ON THE CORRELATIONS OF POLARIZATIONS IN THE SYSTEM OF TWO PHOTONS PRODUCED IN HADRONIC DECAYS

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The theoretical study of correlations of the linear and circular polarizations in the system of two photons has been performed. The polarization of a two-photon state is described by the one-photon Stokes parameters and by the components of the correlation "tensor" in the Stokes space. It is shown that the correlations between the Stokes parameters in the case of the two-photon decays $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$, $K_S^0 \to 2\gamma$ and the cascade process $|0\rangle \to |1\rangle + \gamma \to |0\rangle + 2\gamma$ (here $|0\rangle$ and $|1\rangle$ are states with the spin 0 and 1, respectively) have the purely quantum character – the Bell-type incoherence inequalities for the components of the correlation "tensor", established previously for the case of classical "mixtures", are violated (i.e. there is always one case when the modulus of sum of two diagonal components of the correlation "tensor" exceeds unity). The general analysis of the registration procedure for the system of two correlated photons by two one-photon detectors is performed.

1 Introduction

Previously, in the works [1-5] the spin correlations of two free particles with spin 1/2 [1-4], as well as the angular correlations between the flight directions of decay products of two particles or resonances [5], reflecting the spin correlations in the system of two unstable particles with arbitrary spin, have been analyzed in detail. In doing so, the spin states of each of the particles were set in the respective rest frames, which is possible only at nonzero masses of both the particles.

In the present work we study the correlation properties of the system of two photons. Since the photon mass is equal to zero, the introduction of spin as the internal angular momentum in the rest frame is inapplicable in this case, and, thus, for describing the photon polarization the special consideration is required.

2 Density matrix of the two-photon system

Let us consider the system of two photons with the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$. We introduce two systems of coordinate axes: (x, y, z) with the axis z parallel to the momentum $\mathbf{k}_1$ of the first photon, and $(\tilde{x}, \tilde{y}, \tilde{z})$ with the axis $\tilde{z}$ parallel to the momentum $\mathbf{k}_2$ of the second photon. Let us choose the axes x and $\tilde{x}$ so that they were parallel to each other and perpendicular to the plane passing through the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$.

Analogously to the spin density matrix of two particles with spin 1/2 (see, for example, [4]), we can represent the polarization density matrix of two photons in the form:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \sum_{i=1}^{3} \epsilon_i^{(1)} \hat{\sigma}_i^{(1)} \otimes \hat{I}^{(2)} + \sum_{k=1}^{3} \epsilon_k^{(2)} \hat{I}^{(1)} \otimes \hat{\sigma}_k^{(2)} +$$

$$+ \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik} \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}]. \qquad (1)$$

Here $\hat{\sigma}_i^{(1)}$, $\hat{\sigma}_k^{(2)}$ are the Pauli matrices; $\epsilon_i^{(1)}$ denotes the Stokes parameters of the first photon [6-8], defined in the system of axes (x, y, z), $\epsilon_k^{(2)}$ denotes the Stokes parameters of the second photon, defined in the system of axes $(\tilde{x}, \tilde{y}, \tilde{z})$, T_{ik} is the correlation "tensor" in the Stokes space, describing the correlation of polarizations of the first and second photons. For independent photons we have $T_{ik} = \epsilon_i^{(1)}\epsilon_k^{(2)}$. In the general case such an equality does not hold.

Let $|3, +\rangle$ and $|3, -\rangle$ be the one-photon states with the full linear polarization along the axes x and y, respectively; let $|2, +\rangle$ and $|2, -\rangle$ be the one-photon states with the right (helicity $+1$) and left (helicity -1) circular polarization, respectively, and let $|1, +\rangle$ and $|1, -\rangle$ be the one-photon states with the full linear polarization along the axis directed at the angles $\pi/4$ and $3\pi/4$, respectively, with respect to the axis x.

Then, by definition, the one-photon Stokes parameters are as follows: $\epsilon_i = (1/2)(W_i^{(+)} - W_i^{(-)})$, where $W_i^{(+)}$ and $W_i^{(-)}$ are the probabilities of registering the photon in the states $|i, +\rangle$ and $|i, -\rangle$, respectively ($W_i^{(+)} + W_i^{(-)} = 1$). In so doing, $r = \sqrt{\epsilon_1^2 + \epsilon_3^2}$ is the degree of linear polarization and ϵ_2 is the degree of circular polarization, which are invariant with respect to rotations in the plane (x, y).

Components of the "tensor" T_{ik} can be determined by using the following probabilistic formula (compare with [4]):

$$T_{ik} = \frac{1}{4}\,(W_{i,k}^{(+,+)} - W_{i,k}^{(-,+)} - W_{i,k}^{(+,-)} + W_{i,k}^{(-,-)}). \tag{2}$$

Here $i = 1, 2, 3,\ k = 1, 2, 3$; $W_{i,k}^{(+,+)}$ is the probability of registering the first photon in the state $|i, +\rangle$ and the second photon – in the state $|k, +\rangle$; $W_{i,k}^{(-,+)}$ is the probability of registering the first photon in the state $|i, -\rangle$ and the second photon – in the state $|k, +\rangle$; $W_{i,k}^{(+,-)}$ is the probability of registering the first photon in the state $|i, +\rangle$ and the second photon – in the state $|k, -\rangle$; $W_{i,k}^{(-,-)}$ is the probability of registering the first photon in the state $|i, -\rangle$ and the second photon – in the state $|k, -\rangle$.

In accordance with the normalization condition,

$$W_{i,k}^{(+,+)} + W_{i,k}^{(-,+)} + W_{i,k}^{(+,-)} + W_{i,k}^{(-,-)} = 1.$$

In the case of the entirely unpolarized photons we have: $\epsilon_i^{(1)} = \epsilon_i^{(2)} = 0$, $T_{ik} = 0$ (all the Stokes parameters and all components of the correlation tensor equal zero).

3 Covariant Stokes parameters

The unit vector of photon polarization $\mathbf{e}$ is directed transversely to the photon momentum $\mathbf{k}$ in any frame. In accordance with this, we may consider the unit polarization vector $\mathbf{e}$ as the spatial part of the 4-vector e, which is subjected, at the transition to a new frame, to the gradient transformation in addition to the Lorentz transformation. If in the frame 1 the polarization vector $\mathbf{e}$ satisfies the transversality condition $\mathbf{ek} = 0$, then in the frame 2 the polarization vector is defined as the spatial part of the 4-vector [8]:

$$e' = e - k\,\frac{eu}{ku}. \tag{3}$$

Here u is the 4-velocity of the frame 2, k is the 4-momentum of the photon, e is the 4-vector being orthogonal to k ($ke = 0$), whose components in the frame 1 are equal to $(0, \mathbf{e})$, eu and ku are the scalar products of the respective 4-vectors.

It is easy to see that the 4-vector e' obeys the conditions $e'u = e'k = 0$, which correspond to the property of transversality of the photon: in the frame 2, where $\mathbf{u} = 0$, the components of the 4-vector e' amount to $(0, \mathbf{e}')$; in doing so, $|\mathbf{e}'| = 1$, $\mathbf{e}'\mathbf{k} = 0$.

Let us decompose the 4-dimensional polarization vector e through two 4-dimensional unit vectors χ_1 and χ_2 satisfying the orthogonality conditions $\chi_1\chi_2 = 0$, $\chi_1 k = \chi_2 k = 0$. We may write: $e = c_1\chi_1 + c_2\chi_2$, where $c_1 = e\chi_1$, $c_2 = e\chi_2$ are complex numbers, $|c_1| = |c_2| = 1$. At the gradient transformation (3) we obtain:

$$\chi_1' = \chi_1 - k\,\frac{\chi_1 u}{ku}, \quad \chi_2' = \chi_2 - k\,\frac{\chi_2 u}{ku}. \tag{4}$$

It is easy to see that $e'\chi_1' = e\chi_1 = c_1, \quad e'\chi_2' = e\chi_2 = c_2$. This means that the photon density matrix $\rho_{ik} = \overline{c_i c_k^*}$ is invariant with respect to the Lorentz transformations. Within such a definition, *all* the Stokes parameters [6-8] are Lorentz-invariant:

$$\epsilon_1 = 2\,\mathrm{Re}\,\overline{(c_1 c_2^*)}, \quad \epsilon_2 = 2\,\mathrm{Im}\,\overline{(c_1 c_2^*)}, \quad \epsilon_3 = \overline{|c_1|^2 - |c_2|^2}.$$

One should take into account the change of spatial orientation of polarization unit vectors in accordance with the three-dimensional transversality condition:

$$\boldsymbol{\chi}_1\mathbf{k} = 0, \quad \boldsymbol{\chi}_2\mathbf{k} = 0, \quad \boldsymbol{\chi}_1\boldsymbol{\chi}_2 = 0.$$

It is clear that at the transition from the frame 1 to the frame 2, moving with the velocity $\mathbf{v}$ with respect to the frame 1, the polarization unit vectors turn around the axis parallel to the vector $[\mathbf{vk}]$ by the angle θ equaling the angle between the photon momentum $\mathbf{k}$ in the frame 1 and the photon momentum $\mathbf{k}'$ in the frame 2.

In particular, if in the frame 1 the vector $\boldsymbol{\chi}_1$ is chosen to be parallel to the vector $[\mathbf{vk}]$, then at the transition to the frame 2 the direction of the vector $\boldsymbol{\chi}_1$ coincides with $\boldsymbol{\chi}_1{}'$ and, meantime, the angle between the vectors $\boldsymbol{\chi}_2$ and $\boldsymbol{\chi}_2{}'$ in the plane $(\mathbf{v}, \mathbf{k})$ is equal to the angle θ between the vectors $\mathbf{k}$ and $\mathbf{k}'$. Let us note that, as a result of the Lorentz transformation along the velocity $\mathbf{v}$, the angle between the photon momentum and the transfer velocity increases, which corresponds to the positive angle of rotation θ around the vector $[\mathbf{vk}]$.

4 Correlations between the Stokes parameters of two photons

In the case of a system of two photons (see Section 2) we have chosen the pair of transverse unit vectors of polarization of the first and second photons in the same direction $[\mathbf{k}_1\mathbf{k}_2]$, which is perpendicular to the plane passing through the momenta of two photons $\mathbf{k}_1$ and $\mathbf{k}_2$ ($\boldsymbol{\chi}_1 = \widetilde{\boldsymbol{\chi}}_1$). Two other unit vectors of polarization of the first and second photons satisfy the equalities:

$$\boldsymbol{\chi}_2\boldsymbol{\chi}_1 = \boldsymbol{\chi}_2\mathbf{k}_1 = 0, \quad \widetilde{\boldsymbol{\chi}}_2\widetilde{\boldsymbol{\chi}}_1 = \widetilde{\boldsymbol{\chi}}_2\mathbf{k}_2 = 0, \quad \widetilde{\boldsymbol{\chi}}_2\boldsymbol{\chi}_2 = \cos\beta,$$

where β is the angle between the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$.

We will consider the transverse unit vectors as spatial parts of the unit 4-vectors χ_1 and χ_2, $\widetilde{\chi}_1$ and $\widetilde{\chi}_2$; let us introduce further the gradient transformations at the transition to the frame moving with the 4-velocity u:

$$\chi_1' = \chi_1 - k_1\frac{\chi_1 u}{k_1 u}, \qquad \chi_2' = \chi_2 - k_1\frac{\chi_2 u}{k_1 u};$$

$$\widetilde{\chi}_1' = \widetilde{\chi}_1 - k_2\frac{\widetilde{\chi}_1 u}{k_2 u}, \qquad \widetilde{\chi}_2' = \widetilde{\chi}_2 - k_2\frac{\widetilde{\chi}_2 u}{k_2 u}. \tag{5}$$

In the basis of the 4-vectors (5) the polarization density matrix of two photons (1) is invariant with respect to the Lorentz transformations. In accordance with this, the Stokes parameters of the first and second photons $\epsilon_i^{(1)}, \epsilon_k^{(2)}$ and all the components of the correlation tensor T_{ik} $(i,k = 1,2,3)$ are Lorentz-invariant.

Due to the transversality of polarization unit vectors in any frame, at the transition from the initial frame 1 to the frame 2, moving with the velocity $\mathbf{v}$ with respect to the frame 1, their spatial orientation changes: the unit vectors of polarization of the first photon $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$ turn around the vector $[\mathbf{v}\,\mathbf{k}_1]$ by the angle

$$\theta_1 = \arcsin\left[\frac{|[\mathbf{k}_1\mathbf{k}_1']|}{k_1k_1'}\right] = \arcsin\left[\frac{(\gamma-1)\cos\alpha_1 - (v/c)}{1-(v/c)\cos\alpha_1}\sin\alpha_1\right], \tag{6}$$

and the unit vectors of polarization of the second photon $\widetilde{\boldsymbol{\chi}}_1$ and $\widetilde{\boldsymbol{\chi}}_2$ turn around the vector $[\mathbf{v}\,\mathbf{k}_2]$ by the angle θ_2 determined analogously to (6), with replacing α_1 by α_2. Here α_1 and α_2 are the angles between the velocity $\mathbf{v}$ and the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively; γ is the Lorentz-factor.

Let us introduce now the frame of the center-of-inertia of two photons. This frame always exists, if the momenta of two photons are not parallel to each other. The velocity of the center-of-inertia frame with respect to the given frame is determined by the formula $\mathbf{v} = (\mathbf{k}_1 + \mathbf{k}_2)/(k_1 + k_2)$, where $k_1 = |\mathbf{k}_1|$, $k_2 = |\mathbf{k}_2|$. It is clear that at the transition to the c.i. frame the unit vectors of polarization of the first and second photons turn in opposite directions around the axis being parallel to the vector $[\mathbf{k}_1\,\mathbf{k}_2]$. In doing so, in the c.i. frame $\mathbf{k}_1 = -\mathbf{k}_2, \quad \boldsymbol{\chi}_1{}' = \widetilde{\boldsymbol{\chi}}_1{}', \quad \boldsymbol{\chi}_2{}' = -\widetilde{\boldsymbol{\chi}}_2{}'$.

Let us consider, as an example, the decay $\pi^0 \to 2\gamma$. In the π^0-meson rest frame (coinciding with the c.i. frame of two γ-quanta) the decay amplitude has the structure: $A_{\gamma\gamma} \sim ([\,\mathbf{e}^{(1)*}\,\mathbf{e}^{(2)*}\,]\mathbf{n})$, where $\mathbf{n}$ is the unit vector directed along the momentum of one of the photons, $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are complex unit vectors of polarization of the first and second photon, respectively, being perpendicular to the vector $\mathbf{n}$.

So, we find that in this case all the Stokes parameters of the first and second photon are equal to zero (thus, the single-photon states are unpolarized: $\epsilon_i^{(1)} = \epsilon_k^{(2)} = 0, \quad i,\, k = 1,\, 2,\, 3$). Meantime, according to (2), the two-photon system is correlated: $T_{11} = +1, \quad T_{22} = +1, \quad T_{33} = -1$, all the non-diagonal components of the correlation tensor T_{ik} equaling zero.

Let us remark that the equality $T_{22} = +1$, according to which the helicities of two photons at the decay $\pi^0 \to 2\gamma$ coincide, follows from the fact that the π^0-meson has zero spin. Meantime, the equality $T_{33} = -1$, according to which the linear polarizations of two γ-quanta are mutually perpendicular, is the consequence of the negative internal parity of the π^0-meson.

Taking into account the above-considered changes of spatial orientation of polarization unit vectors, the values of polarization parameters of two γ-quanta at the decay $\pi^0 \to 2\gamma$, indicated above, remain valid in any frame (in particular, in the laboratory frame, where the decaying π^0-meson is moving). It is clear that the same holds also for the decays $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$, [1] as well as for the para-positronium decay into two γ-quanta.

5 Registration of the system of two correlated photons

The probability of registration of a system of two photons with two one-photon detectors, selecting the state of the first photon with the Stokes parameters $\xi_1^{(1)}, \xi_2^{(1)}, \xi_3^{(1)}$, being specified in the representation of above-indicated unit vectors $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$ and the state of the second photon with the Stokes parameters $\xi_1^{(2)}, \xi_2^{(2)}, \xi_3^{(2)}$, being specified in the representation of the unit vectors $\widetilde{\boldsymbol{\chi}}_1$ and $\widetilde{\boldsymbol{\chi}}_2$, is described, according to the density matrix (1), by the following correlation formula:

$$W \sim 1 + \sum_{i=1}^{3} \epsilon_i^{(1)} \xi_i^{(1)} + \sum_{k=1}^{3} \epsilon_k^{(2)} \xi_k^{(2} + \sum_{i=1}^{3} \sum_{k=1}^{3} T_{ik}\, \xi_i^{(1)} \xi_k^{(2)}. \qquad (7)$$

The “final” Stokes parameters have the meaning of analyzing powers. In particular, the Compton scattering on an unpolarized electron, selecting the states with the polarization vector being perpendicular to the scattering plane and the states with the polarization vector lying in the scattering plane, is a characteristic analyzer of the photon linear polarization. In the representation of these states the analyzing power is determined by one parameter, namely, by the coefficient of left-right azimuthal asymmetry at the Compton scattering of a linearly polarized photon:

$$r(\omega, \theta_{\rm sc}) = \frac{\sin^2\theta_{\rm sc}}{(\omega_f/\omega) + (\omega/\omega_f) - \sin^2\theta_{\rm sc}}, \qquad (8)$$

where $\theta_{\rm sc}$ is the angle of the photon scattering in the laboratory frame, ω and ω_f are the photon energies before and after the Compton scattering, respectively. In the representation of the polarization unit vectors $\boldsymbol{\chi}_1$,

[1] Neglecting the effects of CP- invariance violation, the CP-parity of the long-lived neutral kaon K_L^0 is negative. Meantime, the amplitude of the two-photon decay of the short-lived neutral kaon K_S^0 with the positive CP-parity has the structure: $A_{K_S^0 \to 2\gamma} \sim (\mathbf{e}^{(1)*}\, \mathbf{e}^{(2)*})$. In this case the linear polarizations of the first and second photons, as well as their helicities, are mutually equal: $T_{11} = -1, T_{22} = +1, T_{33} = +1$.

$\boldsymbol{\chi}_2$ and $\widetilde{\boldsymbol{\chi}}_1$, $\widetilde{\boldsymbol{\chi}}_2$, which have been introduced earlier for describing the polarization properties of the system of two γ-quanta, the analyzing powers are related with the "vectors" in the Stokes space $\vec{\xi}^{(1)} = (\xi_1^{(1)}, 0, \xi_3^{(1)})$ and $\vec{\xi}^{(2)} = (\xi_1^{(2)}, 0, \xi_3^{(2)})$, where $(j = 1, 2)$:

$$\xi_1^{(j)} = r(\omega_j, \theta_{\rm sc}^{(j)}) \sin 2\psi_{\rm sc}^{(j)}, \quad \xi_3^{(j)} = r(\omega_j, \theta_{\rm sc}^{(j)}) \cos 2\psi_{\rm sc}^{(j)}; \tag{9}$$

Here $\psi_{\rm sc}^{(1)}$ ($\psi_{\rm sc}^{(2)}$) is the angle between the plane of Compton scattering of the first (second) photon and the plane $(\mathbf{k}_1, \mathbf{k}_2)$, passing through the momenta of two photons. Taking into account the values of components of the correlation tensor (see above), it follows from the relations (7) and (9) that the correlation of the planes of Compton scattering of two γ-quanta, produced in the decay $\pi^0 \to 2\gamma$, will have the form:

$$d^2W = \frac{d\psi_{\rm sc}^{(1)} d\psi_{\rm sc}^{(2)}}{4\pi^2}[1 - r(\omega_1, \theta_{\rm sc}^{(1)})\, r(\omega_2, \theta_{\rm sc}^{(2)}) \cos 2(\psi_{\rm sc}^{(1)} + \psi_{\rm sc}^{(2)})]. \tag{10}$$

In the c.i. frame of two photons ($\mathbf{k}_1 = -\mathbf{k}_2$) the angle $\psi = \psi_{\rm sc}^{(1)} + \psi_{\rm sc}^{(2)}$ is equal to the angle between the planes of Compton scattering of two photons, and we have:

$$dW = \frac{1}{2\pi}[1 - r(\omega_1, \theta_{\rm sc}^{(1)})\, r(\omega_2, \theta_{\rm sc}^{(2)}) \cos 2\psi] d\psi. \tag{11}$$

Apart from the Compton scattering, any processes of photon absorption may serve as analyzers of the photon linear polarization, for example – the deuteron photodisintegration, the meson photoproduction on nucleons and nuclei, the formation of electon-positron pairs in the Coulomb field of a nucleus.

If, in doing so, one deals with the c.i. frame of two γ-quanta generated in the decay processes $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, $K_L^0 \to 2\gamma$, in the parapositronium decay, in the low-energy electron-positron annihilation, then the correlations of photoproduction planes are described by the formula being similar to (11) with $r(\omega_1, \theta_{\rm sc}^{(1)}) \to r_1$, $r(\omega_2, \theta_{\rm sc}^{(2)}) \to r_2$, where r_1, r_2 are the analyzing powers. The positive sign of r corresponds to the photoproduction mainly in the plane that is perpendicular to the vector of photon polarization, the negative sign – to the photoproduction in the plane that is parallel to the vector of photon polarization.

In particular, the processes of the π^0-meson photoproduction on spinless nuclei ($\gamma + {}^4He \to \pi^0 + {}^4He$, $\gamma + {}^{12}C \to \pi^0 + {}^{12}C$) are ideal analyzers of the photon linear polarization. In this case the amplitudes of the reactions have the structure $A_\pi \sim (\mathbf{e}[\mathbf{k}\mathbf{p}_\pi])$, and, as it is easy to see, the analyzing power takes the maximum value $r = 1$.

6 Quantum character of the two-photon correlations

Analogously to the results of the works [4,5] for the correlation properties of a system of two particles with spin 1/2, in the case of incoherent ("classical") mixtures of factorizable two-photon states the modulus of the sum of any two components of the correlation "tensor" cannot exceed unity. We see that in the case of decays like $\pi^0 \to 2\gamma$ these incoherence inequalities are violated: the correlations of polarizations of two photons have the strongly pronounced quantum character. Indeed, in the c.i. frame of two γ-quanta we have:

$$T_{11} + T_{22} = 2 > 1.$$

Let us consider, from this viewpoint, the cascade decay

$$|0\rangle \to |1\rangle + \gamma; \quad |1\rangle \to |0\rangle + \gamma$$

with the emission of two photons (the spins of the initial and final states equal zero, and the spin of the intermediate state equals 1). Let us denote by $\mathbf{B}_m$ the complex vector, normalized to unity, corresponding to the intermediate state with the spin projection m onto the quantization axis. It is obvious that the amplitude of the cascade transition has the structure:

$$A_{\gamma\gamma} \sim \sum_{m=0,\pm 1} (\mathbf{e}^{(1)*}\mathbf{B}_m^*)(\mathbf{B}_m\mathbf{e}^{(2)*}) \sim (\mathbf{e}^{(1)*}\mathbf{e}^{(2)*}),$$

where $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are the vectors of polarization of two cascade photons, respectively. In the representation of basis unit vectors, introduced above for the description of the polarization properties of a two-photon system $((\boldsymbol{\chi}_1 = \widetilde{\boldsymbol{\chi}}_1) \parallel [\mathbf{k}_1\mathbf{k}_2]$, the unit vectors $\boldsymbol{\chi}_2$ and $\widetilde{\boldsymbol{\chi}}_2$ are parallel to the plane passing through the momenta of two photons $\mathbf{k}_1$ and $\mathbf{k}_2$), the Stokes parameters and the components of the correlation "tensor" (2) have the values:

$$\epsilon_1^{(1)} = \epsilon_2^{(1)} = \epsilon_3^{(1)} = 0; \qquad \epsilon_1^{(2)} = \epsilon_2^{(2)} = \epsilon_3^{(2)} = 0;$$

$$T_{12} = T_{21} = T_{13} = T_{31} = T_{23} = T_{32} = 0;$$

$$T_{11} = \frac{2\cos\theta}{1+\cos^2\theta}, \qquad T_{22} = -\frac{2\cos\theta}{1+\cos^2\theta}, \qquad T_{33} = 1, \tag{12}$$

where θ is the angle between the momenta of two photons, as before.

At $\theta = 0$, when the photon momenta are parallel, we have: $T_{22} = -1$ (the photon helicities are mutually opposite, which follows directly from the fact of conservation of the projection of angular momentum onto the coordinate axis in the cascade decay). At $\theta = \pi$, when the photon momenta are antiparallel, $T_{22} = +1$ (the photon helicities are the same).

According to (12), within the interval of angles $\pi/2 > \theta > 0$

$$T_{33} + T_{11} > 1,$$

and within the interval of angles $\pi > \theta > \pi/2$

$$T_{33} + T_{22} > 1.$$

So, in this case the incoherence inequalities are also violated.

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