

TOWARDS HIGHER-ORDER ACCURACY IN LCPT

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Work in progress
see also T. Lappi, RP **Annals of Physics** (2017) 379, 34-66

Initial Stages, September 20, 2017

Outline of the talk

Motivation

Develop an approach to automatize higher-order LCPT computations in CGC framework

A test bench

Full NLO correction to the (DIS) $\gamma_{T/L}^* \rightarrow q\bar{q}$ LC wave-functions

DIS cross-section at NLO

Example: full result for γ_L^* case

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- ▶ LCPT¹ computations of small-x scattering processes in the CGC picture slowly approaching the NLO
 - ▶ The inclusive DIS cross-section (Beuf: arXiv:1606.00777)
 - ▶ Small-x evolution equations (Balitsky, Chirilli: arXiv:0710.4330)
 - ▶ ...

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- ▶ Problem in LCPT approach
 - ▶ Breaking of manifest rotational invariance -> "covariant style" perturbative computations are tedious (hard to automatize)
- ▶ Solution: a new formulation for LCPT computations
 - ▶ Easy to implement and can be fully automatized (in all orders!)

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NLO correction to the γ^* LC wave-functions

- ▶ Full perturbative Fock state decomposition for γ^*

$$|\gamma^*\rangle_i = \sqrt{Z_{\gamma^*}} \left[|\gamma^*\rangle_b + PS_{(2)} \psi^{\gamma^* \rightarrow q\bar{q}} |q\bar{q}\rangle + PS_{(3)} \psi^{\gamma^* \rightarrow q\bar{q}g} |q\bar{q}g\rangle + \dots \right]$$

NLO correction to the γ^* LC wave-functions

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- ▶ Compute the NLO i.e. $\mathcal{O}(eg^2)$ QCD corrections to $|\gamma^*\rangle_i$ in momentum (p^+, \mathbf{p}) -space
 - ▶ $q\bar{q}$ -sector: one-loop virtual corrections
 - ▶ $q\bar{q}g$ -sector: radiative corrections

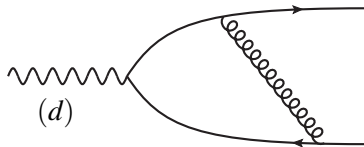
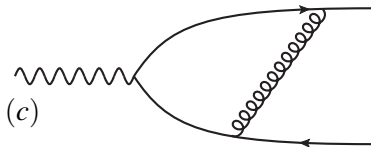
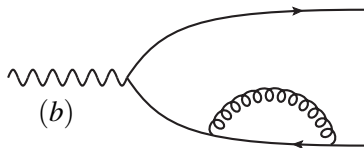
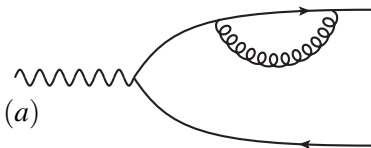
NLO correction to the γ^* LC wave-functions

- ▶ Full perturbative Fock state decomposition for γ^*

$$|\gamma^*\rangle_i = \sqrt{Z_{\gamma^*}} \left[|\gamma^*\rangle_b + PS_{(2)} \psi^{\gamma^* \rightarrow q\bar{q}} |q\bar{q}\rangle + PS_{(3)} \psi^{\gamma^* \rightarrow q\bar{q}g} |q\bar{q}g\rangle + \dots \right]$$

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 - ▶ $q\bar{q}$ -sector: one-loop virtual corrections
 - ▶ $q\bar{q}g$ -sector: radiative corrections
- ▶ Note: no UV renormalization at this order $\Rightarrow \sigma^{\gamma^*_{T/L}}$ is scheme independent!

- $q\bar{q}$ -sector: LC diagrams contributing to γ_T^* LCWF at NLO



Computation of (a)-(d) in practice:

²Unobserved states are treated as $d_s > d$. Once the tensoral algebra is done the limit $d_s \rightarrow 4$ can be taken!

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Computation of (a)-(d) in practice:

- ▶ Construct the LCWF using LCPT rules
- ▶ Handle the UV-divergent \mathbf{k} -integrals and numerator γ -algebra
 - ▶ **Conventional Dim. Reg.** (CDR) scheme: both observed and unobserved particles and momenta are continued to d dimension
 - ▶ **Four Dimensional Helicity** (FDH) scheme: momenta of unobserved particles² are continued to $d > 4$ and observed particles are kept in $4d$

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 - ▶ **Four Dimensional Helicity** (FDH) scheme: momenta of unobserved particles² are continued to $d > 4$ and observed particles are kept in 4d
- ▶ Regulate the longitudinal k^+ -integrals with the momentum cut-off

$$k^+ > \alpha q^+, \quad \text{where } \alpha > 0$$

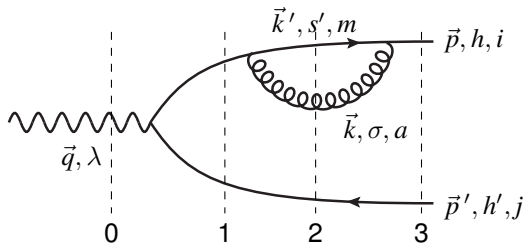
²Unobserved states are treated as $d_s > d$. Once the tensoral algebra is done the limit $d_s \rightarrow 4$ can be taken!

$$k^+/q^+ = z'$$

$$p^+/q^+ = z$$

$$p'^+ = (1-z)q^+$$

$$\bar{Q}^2 = z(1-z)Q^2$$



$$\psi_{(a)}^{\gamma_{\Gamma}^* \rightarrow q\bar{q}} = \frac{-ee_f g^2 C_F \delta^{ij}}{16\pi (q^+)^2} \int_{\alpha}^z \frac{dz'}{zz'(z-z')} \int \frac{d^{d-2}\mathbf{k}}{(2\pi)^{d-2}} \frac{\text{num}|_a}{\Delta_{01}^- \Delta_{02}^- \Delta_{03}^-}$$

► The LC energy denominators

$$\Delta_{01}^- = \Delta_{03}^- = \frac{1}{(-2q^+)z(1-z)} (\mathbf{p}^2 + \bar{Q}^2)$$

$$\Delta_{02}^- = \frac{z}{(-2q^+)z'(z-z')} \left[\mathbf{m}^2 + \frac{z'(z-z')}{z^2(1-z)} (\mathbf{p}^2 + \bar{Q}^2) \right]$$

- ▶ The numerator

$$\text{num}|_a = \sum_{\sigma, s', s} \varepsilon_{\sigma}^{\mu}(k) \varepsilon_{\sigma}^{*\nu}(k) \left[\bar{u}_h(p) \gamma_{\mu} \underbrace{u_{s'}(k') \bar{u}_{s'}(k')}_{=k'} \gamma_{\nu} \underbrace{u_s(p) \bar{u}_s(p)}_{=p} \not{x}_{\lambda}(q) v_{h'}(p') \right]$$

- ▶ 4-momentum not conserved $\Rightarrow k' = p - k + (k^- + k'^- - p^+) \gamma^+$
- ▶ Light-cone γ -algebra is difficult to automatize

- ▶ The numerator

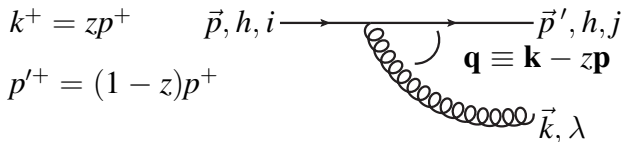
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- ▶ 4-momentum not conserved $\Rightarrow k' = p - k + (k^- + k'^- - p^+) \gamma^+$
- ▶ Light-cone γ -algebra is difficult to automatize
 - ▶ \Rightarrow Full "(a) + (b) + (c) + (d)": long and complicated "brute force" calculation

Urgent need for a simpler approach!

► Systematic approach to automatise LCPT computations 

- Decompose the vertices to symmetric and anti-symmetric part.

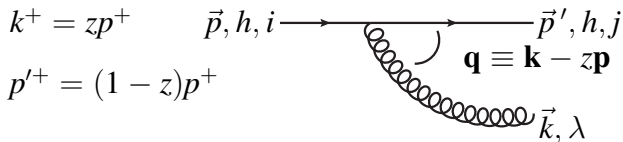


$$\bar{u}_h(p') \not{\epsilon}_\lambda^*(k) u_h(p) \stackrel{d=4}{=} \frac{2}{z\sqrt{1-z}} \left[\left(1 - \frac{z}{2}\right) \delta^{ij} + ih \frac{z}{2} \epsilon^{ij} \right] \mathbf{q}^i \epsilon_\lambda^{*j} \quad (1)$$

$$\bar{u}_h(p') \not{\epsilon}_\lambda^*(k) u_h(p) \stackrel{d \neq 4}{=} \left[a(z) \bar{u}_h \gamma^+ u_h \delta^{ij} + b(z) \bar{u}_h \gamma^+ [\gamma^i, \gamma^j] u_h \right] \mathbf{q}^i \epsilon_\lambda^{*j} \quad (2)$$

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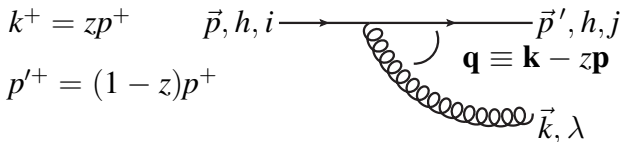
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- Step1: Compute $d = 4$ part with (1)

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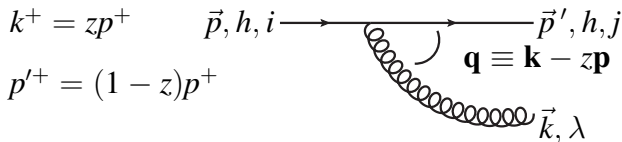
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- Step2: Compute $d \neq 4$ rational parts with (2)

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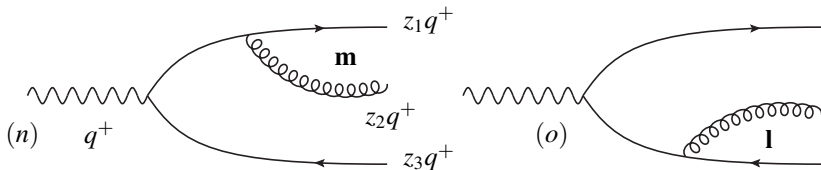
- Step1: Compute $d = 4$ part with (1)
- Step2: Compute $d \neq 4$ rational parts with (2)
- Step3: After Step2 use $[\gamma^i, \gamma^j] u_h(p) \stackrel{d=4}{=} -2ih\epsilon^{ij} u_h(p)$

$$\psi_{(\text{NLO})}^{\gamma_T^* \rightarrow q\bar{q}} \Big|_{\text{FDH}} = \psi_{\text{LO}}^{\gamma_T^* \rightarrow q\bar{q}}(\mathbf{p}, z) \left(\frac{g_R^2 C_F}{8\pi^2} \right) \left\{ \left[\frac{3}{2} + \log\left(\frac{\alpha}{z}\right) + \log\left(\frac{\alpha}{1-z}\right) \right] C_{\text{full}} \right. \\ \left. + \frac{1}{2} \log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{3} + \frac{5}{2} \right\} + \mathcal{O}(\varepsilon)$$

$$C_{\text{full}} = \frac{1}{\varepsilon_{\overline{\text{MS}}}} + \log\left(\frac{\mu^2}{Q^2}\right) + \left(\frac{\bar{Q}^2 - \mathbf{p}^2}{\mathbf{p}^2}\right) \log\left(\frac{\mathbf{p}^2 + \bar{Q}^2}{\bar{Q}^2}\right)$$

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- ▶ $q\bar{q}g$ -sector: straightforward computation for $\psi^{\gamma_{T/L}^* \rightarrow q\bar{q}g}$
- ▶ Example: full result for γ_L^* (in FDH scheme)



$$\psi_{\text{NLO}}^{\gamma_L^* \rightarrow q\bar{q}g} \Big|_{\text{FDH}} = 8q^+ e e_f Q g^2 t_{\alpha\beta}^a z_1 z_3 \left[\Sigma_{(n)}^{ij} \frac{\mathbf{m}^i}{\left[\mathbf{p}'^2 + \bar{Q}_{(n)}^2 \right] \left[\mathbf{m}^2 + \omega_{(n)} \left(\mathbf{p}'^2 + \bar{Q}_{(n)}^2 \right) \right]} - \Sigma_{(o)}^{ij} \frac{\mathbf{l}^i}{\left[\mathbf{p}^2 + \bar{Q}_{(o)}^2 \right] \left[\mathbf{l}^2 + \omega_{(o)} \left(\mathbf{p}^2 + \bar{Q}_{(o)}^2 \right) \right]} \right] \epsilon_{\sigma}^{*j}$$

where

$$\Sigma_{(n)}^{ij} = (z_1 + z_2) \left(\frac{z_3}{z_1} \right)^{1/2} \left[\left(1 - \frac{1}{2} \left(\frac{z_2}{z_1 + z_2} \right) \right) \delta^{ij} + i h \frac{1}{2} \left(\frac{z_2}{z_1 + z_2} \right) \epsilon^{ij} \right]$$

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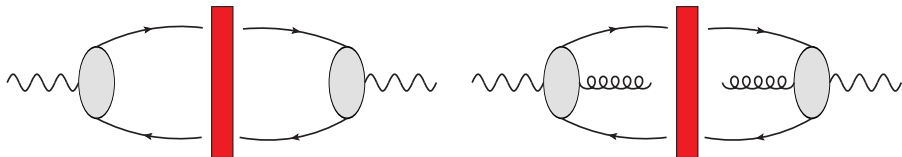
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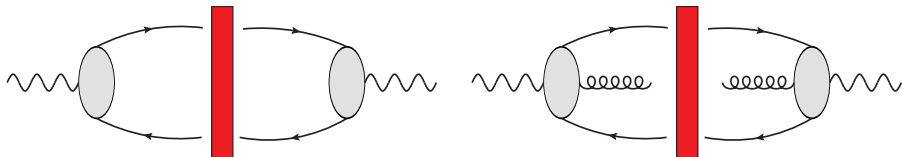
DIS cross-section at NLO



The total cross-section for γ^* scattering from a classical gluon field:

$$\sigma^{\gamma^*_{T/L}}[A] = \frac{2}{2q^+(2\pi)\delta(q'^+ - q^+)} \left\{ q\bar{q} \langle \gamma^* | 1 - \hat{S}_E | \gamma^* \rangle_{q\bar{q}} + q\bar{q}g \langle \gamma^* | 1 - \hat{S}_E | \gamma^* \rangle_{q\bar{q}g} \right\}$$

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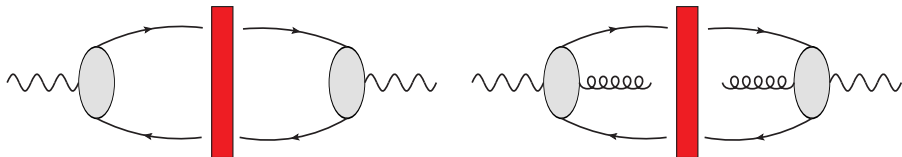


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- ▶ Step1: Fourier transform $|\gamma^*\rangle_{q\bar{q}}$ and $|\gamma^*\rangle_{q\bar{q}g}$ into the mixed (p^+, \mathbf{x}) -space

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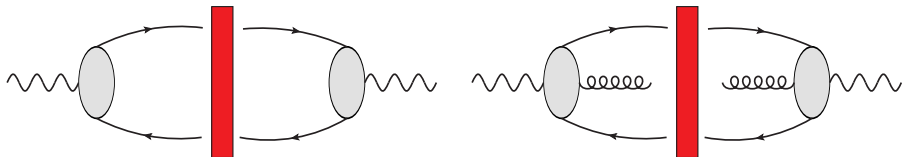


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- ▶ Step2: compute the amplitude square with eikonal target $(1 - \hat{S}_E)|\gamma^*\rangle_{q\bar{q}}$

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- ▶ Step2: compute the amplitude square with eikonal target $(1 - \hat{S}_E)|\gamma^*\rangle_{q\bar{q}}$
- ▶ Step3: UV subtraction between $q\bar{q}$ and $q\bar{q}g$ sectors

- Example: full result for γ_L^* (similar result for γ_T^*)

$$\sigma^{\gamma_L^*} \Big|_{q\bar{q}} = 4N_c \frac{4\alpha_{em}e_f^2 Q^2}{(2\pi)^2} \int d^2\mathbf{x} \int d^2\mathbf{y} \int_0^1 dz z^2 (1-z)^2 [K_0(Q\sqrt{z(1-z)}|\mathbf{r}_{xy}|)]^2$$

$$\times \left\{ 1 + \left(\frac{\alpha_s C_F}{\pi} \right) \left[\frac{1}{2} \log^2 \left(\frac{z}{1-z} \right) - \frac{\pi^2}{6} + \frac{5}{2} \right] \right\} (1 - \mathcal{S}_{xy})$$

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$$\sigma^{\gamma_L^*} \Big|_{q\bar{q}g} = 4N_c \frac{4\alpha_{em}e_f^2 Q^2}{(2\pi)^3} \left(\frac{\alpha_s C_F}{\pi} \right) \int d^2\mathbf{x} \int d^2\mathbf{y} \int d^2\mathbf{z} \int_0^1 dz_1 \int_0^{1-z_1} \frac{dz_2}{z_2} \left\{ \right.$$

$$z_3^2 \left(2z_1(z_1 + z_2) + z_2^2 \right) \frac{1}{\mathbf{r}_{zx}^2} \left([K_0(Q|\mathbf{R}|)]^2 (1 - \mathcal{S}_{xyz}) - [K_0(\bar{Q}_{(n)}|\mathbf{r}_{xy}|)]^2 e^{-\mathbf{r}_{zx}^2/(\mathbf{r}_{xy}^2 e^{\gamma_E})} (1 - \mathcal{S}_{xy}) \right)$$

$$+ z_1^2 \left(2z_3(z_2 + z_3) + z_3^2 \right) \frac{1}{\mathbf{r}_{zy}^2} \left([K_0(Q|\mathbf{R}|)]^2 (1 - \mathcal{S}_{xyz}) - [K_0(\bar{Q}_{(o)}|\mathbf{r}_{xy}|)]^2 e^{-\mathbf{r}_{zy}^2/(\mathbf{r}_{xy}^2 e^{\gamma_E})} (1 - \mathcal{S}_{xy}) \right)$$

$$\left. - 2 \left((z_1 + z_2)z_1 z_3^2 + (z_2 + z_3)z_3 z_1^2 \right) \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{zy}}{(\mathbf{r}_{zx}^2)(\mathbf{r}_{zy}^2)} [K_0(Q|\mathbf{R}|)]^2 (1 - \mathcal{S}_{xyz}) \right\}$$

- ▶ Cancellation of UV reg. scheme dependence
- ▶ Result numerically consistent with Beuf arXiv:1708.06557

Outlook

- ▶ Automatization of higher-order LCPT computation for QCD/QED
 - ▶ replace the complicated light-cone γ -algebra and spin sums by simple tensor algebra (easy to implement - FORM)
 - ▶ Test bench: Computation of massless DIS cross-sections at NLO accuracy (results fully agree with Beuf arXiv:1708.06557)
- ▶ To-Do List:
 - ▶ DIS: include the mass corrections
 - ▶ NLO computations for other processes
 - ▶ Higher-order (NNLO) corrections