PSEUDOSCALAR CURRENT TRANSITION FORM FACTORS OF THE DELTA RESONANCE IN BARYON CHIRAL PERTURBATION THEORY

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INTRODUCTION

CHIRAL PERTURBATION THEORY (CHPT)

- Approximate chiral symmetry is spontaneously broken in QCD
  $\Rightarrow$ Existence of light weakly interacting Goldstone bosons
- ChPT: Effective field theory of QCD in low energy sector
- Typical scale: $p, M_\pi \ll \Lambda_\chi \approx 1\,\text{GeV}$
- Expansion in low momenta and masses of Goldstone bosons
- Interaction of pions with nucleons and external fields

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \ldots \\ =& \mathcal{L}_2 + \mathcal{L}_4 + \ldots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \ldots\end{aligned}$$

POWER COUNTING (PC)

- The $\Delta(1232)$ is a well separated baryon resonance in the πN system.
- Chiral expansion, $M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\text{MeV}$
- Employ the so-called small scale expansion (SSE) or ϵ expansion with the expansion parameter being defined as

$$\epsilon \in \left\{ \frac{p}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right\},$$

the delta-nucleon mass splitting is treated on the same footing as the pion mass

- Small scale expansion, $M_\pi \sim \Delta \ll \Lambda_\chi$

COMPLEX-MASS RENORMALIZATION SCHEME (CMS)

- Resonances have complex properties; mass, width, ...
- CMS is the generalization of the on-mass-shell scheme to unstable particles,
  $\rightarrow m_0 = (m_R + i\frac{\Gamma_R}{2}) + \delta m \equiv z_R + \delta m$
- z_R, pole of the propagator in the chiral limit

PSEUDOSCALAR $\Delta \to \Delta$ TRANSITION FORM FACTORS

- Consider the reaction $\pi N \to \pi\pi N$
- Provide an important test to describe the understructure of the nucleon and the underlying dynamics
- E.g., Goldberger-Treiman relation relates the Δ axial charge with the $G_{\pi\Delta\Delta}$
- Less theoretical and experimental work

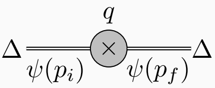

Figure 1: Pseudoscalar current $\Delta - \Delta$ transition represented by a class of diagram

- The structure of a particle is encoded in a several form factors.
- For the $\Delta \to \Delta$ transition: the two form factors of the pseudoscalar current, $\tilde{g}(q^2)$ and $\tilde{h}(q^2)$.
- They encode the structure of the matrix elements of the pseudoscalar density operator $P(x)$ which, in the SU(2) case, is given by

$$P^i(x) \equiv i\bar{q}(x)\gamma_5 \frac{\tau^i}{2} q(x), \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad i = 1, 2, 3.$$

- The corresponding matrix element of the pseudoscalar current between initial and final delta (Δ) states is parameterized as

$$\left\langle \Delta(p_f) \,\middle|\, P \,\middle|\, \Delta(p_i) \right\rangle = -\frac{1}{2}\bar{u}_\alpha(p_f)\left[g^{\alpha\beta}(\tilde{g}(q^2)\gamma^5) + \frac{q^\alpha q^\beta}{4m_\Delta^2}(\tilde{h}(q^2)\gamma^5)\right]u_\beta(p_i),$$

- $G_{\pi\Delta\Delta}(q^2) \sim \tilde{g}(q^2), \quad H_{\pi\Delta\Delta}(q^2) \sim \tilde{h}(q^2).$

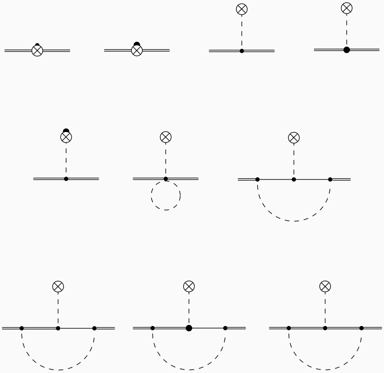

Figure 2: Tree level and one-loop contributions to the pseudoscalar Δ transition form factors at $\mathcal{O}(p^4)$. The cross represent the pseudoscalar source. The light dots represent lower order vertices while the heavy dot characterizes higher orders.

$$\mathcal{L}_2 = \frac{F^2}{4}(Tr[D_\mu U(D^\mu U)^\dagger] + Tr[\chi U^\dagger + U\chi^\dagger]),$$

$$\begin{aligned}\mathcal{L}_4^{GL} = & L_4 Tr[D_\mu U(D^\mu \chi)^\dagger]Tr(\chi U^\dagger + U\chi^\dagger) + L_5 Tr[D_\mu U(D^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)] \\ & L_6[Tr(\chi U^\dagger + U\chi^\dagger)]^2 + L_7[Tr(\chi U^\dagger - U\chi^\dagger)]^2 \\ & L_8 Tr(U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger)]^2 + H_2 Tr(\chi\chi^\dagger) + \ldots\end{aligned}$$

where

$$\chi = 2B_0(s + ip)$$

$$U = u^2 = \exp\frac{i\phi}{F}, \quad \phi = \vec{\tau}.\vec{\phi},$$

$$u_\mu = i[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger a_\mu u + u a_\mu u^\dagger)]$$

$$\mathcal{L}^{(1)}_{\pi\Delta} = -\bar{\psi}^{\mu}\ P^{\frac{3}{2}}\ \Big\{ \frac{g_1}{2} \not{u} \gamma_5 g_{\mu\nu} + \frac{g_2}{2}(\gamma_\mu u_\nu + u_\mu \gamma_\nu)\gamma_5$$

$$+ \frac{g_3}{2}\gamma_\mu \not{u} \gamma_5 \gamma_\nu + \ldots \Big\}\ P^{\frac{3}{2}} \psi^{\nu},$$

$$\mathcal{L}^{(3)}_{\pi\Delta} = \bar{\psi}^{\mu}_{i}\ P^{\frac{3}{2}}_{ik} \theta(z)_{\mu\alpha} \big(i c_3 \nabla^{\alpha} \chi_{-,\,j} \tau_j \gamma_5 \gamma_\nu + \frac{1}{2} c_5\ u^{\alpha}_{j} Tr(\chi_+) \tau_j \gamma_5 \gamma_\nu \big) \psi^{\nu}_{k} + \ldots$$

$$\mathcal{L}^{(4)}_{\pi\Delta} = \bar{\psi}^{\mu}_{i}\ P^{\frac{3}{2}}_{ij} \theta(z)_{\mu\alpha} c_8 \det(\chi) \psi^{\alpha}_{j}$$

- Non-trivial constraints among the known coupling constants $g_1 = -g_2 = -g_3$

$$\mathcal{L}^{(2)}_{\pi N\Delta} = \bar{\psi}^i_\mu P^{\frac{3}{2}}_{ij} \theta^{\mu\alpha}[ib_3\, \omega^j_{\alpha\nu}\gamma^\nu - \frac{b_8}{m_N}\omega^j_{\alpha\nu}D^\nu]\Psi,$$

where

$$\theta^{\mu\nu} = g^{\mu\nu} - \gamma^\mu\gamma^\nu,$$
$$\omega^i_{\mu\nu} = \frac{1}{2}Tr(\tau^i[\partial_\alpha, u_\beta])$$

- Studied the reaction $\pi N \to \pi\pi N$ in manifestly covariant BChPT by *Siemens et al., Phys. Rev. C89, 065211*(2014), determination of the LECs using total cross section data

HOW TO EXTRACT FORM FACTORS?

- Diagram$[p_f, p_i, \mu, \nu]$ is the result of the diagrams after one reduces the tensor integrals and simplifies the algebra

$$\text{Diagram}[p_f, p_i, \mu, \nu] := k_1 g^{\mu\nu} + k_2 p_i^{\mu} p_f^{\nu}$$

- FFs$[p_f, p_i, \mu, \nu]$ is the structure of the transition matrix element

$$\text{FFs}[p_f, p_i, \mu, \nu] := -\frac{1}{2}\left(g^{\alpha\beta}\gamma^5 \tilde{g} \;+\; \frac{q^{\alpha} q^{\beta}}{4m_{\Delta}^2}\gamma^5 \tilde{h}\right)$$

- $k_1 \to$ the coefficient of the result corresponding to $g^{\mu\nu}$
  $k_2 \to p_f^{\mu} p_i^{\nu}$
- $\tilde{g} : -2k_1, \quad \tilde{h} : \frac{-8k_2}{(m_{\Delta})^2}$

- Three Unknowns: c_3, c_5, c_8 as free parameters
- Confront with either the experimental or lattice QCD data
- Take the linear combination of c_3, c_5 and c_8 and use for normalization
  $\rightarrow$ Lattice QCD results by
  Alexandrou et al., Phys. Rev. D87, 114513(2013)
- Energy dependence of the $\tilde{g}, \tilde{h}$ and estimation of the numerical values of the couplings $G_{\Delta\Delta\pi}$ and $H_{\Delta\Delta\pi}$

- Study of the pseudoscalar-$\Delta(1232)$ transition form factors up to 4th chiral order in relativistic baryon chiral perturbation theory
- Since Δ is an unstable particle $\rightarrow$ CMS as a renormalization scheme
- Compare the results with Lattice calculation for the unknown free parameters
  - Obtain the q^2 dependence of the $\tilde{g}$ and $\tilde{h}$
  - Estimate the values of the couplings $G_{\Delta\Delta\pi}$ and $H_{\Delta\Delta\pi}$

THANK YOU FOR YOUR ATTENTION