Moving to NLO accuracy in dilute-dense processes in the CGC picture

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HELSINKI INSTITUTE OF PHYSICS

Outline

Trinity of dilute-dense CGC calculations

- Evolution equation (BK)
- Total DIS cross section see also Paatelainen tomorrow
- Single inclusive hadron production in pA-collisions
  Previous talk (Iancu)+ tomorrow (Ducloué)

+ essential question: doing the three consistently.

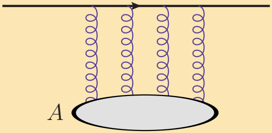
Recent progress in topics not covered here:

- NLO JIMWLK equation
- Exclusive processes
- Dihadron correlations

Outline of this talk

- CGC/eikonal description of dilute-dense process
- Status of NLO evolution equations
- Status of NLO DIS cross section

Eikonal scattering off target of glue



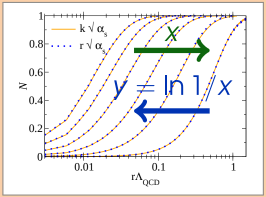
- Dilute probe through target color field
- At high energy interaction is eikonal

Eikonal scattering amplitude: Wilson line V

$$V = \mathbb{P}\exp\left\{-ig\int^{x^+}\mathrm{d}y^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_\mathrm{c})$$

- Initial gluon field in AA: same $V(\mathbf{x})$
- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

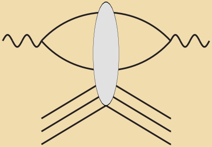


from color transparency to saturation

Dilute-dense process at LO

Physical picture at small x

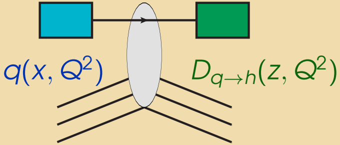
DIS



- $\gamma^* \to q\bar{q}$ dipole interacts with target color field
- Total cross section is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991
Fits to HERA data:
e.g. Golec-Biernat, Wüsthoff 1998

Forward hadrons



- q/g from probe: collinear pdf
- $|\text{amplitude}|^2 \sim$ dipole
- Indep. fragmentation

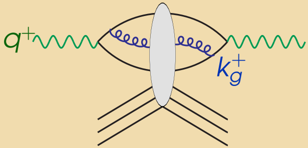
"Hybrid formalism";
Dumitru, Jalilian-Marian 2002

Both involve same dipole amplitude $\mathcal{N} = 1 - S$

Dilute-dense process at LL

Add one **soft** gluon: large logarithm of energy/x

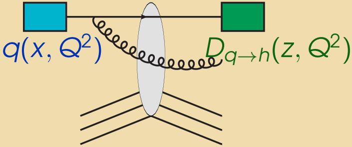
DIS



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Forward hadrons



- Soft gluon $k^+ \to 0$: same large log
- Collinear gluon $k_T \to 0$: DGLAP evolution of pdf, FF

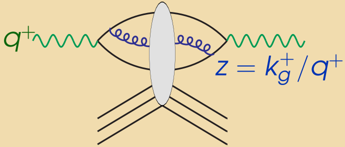
Dumitru et al 2005

Absorb large log into renormalization of target:
BK equation Balitsky 1995, Kovchegov 1999

Dilute-dense process at NLO

Add one gluon, but **not** necessarily soft

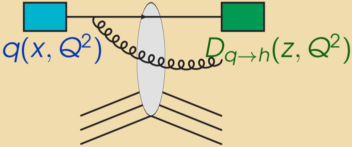
DIS



- DIS impact factor

  Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons



- NLO single inclusive

  Chirilli et al 2011

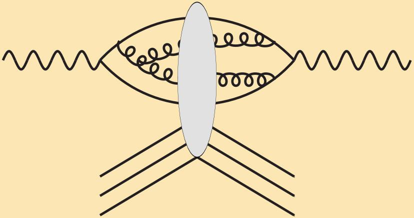
- Leading small-k^+ gluon already in BK-evolved target
- Need to **subtract** leading log from cross section:

$$\sigma_{NLO} = \int \mathrm{d}z \Big[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}} \Big] \qquad z = \frac{k_g^+}{P_{\text{tot}}^+}$$

NLO to NLL

NLO evolution equation:

- Consider NNLO DIS
- Extract leading soft logarithm
- Lengthy calculation:
  Balitsky & Chirilli 2007
- But additional resummations needed for practical phenomenology



(+ many diagrams at same order)

- $\alpha_s^2 \ln^2(1/x)$: two iterations of LO BK
- $\alpha_s^2 \ln 1/x$: NLO BK
- α_s^2: part of NNLO impact factor (not calculated)

Summary: power counting

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \underbrace{\mathcal{O}(\alpha_s \ln 1/x)}_{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \underbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}_{\text{NLL}}$$

- Current phenomenology LL
- Theory recently becoming understood at NLO & NLL
  talks Iancu, Paatelainen
- Moving to phenomenology, numerical implementations:
  - Fit to DIS data with (approx) NLL evolution (but not NLO) : Albacete 2015, Iancu et al 2015
  - Single inclusive hadrons at NLO (but not NLL) : Stasto et al 2013, Ducloué et al 2015 $\Longrightarrow$ talk Ducloué
  - Full NLL evolution (Not yet NLO) Mäntysaari 2015
  - NLO DIS cross section (Not yet NLL) Ducloué et al 2017

  } next

NLL evolution

The NLO BK equation

as derived by Balitsky and Chirilli, 2007

Equation: $y = \ln 1/x$-dependence from

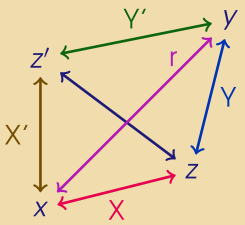
$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z - z')S(Y') - S(X)S(Y)]$$

Notations & approximations

- $S(x - y) \equiv (1/N_c) \langle \operatorname{Tr} V^\dagger(x) V(y) \rangle$
- $\otimes = \int d^2z$ or $\int d^2z\, d^2z'$
- Here large N_c & mean field:

$$\langle \operatorname{Tr} V^\dagger V \operatorname{Tr} V^\dagger V \rangle \to \langle \operatorname{Tr} V^\dagger V \rangle \langle \operatorname{Tr} V^\dagger V \rangle$$

Coordinates



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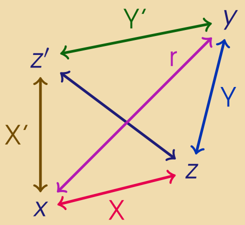

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4 (X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2 (X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

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- Leading order

Kernels

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- Leading order
- Running coupling (Terms with β function coefficient)

Kernels

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- Leading order
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- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)

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- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)
- Nonconformal double log $\Longrightarrow$ blows up for $r = 0$

Resummations

Following Iancu et al 2015

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

Resum:

- β-function terms in K_1 into running coupling: K_{Bal}
- Double transverse logarithms in K_1 into $K_{\mathrm{DLA}} \sim J_1(\ln r^2)/\ln r^2$.
- Single transverse logs in K_2 into $K_{\mathrm{STL}} \sim r^{\alpha_s A_1}$ with DGLAP anomalous dimension A_1
- Subtract double counting K_{sub}, include rest of NLO K_1^{fin}

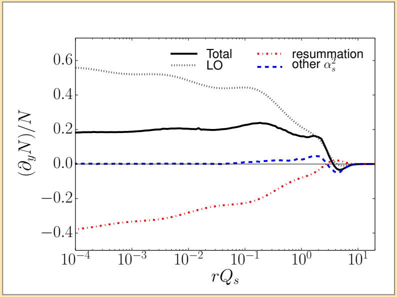
$\Longrightarrow$ Solve equation Mäntysaari, T.L. 2016 :

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \left[K_{DLA} K_{STL} K_{Bal} - K_{\mathrm{sub}} + K_1^{\mathrm{fin}} \right] \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

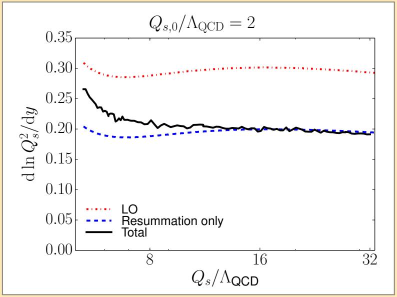
NLL evolution with resummation

Mäntysaari, T.L. 2016

Evolution speed vs r



Evolution speed of Q_s

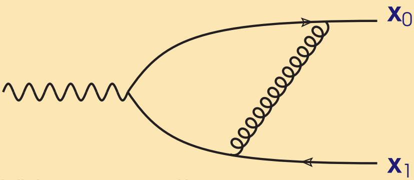


- Resummations essential to get stable results
  $\Longrightarrow$ good HERA fit with "resummation only" Iancu et al 2015
- Importance of non-resummed terms can be tuned
  (choice of 'constant under log' in resummation)
- Here simple rapidity-local resummation Iancu et al 2015
  Alternative: impose cumbersome but better defined kinematical constraint Beuf 2014, implementation Albacete 2015

DIS cross section at NLO

DIS at NLO: impact factor

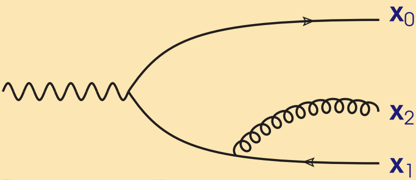
Beuf 2016, 2017, more details in talk of Paatelainen



Virtual corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

+ UV divergence in loop



Real corrections,
interaction with target

$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$$

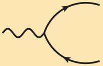
UV divergence in $\mathbf{x}_2$-integral

UV-divergence cancels because

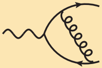
$$\mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_1) = \mathcal{N}(\mathbf{x}_0, \mathbf{x}_1)$$

DIS at NLO: subtraction of BK

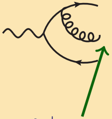
Evaluate cross section as $\sigma^{\mathrm{NLO}}_{L,T} = \sigma^{\mathrm{LO}}_{L,T} + \sigma^{\mathrm{dip}}_{L,T} + \sigma^{qg}_{L,T,\mathrm{sub.}}$



$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\mathrm{dip}} \sim \alpha_s C_\mathrm{F} \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2))$$

- LL

$k_g^+ \sim z_2$

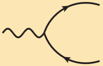
$$- \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

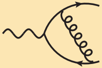
- LL: subtract leading log, already in BK-evolved $\mathcal{N}$

DIS at NLO: subtraction of BK

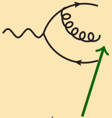
Evaluate cross section as $\sigma^{\text{NLO}}_{L,T} = \sigma^{\text{LO}}_{L,T} + \sigma^{\text{dip}}_{L,T} + \sigma^{qg}_{L,T,\text{sub.}}$



$$\Rightarrow \quad \sigma^{\text{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$$- * \Rightarrow \quad \sigma^{\text{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



- LL

$k_g^+ \sim z_2$

$$+ * \Rightarrow \quad \sigma^{qg}_{\text{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

- LL: subtract leading log, already in BK-evolved $\mathcal{N}$
- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!

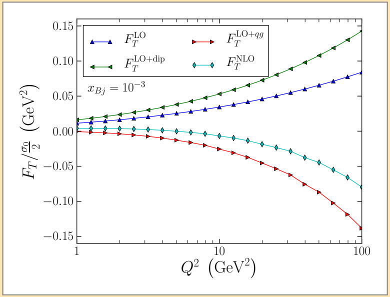
  ($X(z_2)$ =momentum fraction to which the target is evolved)

Numerical implementation

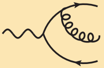
Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz_2}{z_2} \left[\mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2)) \right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable
    (like single inclusive)



$X(z_2) = x_{Bj}$



$k_g^+ \sim z_2$

Numerical implementation

Ducloué, Hänninen, T.L., Zhu 2017

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- Target fields at scale $X(z_2)$:
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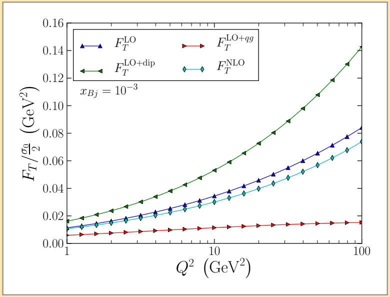
  - $X(z_2) = x_{Bj}/z_2$ OK



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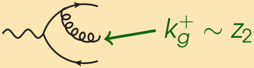
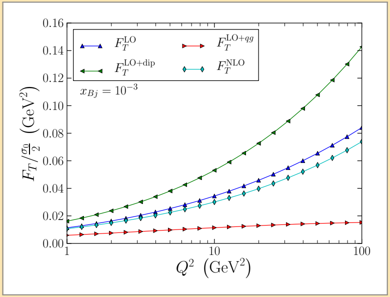


Numerical implementation

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\mathrm{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{\mathrm{d}z_2}{z_2} \left[\mathcal{K}_{L,T}^{\mathrm{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\mathrm{NLO}}(0, X(z_2)) \right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK
- Lower limit of z_2
  - $z_2 > \frac{x_{Bj}}{x_0}$ from target k^- (assuming $k_T{}^2 \sim Q^2$)
  - Strict k^+ factorization: $z_2 > \frac{x_{Bj}}{x_0}\frac{M_p^2}{Q^2}$ $\Longrightarrow$ would require kinematical constraint
  - For "dipole" term integrate to $z_2 = 0$



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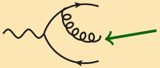
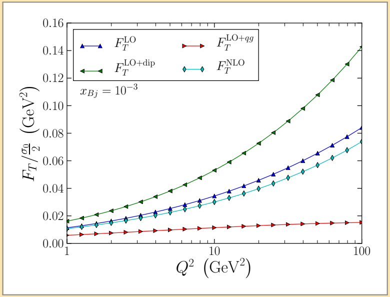


$k_g^+ \sim z_2$

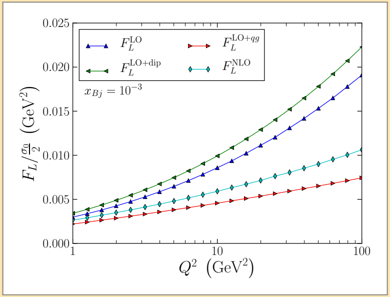
Numerical implementation: general features

- Major cancellation between different NLO terms



Numerical implementation: general features

- Major cancellation between different NLO terms (similar for F_L)



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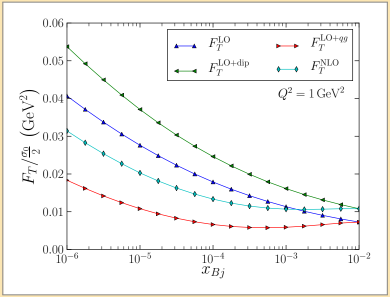
- qg-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect



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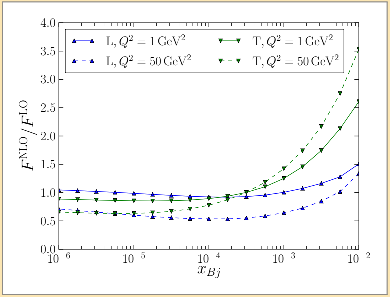
- qg-term explicitly zero at $x_{Bj} = x_0 \implies$ transient effect
- Running coupling (parent dipole)
  - Transient effect larger



NLO/LO ratio

Numerical implementation: general features

- Major cancellation between different NLO terms (similar for F_L)
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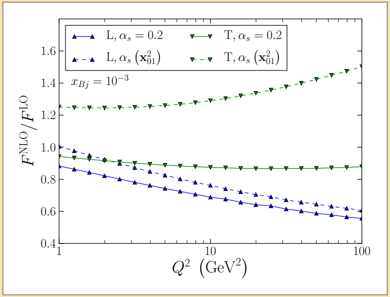
  - But Q^2-dependence stable



NLO/LO ratio

Relative NLO corrections of the magnitude one would expect

Instead off conclusions: to do

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \overbrace{\mathcal{O}(\alpha_s \ln 1/x)}^{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \overbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}^{\text{NLL}}$$

- Next: fit to HERA data with NLO impact factor
  (with LL or NLL evolution)
- Needs implementation (both DIS and single inclusive) :
  match NLL evolution with NLO cross section:
  - Evolution variable k^+ vs k^-
  - Kinematical constraint vs
    rapidity local resummation of double logs
  - Corresponding different subtractions from cross sections
- Needs loop calculation: quark masses
- Other:
  - Exclusive processes
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Thank you!