SPECTRUM OF RELATIVE MOMENTA OF THE NEUTRON AND PROTON AT THE DEUTERON PERIPHERAL BREAKUP IN THE LIMIT OF VERY LOW MOMENTUM TRANSFER

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In the limit of very low momentum transfer to one of the nucleons, the analytical expression for the spectrum $dW(k)$ of relative momenta $\mathbf{k}$ of the neutron and proton produced at the deuteron peripheral breakup is obtained taking into account the S-wave function of the deuteron. Just this formula for $dW(k)$ describes the spectrum of relative momenta of nucleons at the deuteron dissociation in the Coulomb field of charged particles (in particular - heavy nuclei). For the well-known Hulthen form of the deuteron S-wave function, the spectrum $dW(k)$ has been explicitly calculated. Corrections due to the deuteron D-wave state are briefly analyzed .

1 Basic formula for the momentum spectrum

In the framework of the impulse approach, the spectrum of relative momenta of the neutron and proton at the deuteron peripheral breakup is described by the following general formula:

$$d^3W = N\left|\int \Psi_d(r)\Psi^{*(-)}_{\mathbf{k}}(\mathbf{r})e^{i\mathbf{qr}/2}d^3\mathbf{r}\right|^2 \frac{d^3\mathbf{k}}{(2\pi)^3}\,. \tag{1}$$

Here $\Psi_d(r)$ is the normalized wave function of the deuteron ($4\pi\int_0^\infty \Psi_d^2(r)r^2dr = 1$), N is the normalization constant, $\mathbf{k}$ is the momentum of one of the nucleons in the deuteron rest frame, $\mathbf{q}$ is the momentum transferred to one of the nucleons in the deuteron in the same frame, $\Psi^-_{\mathbf{k}}(\mathbf{r})$ is the wave function of relative motion of the neutron and proton, corresponding to the scattering problem and having the asymptotics in the form of a superposition of a plane wave and a converging spherical wave.

2 The case of very small transferred momenta

Now let us consider the case of very small transferred momenta q (namely, $q \ll 1/\rho$, where ρ is the deuteron radius). In this case only the first-order term in the factor $e^{i\mathbf{qr}/2}$ (1) contributes in fact to the spectrum ; meantime, it should be stressed that, due to the mutual orthogonality of the wave functions $\Psi_d(r)$ and $\Psi^-_{\mathbf{k}}(\mathbf{r})$, the contribution of the zero-order term turns to zero identically:

$$\int \Psi_d(r)\Psi^{*(-)}_{\mathbf{k}}(\mathbf{r})d^3\mathbf{r} \equiv 0.$$

As a result, expression (1) takes the form:

$$d^3W = \frac{1}{4}N\left|\int \Psi_d(r)(\mathbf{qr})\Psi^{*(-)}_{\mathbf{k}}(\mathbf{r})d^3\mathbf{r}\right|^2 \frac{d^3\mathbf{k}}{(2\pi)^3}\,. \tag{2}$$

In doing so, the normalization constant N in (1,2) can be determined in the following way:

a) Taking into account the completeness condition

$$\int \Psi^{(-)}_{\mathbf{k}}(\mathbf{r})\Psi^{*(-)}_{\mathbf{k}}(\mathbf{r}')d^3\mathbf{k} = (2\,\pi)^3\delta(\mathbf{r}-\mathbf{r}')\,,$$

the integration of d^3W over $d^3\mathbf{k}$ gives:

$$W = \frac{1}{4}N\int \Psi_d^2(r)(\mathbf{q\,r})^2d^3\mathbf{r} = \frac{1}{12}N\mathbf{q}^2\langle r^2\rangle\,, \tag{3}$$

where $\langle r^2 \rangle = 4\,\pi \int \Psi_d^2(r) r^4 dr$ is the mean square of the distance between the neutron and proton in the deuteron.

b) The spectrum of relative momenta should be normalized to unity, so that $W = 1$. Hence, according to (3):

$$N = \frac{12}{\mathbf{q}^2 \langle r^2 \rangle}\,. \tag{4}$$

Further, it should be noted that, neglecting a small contribution of the D-wave state, the neutron and proton in the deuteron are in the state with zero orbital momentum ($l = 0$). Thus, one may explicitly see from Eq. (2) that only P-states of the final neutron-proton system ($l = 1$) give contribution into the integral determining the spectrum of relative momenta d^3W.

Meantime, at low relative momenta ($k = |\mathbf{k}| < 1/r_0$, where r_0 is the radius of action of nuclear forces) the role of neutron-proton interaction in the P-states is negligibly small and, hence, the wave function $\Psi_{\mathbf{k}}^{(-)}(\mathbf{r})$ in Eq. (2) may be replaced with good precision by the plane wave $e^{i\,\mathbf{kr}}$.

As a result, we obtain:

$$d^3W = \frac{1}{4} N \left| \mathbf{q} \int \Psi_d(r) \mathbf{r} e^{-i\,\mathbf{kr}} d^3\mathbf{r} \right|^2 \frac{d^3\mathbf{k}}{(2\pi)^3}\,, \tag{5}$$

or, introducing the function

$$F(\mathbf{k}) = \int \Psi_d(r) e^{-i\,\mathbf{kr}} d^3\mathbf{r} :$$

$$d^3W = \frac{1}{4} N \left| \mathbf{q} \frac{d}{d\mathbf{k}} F(\mathbf{k}) \right|^2 \frac{d^3\mathbf{k}}{(2\pi)^3}\,. \tag{6}$$

Since the deuteron wave function $\Psi_d(r)$ is a real function, and on account of spherical symmetry, the function $F(\mathbf{k})$ proves to be a real function having no dependence on the direction of $\mathbf{k}$. Thus, Eq. (6) can be rewritten in the form:

$$d^3W = \frac{1}{4} \frac{N\,(\mathbf{qk})^2}{k^2} \left(\frac{d}{d\,k} F(k) \right)^2 \frac{d^3\mathbf{k}}{(2\pi)^3}\,. \tag{7}$$

Finally, integrating Eq. (7) over the directions of $\mathbf{k}$, we arrive at the following expression for the spectrum of relative momenta $d\,W(k)$, normalized to unity:

$$d\,W(k) = \frac{1}{\langle r^2 \rangle} \left(\frac{d}{d\,k}\,F(k) \right)^2 \frac{k^2\,dk}{2\pi^2}\,. \tag{8}$$

(indeed, we have:

$$\int d\,W(k) = \frac{1}{2\pi^2 \langle r^2 \rangle} \int_0^\infty \left(\frac{d}{d\,k}\,F(k) \right)^2 k^2 dk = 1\,.)$$

It should be emphasized that namely this formula for $d\,W(k)$ (Eq. (8)) describes the spectrum of relative momenta of the neutron and proton at the deuteron disintegration in the Coulomb field of a nucleus. Analogous formulas describe the spectrum of relative momenta of final clusters at the Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei [1-4] .

3 Calculation of the spectrum of relative momenta with the S-wave Hulthen deuteron wave function

Further we will use the well-known Hulthen expression for the S-wave deuteron wave function $\Psi_d(r)$:

$$\Psi_d(r) = \frac{1}{\sqrt{2\pi(\rho - d)}} \frac{\exp(-r/\rho) - \exp(-\alpha r/\rho)}{r} \tag{9}$$

Here $\rho = 4.31$ Fm is the deuteron radius, $d = 1.7$ Fm is the effective radius of low-energy neutron-proton interaction in the triplet state, $\alpha = 6.25$ is the constant determined from the normalization condition for $\Psi_d(r)$ ($4\pi \int_0^\infty \Psi_d^2(r) r^2 dr = 1$) .

In the considered case the function $F(k)$, determining the spectrum of relative momenta, takes the following form [5,6]:

$$F(k) = \sqrt{\frac{8\pi}{\rho - d}}\,\rho^2 \left[\frac{1}{1 + (k\rho)^2} - \frac{1}{\alpha^2 + (k\rho)^2} \right]\,. \tag{10}$$

So, the spectrum of relative momenta $d\,W(k)$ (8) is described by the expression:

$$d\,W(k) = \frac{1}{\langle r^2 \rangle} \left(\frac{d}{d\,k}\,F(k) \right)^2 \frac{k^2\,dk}{2\pi^2} =$$

$$= \frac{16}{\pi} \frac{\rho}{\rho - d} \frac{\rho^7}{\langle r^2 \rangle} \left[\left(\frac{1}{1 + (k\rho)^2} \right)^2 - \left(\frac{1}{\alpha^2 + (k\rho)^2} \right)^2 \right]^2 k^4\,dk\,. \tag{11}$$

Meantime, for the case of Hulthen wave function (9) the quantity $\langle r^2 \rangle$ - mean square of the neutron-proton distance in the deuteron - takes the following form:

$$\langle r^2 \rangle = \frac{\rho^3}{\rho - d} \left(\frac{1}{2} + \frac{1}{2\,\alpha^3} - \frac{8}{(1+\alpha)^3} \right) . \tag{12}$$

Thus, substituting (12) into Eq. (11) and introducing the dimensionless variable $x = k\rho$ ($k = 45.8\,x$ MeV/c), finally we obtain the following expression for the distribution of relative momenta of the neutron and proton in the deuteron rest frame:

$$dW(x) = \frac{32}{\pi} \beta \left[\left(\frac{1}{1+x^2} \right)^2 - \left(\frac{1}{\alpha^2 + x^2} \right)^2 \right]^2 x^4 dx \,, \tag{13}$$

where

$$\beta = \left(1 + 1/\alpha^3 - 16/(1+\alpha)^3 \right)^{-1} \approx 1.04 \,. \tag{14}$$

This distribution satisfies the normalization condition

$$\int d\,W(x) = \int_0^\infty \frac{d\,W(x)}{dx}\, dx = 1.$$

As it can be seen from Fig.1, where the calculated spectrum $\frac{d\,W(x)}{dx}$ (13) is presented, the maximum of $\frac{d\,W(x)}{dx}$ is observed at $x \approx 1$, which corresponds to $k \approx 45.8$ MeV/c and the value of relative momentum of the neutron and proton $2\,k \approx 91.6$ MeV/c .

4 Contribution of the deuteron D-wave state

In the case of an unpolarized deuteron, the contribution of the D-wave function of the deuteron to the spectrum of relative momenta of the neutron and proton is summed incoherently with the contribution of the S-wave function:

$$d\,W(k) = \frac{1}{2\pi^2 \langle r^2 \rangle} \left[(1 - a_D) \left(\frac{d}{d\,k} F^{(S)}(k) \right)^2 + a_D \left(\frac{d}{d\,k} F^{(D)}(k) \right)^2 \right] k^2\, dk \,, \tag{15}$$

where a_D is the relative fraction of the deuteron D-wave state ($a_D \approx 0.04$) .

The quantities $F^{(S)}(k)$, $F^{(D)}(k)$ in Eq. (15) are given by the expressions:

$$F^{(S)}(k) = \int \Psi_d^{(S)}(r) e^{-i\,\mathbf{kr}} d^3\mathbf{r}\,, \qquad F^{(D)}(k) = \int \Psi_d^{(D)}(r) e^{-i\,\mathbf{kr}} d^3\mathbf{r}\,, \qquad (16)$$

where $\Psi_d^{(S)}(r)$ is the normalized wave function of the S-state and $\Psi_d^{(D)}(r)$ is the normalized radial wave function of the D-state

$$(\,4\pi \int_0^\infty (\Psi_d^S(r))^2\, r^2 dr = 4\pi \int_0^\infty (\Psi_d^D(r))^2\, r^2 dr = 1\,)\,.$$

In doing so, the mean square of the distance between the neutron and proton in the deuteron in Eq. (15), taking into account the D-wave, is as follows:

$$\langle r^2 \rangle = 4\,\pi \int_0^\infty [(1 - a_D)(\Psi_d^{(S)}(r))^2 + a_D(\Psi_d^{(D)}(r))^2] r^4 dr\,.$$

5 Summary

1. The analytical expressions for the spectrum of relative momenta of the neutron and proton, produced at the deuteron peripheral breakup, are derived in the limit of very small transferred momenta.

2. Calculations of the relative momentum spectrum are performed for the case of the Hulthen form of the deuteron S-wave function.

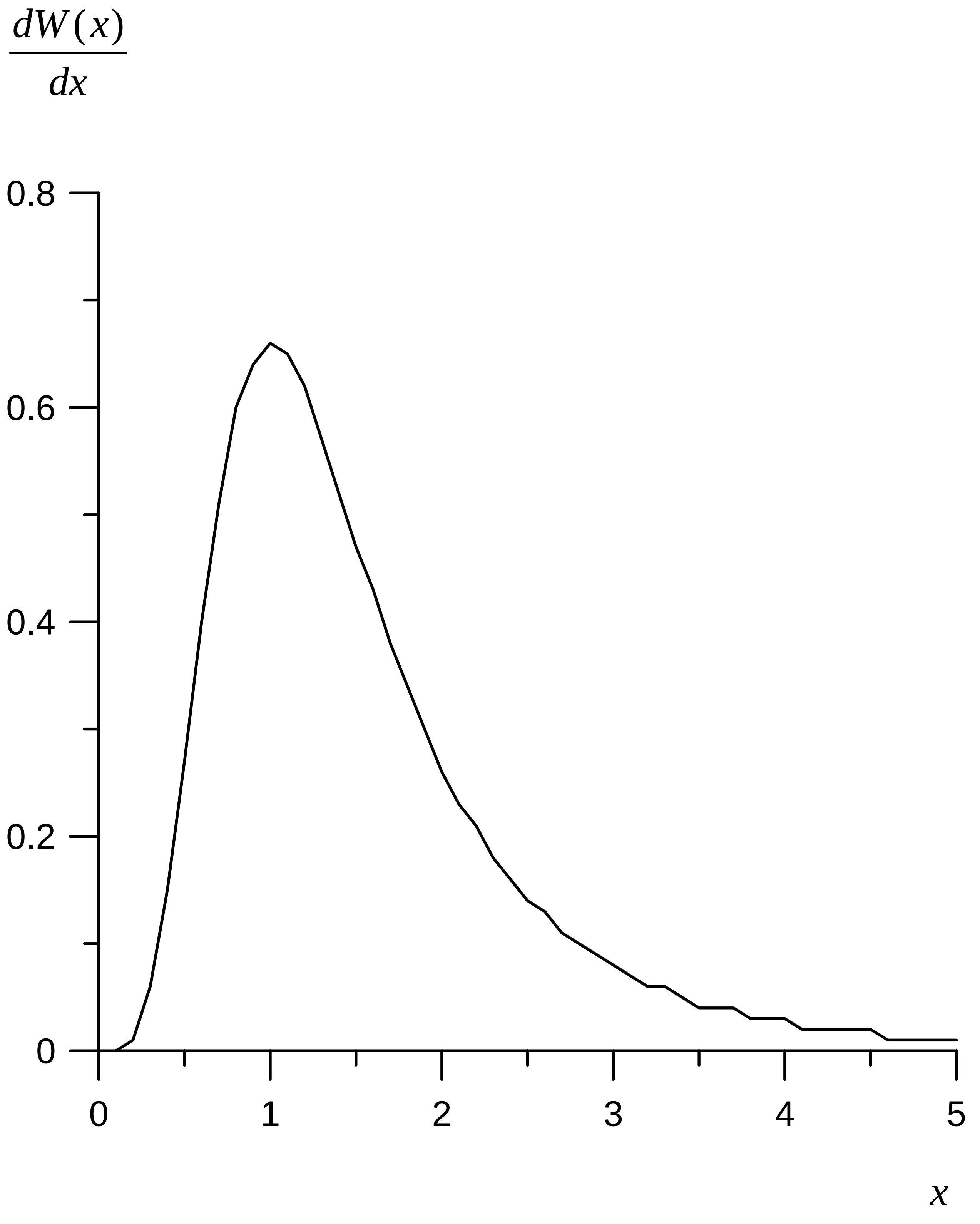

Fig. 1 . Spectrum of relative momenta of the neutron and proton for the case of the *S*-wave Hulthen deuteron wave function .

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