Smooth matching of pre-equilibrium evolution to hydrodynamics in heavy ion collisions

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L. Keegan, A. Kurkela, A.M., and D. Teaney, JHEP 08, 171 (2016)[1].

A. Kurkela, A.M., J.-F. Paquet, S. Schlichting, and D. Teaney, *in preparation*.

see also Teaney's talk yesterday

ISOQUANT SFB1225

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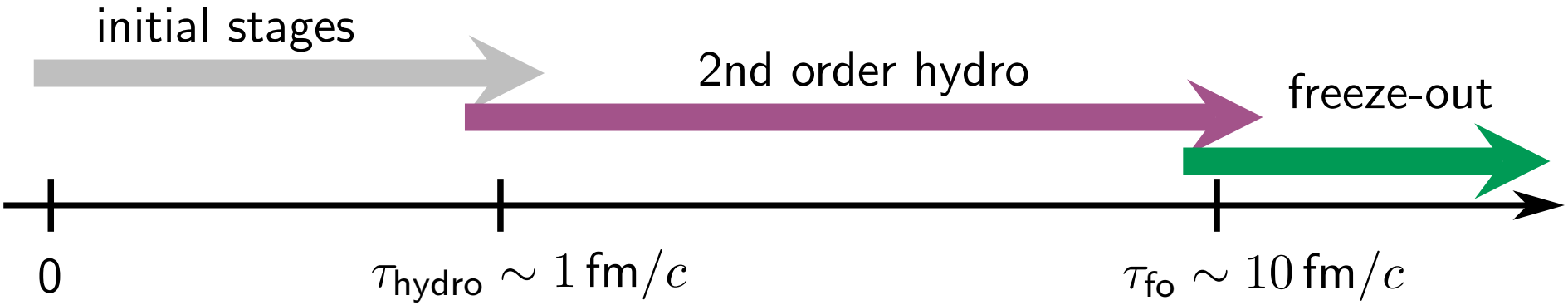
Ultra-relativistic heavy ion collisions in a nutshell



Two examples of initial stages:

A MC-Glauber:
✓ e-by-e fluctuations, ✗ dynamics, ✗ flow, ✗ hydrodynamization

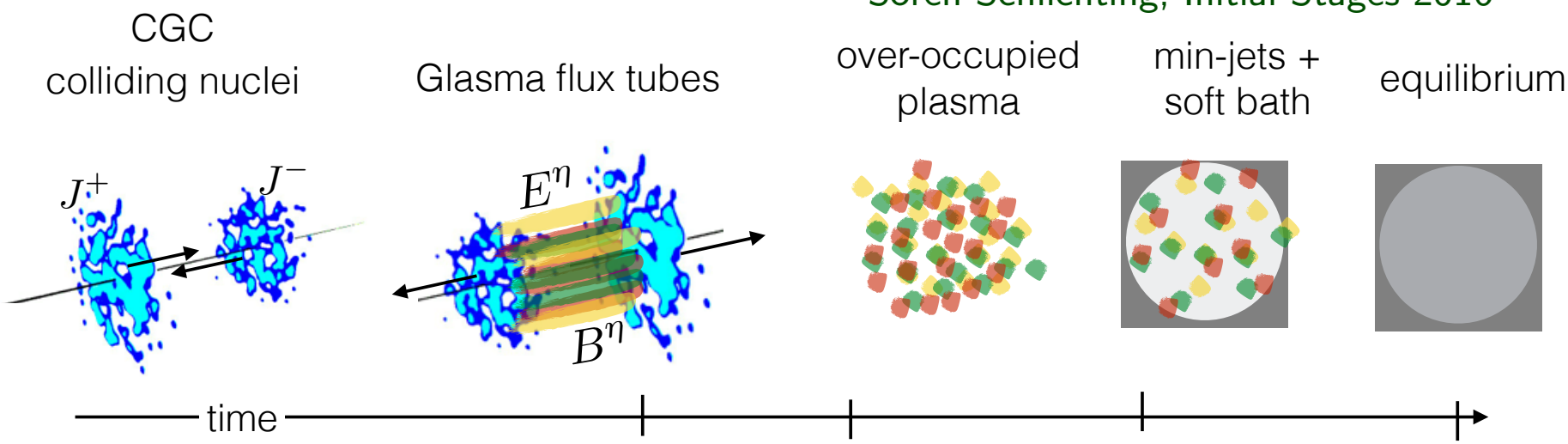
B IP-Glasma: see Schenke's talk yesterday
✓ e-by-e fluct., ✓ 2+1D Yang-Mills, ✓ flow, ✗ hydrodynamization

Our goal: consistent theoretical overlap of initial stages and the onset of hydrodynamics.

Motivation: weak coupling picture

At high energies and densities $\alpha_s \ll 1$

Sören Schlichting, Initial Stages 2016



strong fields

quasi particles

see Boguslavski's talk
and Gelis' talks yesterday

classical-statistical
lattice gauge theory
$f \sim \frac{1}{\alpha_s} \gg 1$

eff. kinetic theory
$1 \ll f \ll \frac{1}{\alpha_s}$

hydro
$f \sim 1$

We will use QCD kinetic theory to map the IP-Glasma initial conditions to hydrodynamics.

Thermalization in QCD effective kinetic theory

see Teaney's and Boguslavski's talks yesterday

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[3]

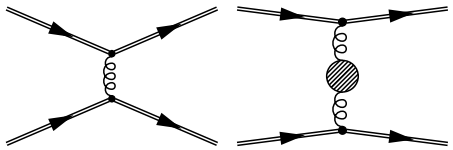
Universal-attractor — Berges, Boguslavski, Schlichting, Venugopalan (2014)[4]

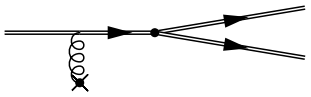
Numerical realization — Kurkela and Zhu (2015)[5]

QCD effective kinetic theory — Arnold, Moore, Yaffe (2003)[2]

Solve Boltzmann equation for $f(\mathbf{p})$—gluon distribution function

$$\partial_\tau f + \frac{\mathbf{p}}{|p|} \cdot \nabla f - \frac{p_z}{\tau} \partial_{p_z} f = - \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]}_{} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}_{}$$





- transverse gradients, boost invariant expansion
- elastic $2 \leftrightarrow 2$ and *inelastic* $1 \leftrightarrow 2$ QCD processes
- equilibrates to conformal hydrodynamics: $c_s^2 = \frac{1}{3}$, $\eta(\lambda)/s = \text{const}$

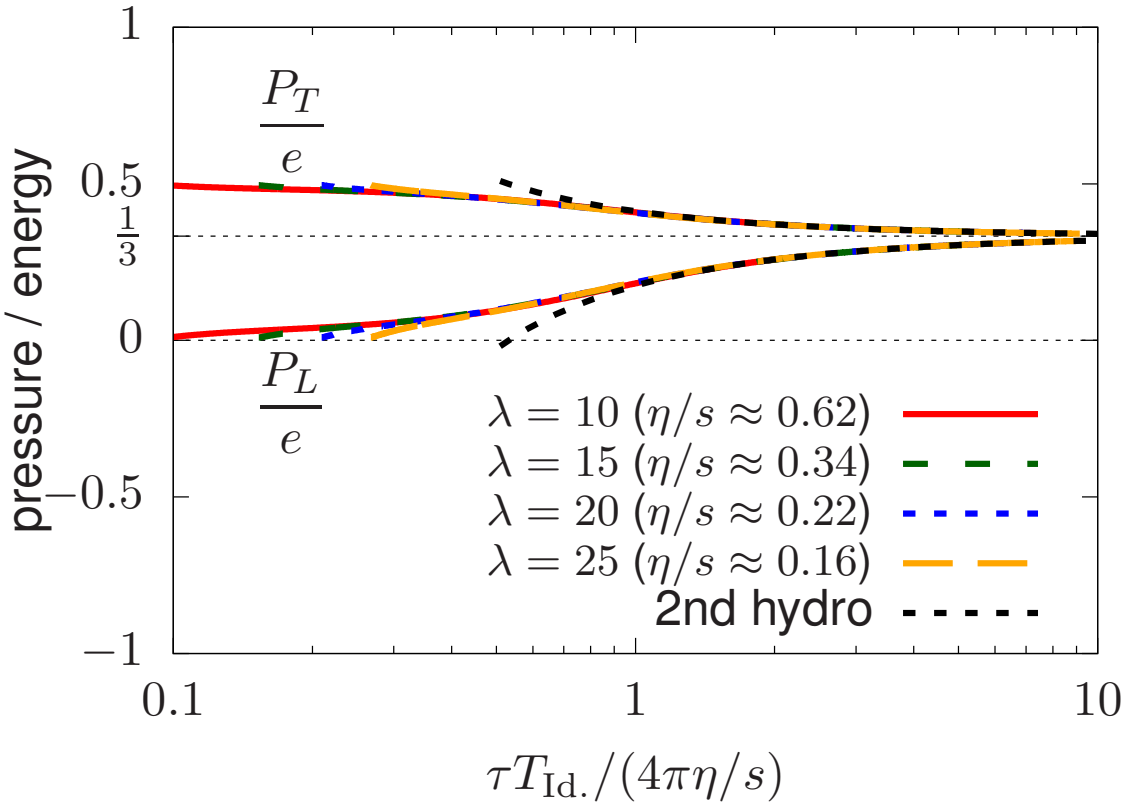
In practice extrapolate to realistic values of the coupling $\lambda = 4\pi\alpha_s N_c$.

QCD kinetic theory $\Rightarrow$ conformal hydrodynamics

Scaling of pressure evolution for boost invariant system

First moments of distribution function

$$e = P_T + P_L, \quad P_L = \nu_g \int_{\mathbf{p}} \frac{p_z^2}{p} f(\mathbf{p}), \quad P_T = \nu_g \int_{\mathbf{p}} \frac{p_\perp^2}{p} f(\mathbf{p})$$

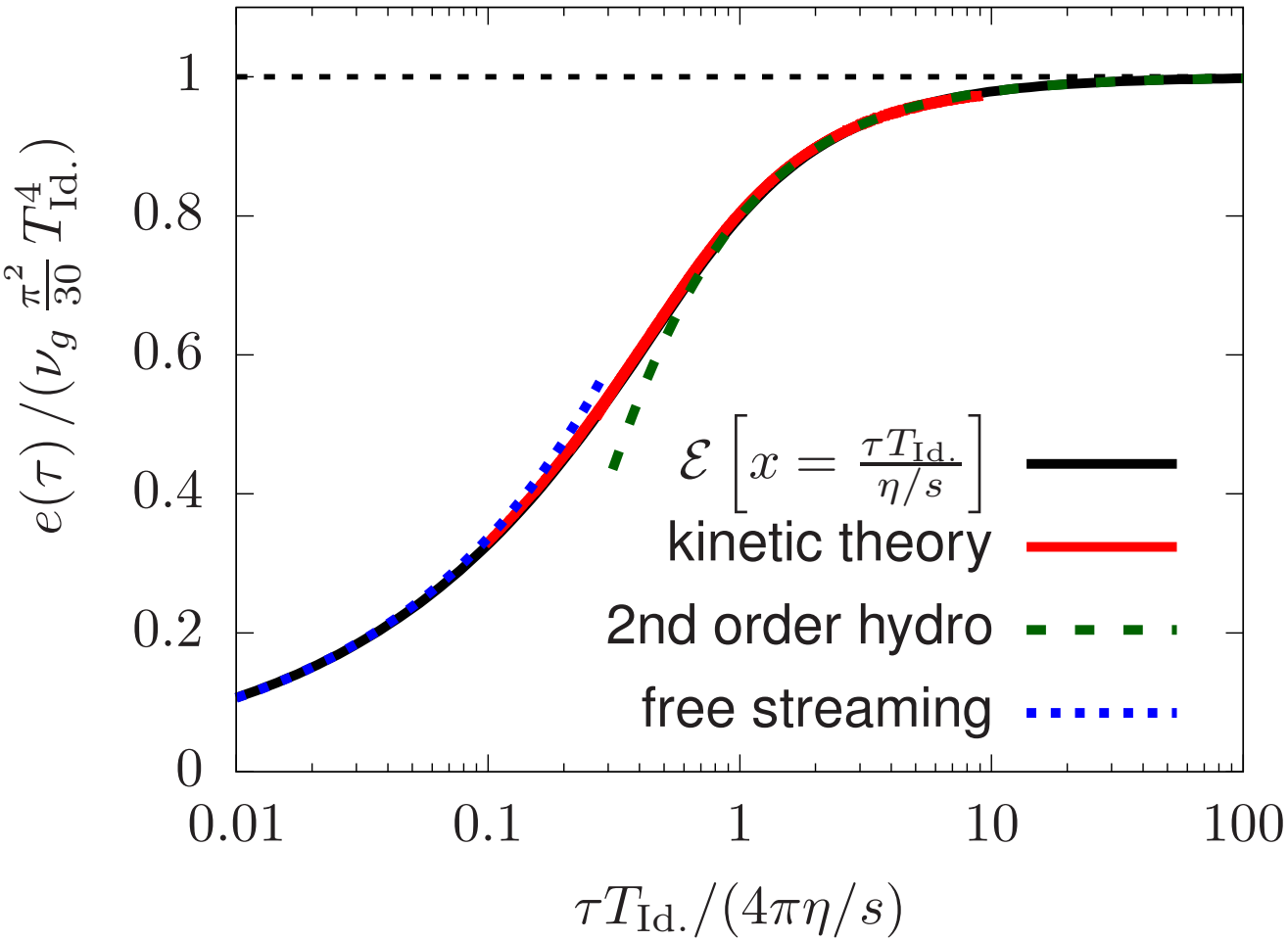


see Strickland's talk yesterday
see Romatschke's talk tomorrow

Collapse to scaling solution for time $\frac{\tau T_{\mathrm{Id.}}}{\eta/s} \propto \frac{\tau}{\tau_R}$ *in units of relaxation time.*

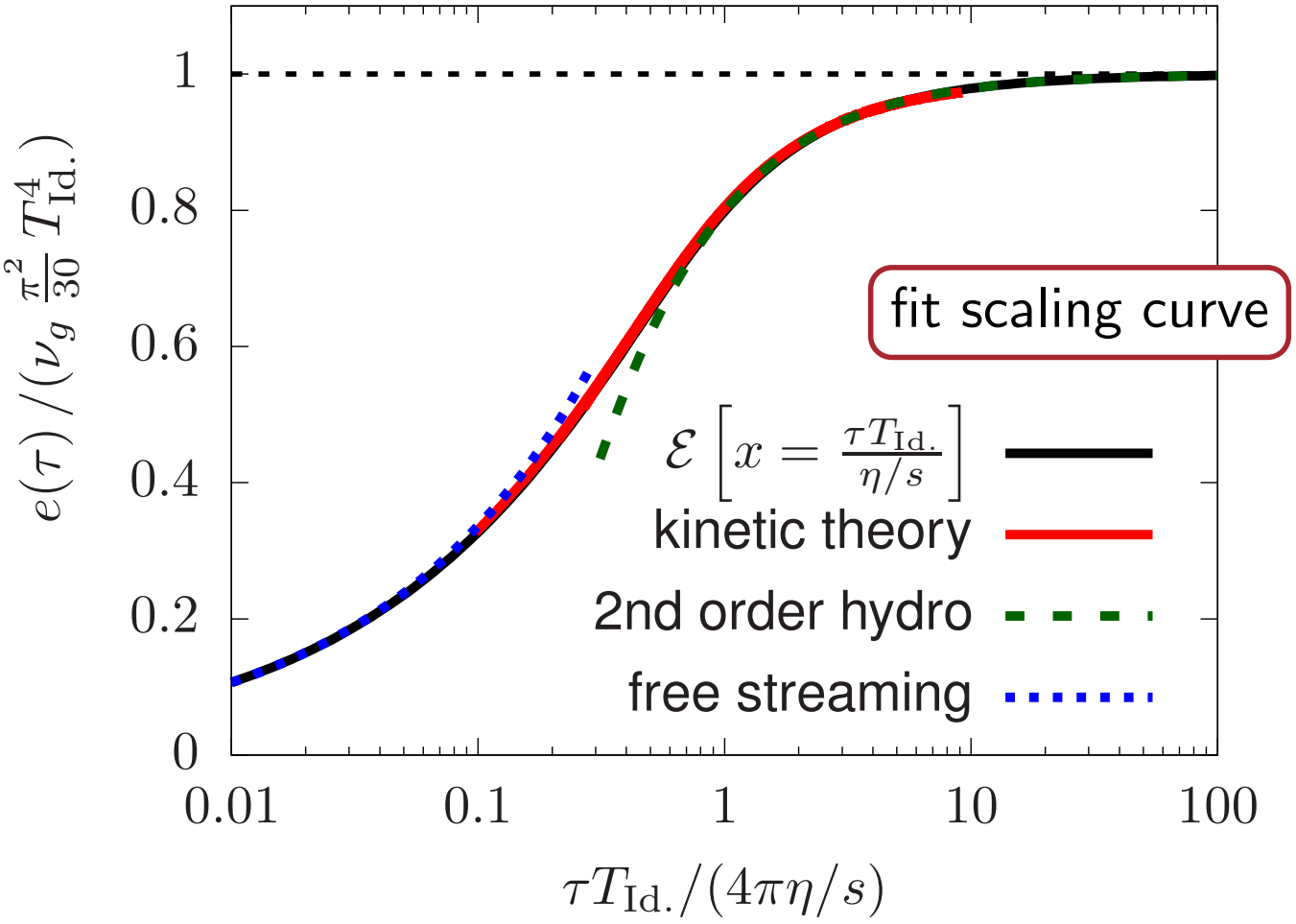
Out-of-equilibrium energy evolution for boost invariant system

scaling in hydro: $$e(\tau) = \underbrace{\nu_g \frac{\pi^2}{30} T_{\text{Id.}}^4}_{\text{“ideal” temp.}} \left(\underbrace{1}_{\text{ideal}} - \underbrace{\frac{8}{3} \frac{\eta/s}{\tau T_{\text{Id.}}}}_{\text{viscous}} + \underbrace{C_2 \left(\frac{\eta/s}{\tau T_{\text{Id.}}} \right)^2}_{\text{2nd order hydro}} \right)$$



Out-of-equilibrium energy evolution for boost invariant system

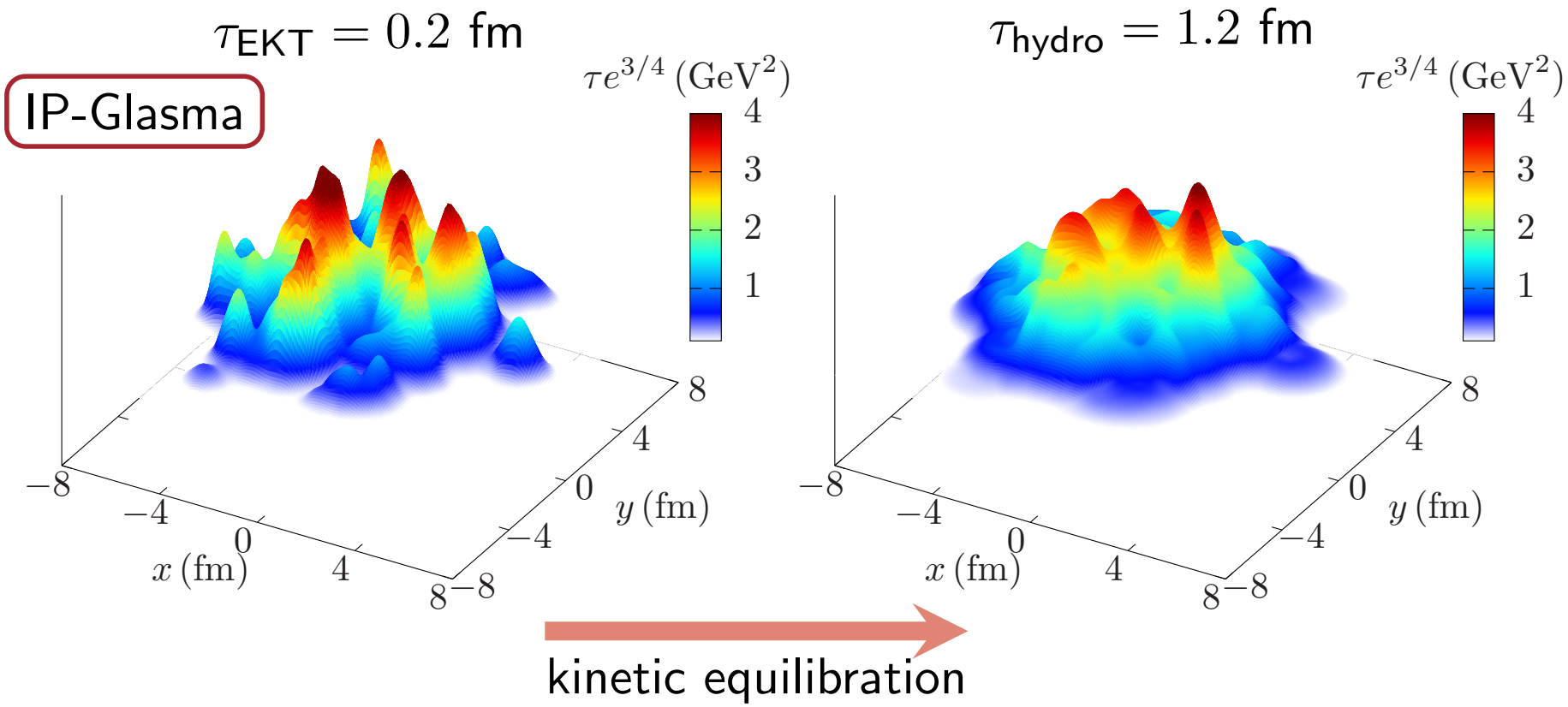
generalized scaling: $$e(\tau) = \nu_g \frac{\pi^2}{30} T^4_{\text{ld.}}(\tau) \times \underbrace{\mathcal{E}\left[x = \frac{\tau T_{\text{ld.}}(\tau)}{\eta/s}\right]}_{\text{non-equilibrium evolution}}$$



Event by event pre-equilibrium dynamics

Evolution of perturbations with kinetic theory

IP-Glasma initial conditions at $\tau_{\mathsf{EKT}} = 0.2\,\mathrm{fm}$



$$\underbrace{e(\boldsymbol{x}, \tau_{\mathsf{EKT}}), \vec{v}(\boldsymbol{x}, \tau_{\mathsf{EKT}})}_{\text{input to EKT}} \implies \underbrace{f(\tau, \mathbf{p}, \boldsymbol{x})}_{\text{kinetic evolution}} \implies \underbrace{e(\boldsymbol{x}, \tau_{\mathsf{hydro}}), \vec{v}(\boldsymbol{x}, \tau_{\mathsf{hydro}})}_{\text{input to hydro}}$$

Plane wave perturbations in transverse plane

Simplify 5+1D problem for $f(\tau, \mathbf{p}, \boldsymbol{x}_\perp)$ by linearization

$$f(\tau, \mathbf{p}, \boldsymbol{x}_\perp) = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}}_{\text{transverse perturbations}} .$$

Linearized Boltzmann equation for perturbations

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp \cdot \boldsymbol{k}_\perp}{p}\right)\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} = -\delta\mathcal{C}[\bar{f}, \delta f] \qquad \boldsymbol{k}_\perp \text{ perturbation}$$

Construct kinetic response functions to energy/momentum perturbations

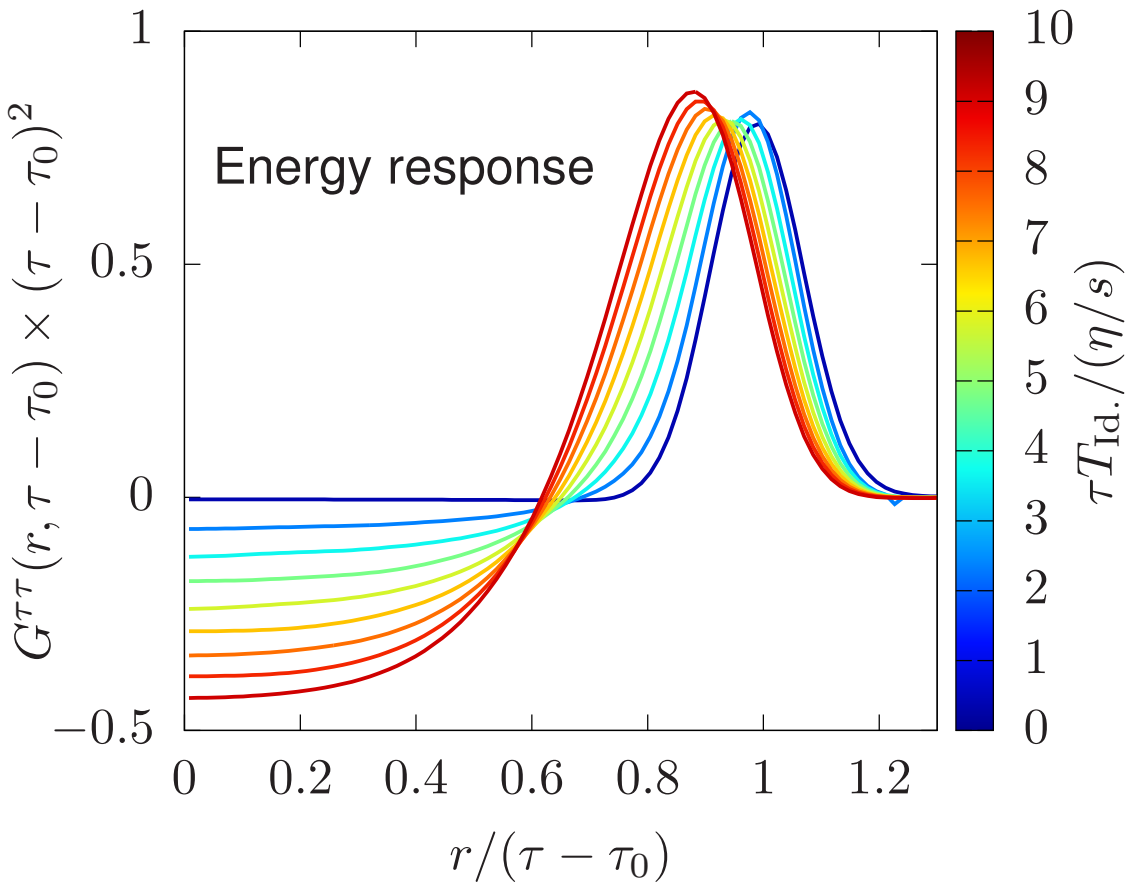
$$\underbrace{\frac{\delta T^{\mu\nu}(\tau, \boldsymbol{k})}{e(\tau)}}_{\text{output}} = \underbrace{\tilde{G}^{\mu\nu}_{(\tau,i)}\left(\boldsymbol{k}, \tau, \tau_0\right) \times}_{\text{non-equilibrium kinetic response}} \underbrace{\left[\frac{\delta T^{\tau\tau}(\tau_0, \boldsymbol{k})}{e(\tau_0)} \text{ or } \frac{\delta T^{\tau i}(\tau_0, \boldsymbol{k})}{e(\tau_0)}\right]}_{\text{input}}$$

Scaling of kinetic theory response functions

- Response to perturbations also collapse to scaling solutions

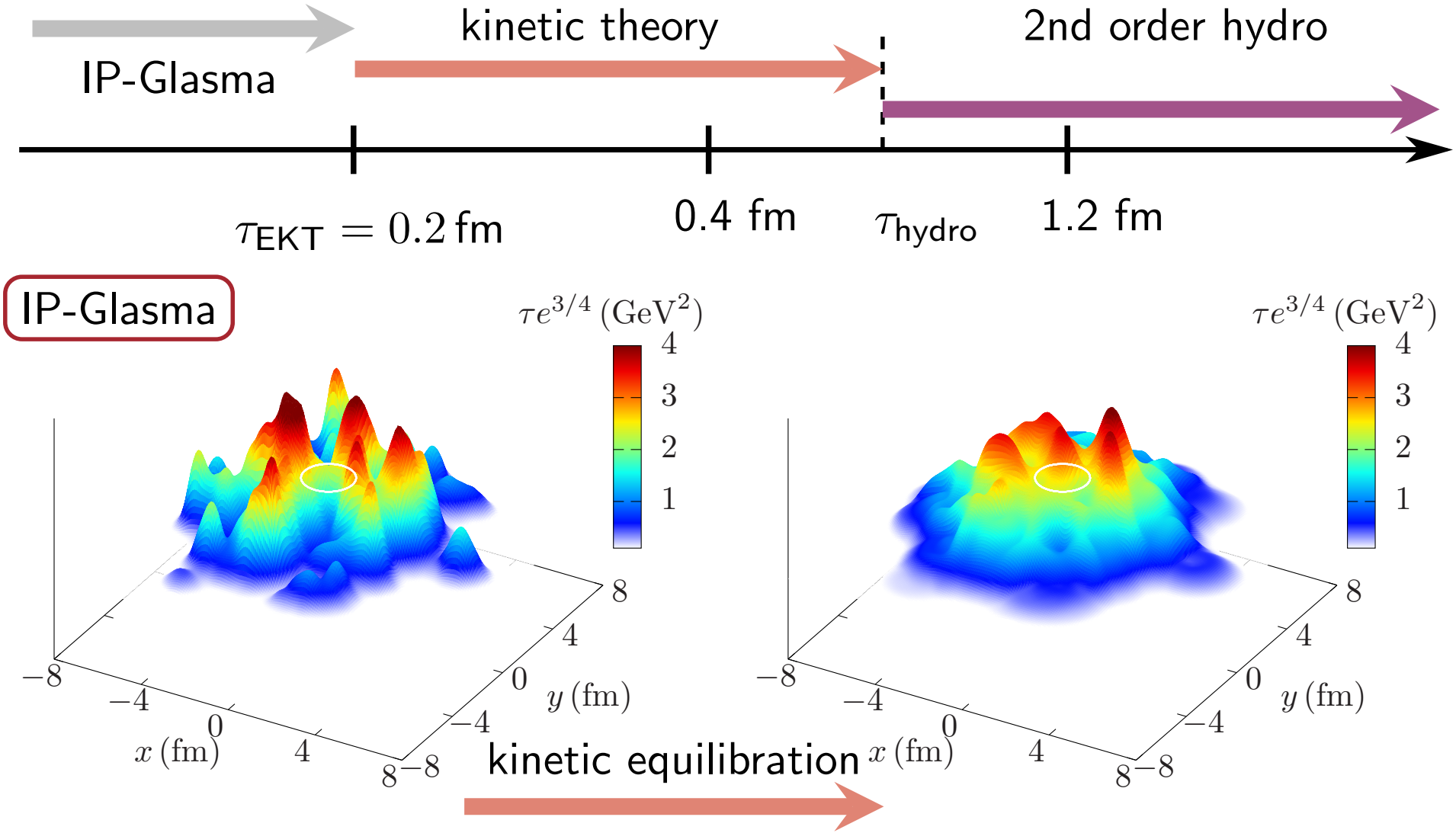
$$\tilde{G}^{\mu\nu}\Big(|\boldsymbol{k}|, \tau, \tau_0, e(\tau_0), \lambda\Big) = \tilde{G}^{\mu\nu,\mathrm{univ}}\left(\frac{\tau T_{\mathrm{ld.}}}{\eta/s}, |\boldsymbol{k}|(\tau - \tau_0)\right)$$

- Agrees with hydrodynamic scaling in $\tau \to \infty, |\boldsymbol{k}|\tau \to 0$ limit.
- Good scaling for a range of $\eta/s = 0.16{-}0.62$ values.



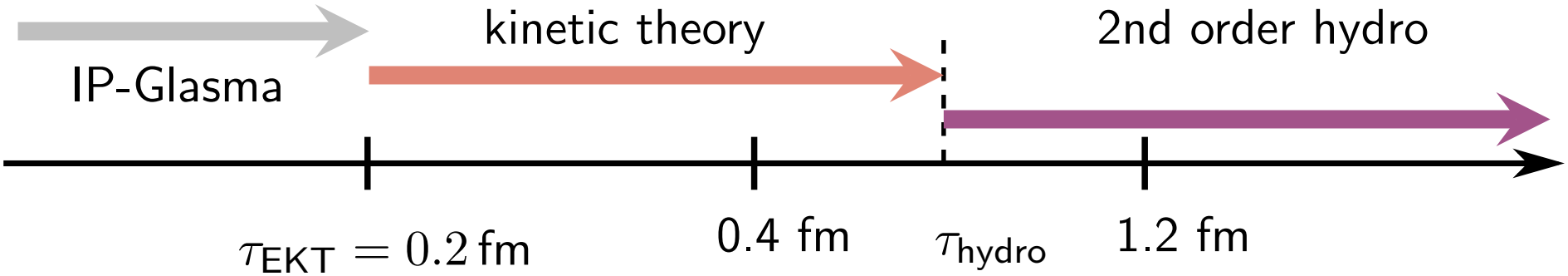
Kinetic evolution with IP-Glasma initial conditions

Equilibration with IP-Glasma initial conditions

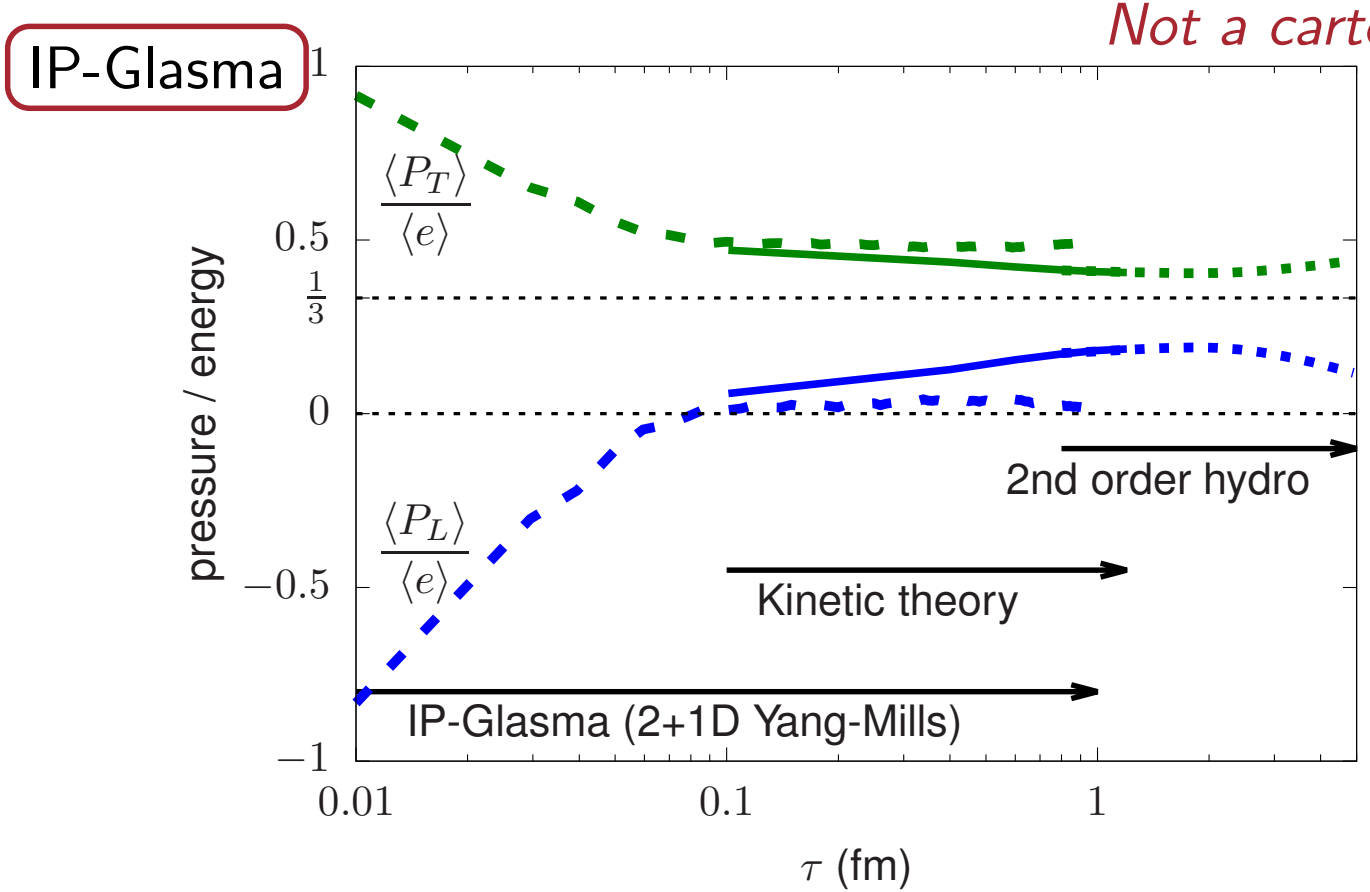


$$\underbrace{e(\boldsymbol{x},\tau_{\mathrm{EKT}}),\ \vec{v}(\boldsymbol{x},\tau_{\mathrm{EKT}})}_{\text{non-zero initial flow}} \Longrightarrow e(\tau)+\delta e(\tau,\boldsymbol{x}),\ \vec{v}(\tau,\boldsymbol{x}) \Longrightarrow \underbrace{T^{\mu\nu}(\boldsymbol{x},\tau_{\mathrm{hydro}})}_{\text{full eng.-mom. tensor}}$$

Average pressure in the transverse plane

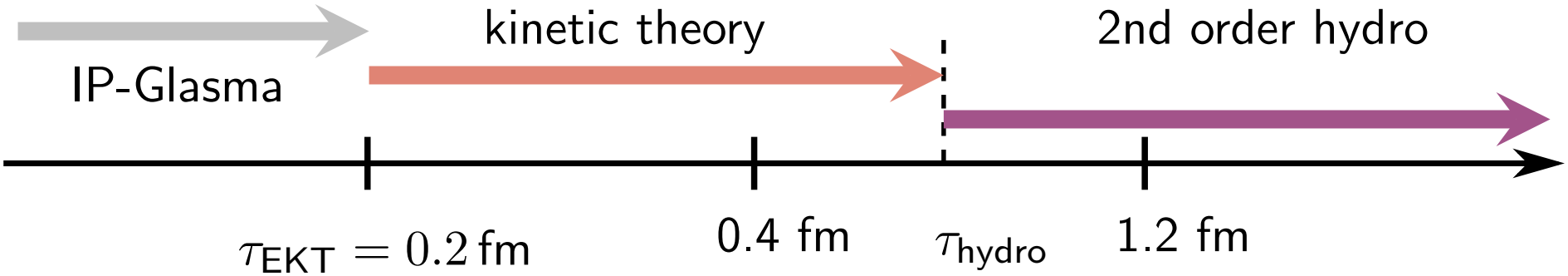


Not a cartoon!

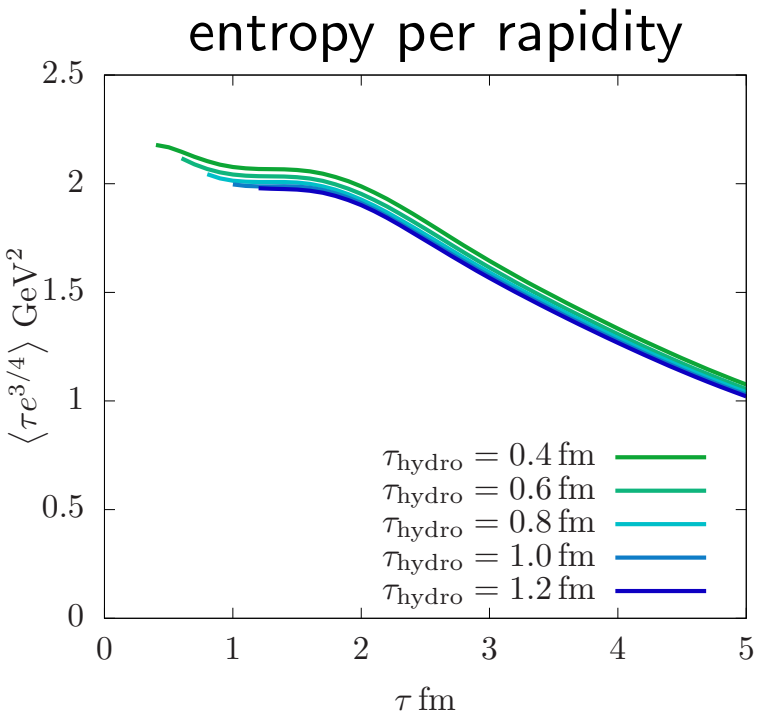


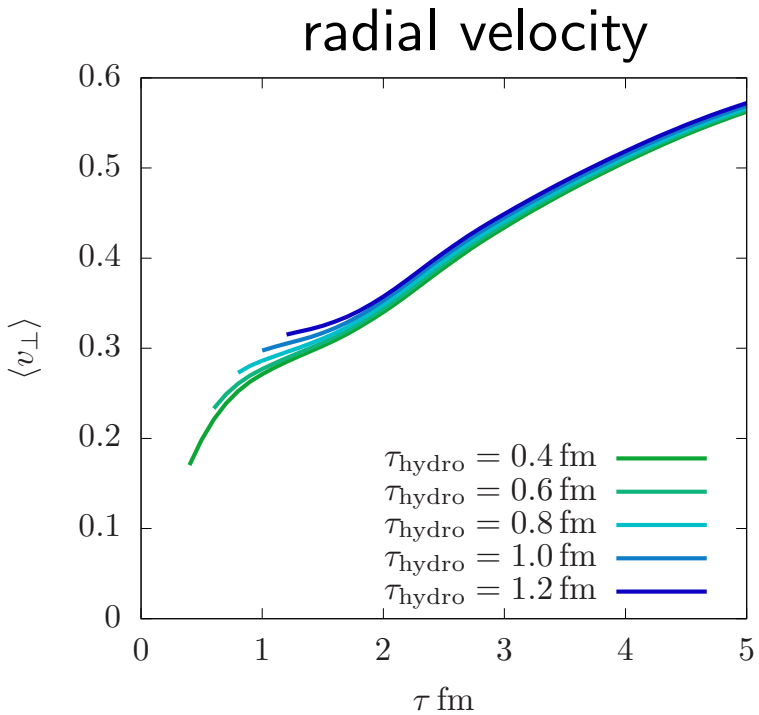
Overlapping and consistent pressure evolution between initial stages!

Transverse averaged density and flow

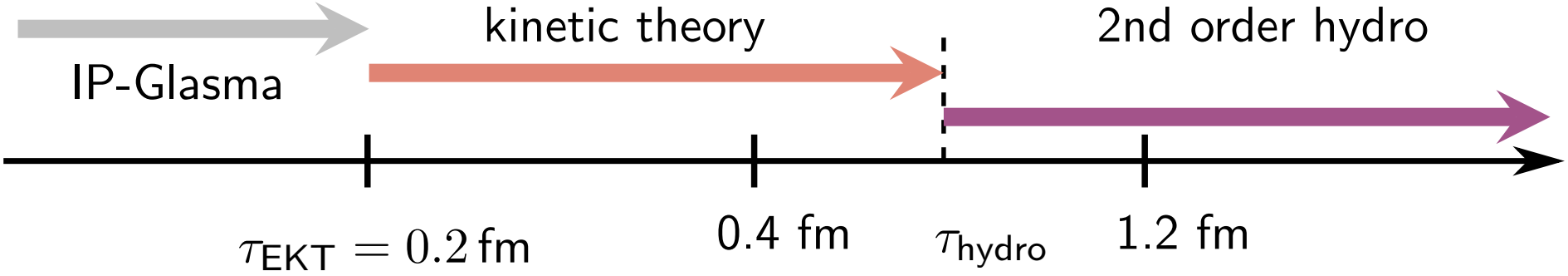


transverse averages $\quad \langle \ldots \rangle = \dfrac{\int d^2\boldsymbol{x}_\perp u^\tau e \ldots}{\int d^2\boldsymbol{x}_\perp u^\tau e}$

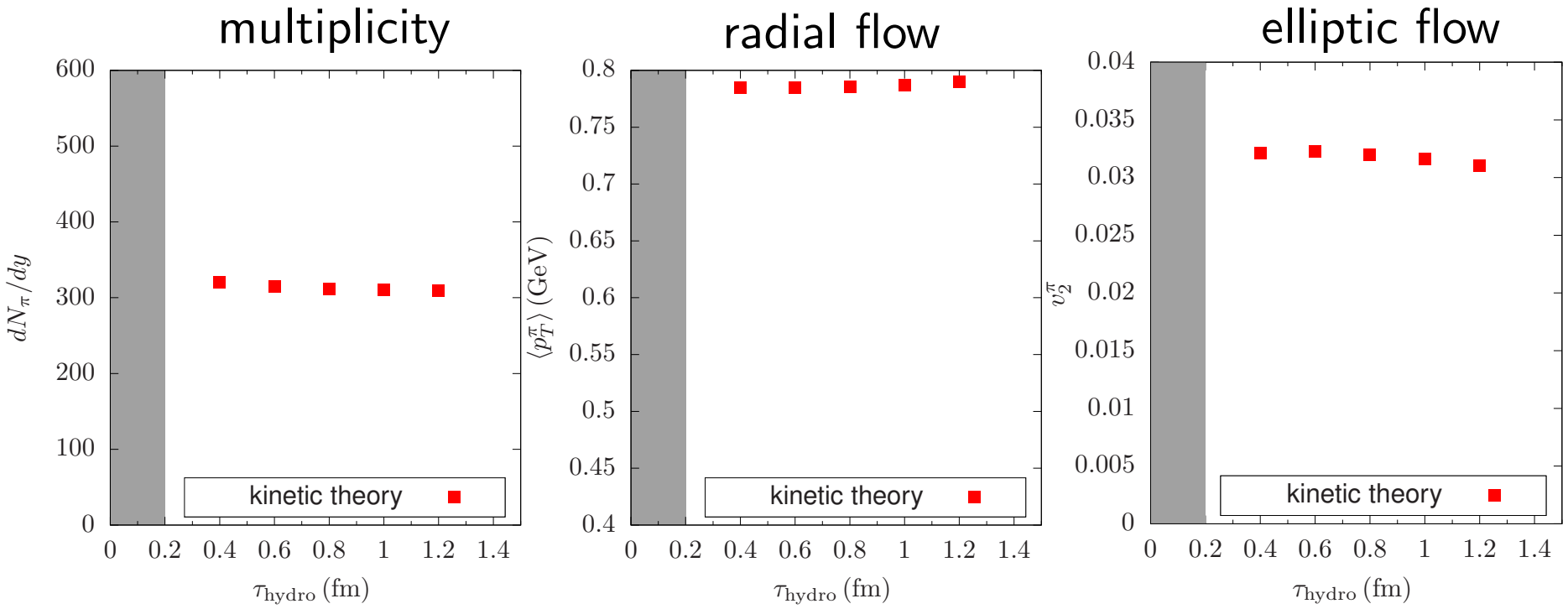




Hadronic observables: IP-Glasma



Thermal pions at freeze-out $T_{\mathrm{fo}} = 145\,\mathrm{MeV}$

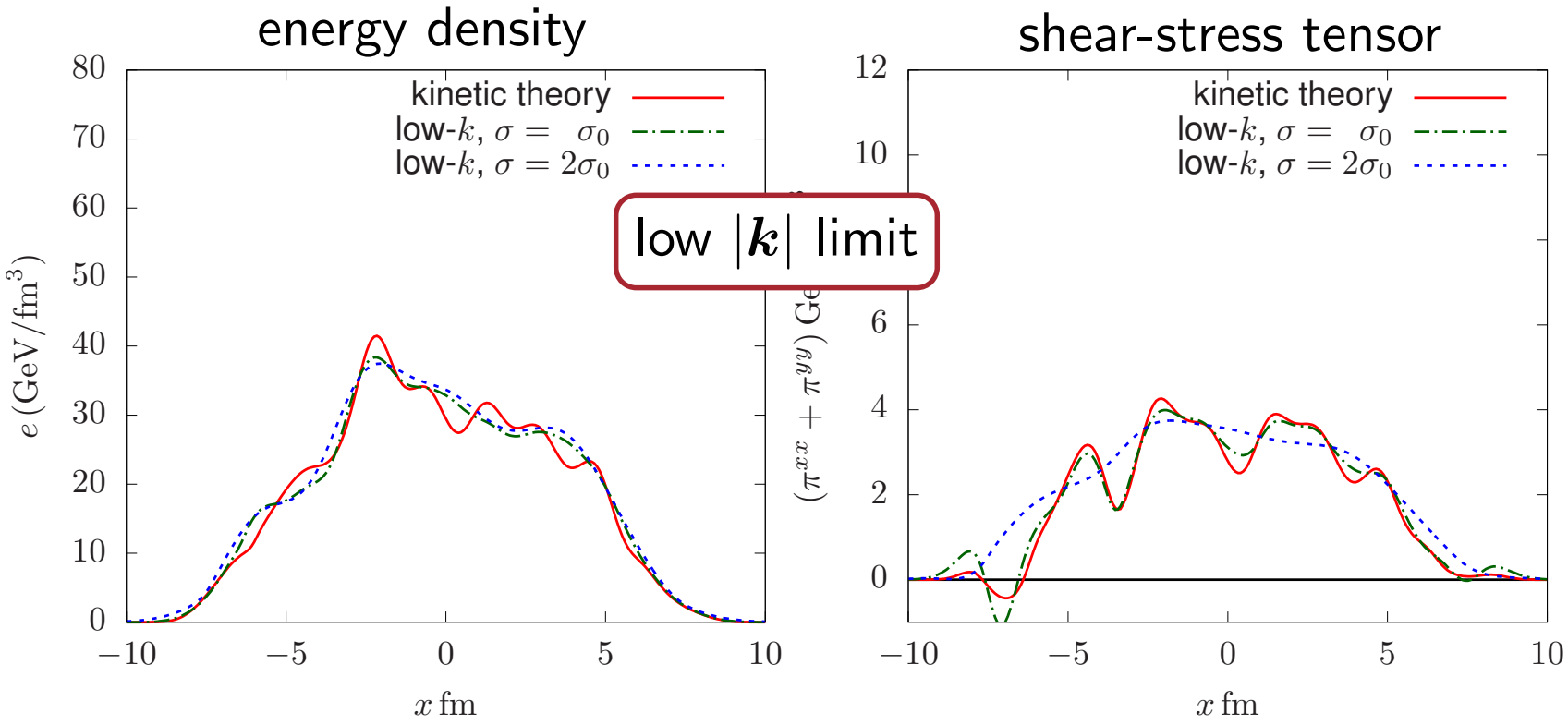


Approximate independence of τ_{hydro} *for kinetic pre-equilibrium evolution!*

Free streaming and low-$|\boldsymbol{k}|$ limits of kinetic theory

Extreme limits of kinetic response:

- small-$|\boldsymbol{k}|$ expansion à la Pratt, c.f. [6, 7]
- free streaming response functions, c.f.[8, 9]



Note: MC-Glauber initial conditions. See backup for more results.

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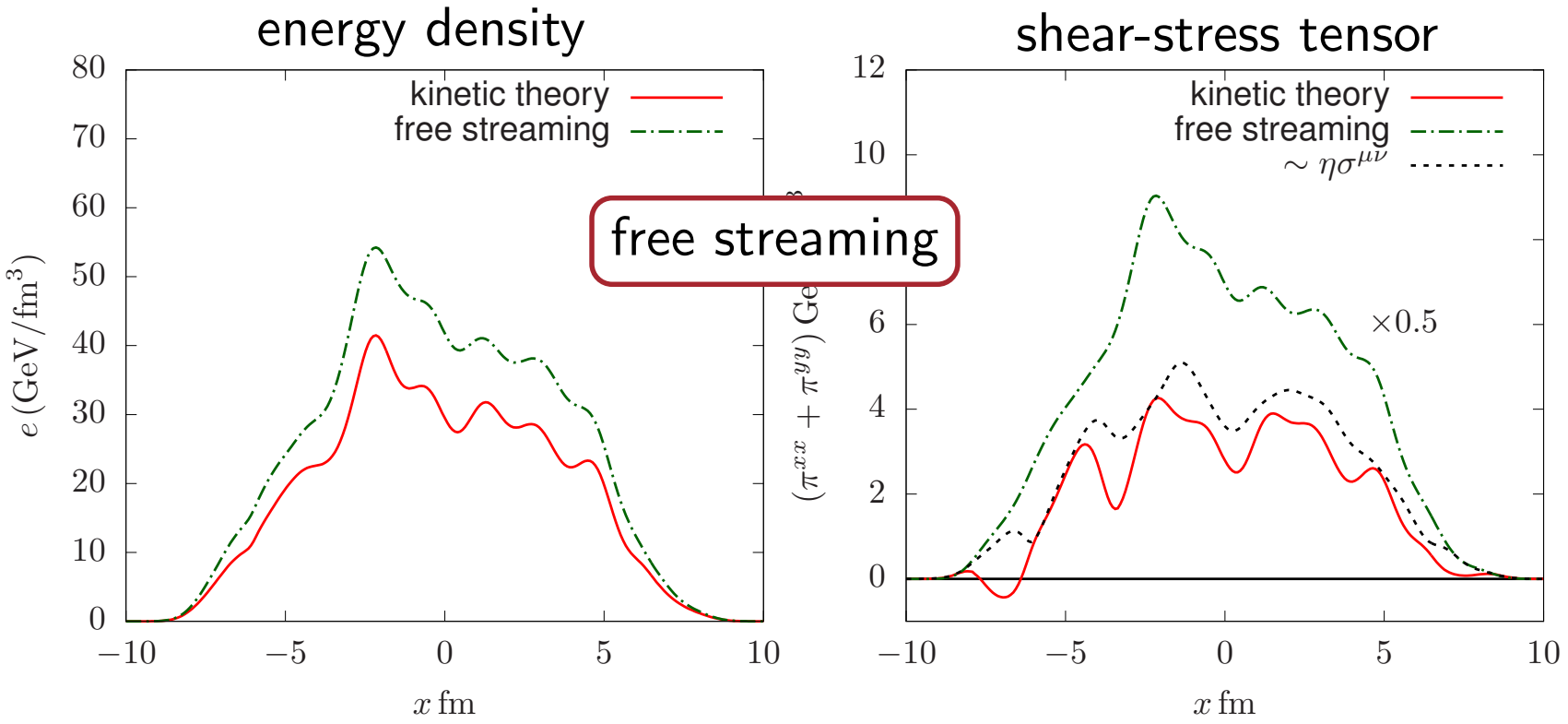
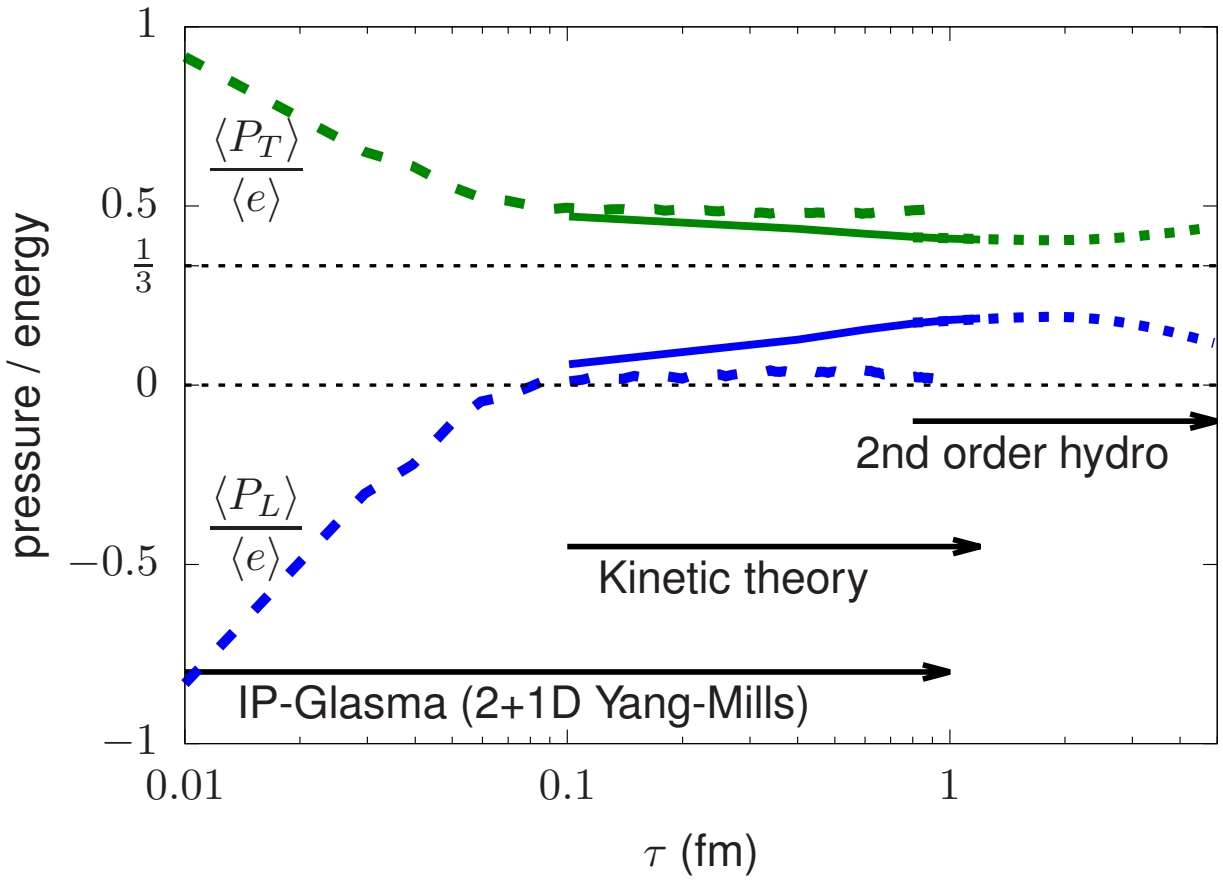


Note: MC-Glauber initial conditions. See backup for more results.

Summary

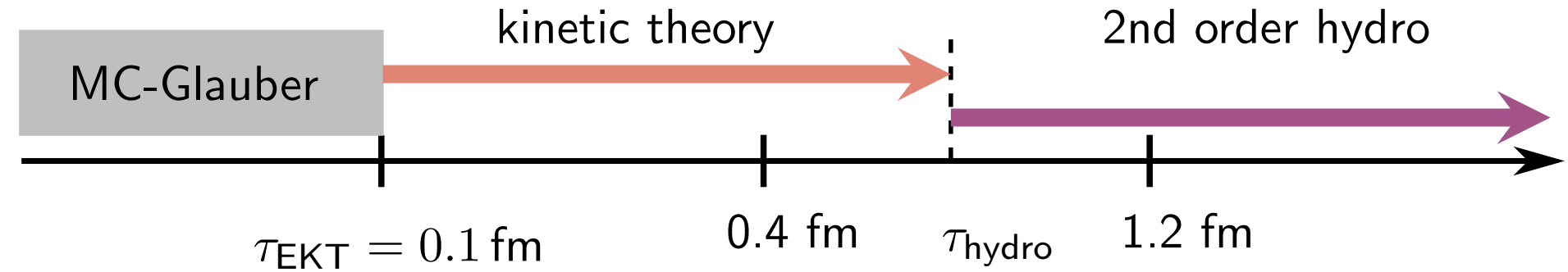
"Bottom-up" equilibration for realistic initial conditions:

- Background equilibration collapses to a scaling solution in $\tau T_{\text{ld.}}/(\eta/s)$
- Scalable kinetic response functions $G^{\mu\nu}(\tau T_{\text{ld.}}/(\eta/s), r/(\tau - \tau_0))$
- Approximate independence of crossover time τ_{hydro}
- *Consistent and smooth matching of initial stages!*

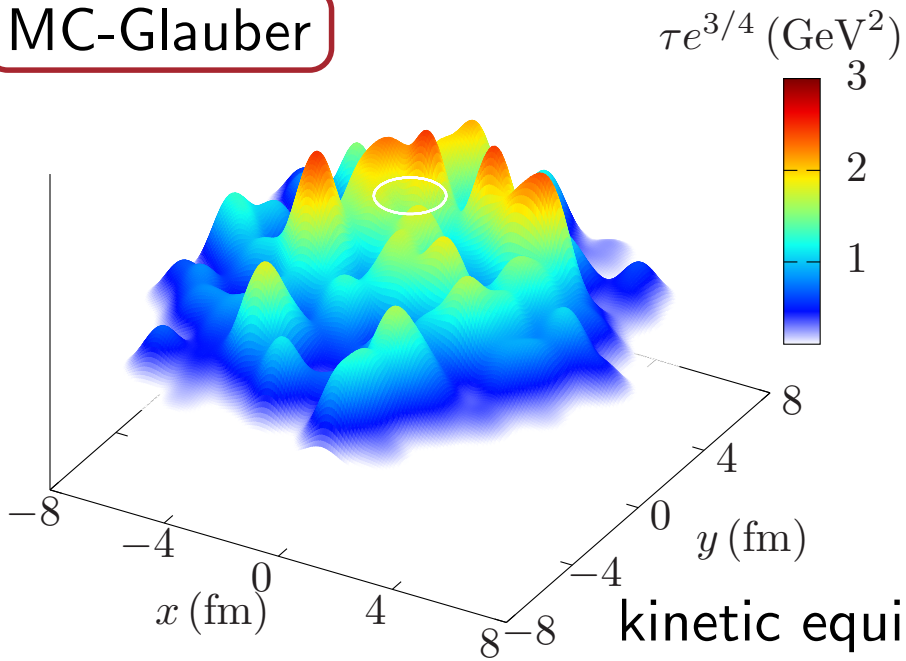


Backup

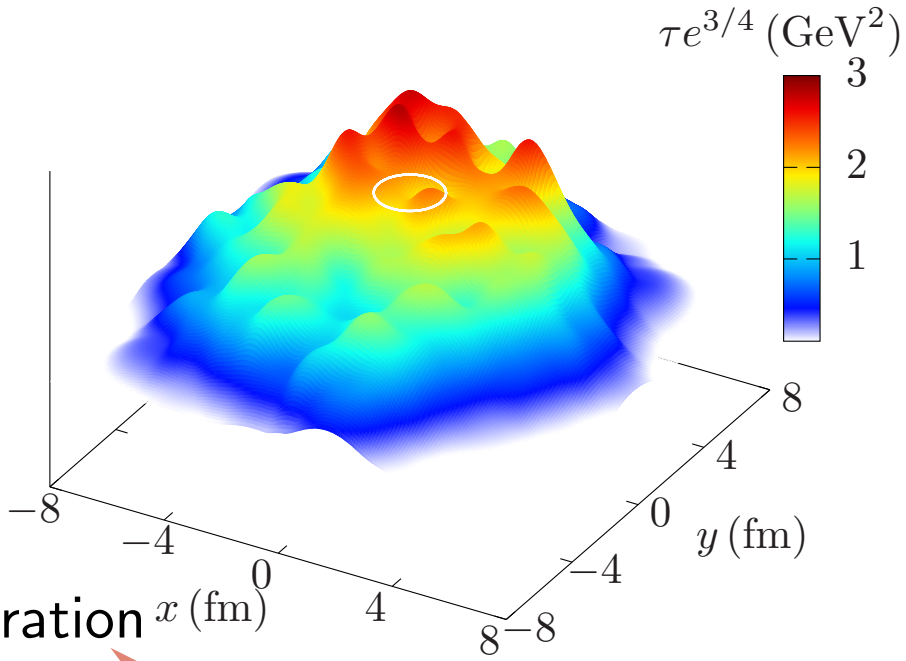
Evolution of transverse energy perturbations



MC-Glauber

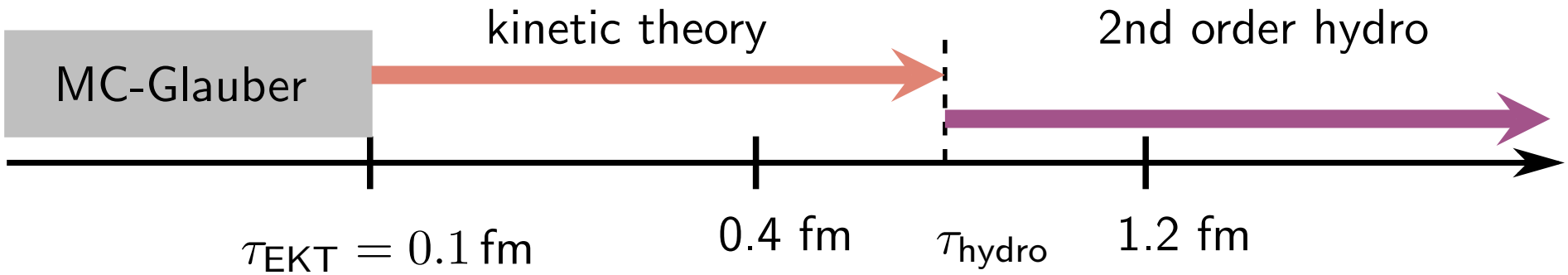


kinetic equilibration



$$e(\boldsymbol{x}, \tau_{\mathsf{EKT}}) \implies \underbrace{e(\tau) + \delta e(\tau, \boldsymbol{x})}_{\text{linearization in causal circle}} \implies \underbrace{T^{\mu\nu}(\boldsymbol{x}, \tau_{\mathsf{hydro}})}_{\text{full energy momentum tensor}}$$

Transversly averaged density and flow



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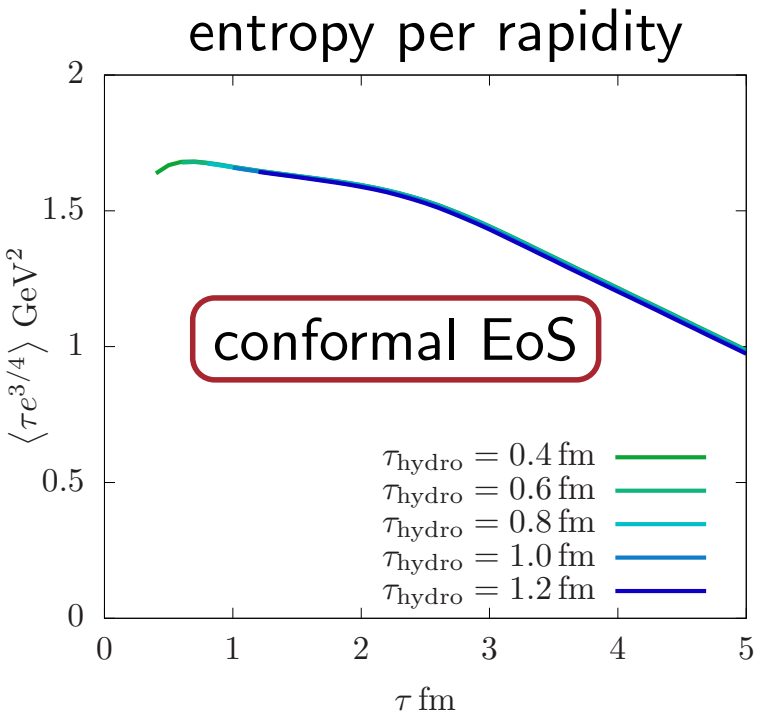

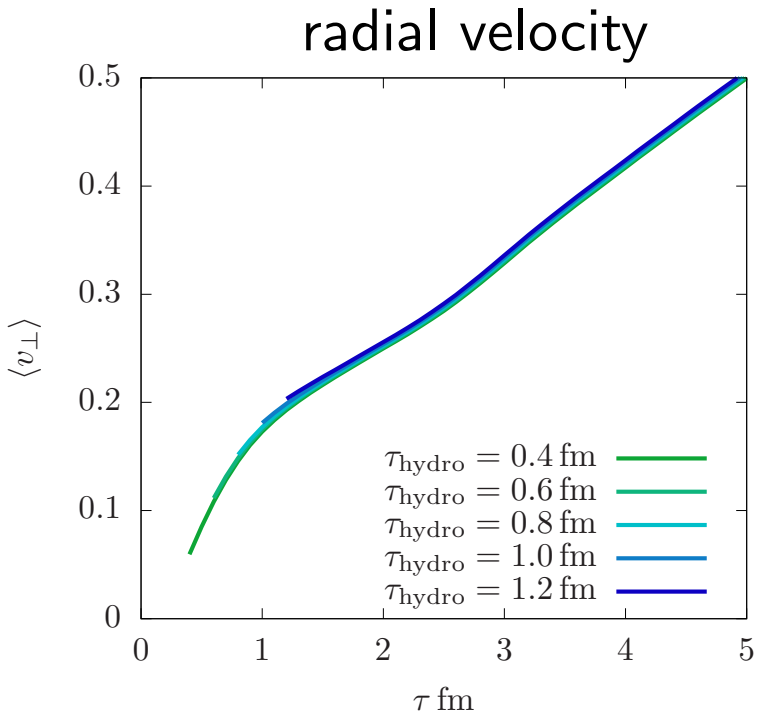

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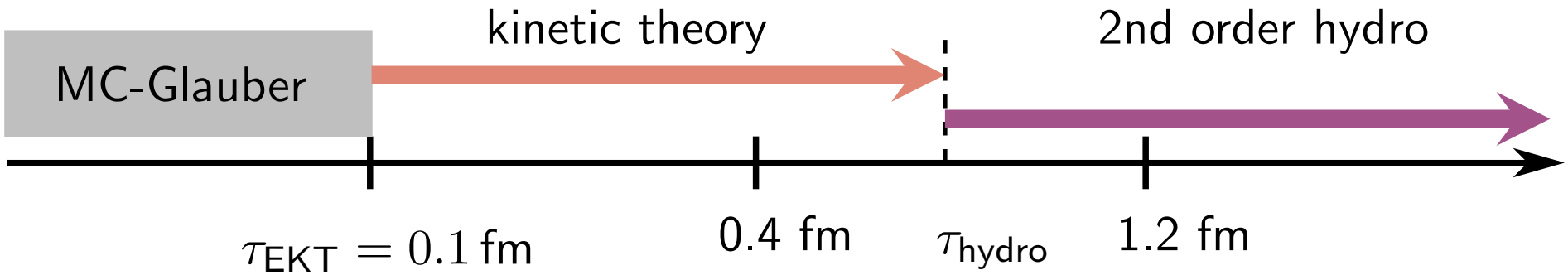

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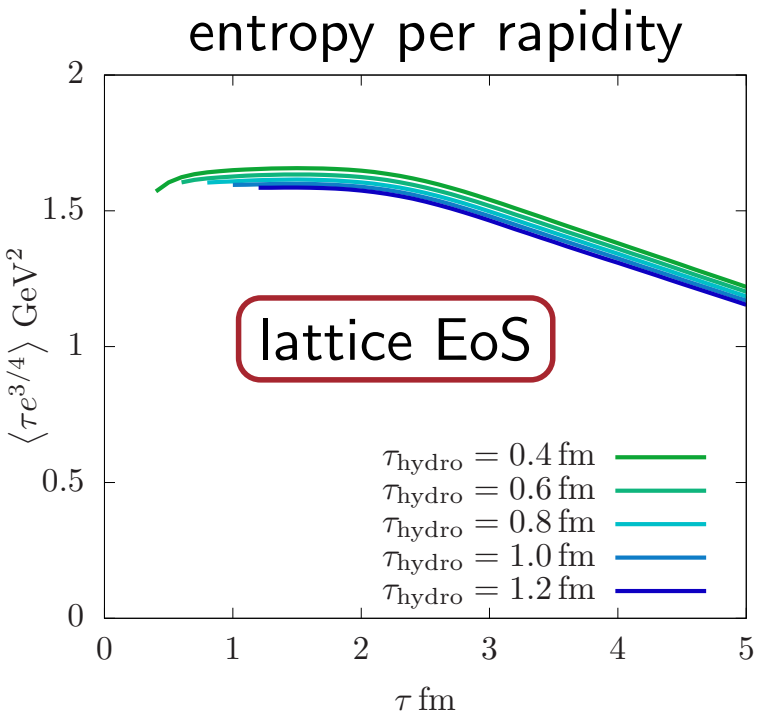
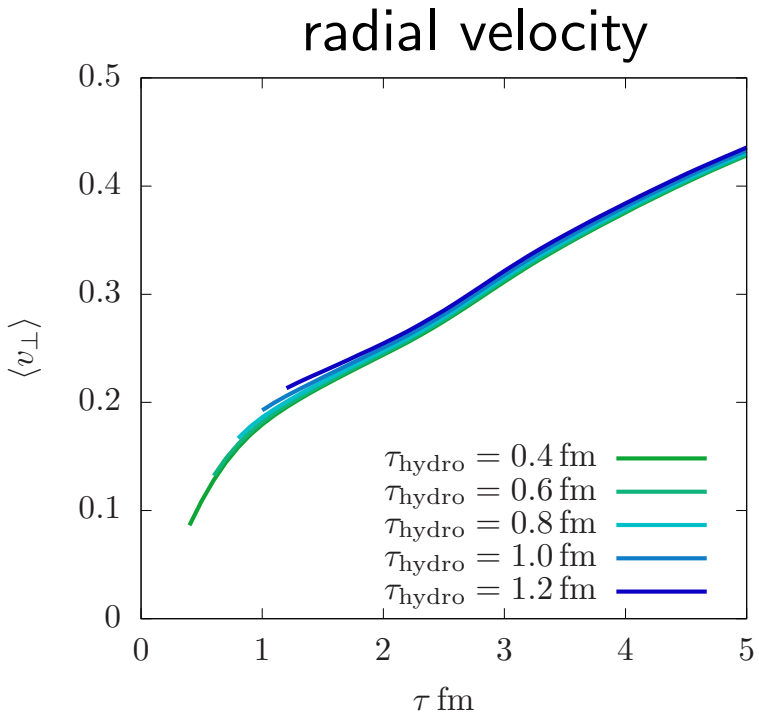
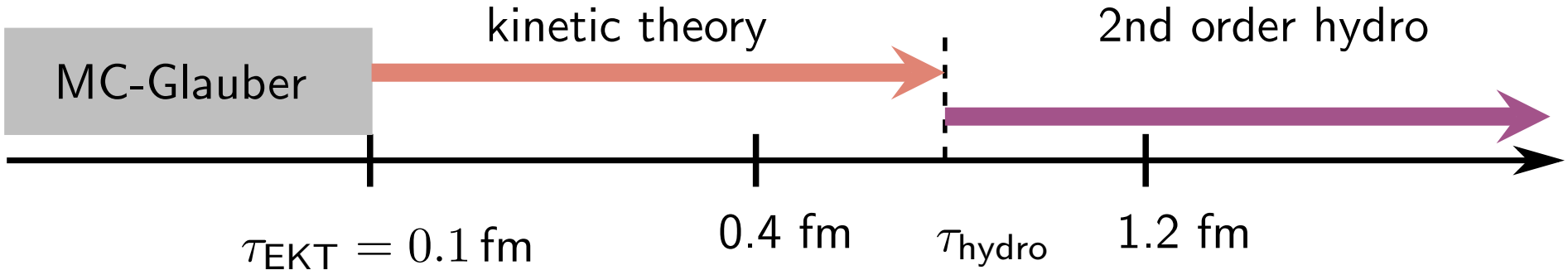




Hadronic observables: MC-Glauber



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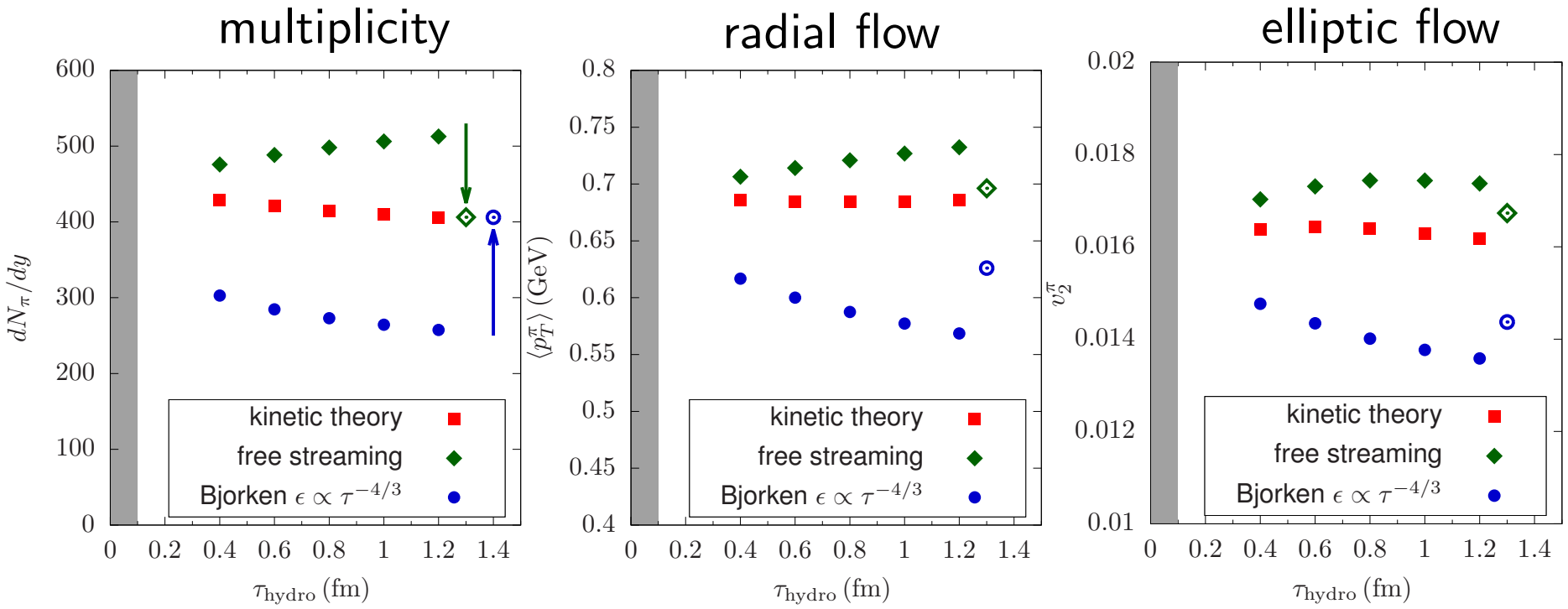
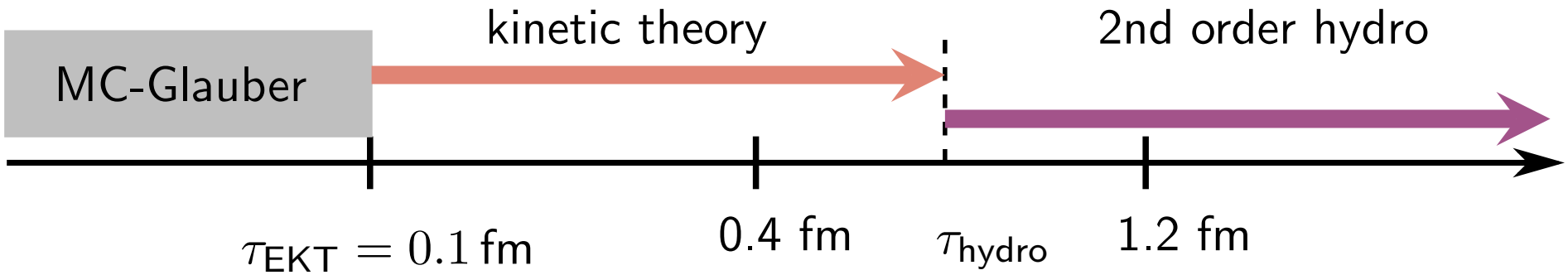
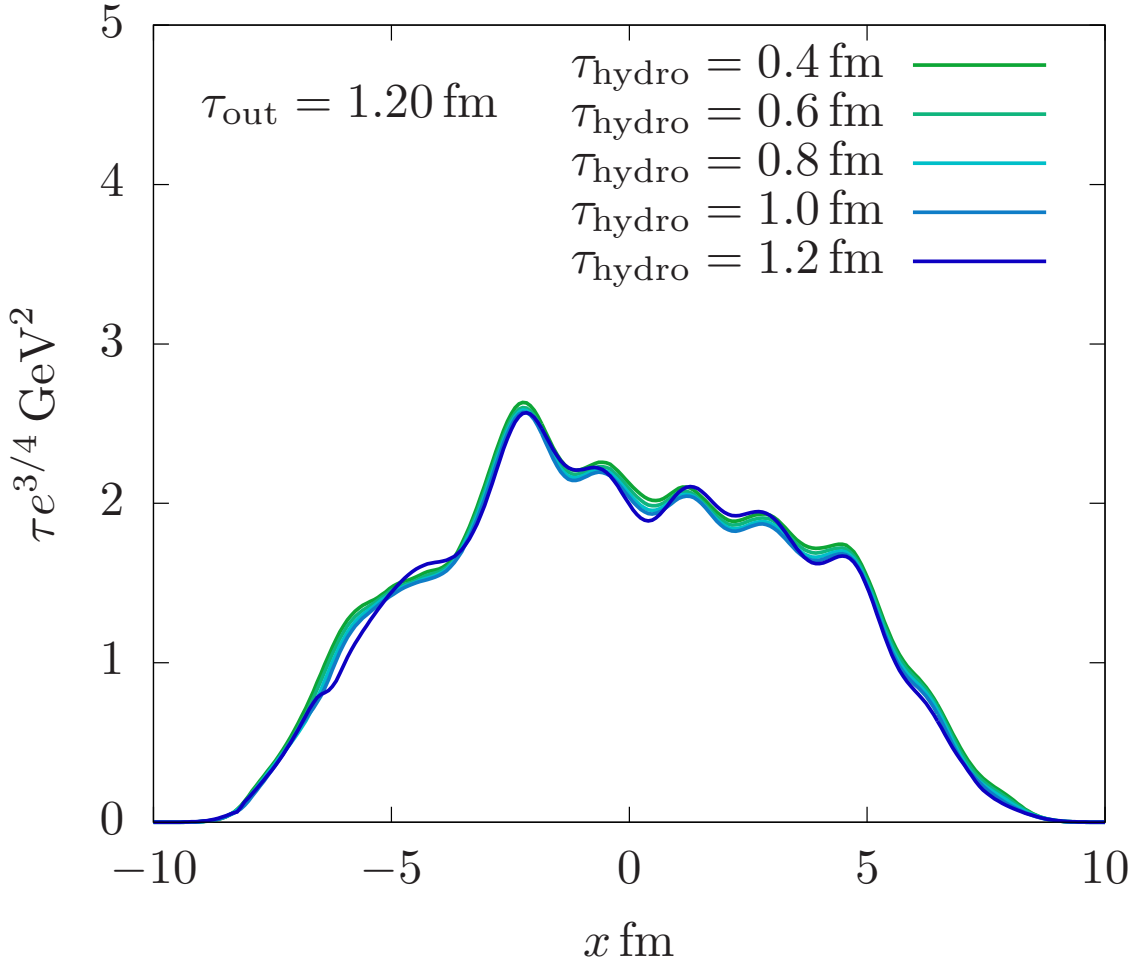


Approximate independence of τ_{hydro} *for kinetic pre-equilibrium evolution!*

Entropy profile evolution for MC-Glauber





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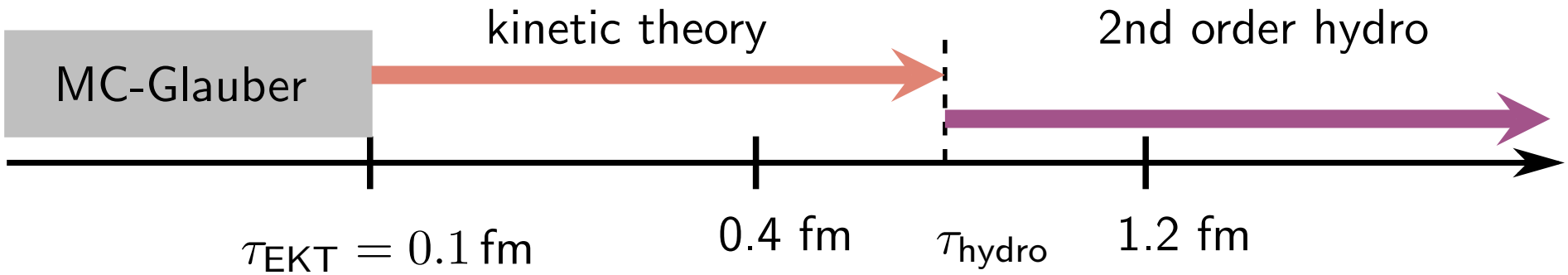

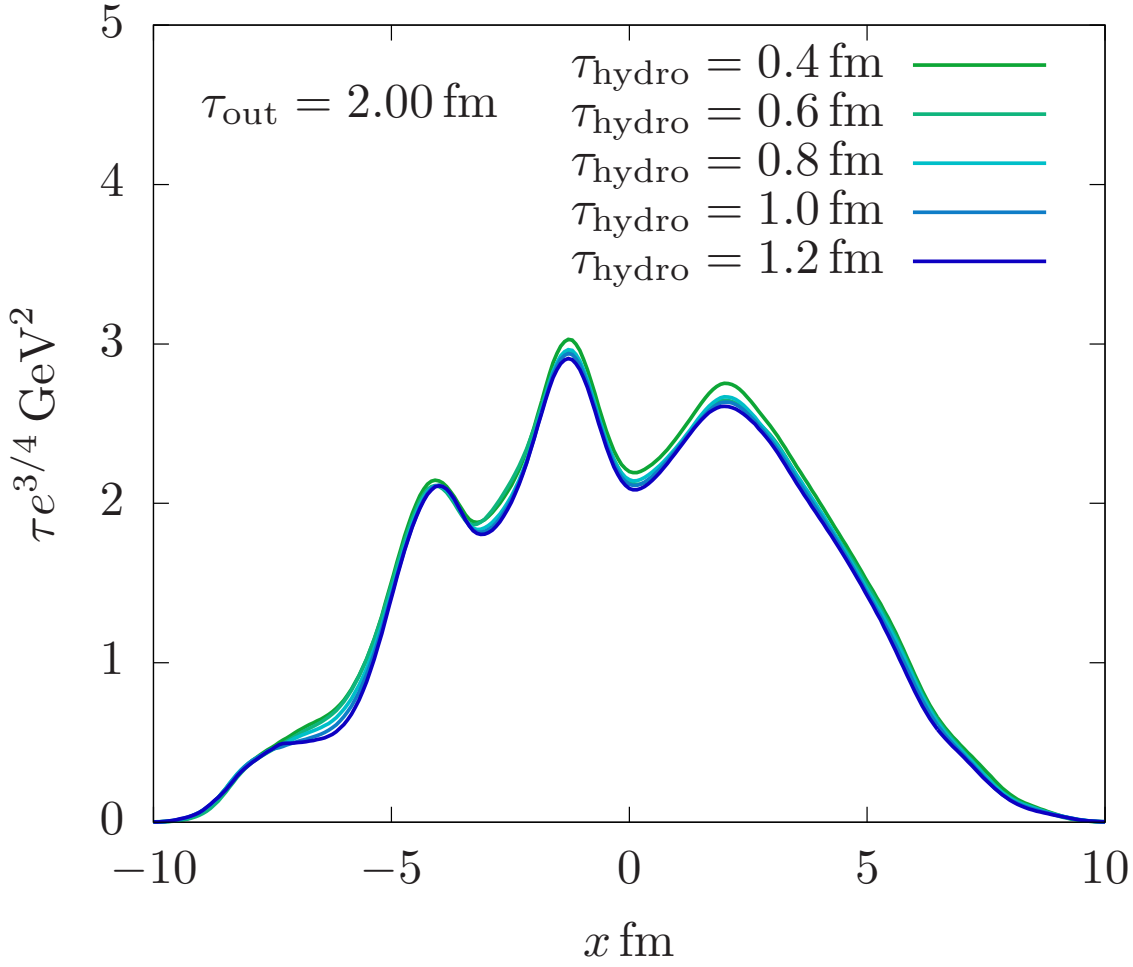

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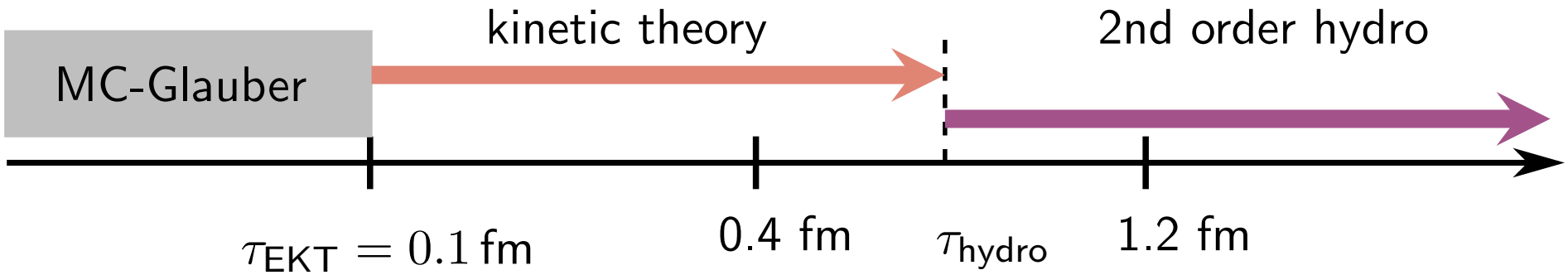
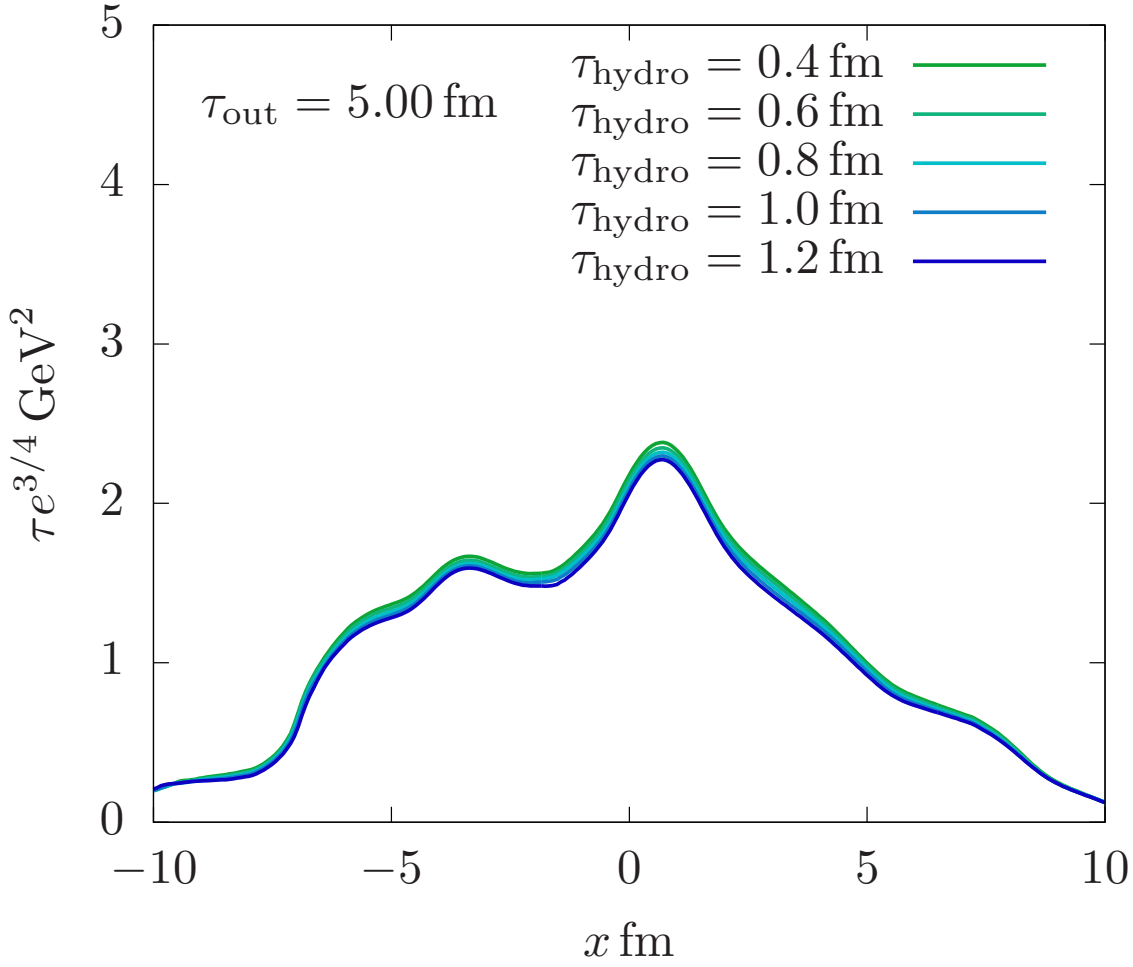
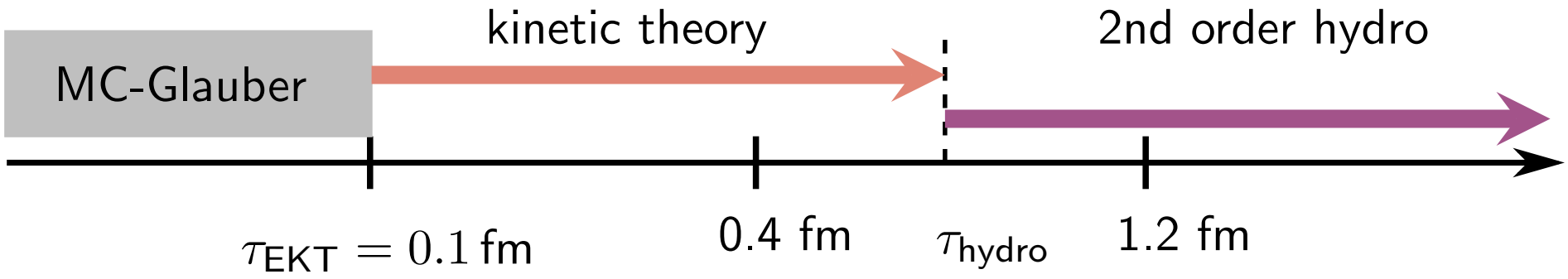
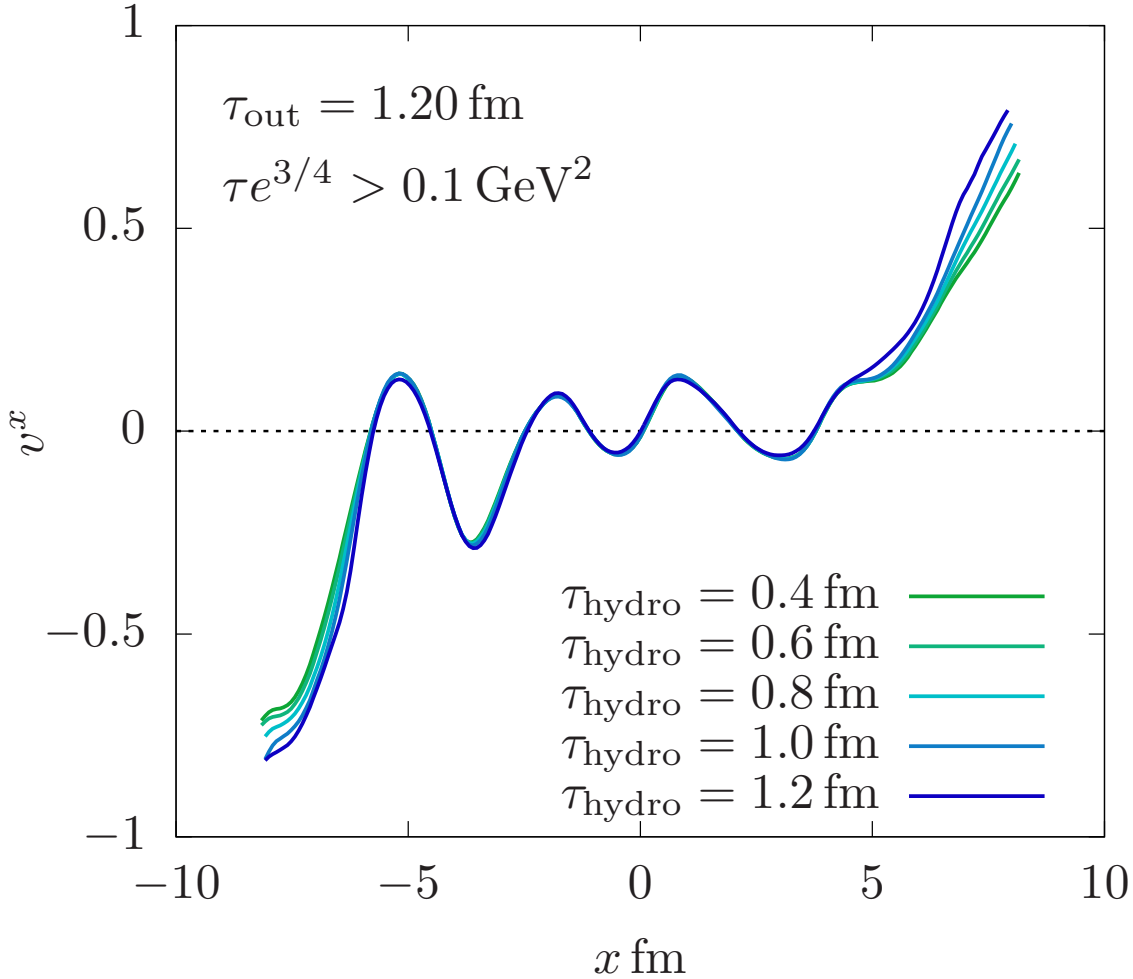


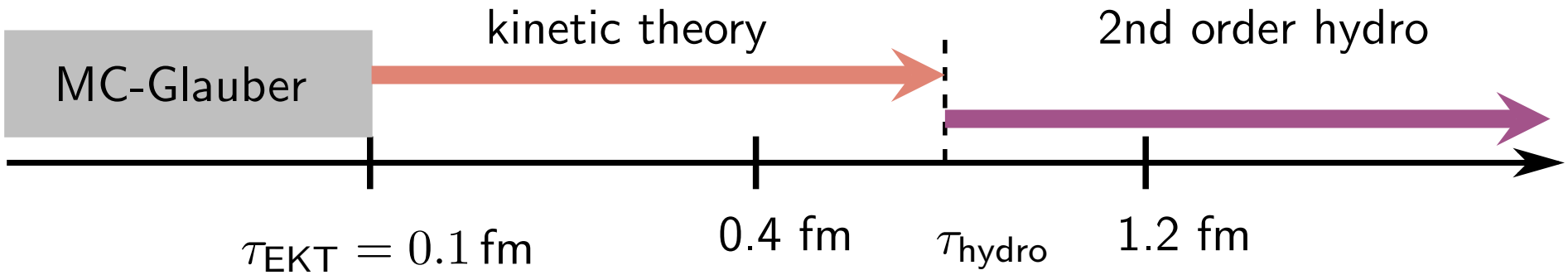


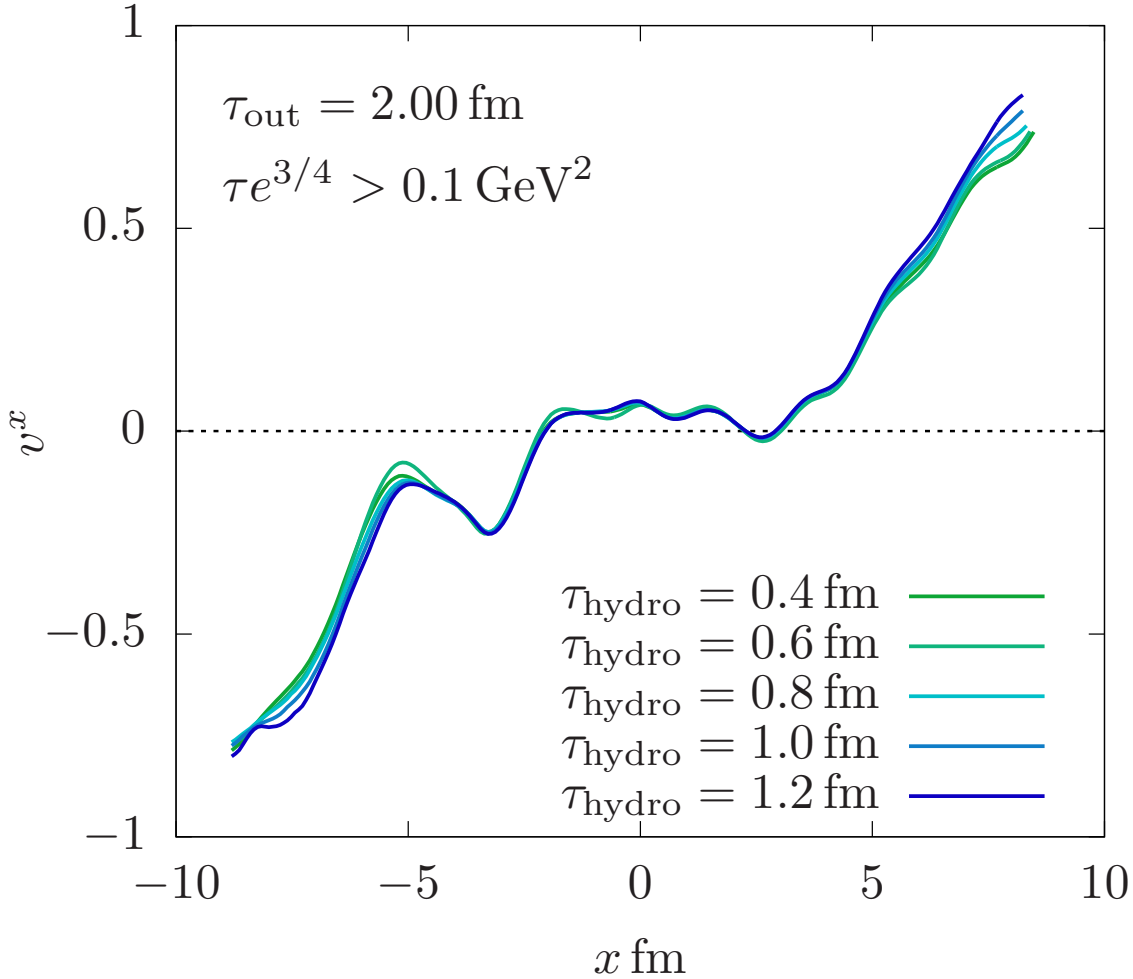
Transverse velocity evolution for MC-Glauber



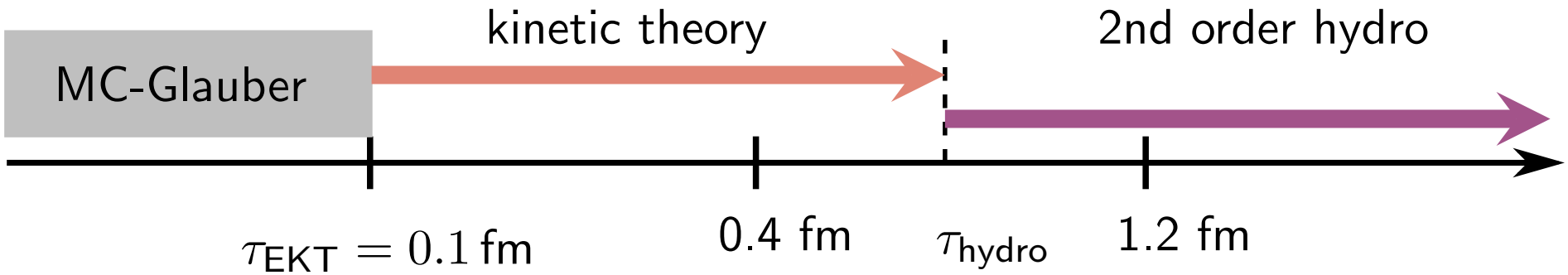


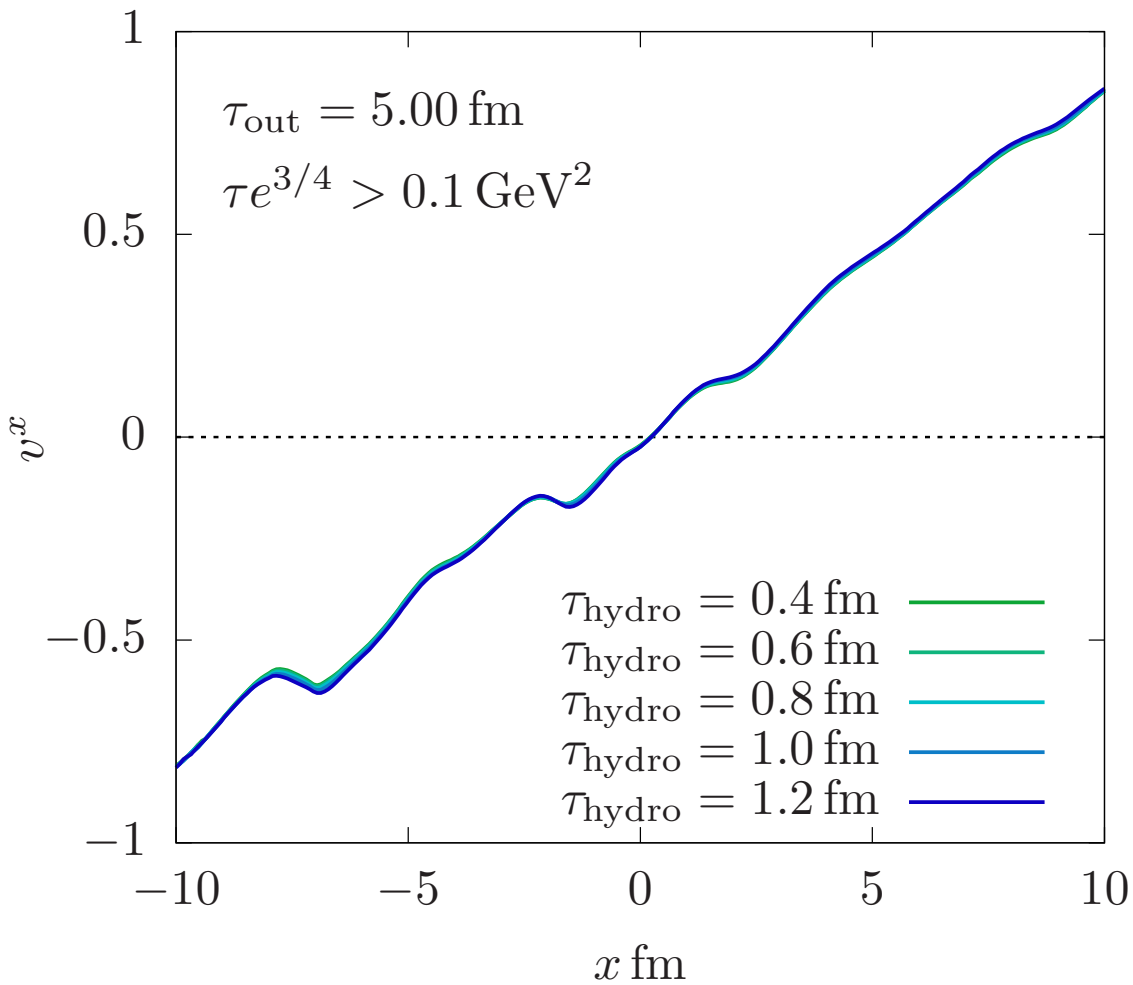
Transverse velocity evolution for MC-Glauber



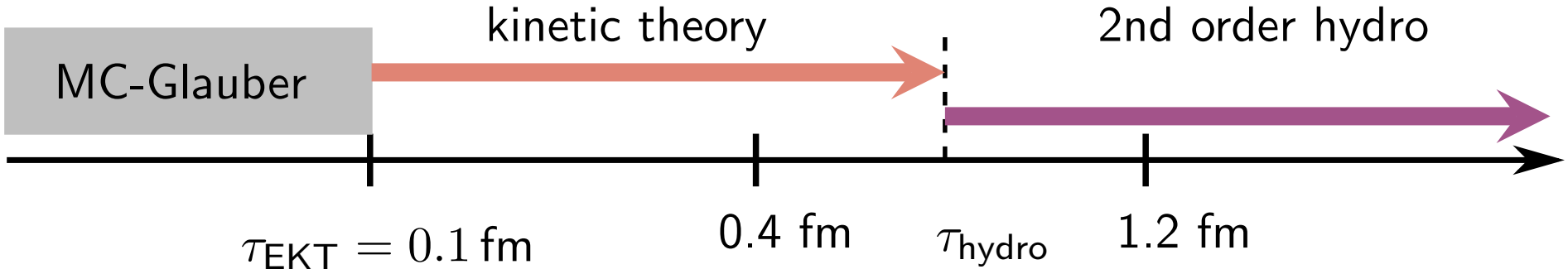


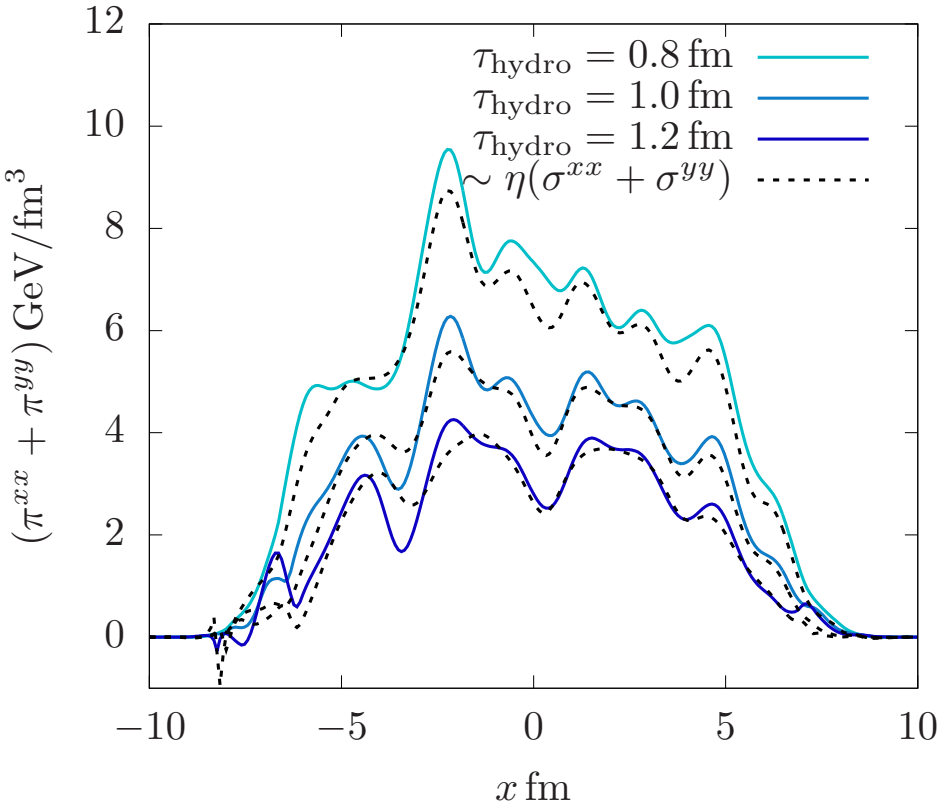
Transverse velocity evolution for MC-Glauber

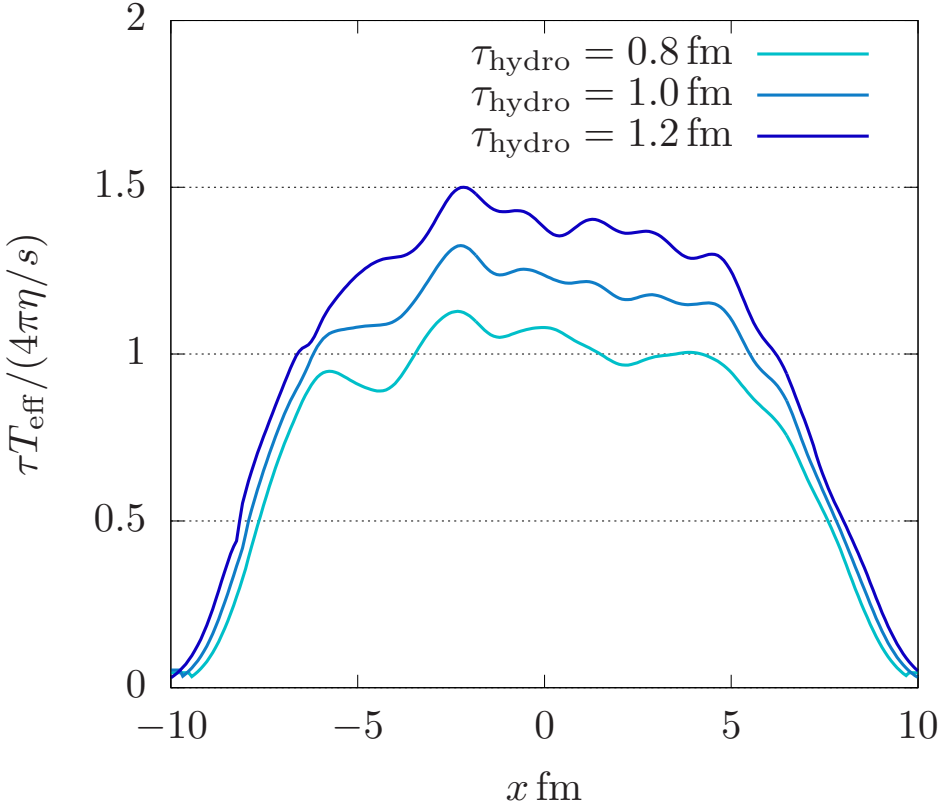




Shear-stress profile evolution for MC-Glauber







Parametrization of distribution function

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[3]

Universal-attractor — Berges, Boguslavski, Schlichting, Venugopalan (2014)[4]

Numerical realization — Kurkela and Zhu (2015)[5]

Initial anisotropic gluon distribution function

$$f_{\mathsf{BG}}(p_\perp, p_z) = \frac{A}{\lambda}\frac{p_0}{\tilde{p}} e^{-\frac{2}{3}\frac{\tilde{p}^2}{p_0^2}}, \quad \tilde{p} = \sqrt{p_\perp^2 + \xi^2 p_z^2}$$

with $\xi = 10$ and A and $p_0 = 1.8 Q_s$ adjusted to match lattice results [5, 10].

c.f. scaling solution for scalar theories [11]

$$f(p_\perp, p_z) = \frac{A}{\lambda}\frac{p}{p_T} e^{-\frac{p_z^2}{2\sigma^2}}$$

Scalar and vector perturbations for background

$f_{\mathsf{BG}}(p_\perp, p_z) \approx f_{\mathsf{BG}}(|\mathbf{p}_\perp|/Q_s)$

$$\delta f_{\boldsymbol{k}}^{(s)} = \frac{\delta Q_s(\boldsymbol{k})}{Q_s}\partial_{Q_s} f_{\mathsf{BG}}\left(\frac{|\mathbf{p}_\perp|}{Q_s}\right), \quad \delta f_{\boldsymbol{k}}^{(v),i} = \partial_{v_i} f_{\mathsf{BG}}\left(\frac{|\mathbf{p}_\perp - \mathbf{v}\mathbf{p}_\perp|}{Q_s}\right)\Bigg|_{\mathbf{v}=0}$$

Thermalization in kinetic theory

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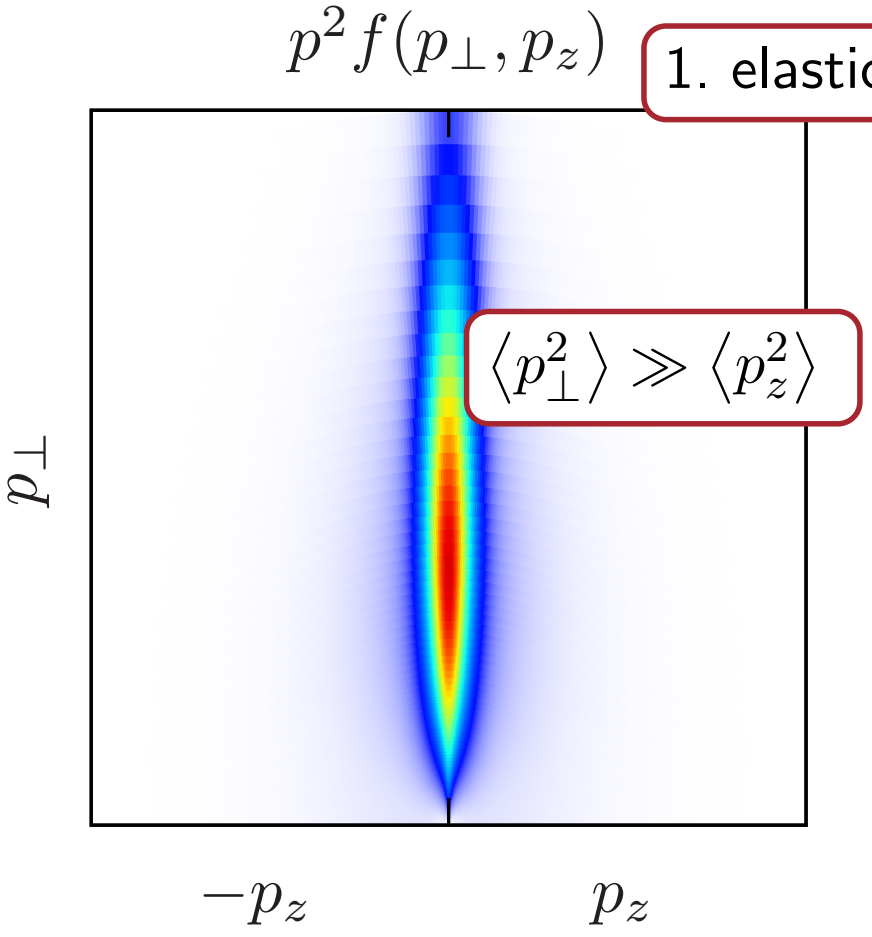
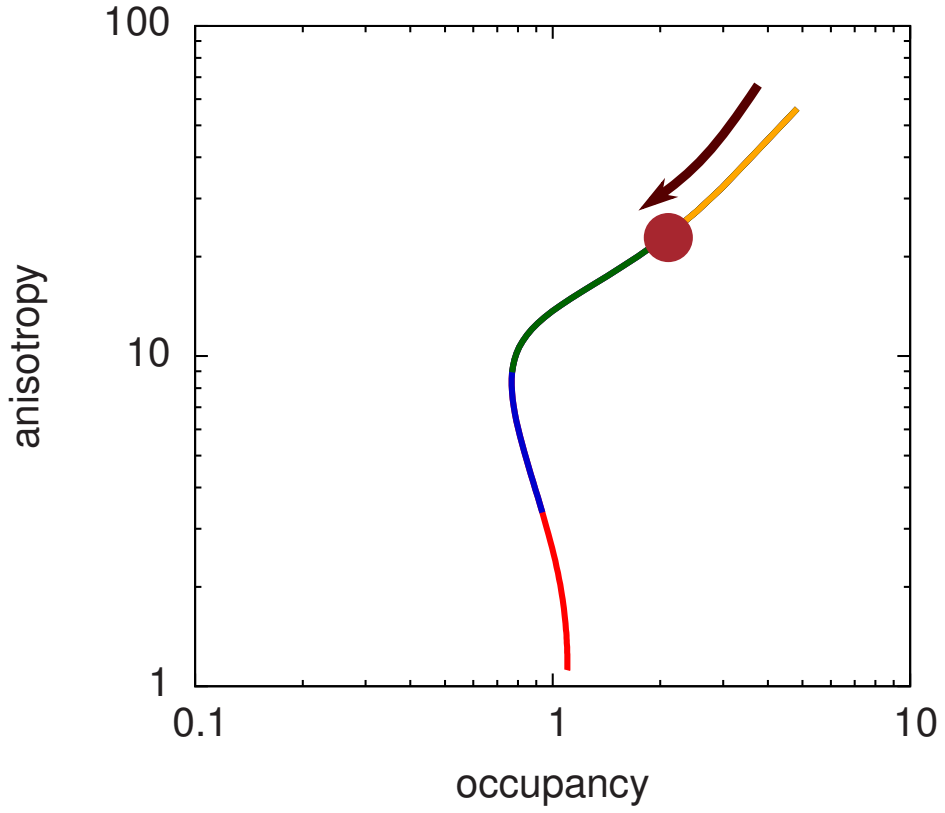


1. elastic $2 \leftrightarrow 2$ scatterings

$\langle p_\perp^2 \rangle \gg \langle p_z^2 \rangle$



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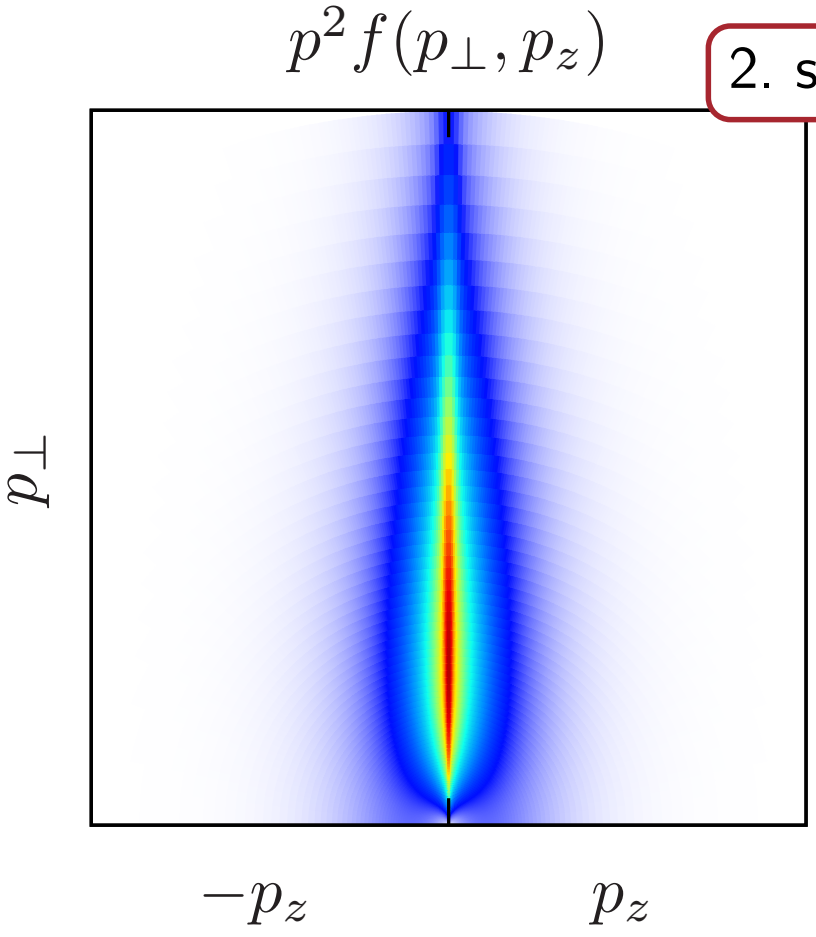
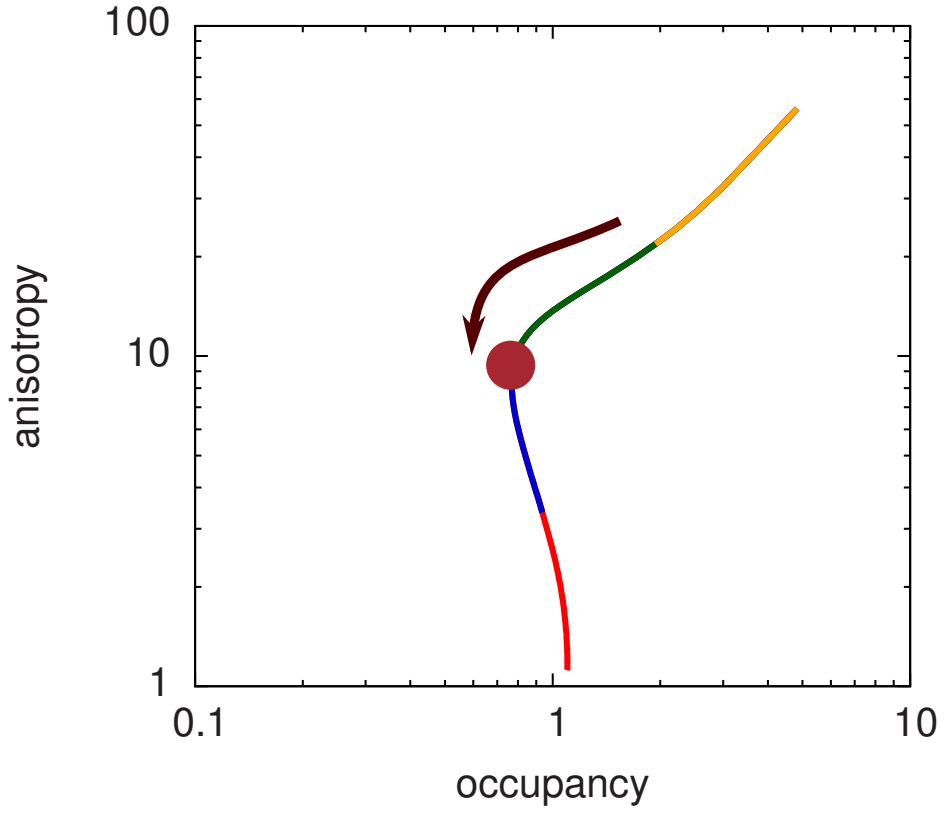


2. soft $1 \leftrightarrow 2$ splitting



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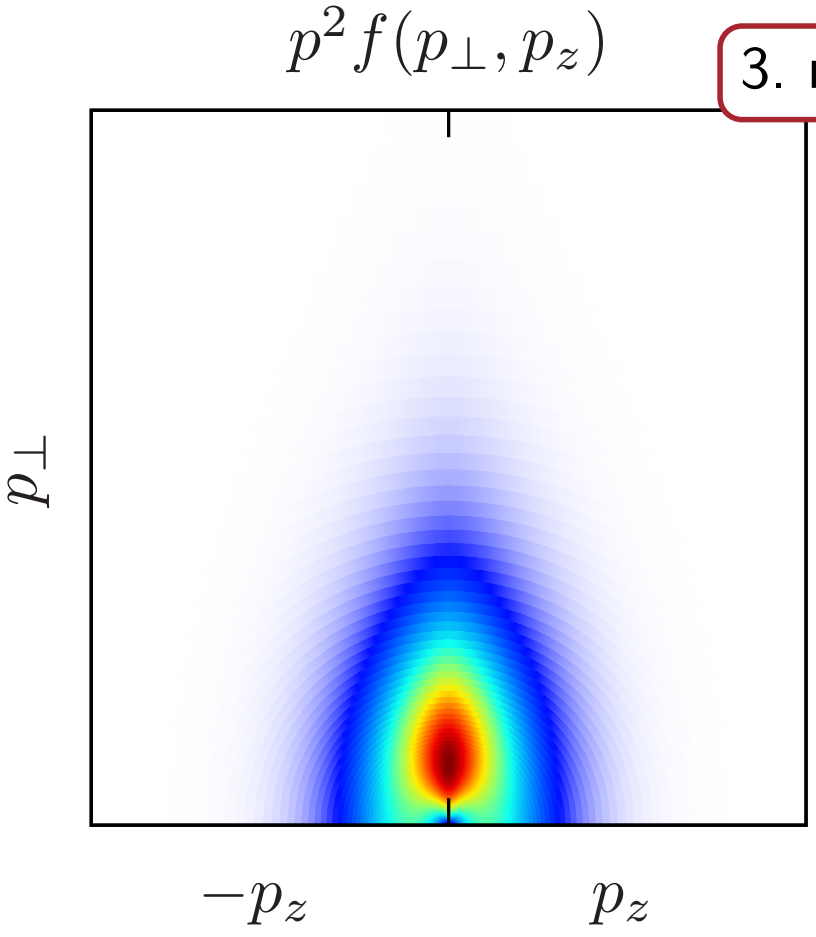
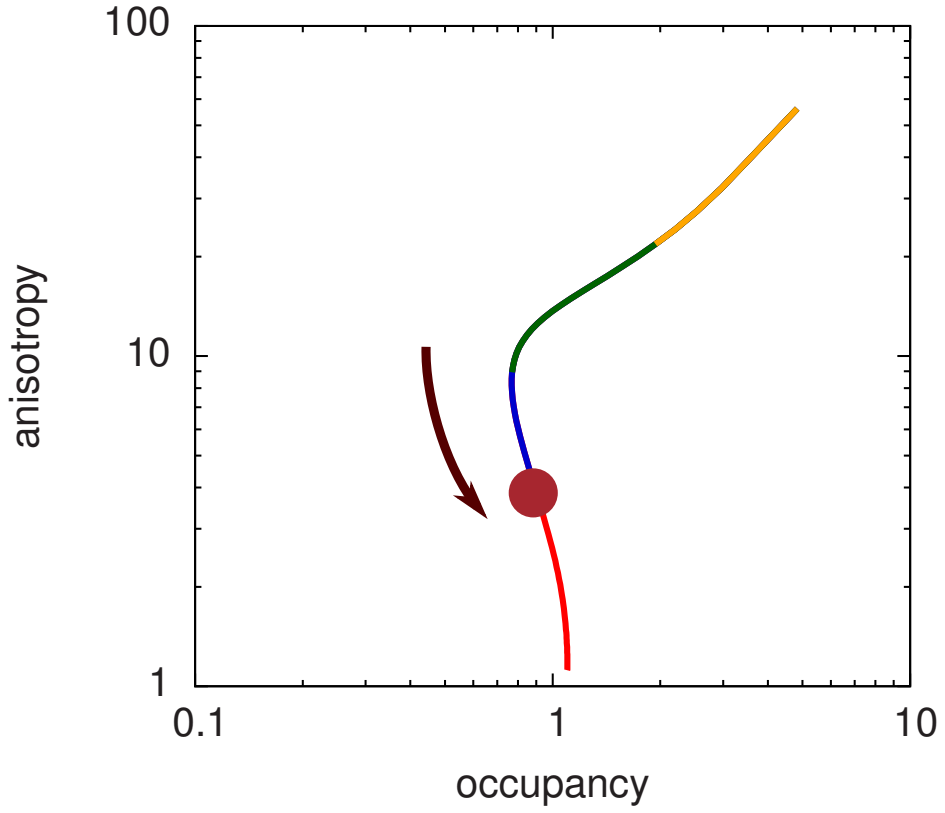


3. mini-jet quenching



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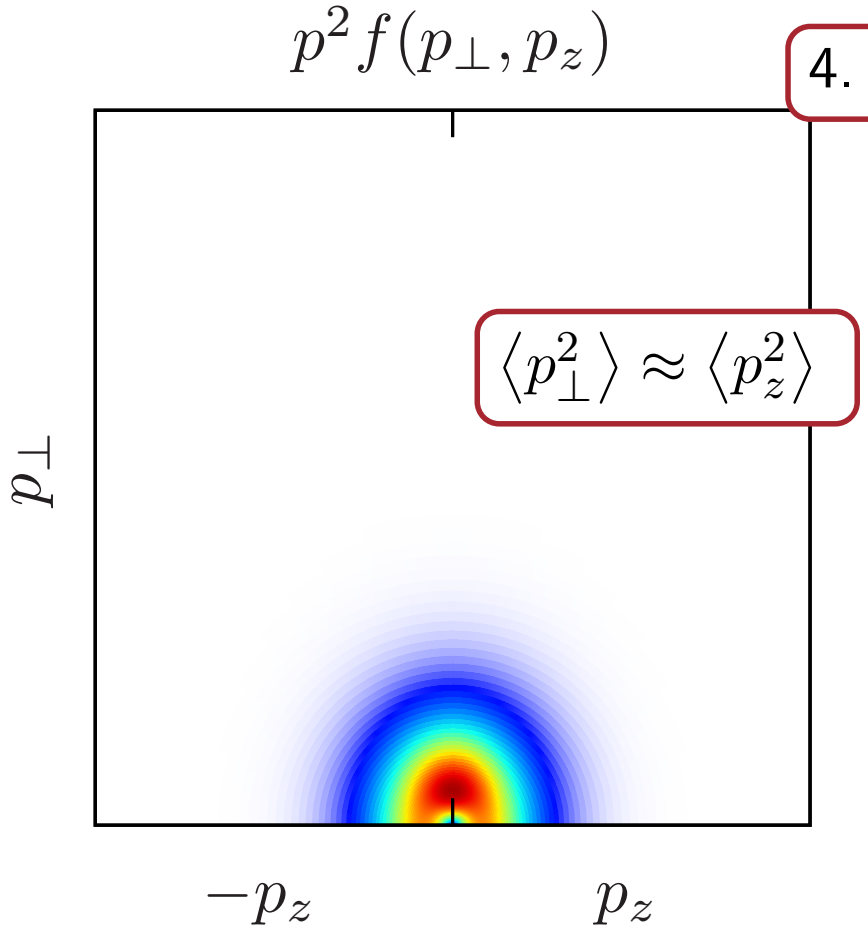
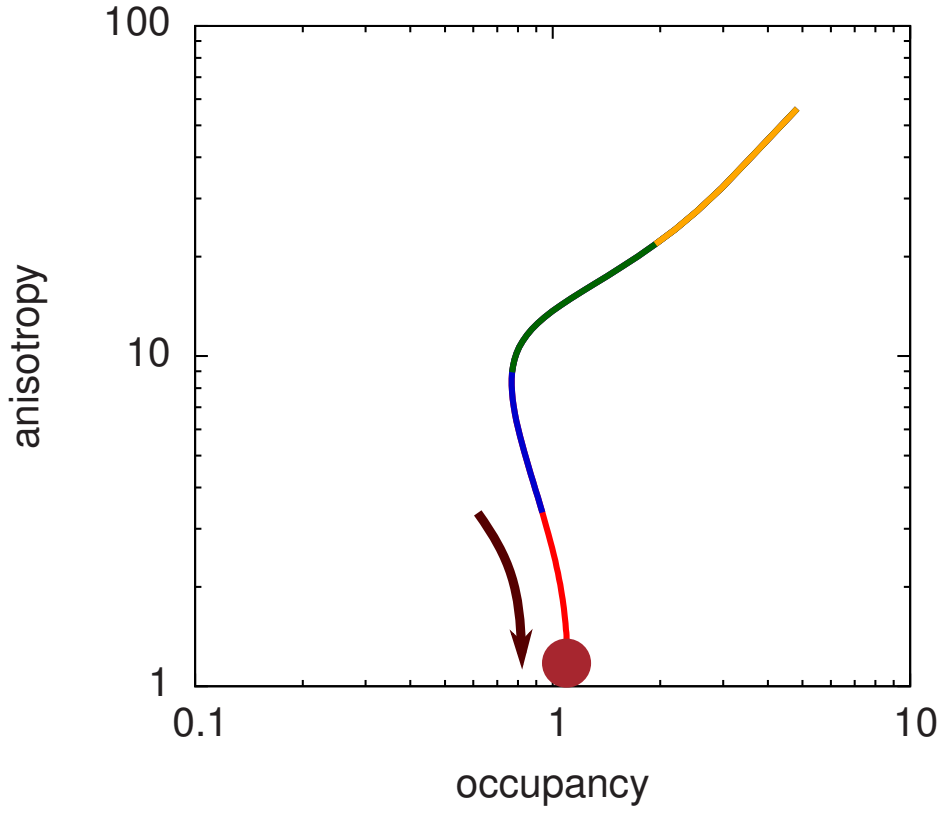


4. isotropization

$\langle p_\perp^2 \rangle \approx \langle p_z^2 \rangle$



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