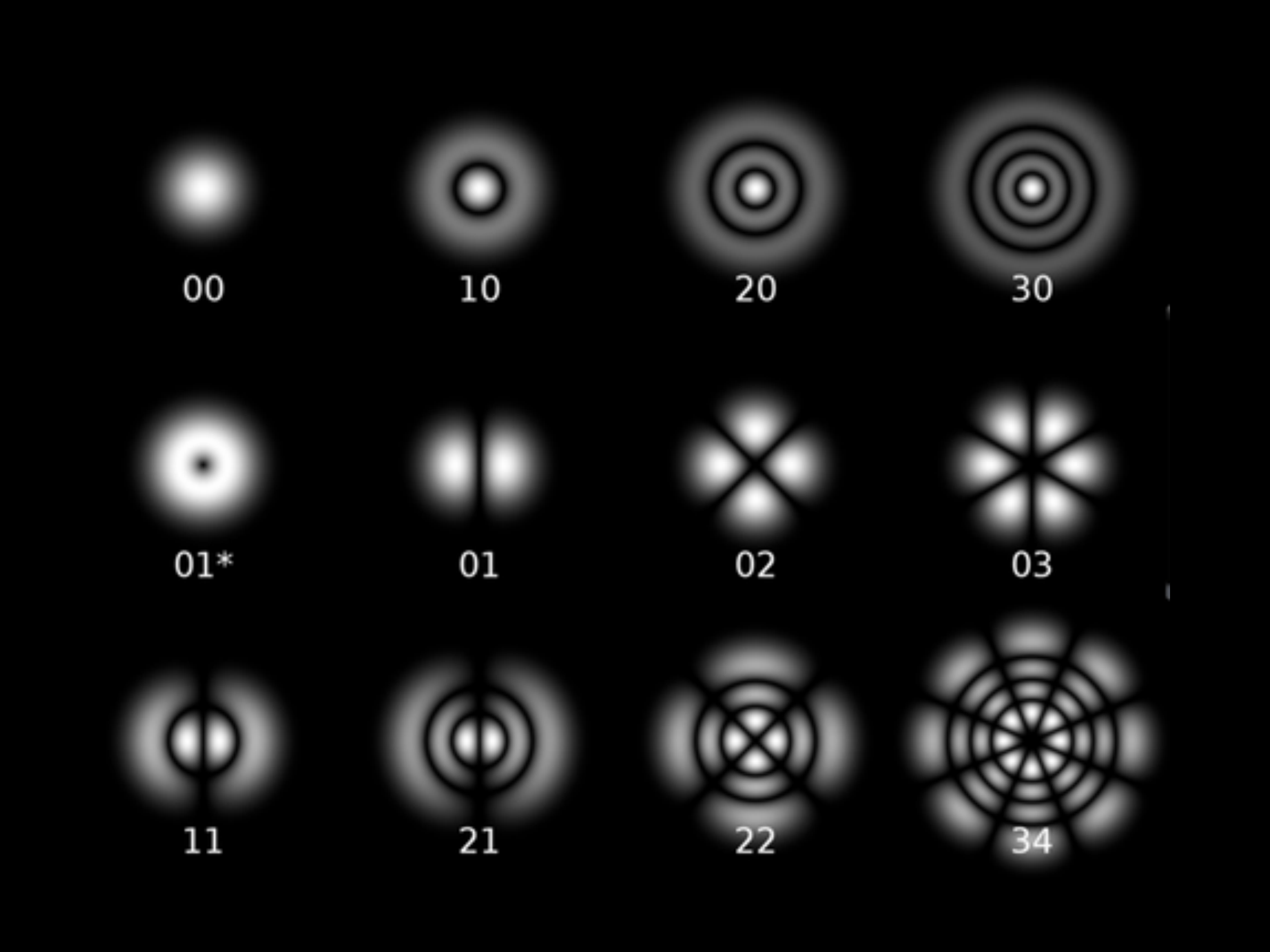
Paraxial Equation in the Flat Space

$$LG_n^l = \frac{a_n^l}{w(z)}\left(\frac{\sqrt{2}\rho}{w(z)}\right)^{|l|} L_n^{|l|}\left(\frac{2\rho^2}{w(z)^2}\right) e^{-\rho^2/w(z)^2} e^{ik\rho^2/2R} e^{il\phi} e^{-i\varphi(z)}$$

Laguerre-Gaussian Modes

$$a_n^l = (2n!/\pi(n+|l|)!)^{1/2} \qquad z_R = \frac{1}{2}kw_0^2$$

$$\varphi(z) = (2n+|l|+1)\tan^{-1}(z/z_R)$$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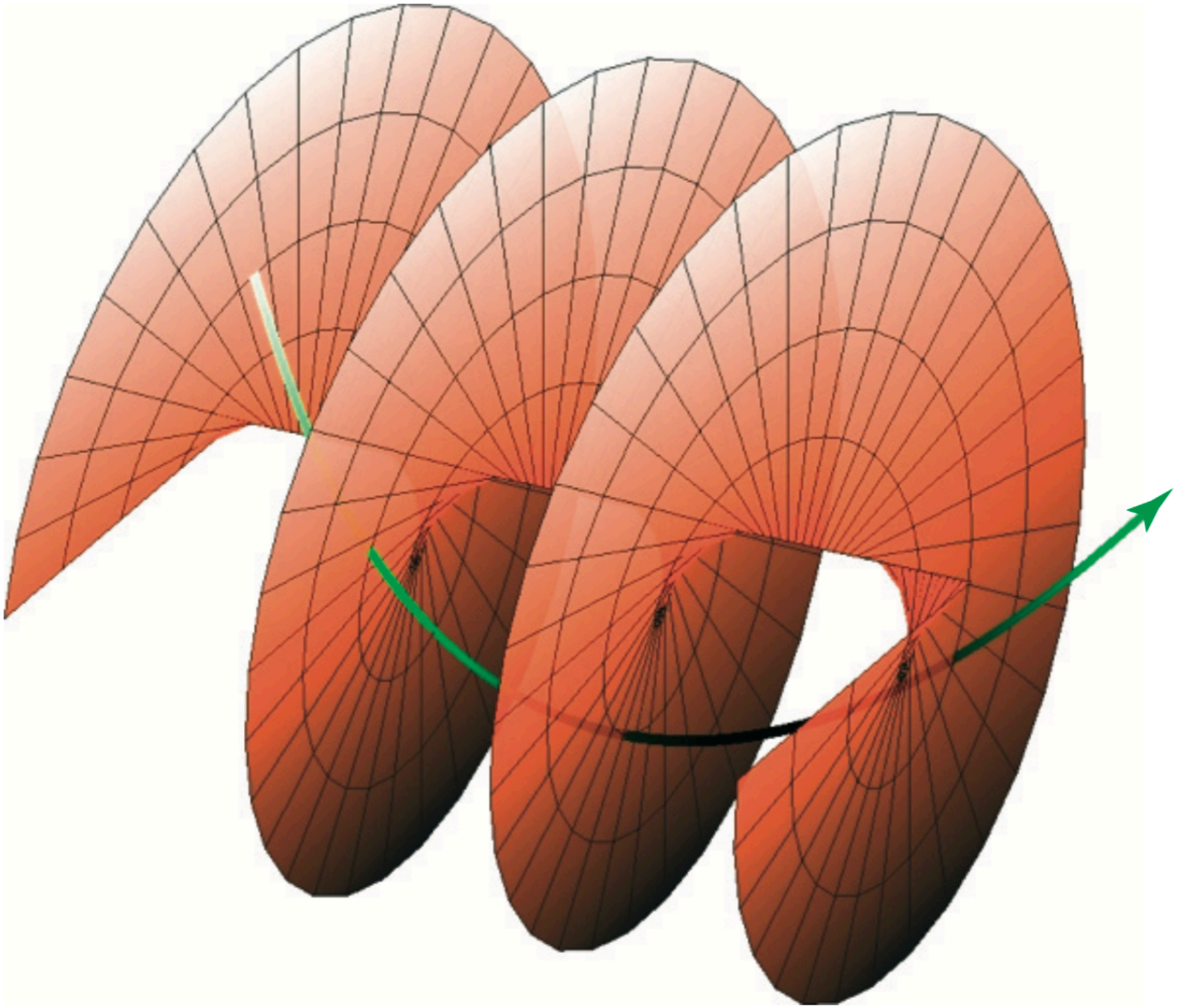
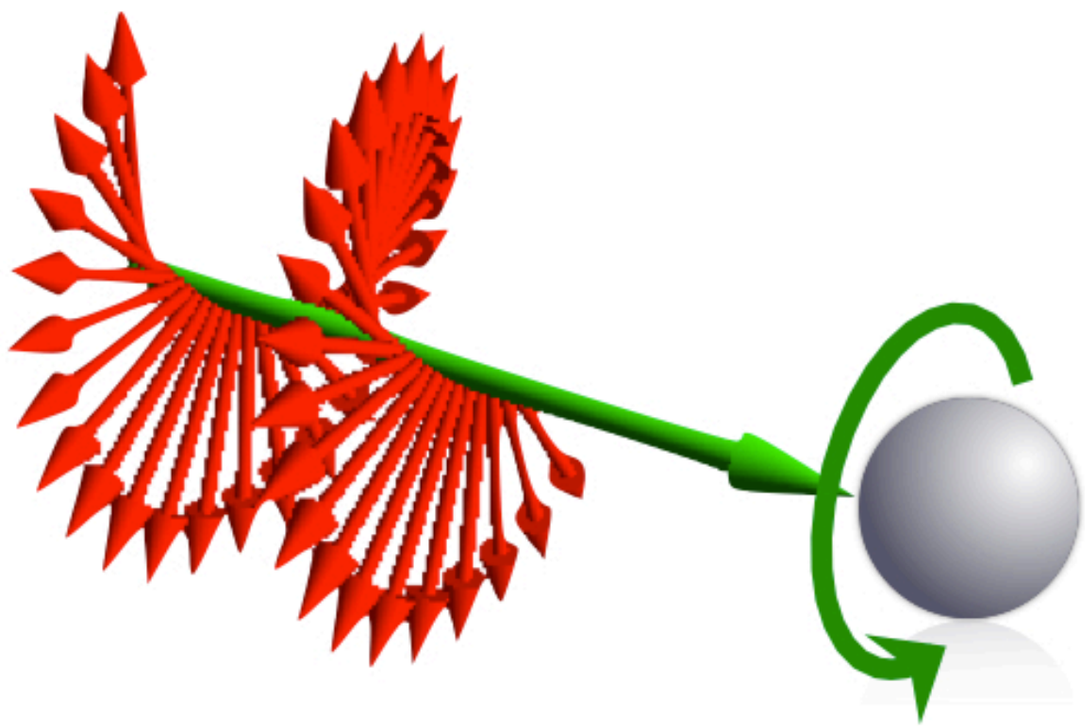
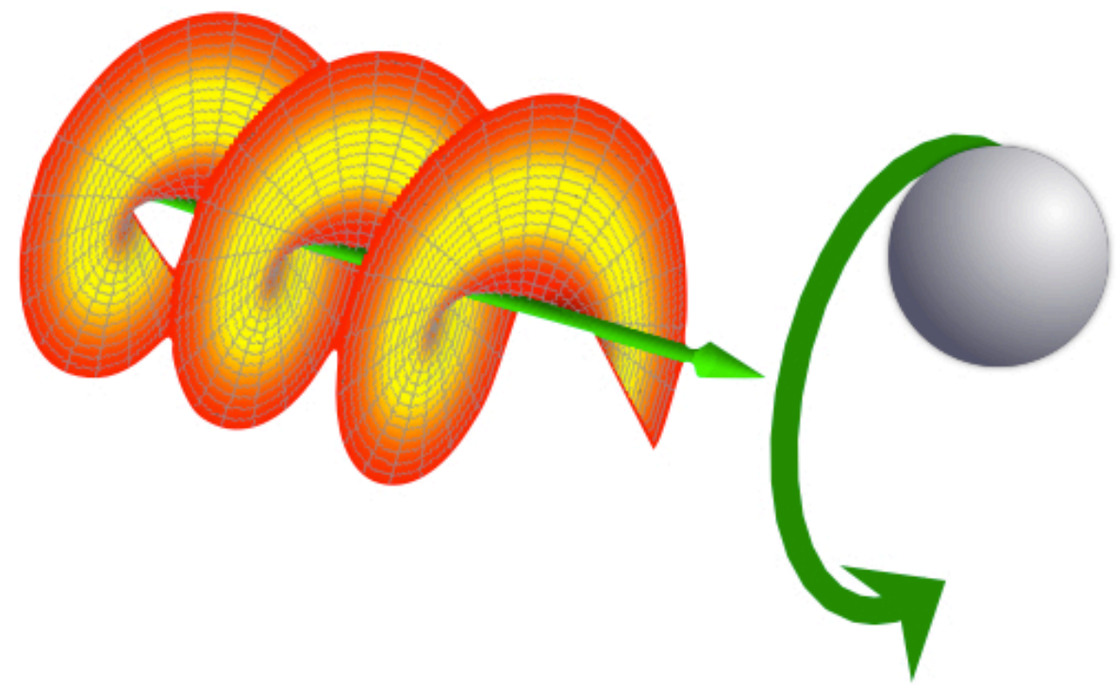


SAM & OAM



SAM interaction

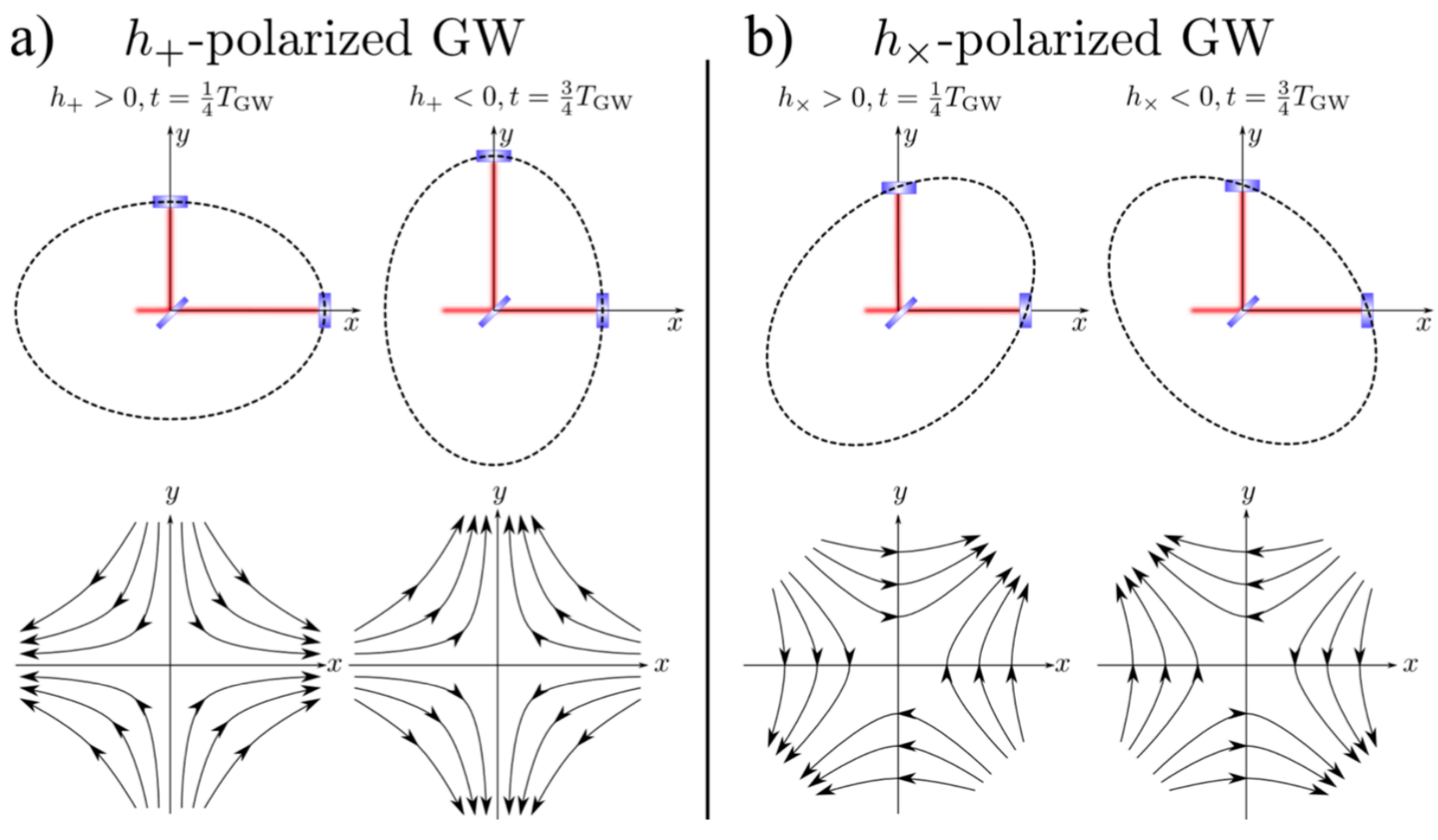


OAM interaction

Dipole Interaction of Photons and Gravitational Waves I

$$H_I = -\mathbf{q} \cdot \mathbf{k}_\perp \qquad \mathbf{k}_\perp = -i\nabla_\perp$$

$$\mathbf{q} = h^+[\mathbf{e}_k - (\mathbf{e}_g \cdot \mathbf{e}_k)\mathbf{e}_g] + h^\times(\mathbf{e}_g \times \mathbf{e}_k)$$



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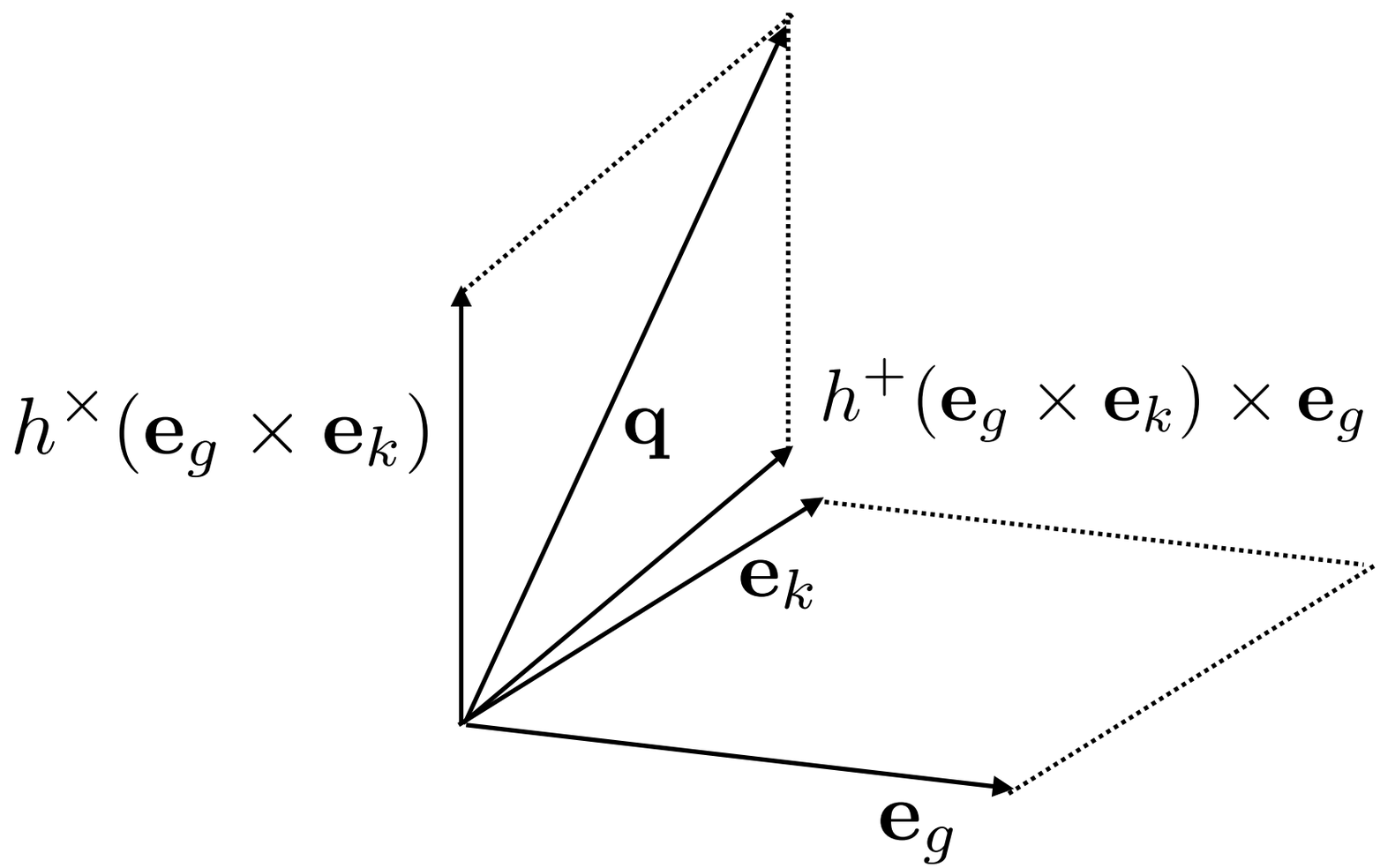


Dipole Transition of Photons

- **Selection Rule**

$$\nabla_{\pm}|n,\pm l\rangle_{LG} = k_w\Big[\sqrt{n+l}|n,\pm(l-1)\rangle_{LG} + \sqrt{n+1}|n+1,\pm(l-1)\rangle_{LG}\Big]$$

$$\nabla_{\mp}|n,\pm l\rangle_{LG} = -k_w\Big[\sqrt{n+l+1}|n,\pm(l+1)\rangle_{LG} + \sqrt{n}|n-1,\pm(l+1)\rangle_{LG}\Big]$$

- **Selection Rule**

$$\Psi_{0+} = \sum_{m,k}\xi_{m.k}(z)|m,k\rangle_{LG}$$

$$\frac{d\xi_{n,l}(z)}{dz} = \sum_{m,k}\langle n,l|Q_{-1}\nabla_{+} + Q_{+1}\nabla_{-}|m,k\rangle$$

Dipole in the Intensity Pattern

- **Input Light - Gaussian Mode**

$$|\mathrm{in}\rangle = |0,0\rangle_{LG}$$

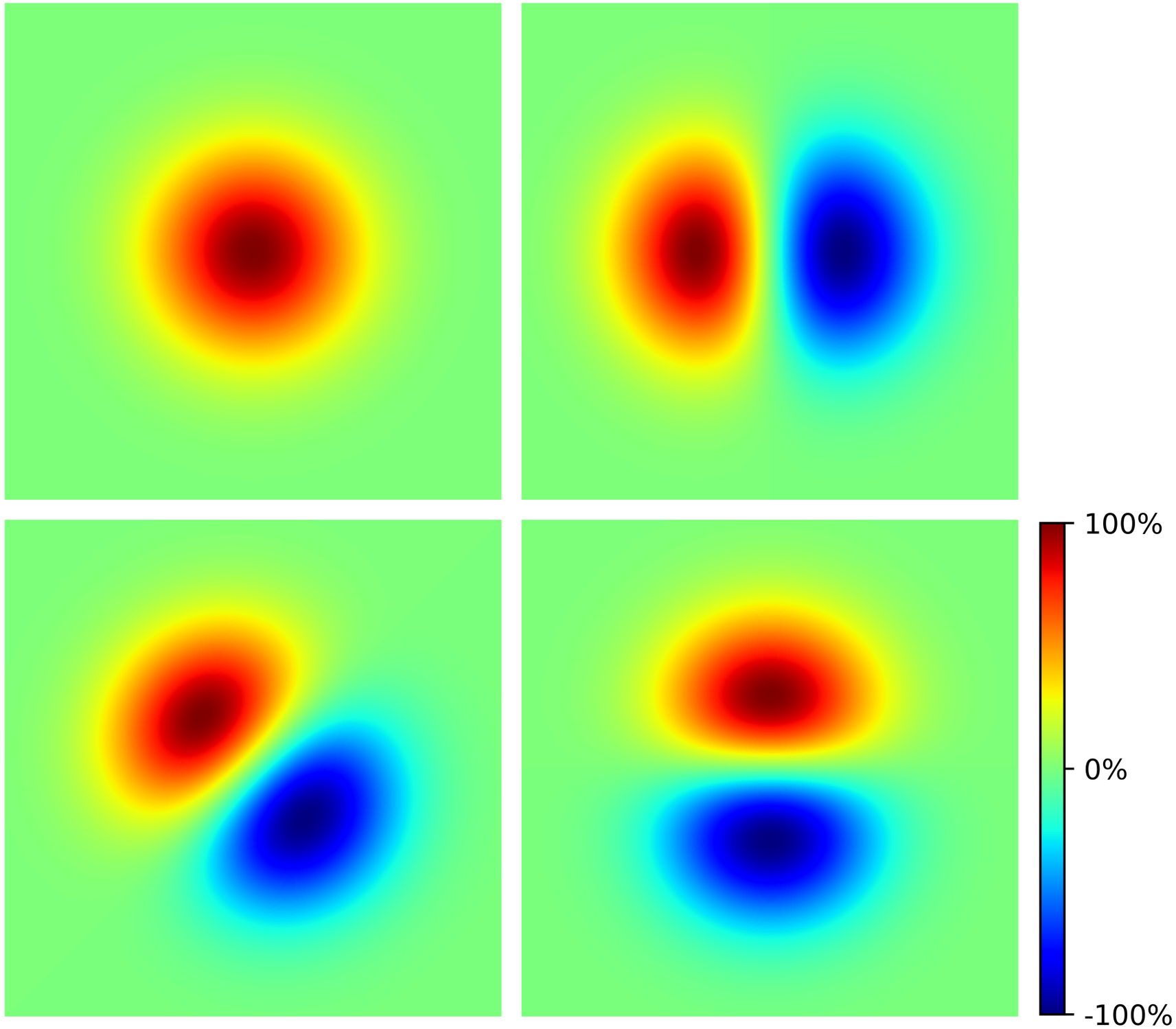
$$|\mathrm{out}\rangle = |\mathrm{in}\rangle - Q_1 k_w L\left[e^{i\alpha}|0,+1\rangle_{LG} + e^{-i\alpha}|0,-1\rangle_{LG}\right]$$

Rotation

$$\mathbf{e}_g \times \mathbf{e}_k$$

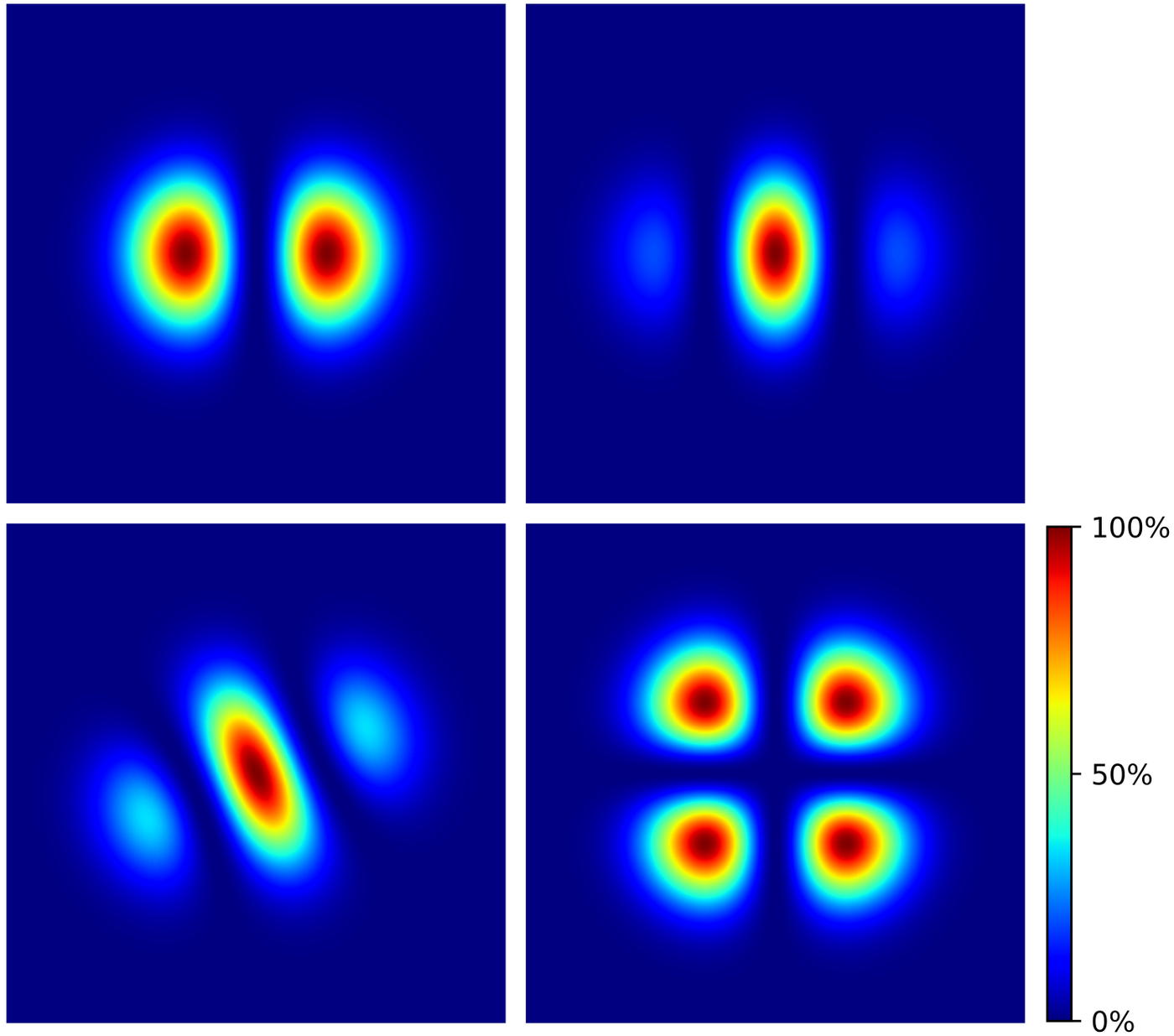
$$Q_{\pm 1} = \sin\theta_g h(\theta_g) e^{\pm i\alpha}$$

$$\alpha + \phi_g = \frac{\pi}{2} + \beta$$

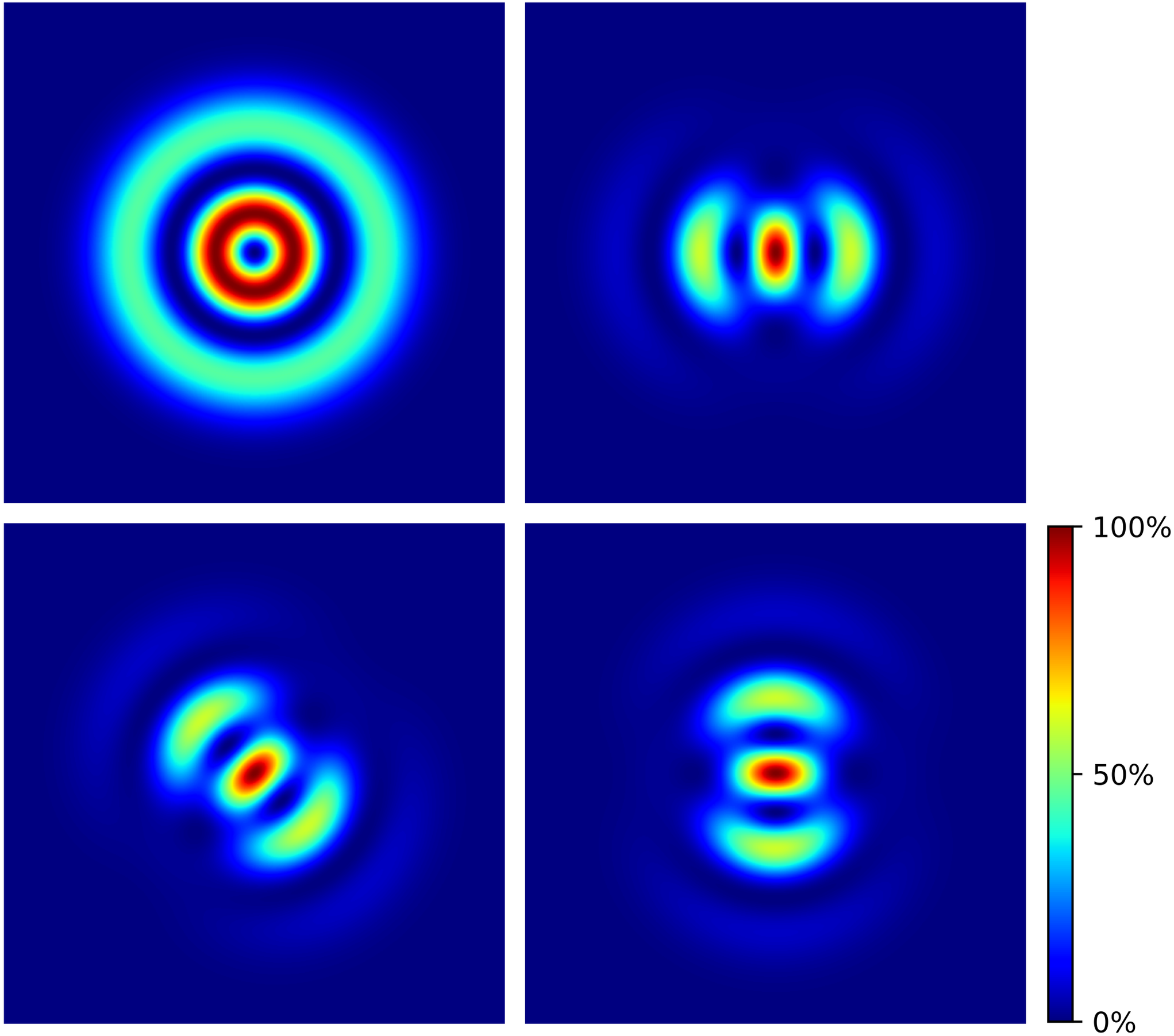


$$I \approx I_0 \left[1 - 4\sqrt{2} h k_w L \frac{\rho}{w(L)} \sin\theta_g \cos\varphi \cos(\phi + \alpha) \right]$$

$$|\text{in}\rangle = HG_{10} = \frac{1}{\sqrt{2}}(|0,+1\rangle_{LG} + |0,-1\rangle)_{LG}$$



$$\begin{aligned}|\text{out}\rangle =& |\text{in}\rangle + Q_1 k_w L \cos\alpha \left[|0,0\rangle_{LG} + |1,0\rangle_{LG}\right] \\ &- \frac{1}{\sqrt{2}} Q_1 k_w L \left[e^{i\alpha}|0,+2\rangle_{LG} + e^{-i\alpha}|0,-2\rangle_{LG}\right]\end{aligned}$$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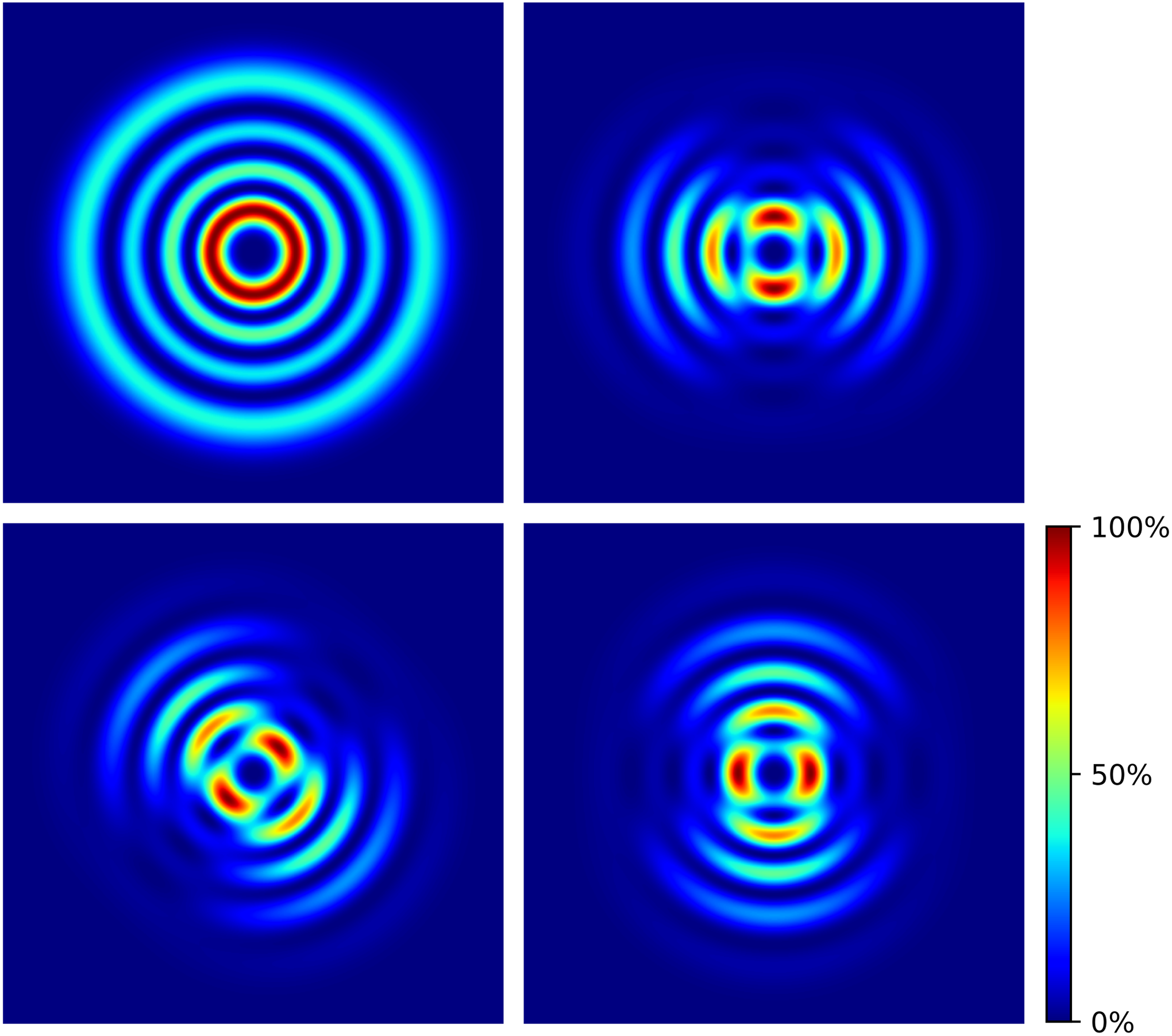
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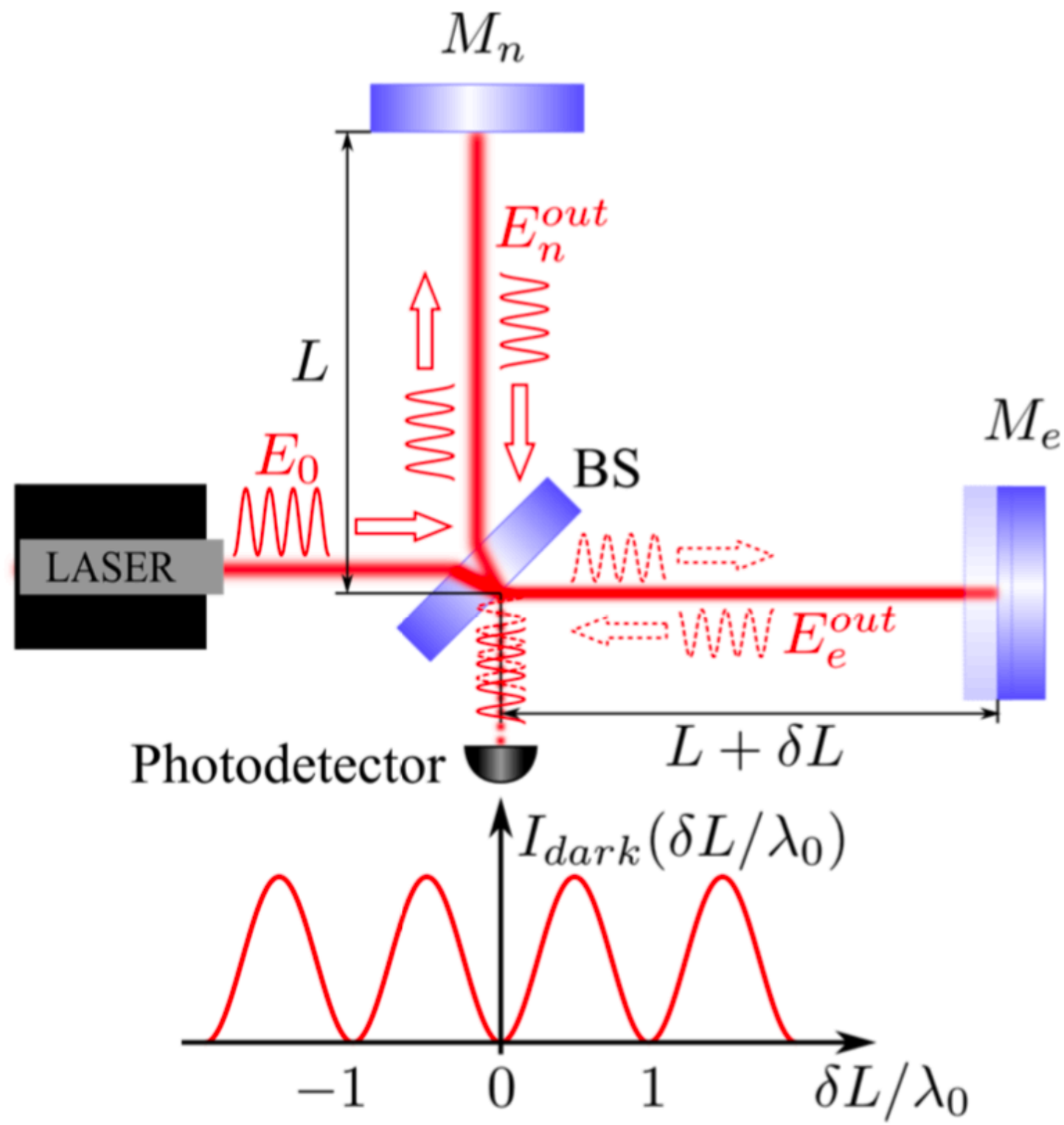
Michelson Interferometer



Summary

- Induced Dipole Structure for the Gaussian Beams
- Macroscopic Rotation of the Intensity Pattern
- Central Intensity Brighten

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Detecting Gravitational Waves in 2D Intensity Space