Implementation of NLO high energy factorization in single inclusive forward hadron production

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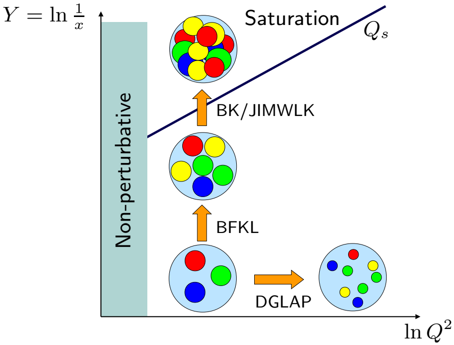
B. D., T. Lappi, Y. Zhu, PRD 93 (2016) 114016 [arXiv:1604.00225]
B. D., T. Lappi, Y. Zhu, PRD 95 (2017) 114007 [arXiv:1703.04962]

Our goal is to study QCD in the saturation regime

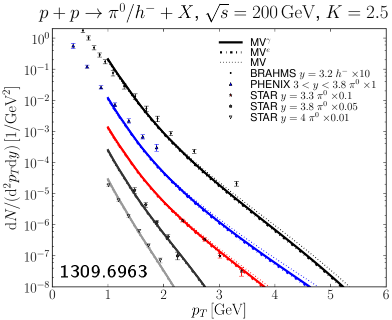


The production of forward particles is a crucial tool to probe small x values

Saturation effects stronger in pA collisions ($Q_s^2 \sim A^{1/3}$)

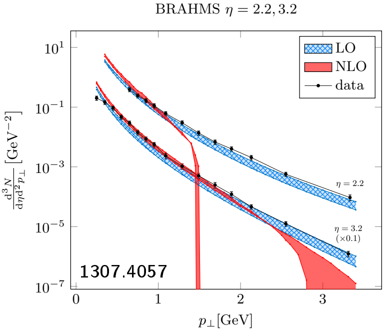
Here we study the inclusive production of forward hadrons in proton-nucleus collisions: $pA \to hX$

Typical calculation at LO:
(Lappi, Mäntysaari)



K factor needed to describe the data

First numerical calculation at NLO:
(Staśto, Xiao, Zaslavsky)

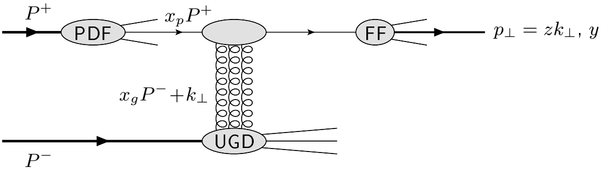


Negative cross section above some $p_\perp$

Several proposals to solve the negativity problem at NLO, for example the kinematical constraint / Ioffe time cutoff (Altinoluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky. Can extend the positivity range but doesn't solve the problem completely.

Formalism

Single inclusive forward hadron production at LO in the $q \to q$ channel:



Dilute projectile: $x_p = \frac{k_\perp}{\sqrt{s}} e^{y}$, described by collinear PDFs

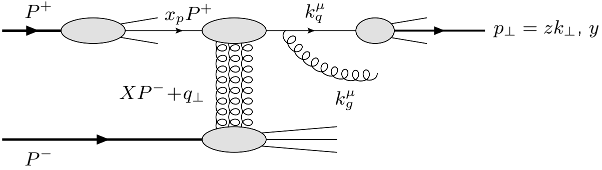
Dense target: $x_g = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{F}$

$$\mathcal{F}(k_\perp) = \int \mathrm{d}^2\mathbf{b}\, \mathcal{S}(k_\perp)\ ,\ \mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})\ ,\ S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

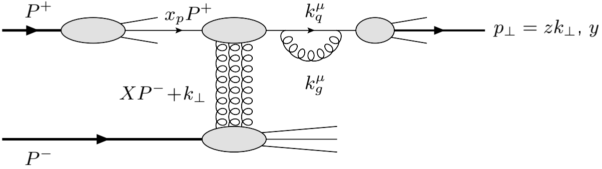
Rapidity (or x) dependence of S : governed by the Balitsky-Kovchegov equation

NLO corrections to the impact factor: Chirilli, Xiao, Yuan

Example of real $q \to q$ contribution:



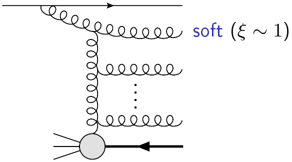
Example of virtual $q \to q$ contribution:



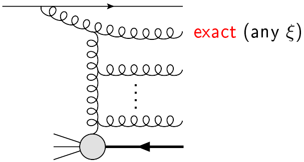
$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

Taking into account Balitsky-Kovchegov (BK) evolution: resummation of any number of soft gluons, already at LO

LO: all the emitted gluons are soft:



NLO impact factor: the first gluon can be hard:



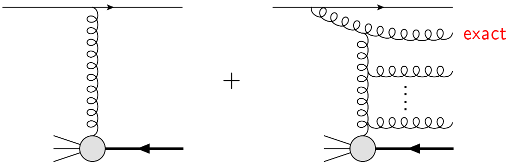
$\Rightarrow$ Need to avoid double counting between LO and NLO

LO vs. NLO

Two possible solutions to avoid double counting:

1) Subtract the case where the gluon in the NLO impact factor is soft
Chirilli, Xiao, Yuan ('CXY')

2) Rearrange the terms to avoid doing a subtraction. The expression for the cross section is explicitly positive
Iancu, Mueller, Triantafyllopoulos



These two choices should be equivalent

The expression for the (quark production) multiplicity at NLO reads

$$\frac{\mathrm{d}N^{pA\to qX}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} \qquad \leftarrow \text{No emission}$$

$$+\frac{\alpha_s}{2\pi^2}\int_{x_p}^{\xi_{\max}}\mathrm{d}\xi\,\frac{1+\xi^2}{1-\xi}\,\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\left\{C_{\mathsf{F}}\mathcal{I}(k_\perp,\xi,X(\xi)) + \frac{N_c}{2}\mathcal{J}(k_\perp,\xi,X(\xi))\right\} \qquad \leftarrow \text{real}$$

$$-\frac{\alpha_s}{2\pi^2}\int_{0}^{\xi_{\max}}\mathrm{d}\xi\,\frac{1+\xi^2}{1-\xi}x_p q\left(x_p\right)\left\{C_{\mathsf{F}}\mathcal{I}_v(k_\perp,\xi,X(\xi)) + \frac{N_c}{2}\mathcal{J}_v(k_\perp,\xi,X(\xi))\right\} \qquad \leftarrow \text{virtual}$$

$$\mathcal{I}(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\left[\frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2} - \frac{\mathbf{k}-\xi\mathbf{q}}{(\mathbf{k}-\xi\mathbf{q})^2}\right]^2\mathcal{S}(q_\perp, X(\xi))$$

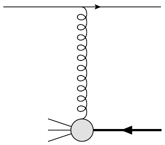
$$\mathcal{J}(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp, X(\xi)) - \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

$$\mathcal{I}_v(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\left[\frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2} - \frac{\xi\mathbf{k}-\mathbf{q}}{(\xi\mathbf{k}-\mathbf{q})^2}\right]^2\mathcal{S}(k_\perp, X(\xi))$$

$$\mathcal{J}_v(k_\perp,\xi,X(\xi)) = \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi)) - \int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

and $\dfrac{\mathrm{d}\sigma^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h} = \displaystyle\int\mathrm{d}^2\mathbf{b}\frac{\mathrm{d}N^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h}$

In the previous expressions:



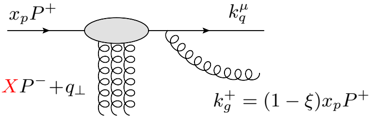
- $x_p q(x_p) \dfrac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2}$ represents the lowest order contribution

  (no BK evolution. x_0: initial condition)

- $X(\xi)$ is the rapidity scale at which the dipole correlators are evaluated

  At LO: the P^- fraction needed from the target is $\dfrac{k_\perp}{\sqrt{s}} e^{-y} \equiv x_g$

  At NLO:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2}\right)$$
$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ when } k_\perp \gtrsim Q_s$$

The limit $\xi < 1 - \frac{x_g}{x_0} \equiv \xi_{\text{max}}$ enforces $X(\xi) < x_0$

C_F terms

The terms proportional to C_F are divergent when the additional gluon at NLO is collinear to the initial or final state quark

These divergences are absorbed in the DGLAP evolution of the PDFs and fragmentation functions

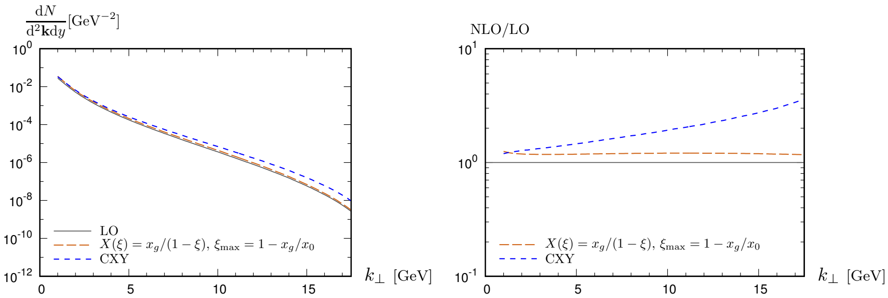
After subtracting the corresponding $1/\varepsilon$ poles, we should replace $\mathcal{I}$ and $\mathcal{I}_v$ by

$$\mathcal{I}^{\text{finite}}(k_\perp, \xi, X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{r}}{4\pi} S(\mathbf{r}, X(\xi)) \ln \frac{c_0^2}{\mathbf{r}^2\mu^2} \left(e^{-i\mathbf{k}\cdot\mathbf{r}} + \frac{1}{\xi^2} e^{-i\frac{\mathbf{k}}{\xi}\cdot\mathbf{r}} \right)$$
$$-2 \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2} \frac{(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2} \mathcal{S}(q_\perp, X(\xi))$$

$$\mathcal{I}_v^{\text{finite}}(k_\perp, \xi, X(\xi)) = \frac{\mathcal{S}(k_\perp, X(\xi))}{2\pi} \left(\ln \frac{k_\perp^2}{\mu^2} + \ln(1-\xi)^2 \right)$$

C_F terms

Results for the LO+C_F NLO corrections at fixed coupling ($\alpha_s = 0.2$):



The 'CXY' approximation corresponds to making the replacements $X(\xi) \to x_g$ and $\xi_{\mathsf{max}} \to 1$

In both cases the NLO corrections proportional to C_F are positive $\to$ not the cause of the negativity

(Initial condition for the BK evolution at $x_0 = 0.01$: MV model

$S(\mathbf{r}, x_0) = \exp\left[-\frac{\mathbf{r}^2 Q_{\mathbf{s},0}^2}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathbf{QCD}}} + e\right)\right]$, $Q_{\mathbf{s},0}^2 = 0.2$ GeV^2 and $\Lambda_{\mathbf{QCD}} = 0.241$ GeV)

N_c terms

We can write the sum of the LO and N_c terms as

$$\frac{dN^{LO+N_c}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\mathcal{K}(k_\perp,\xi,X(\xi)) \equiv \frac{dN^{LO+N_c,unsub}}{d^2\mathbf{k}dy},$$

$$\mathcal{K}(k_\perp,\xi,X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi-x_p)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp,\xi,X) - x_p q(x_p)\mathcal{J}_v(k_\perp,\xi,X)\right].$$

At large $k_\perp$ the function $\mathcal{K}(k_\perp,\xi,X)$ is positive and so is the cross section.

Using the integral BK equation,

$$\mathcal{S}(k_\perp,x_g) = \mathcal{S}(k_\perp,x_0) + 2\alpha_s N_c \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{J}(k_\perp,1,X(\xi)) - \mathcal{J}_v(k_\perp,1,X(\xi))\right],$$

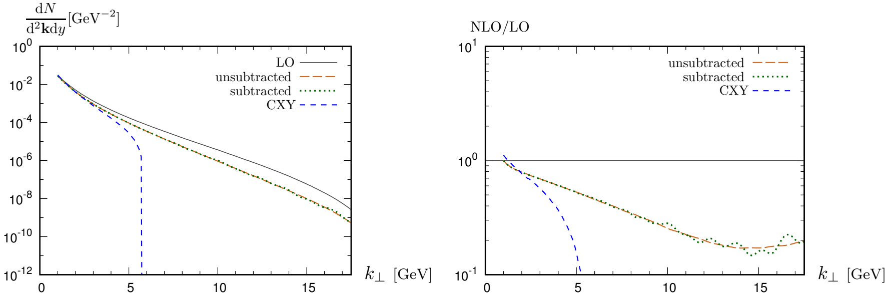
the LO+N_c terms can be rewritten as

$$\frac{dN^{LO+N_c,sub}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{K}(k_\perp,\xi,X(\xi)) - \mathcal{K}(k_\perp,1,X(\xi))\right].$$

The 'CXY' approximation corresponds to making the replacements $X(\xi) \to x_g$ and $\xi_{max} \to 1$ in this subtracted version

N_c terms

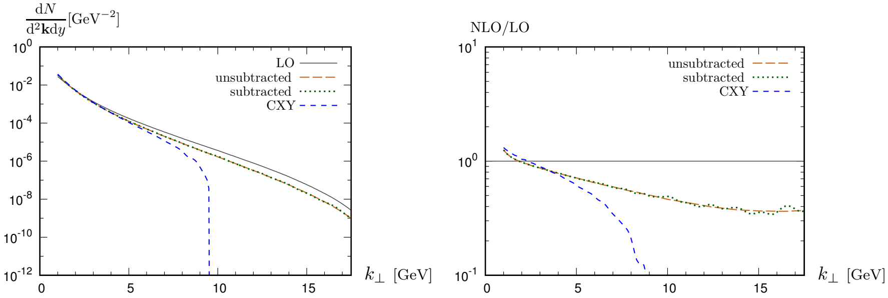
Results for the LO+N_c NLO corrections at fixed coupling ($\alpha_s = 0.2$):



The 'subtracted' and 'unsubtracted' expressions give the same (positive) results

The 'CXY' approximation leads to negative results for $k_\perp \gtrsim 5$ GeV.

Total (LO+C_F+N_c) multiplicity ($\alpha_s = 0.2$):



Similar conclusions as in the LO+N_c case

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

In practice the BK equation is usually solved in coordinate space, with some prescription for the running coupling

Fixed coupling BK equation:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right]$$

BK equation with Balitsky's prescription for the running coupling:

$$\begin{aligned}\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2) N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} &\left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \\ &\times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right) \right. \\ &\quad \left. + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]\end{aligned}$$

Running coupling

We use dipole correlators obtained by solving numerically the LO BK equation with the Balitsky prescription for the running coupling. The initial condition is the 'MV^e' parametrization:

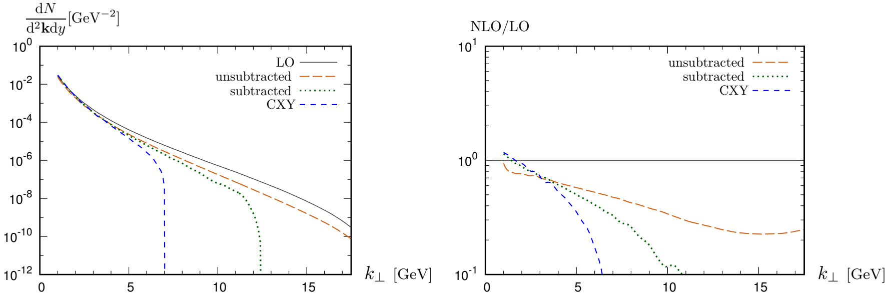
$$S(\mathbf{r}, x_0 = 0.01) = \exp\left[-\frac{\mathbf{r}^2 Q_{\mathsf{s},0}^2}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathsf{QCD}}} + e_c \cdot e\right)\right],$$

and the running coupling is taken as $\alpha_s(\mathbf{r}^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{4C^2}{\mathbf{r}^2 \Lambda_{\mathsf{QCD}}^2}\right)}$.

The values $Q_{\mathsf{s},0}^2 = 0.06$ GeV^2, $C^2 = 7.2$ and $e_c = 18.9$ were obtained by a fit to HERA DIS data (Lappi, Mäntysaari)

It is not possible to use the (coordinate-space) Balitsky prescription using the previously shown momentum-space expressions for the cross section. Here we will use $\alpha_s(k_\perp^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{C_{\mathsf{mom}}^2 k_\perp^2}{\Lambda_{\mathsf{QCD}}^2}\right)}$, with $C_{\mathsf{mom}}^2 = 10^3$

Results with running coupling:



The 'subtracted' and 'unsubtracted' expressions are no longer equivalent

'Subtracted' expression: closer to the 'CXY' result at small $k_\perp$, negative results at large $k_\perp$

Possible way to use consistently a coordinate-space running coupling: rewrite the cross section expression in coordinate space.

We write $\mathcal{J} = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}$ and $\mathcal{J}_v = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}_v$, with

$$\widetilde{\mathcal{J}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{x}\cdot(\mathbf{x}-\mathbf{r})}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}-(1-\xi)\mathbf{x},X) - S(\xi\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right],$$

$$\widetilde{\mathcal{J}}_v(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{1}{\mathbf{x}^2} \left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right].$$

(and similarly for the C_F terms)

In these notations the BK equation reads

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = -2\alpha_s N_\mathsf{c} \left[\widetilde{\mathcal{J}}(\mathbf{r},1,X) - \widetilde{\mathcal{J}}_v(\mathbf{r},1,X)\right]$$

BK equation with Balitsky's prescription for the running coupling:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2)N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right] \times \left[\frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} + \frac{1}{\mathbf{x}^2}\left(\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\mathbf{x})^2)} - 1\right) + \frac{1}{(\mathbf{r}-\mathbf{x})^2}\left(\frac{\alpha_s((\mathbf{r}-\mathbf{x})^2)}{\alpha_s(\mathbf{x}^2)} - 1\right)\right]$$

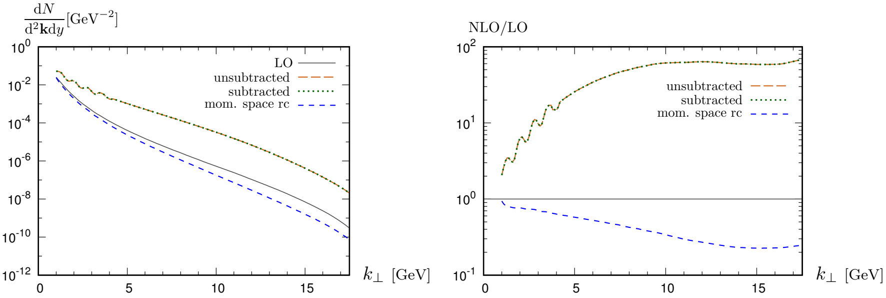
This can be generalized to $\xi \neq 1$ by replacing $\widetilde{\mathcal{J}}_v$ with

$$\widetilde{\mathcal{J}}_v^{\mathrm{rc}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{1}{\mathbf{x}^2}\frac{\alpha_s(\mathbf{x}^2)}{\alpha_s((\mathbf{r}-\xi\mathbf{x})^2)}\left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right],$$

and by replacing the explicit α_s factors by $\alpha_s(\mathbf{r}^2)$. Not a unique choice but:

- $\xi = 1$: recovers Balitsky's prescription
- Fixed coupling results unchanged

Results with this formulation:



The 'subtracted' expression gives the same results as the 'unsubtracted' one

Completely different results compared to fixed coupling or $\alpha_s(k_\perp)$

Similar situation with a simple parent dipole running coupling $\alpha_s(\mathbf{r}^2)$

(reason: behaviour of FT[$\alpha_s(\mathbf{r}^2)S(\mathbf{r})$] completely different from FT[$S(\mathbf{r})$])

Using instead a daughter dipole prescription $\alpha_s(\mathbf{x}^2)$ seems to alleviate the issue

Conclusions

We have studied a recent proposal for the implementation of NLO factorization in single inclusive forward hadron production

- Change of the rapidity scale in the NLO terms: large effect numerically
- Fixed coupling: positive cross sections at all transverse momenta
- Running coupling: mismatch between the subtracted and unsubtracted formulations in momentum space

Directions for future work:

- Better understanding of how to deal with the running of the coupling
- Add the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions
- Use NLO BK for the rapidity evolution of the dipole correlators
- The initial condition for the BK evolution of the target must be obtained by a fit (e.g. to HERA DIS data) also performed at NLO accuracy

Coordinate-space running coupling: $\alpha_s(\mathbf{r}^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{4C^2}{\mathbf{r}^2\Lambda^2_{\text{QCD}}}\right)}$

Momentum-space running coupling: $\alpha_s(k_\perp^2) = \dfrac{4\pi}{\beta_0 \ln\left(\frac{C^2_{\text{mom}} k_\perp^2}{\Lambda^2_{\text{QCD}}}\right)}$

$C^2_{\text{mom}} = 10^3$ is fixed by comparing the LO limits of the 'subtracted' ($\alpha_s \to 0$) and 'unsubtracted' ($\xi \to 1$, $\alpha_s \to \alpha_s(k_\perp^2)$) expressions: