Probing saturation with dijets at LHC

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Motivation & Plan

Motivation

Study saturation effects in forward production of relatively hard jets (~25 GeV) which can be measured at LHC.

Plan

1. Dijet production in CGC and its limiting cases
   1. Leading power limit $\Rightarrow$ TMD factorization at small x
   2. Dilute limit $\Rightarrow$ High Energy (or k_T) Factorization (HEF)
2. Unified approach
3. Numerical results for pA and UPC
4. Towards NLO
   1. methods for multi-leg gauge invariant off-shell amplitudes
   2. on loop and real corrections
5. Summary

Forward dijets in pA collisions within CGC

Example: $qA \to qg$ channel

[C. Marquet, Nucl. Phys. A 796 (2007) 41]

$$\frac{d\sigma_{qA\to 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2}\, e^{-i\vec{p}_{T1}\cdot\left(\vec{x}_T-\vec{x}'_T\right)} e^{-i\vec{p}_{T2}\cdot\left(\vec{y}_T-\vec{y}'_T\right)} \psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$

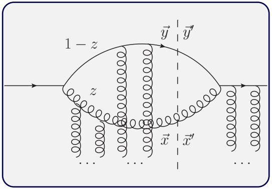
$$\Big\{ S_{x_g}^{(6)}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S_{x_g}^{(4)}\left(\vec{y}_T,\vec{x}_T,(1-z)\,\vec{y}'_T+z\vec{x}'_T\right) - S_{x_g}^{(4)}\left((1-z)\,\vec{y}_T+z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)$$

$$-S_{x_g}^{(2)}\left((1-z)\,\vec{y}_T+z\vec{x}_T,(1-z)\,\vec{y}'_T+z\vec{x}'_T\right)\Big\}$$

$\psi_z\left(\vec{x}_T\right)$ – quark wave function

$S_{x_g}^{(i)}$ – correlators of Wilson line operators

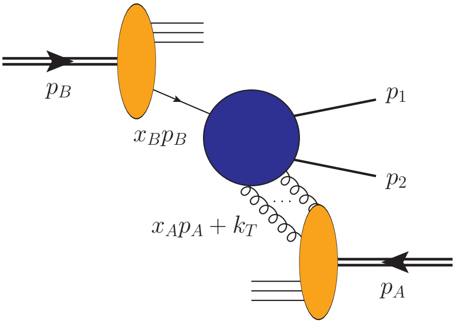
$$S_{x_g}^{(2)}\left(\vec{y}_T,\vec{x}_T\right) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U\left(\vec{y}_T\right)U^\dagger\left(\vec{y}'_T\right)\right]\right\rangle_{x_g}$$



$$S_{x_g}^{(4)}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right) = \frac{1}{2C_F N_c}\left\langle \mathrm{Tr}\left[U\left(\vec{z}_T\right)U^\dagger\left(\vec{y}_T\right)\right]\mathrm{Tr}\left[U\left(\vec{y}_T\right)U^\dagger\left(\vec{x}_T\right)\right]\right\rangle_{x_g} - S_{x_g}^{(2)}\left(\vec{z}_T,\vec{x}_T\right) \text{ etc.}$$

where $U(\vec{x}_T) = U(-\infty,+\infty;\vec{x}_T)$ and $\langle\ldots\rangle_{x_g}$ denotes the average over color sources.

Kinematics in the hybrid approach



forward dijets with transverse momentum imbalance:

$$\left|\vec{p}_{T1} + \vec{p}_{T2}\right| = \left|\vec{k}_T\right| = k_T$$

asymmetric kinematics:

$$x_B \gg x_A$$

Three-scale problem

1. hard scale P_T (of the order of the average transverse momentum of jets)
2. transverse momentum imbalance k_T
3. saturation scale $\Lambda_{\mathrm{QCD}} \ll Q_s$

Relation to (generalized) TMD factorization (1)

Leading power limit of the CGC expression

[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

- Take the limit $k_T \sim Q_s \ll P_T$ of CGC expressions (back-to-back dijets)
- Replace the color averages by the hadronic ME: $\langle \ldots \rangle_{x_g} \to \langle p \,|\ldots|\, p\rangle \,/\, \langle p|p\rangle$

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$$\frac{d\sigma_{AB\to 2j}}{dy_1 d^2p_{T1} dy_2 d^2p_{T1}} \sim \sum_{a,c,d} f_{a/B}\left(x_B, P_T^2\right) \sum_i \mathcal{F}_{ag}^{(i)}\left(x_A, k_T^2\right) H_{ag\to cd}^{(i)}$$

$H^{(i)}$ – hard on-shell factors

$f_{b/B}\left(x_B, \mu^2\right)$ – collinear PDF

$\mathcal{F}_{ag}^{(i)}$ – TMD Gluon Distributions with the operator definitions of the type

$$\int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_A p_A^- \xi^+ - i\vec{k}_T\cdot\vec{\xi}_T} \langle p_A | \operatorname{Tr}\left\{F^{+i}(\xi)\,[\xi, 0]_{C_1} F^{+i}(0)\,[0, \xi]_{C_2}\right\} |p_A\rangle$$

where the Wilson lines $[\xi, 0]_{C_i}$ depend on the particular diagram it accompanies. The operator position ξ is off the light-cone.

Relation to (generalized) TMD factorization (2)

TMD Gluon distributions

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]

$$\mathcal{F}_{qg}^{(1)} \sim \langle p_A | \, \mathrm{Tr}\{F^{+i}(\xi)\, \mathcal{U}^{[-]\dagger} F^{+i}(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle \,,$$

$$\mathcal{F}_{qg}^{(2)} \sim \langle p_A | \, \mathrm{Tr}\{F^{+i}(\xi)\, \frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c} \mathcal{U}^{[+]\dagger} F^{+i}(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle p_A | \, \mathrm{Tr}\{F^{+i}(\xi)\, \frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c} \mathcal{U}^{[-]\dagger} F^{+i}(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(2)} \sim \frac{1}{N_c} \langle p_A | \, \mathrm{Tr}\left\{F^{+i}(\xi)\, \mathcal{U}^{[\Box]\dagger}\right\} \mathrm{Tr}\left\{F^{+i}(0)\, \mathcal{U}^{[\Box]}\right\} | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle p_A | \, \mathrm{Tr}\left\{F^{+i}(\xi)\, \mathcal{U}^{[+]\dagger} F^{+i}(0)\, \mathcal{U}^{[+]}\right\} | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(4)} \sim \langle p_A | \, \mathrm{Tr}\left\{F^{+i}(\xi)\, \mathcal{U}^{[-]\dagger} F^{+i}(0)\, \mathcal{U}^{[-]}\right\} | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle p_A | \, \mathrm{Tr}\left\{F^{+i}(\xi)\, \mathcal{U}^{[\Box]\dagger} \mathcal{U}^{[+]\dagger} F^{+i}(0)\, \mathcal{U}^{[\Box]} \mathcal{U}^{[+]}\right\} | p_A \rangle \,,$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle p_A | \, \mathrm{Tr}\left\{F^{+i}(\xi)\, \mathcal{U}^{[+]\dagger} F^{+i}(0)\, \mathcal{U}^{[+]}\right\} \left(\frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c} \right)^2 | p_A \rangle$$

where the Wilson lines and Wilson loops are:

$$\mathcal{U}^{[\pm]} = U(0, \pm\infty; 0_T) U(\pm\infty, \xi^+; \xi_T), \;\; \mathcal{U}^{[\Box]} = \mathcal{U}^{[+]} \mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]} \mathcal{U}^{[+]\dagger}$$

Relation to (generalized) TMD factorization (3)

Two most basic TMD distributions:

$$\langle p|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_1 \qquad \text{Weizsacker}-\text{Williams (WW)}$$

$$\langle p|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p\rangle \sim xG_2 \qquad \text{dipole}$$

It is possible to choose a gauge to eliminate Wilson lines in xG_1 so that it has an interpretation as a gluon number density. This is not possible for xG_2.

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How to obtain the rest?

- Full evolution equations of the hierarchy of the operators[1,2]
- Approximations:
  - At large N_c some TMD gluon distributions are suppressed $\Rightarrow$ 5 left.
  - In the leading power limit we recover CGC and thus we may assume the Gaussian distribution of color sources known from CGC

$\Rightarrow$ All TMD gluons can be calculated from xG_2

[1] I. Balitsky, A. Tarasov, JHEP 1510 (2015) 017
[2] C. Marquet, E. Petreska, C. Roiesnel, JHEP 1610 (2016) 065

Relation to k_T-factorization

Dilute limit of the CGC expression $Q_s \ll k_T \sim P_T$

[E. Iancu and J. Laidet, Nucl. Phys. A 916 (2013) 48]
[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$d\sigma_{AB\to 2j} = \sum_b \int dx_A dx_B \int dk_T^2\, f_{b/B}\left(x_B, \mu^2\right) \mathcal{F}_{g^*/A}\left(x_A, k_T^2, \mu^2\right)\, d\hat{\sigma}_{g^* b\to 2j}\left(x_A, x_B, k_T^2, \mu^2\right)$$

$\mathcal{F}\left(x_A, k_T^2, \mu^2\right)$ – Unintegrated Gluon Distribution (UGD) with BFKL evolution or similar

$f_{b/B}\left(x_B, \mu^2\right)$ – collinear PDF

$\hat{\sigma}_{g^* b\to 2j}$ – partonic cross section, computed with off-shell incoming gluon in a gauge invariant way (the Lipatov vertexes).

Meinly linear regime.

The form of the High Energy (or k_T) Factorization (HEF).

[L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. 100 (1983) 1-150]
[J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3-30]
[S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188]

Unified approach for hard dijets (1)

Formula for $P_T \gg Q_s$, any k_T

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$$\frac{d\sigma_{AB\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim \sum_{a,c,d} f_{a/B}\left(x_B,\mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A,k_T^2,\mu^2\right) K^{(i)}_{ag\to cd}\left(k_T^2,\mu^2\right)$$

$\Phi^{(i)}_{ag\to cd}$ – small-x TMD Gluon Distributions, linear combinations of $\mathcal{F}^{(i)}_{ag}$
$K^{(i)}$ – hard factors calculated from gauge invariant off-shell amplitudes

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$\Phi^{(i)}_{ag\to cd}$ – small-x TMD Gluon Distributions, linear combinations of $\mathcal{F}^{(i)}_{ag}$
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The formula contains the two limiting cases:

1. $Q_s \ll k_T \sim P_T$ – High Energy Factorization (HEF)/k_T-factorization
   - All $\Phi^{(i)}_{ag\to cd}$ become equal to xG_2.
   - Off-shell factors combine to appropriate Lipatov vertex.
2. $k_T \sim Q_s \ll P_T$ – leading power limit of CGC
   - Off-shell factors become on-shell
   - TMD gluon distributions can be identified with color averages in CGC

Unified approach for hard dijets (2)

Comments

1. Factorized form
   - clear distinction of what is a hard process and what is a gluon distribution
   - suitable for attacking the NLO

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   - complete phase space and momentum conservation
   - two existing independent implementations

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3. At phenomenological level, the k_T dependence mimics initial state parton shower[1].

[1] see eg. M. Bury, M. Deak, K. Kutak, S. Sapeta, Phys.Lett. B760 (2016) 594-601

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3. At phenomenological level, the k_T dependence mimics initial state parton shower[1].
4. In the region $P_T \gg k_T \sim Q_s$ one should resum the Sudakov logs.
   - formally this should be done by considering a complete evolution equation for TMD gluons.
   - however, at lowest order the Sudakov form factor has a simple probabilistic interpretation.
   - this can be used to model the resummation at the Monte Carlo level[2].

[1] see eg. M. Bury, M. Deak, K. Kutak, S. Sapeta, Phys.Lett. B760 (2016) 594-601
[2] A. van Hameren, P.K., K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340

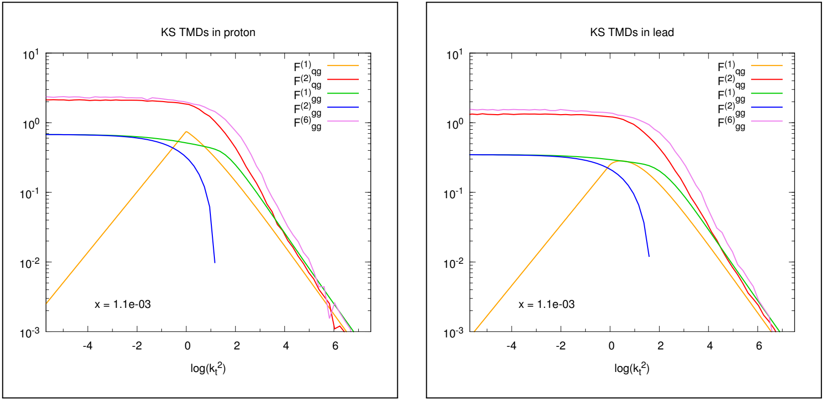
Phenomenology (1)

Small x TMD gluon distributions from data

[A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]

At karge N_c the non-vanishing $\mathcal{F}^{(i)}$-s can be calculated in the gaussian approximation from the dipole gluon xG_2.

xG_2 was fitted to HERA data by Kutak and Sapeta[1] (KS) using nonlinear extension[2] of Kwiecinski-Martin-Stasto[3] (KMS) evolution equation.

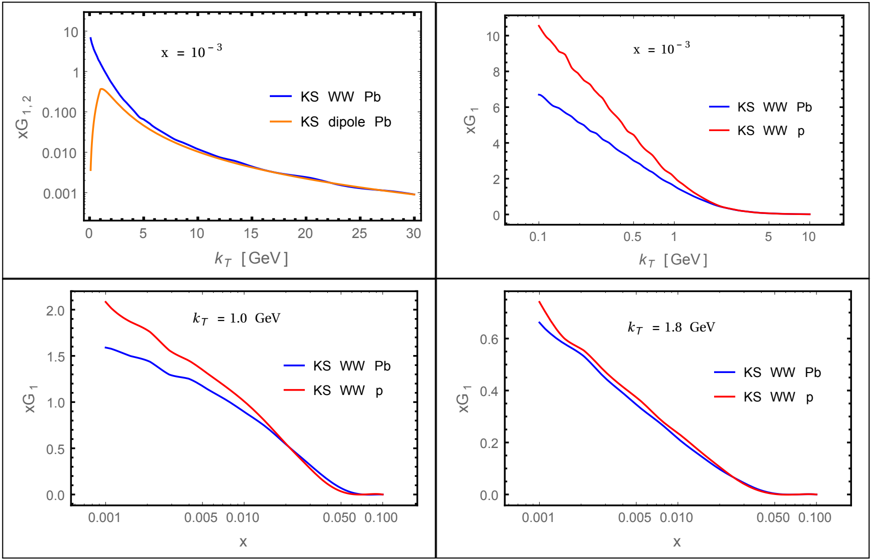


All gluons merge for large k_T (except $\mathcal{F}_{gg}^{(2)}$ which vanishes) $\Rightarrow$ correct HEF limit.

Phenomenology (2)

The Weizsacker-Williams gluon distribution from data

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur. Phys. J. C77 (2017) no.5, 353]



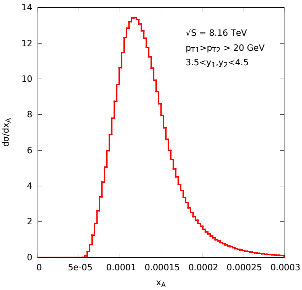
Phenomenology (3)

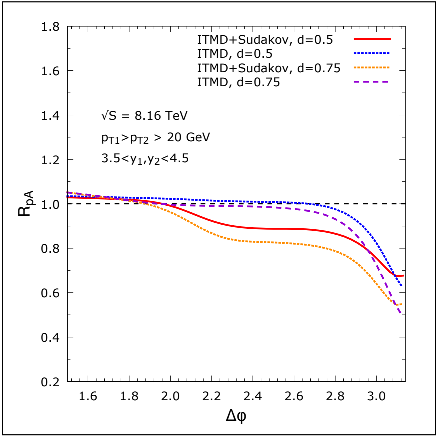
Nuclear modification ratio for azimuthal decorrelations in $pA \to 2j$

[A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]

- CM energy: $\sqrt{S} = 8.16\,\mathrm{TeV}$
- require two jets with $(\Delta\phi)^2 + (\Delta\eta)^2 > R^2, R = 0.5$
- transverse momenta cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$
- rapidity cuts: $3.5 < y_1, y_2 < 4.5$

x fractions probed





Phenomenology (4)

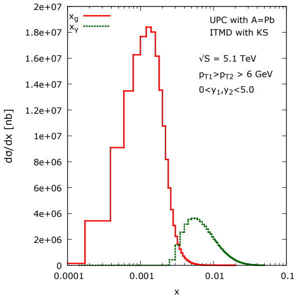
Nuclear modification ratio for azimuthal decorrelations in Ultra-Pheripheral Collisions (UPC) $AA(pA) \to 2j$

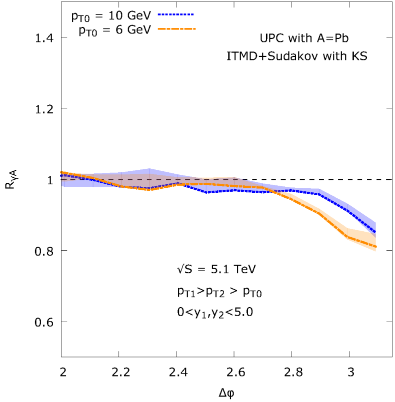
- CM energy: $\sqrt{S} = 5.1\,\mathrm{TeV}$
- require two jets
- transverse momenta cuts: $p_{T1} > p_{T2} > 6 \div 10\,\mathrm{GeV}$
- rapidity cuts: $0 < y_1, y_2 < 5$

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur. Phys. J. C77 (2017) no.5, 353]

Direct component probes the WW gluons directly

x fractions probed





Towards NLO (1)

Tree-level multi-leg off-shell gauge invariant amplitudes

Several methods available:

- Lipatov's effective action [E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135]
- Slavnov-Taylor identities [A. van Hameren, P.K., K. Kutak, JHEP 1212 (2012) 029]
- Extraction from on-shell process [A. van Hameren, P.K., K. Kutak, JHEP 1301 (2013) 078]
- ME of Wilson lines [P.K., JHEP 1407 (2014) 128]
- Britto-Cachazo-Feng-Witten (BCFW) [A. van Hameren, JHEP 1407 (2014) 138]

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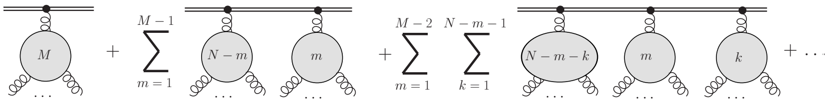
Example for $g^* g \to g \ldots g$ using the Wilson lines

For an off-shell gluon with momentum $k = xp + k_T$ take the following diagrams:



where the Wilson line slope is p and the momentum k. (arbitrary gauge)

Towards NLO (2)

Example: maximally helicity violating (MHV) amplitudes

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

Towards NLO (2)

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Color decomposition of amplitudes

$$\mathcal{M}^{a_1 \dots a_N}\left(\varepsilon_1^{\lambda_1}, \dots, \varepsilon_N^{\lambda_N}\right) = \sum_{\sigma \in S_{N-1}} \mathrm{Tr}\left(t^{a_1} t^{a_{\sigma_2}} \dots t^{a_{\sigma_N}}\right) \mathcal{M}\left(1^{\lambda_1}, \sigma_2^{\lambda_{\sigma 2}} \dots, \sigma_N^{\lambda_{\sigma N}}\right)$$

a_i- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i, S_{N-1} - set of noncyclic permutations.

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The color-ordered off-shell gauge invariant MHV amplitudes read:

$$\mathcal{M}_{g^*g\to g\ldots g}\left(1^*,2^-,3^+,\ldots,n^+\right)\sim\frac{\langle 1^*2\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\ldots\langle n-1n\rangle\langle n1^*\rangle}$$

where $\langle ij\rangle=\langle k_i-|k_j+\rangle$ with spinors defined as $|k_i\pm\rangle=\frac{1}{2}\left(1\pm\gamma_5\right)u\left(k_i\right)$.
Spinor products for off-shell states involve only longitudinal component of the off-shell momentum $\langle 1^*i\rangle=\langle p_1 i\rangle$, where $k_1=p_1+k_{T1}$, $k_1^2\neq 0$, $p_1^2=0$.

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Why the MHV form?

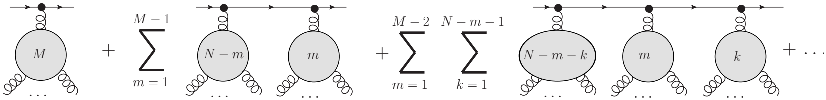
A general feature of certain off-shell gauge invariant currents, not related to high energy amplitudes. [P.K., A. Stasto, JHEP 1709 (2017) 047]

Presence of non-light like Wilson lines in the light-front MHV Lagrangian for the Cachazo-Svrcek-Witten (CSW) method of calculating amplitudes.

Towards NLO (3)

Eikonization

Off-shell gauge invariant amplitudes can be extracted from an auxiliary (unphysical) on-shell process



When the quark becomes eikonal, we get the desire result.
The whole procedure can be automatized to get any amplitude

[A. van Hameren, P.K., K. Kutak, JHEP 1301 (2013) 078]

Towards NLO (3)

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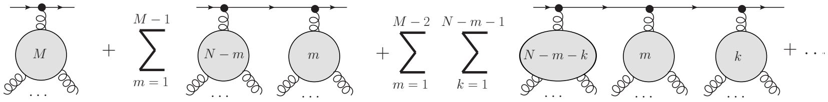

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[A. van Hameren, P.K., K. Kutak, JHEP 1301 (2013) 078]

Loop corrections

For on-shell amplitudes the loop corrections can be calculated automatically for any process.

The idea is to use the eikonization at the loop level.

The main issues are related to the strict eikonal limit and finding the coefficients of the master integrals (boxes, triangles, bubbles), but can be dealt with.

[A. van Hameren, in preparation]

Towards NLO (4)

Real corrections

Work in (slow) progress...

The idea is to treat the incoming off-shell gluon as a massive parton and use elements of the Catani-Seymour dipole subtraction method extended for incoming massive partons. [P.K., W. Slominski, Phys.Rev. D86 (2012) 094008]

Towards NLO (4)

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Warning

Our concern here was NLO correction to an arbitrary hard off-shell process.

The issue of factorization (or factorization breaking) is a separate story.

Summary

1. I reviewed selected approaches to forward production of hard dijet in the context of saturation.
2. Numerical results in a model which is an approximation to CGC when the dijet system is hard:
   - at LHC, forward dijet production in *pA* shows about 40% of suppression due to saturation with jets $p_T > 20\,\mathrm{GeV}$.
   - inn forward dijets in UPC at LHC, the suppression is at most around 20% with jets $p_T > 6\,\mathrm{GeV}$.
3. Current status of efficient methods for calculating any off-shell amplitudes, including some recent progress at NLO

BACKUP

Improved small x TMD factorization (ITMD)

Factorization formula for forward dijets in pA ($\mu = \bar{p}_T \gg Q_s$)

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim \sum_{a,c,d} f_{a/B}\left(x_B,\mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A,k_T^2,\mu^2\right) K^{(i)}_{ag\to cd}\left(k_T^2,\mu^2\right)$$

TMD gluon distributions:

$$\Phi^{(1)}_{qg\to gq} = \mathcal{F}^{(1)}_{qg},\ \Phi^{(2)}_{qg\to gq} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(2)}_{qg} - \mathcal{F}^{(1)}_{qg}\right),\ \Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg} - \mathcal{F}^{(3)}_{gg}\right),\ \Phi^{(2)}_{gg\to q\bar{q}} = \mathcal{F}^{(3)}_{gg} - N_c^2\mathcal{F}^{(2)}_{gg}$$

$$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right),\ \Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$$\mathcal{F}^{(1)}_{qg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle,\quad \mathcal{F}^{(2)}_{qg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle,$$

$$\mathcal{F}^{(1)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\{F^{+i}(\xi)\,\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle,\quad \mathcal{F}^{(2)}_{gg} \sim \frac{1}{N_c}\langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[\square]\dagger}\right\}\mathrm{Tr}\left\{F^{+i}(0)\,\mathcal{U}^{[\square]}\right\}|p_A\rangle,$$

$$\mathcal{F}^{(3)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\right\}|p_A\rangle,\quad \mathcal{F}^{(4)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[-]\dagger}F^{+i}(0)\,\mathcal{U}^{[-]}\right\}|p_A\rangle,$$

$$\mathcal{F}^{(5)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right\}|p_A\rangle,\ \mathcal{F}^{(6)}_{gg} \sim \langle p_A|\,\mathrm{Tr}\left\{F^{+i}(\xi)\,\mathcal{U}^{[+]\dagger}F^{+i}(0)\,\mathcal{U}^{[+]}\right\}\left(\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\right)^2|p_A\rangle$$

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Off-shell hard factors:

$$K^{(1)}_{qg\to gq} = -\frac{\overline{u}\left(\overline{s}^2+\overline{u}^2\right)}{2\overline{t}\hat{t}\hat{s}}\left(1+\frac{\overline{s}\hat{s}-\overline{t}\hat{t}}{N_c^2\,\overline{u}\hat{u}}\right),\; K^{(2)}_{qg\to gq} = -\frac{C_F}{N_c}\,\frac{\overline{s}\left(\overline{s}^2+\overline{u}^2\right)}{\overline{t}\hat{t}\hat{u}},\; K^{(1)}_{gg\to q\bar{q}} = \frac{1}{2N_c}\,\frac{\left(\overline{t}^2+\overline{u}^2\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}\right)}{\overline{s}\hat{s}\hat{t}\hat{u}}$$

$$K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2 C_F}\,\frac{\left(\overline{t}^2+\overline{u}^2\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}-\overline{s}\hat{s}\right)}{\overline{s}\hat{s}\hat{t}\hat{u}},\; K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2 C_F}\,\frac{\left(\overline{t}^2+\overline{u}^2\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}-\overline{s}\hat{s}\right)}{\overline{s}\hat{s}\hat{t}\hat{u}},$$

$$K^{(1)}_{gg\to gg} = \frac{N_c}{C_F}\,\frac{\left(\overline{s}^4+\overline{t}^4+\overline{u}^4\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}\right)}{\overline{t}\hat{t}\overline{u}\hat{u}\overline{s}\hat{s}},\; K^{(2)}_{gg\to gg} = -\frac{N_c}{2C_F}\,\frac{\left(\overline{s}^4+\overline{t}^4+\overline{u}^4\right)\left(\overline{u}\hat{u}+\overline{t}\hat{t}-\overline{s}\hat{s}\right)}{\overline{t}\hat{t}\overline{u}\hat{u}\overline{s}\hat{s}}$$

$\hat{s}, \hat{t}, \hat{u}$ – ordinary Mandelstam variables, $\hat{s}+\hat{t}+\hat{u} = k_T^2$.

$\overline{s}, \overline{t}, \overline{u}$ off-shell momentum is replaced by its longitudinal component of off-shell momentum, $\overline{s}+\overline{u}+\overline{t} = 0$

.

Gaussian approximation for TMD gluons

In the large N_c limit all UGDs can be expressed by only two fundamental UGDs:

$$\mathcal{F}_{qg}^{(2)}\left(x,k_T^2\right)\sim\int\frac{d^2q_T}{q_T^2}\,\mathcal{F}_{gg}^{(3)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(1)}\left(x,k_T^2\right)\sim\int\frac{d^2q_T}{q_T^2}\,\mathcal{F}_{qg}^{(1)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(2)}\left(x,k_T^2\right)\sim\int\frac{d^2q_T}{q_T^2}\,\left(q_T-k_T\right)\cdot q_T\,\mathcal{F}_{qg}^{(1)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(1)}\left(x,k_T^2\right)\sim\int\frac{d^2q_T\,d^2q_T'}{q_T^2}\,\mathcal{F}_{gg}^{(3)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,q_T'^2\right)\mathcal{F}_{qg}^{(1)}\left(x,\left|k_T-q_T-q_T'\right|^2\right)$$

1. dipole: $\mathcal{F}_{qg}^{(1)}=xG^{(2)}\sim\langle p_A|\,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle$
   appears directly in: inclusive DIS, inclusive jet in *pA*
2. Weizsacker-Williams (WW):
   $\mathcal{F}_{gg}^{(3)}=xG^{(1)}\sim\langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p_A\rangle$
   appears directly in dijets in DIS

The WW gluon distribution from data

Relation between xG_1 and xG_2 in gaussian approximation

$$\nabla^2_{k_T} G^{(1)}(x, k_T) = \frac{4\pi^2}{N_c S_\perp} \int \frac{d^2 q_T}{q_T^2} \frac{\alpha_s}{(k_T - q_T)^2} G^{(2)}(x, q_T)\, G^{(2)}(x, |k_T - q_T|)$$

Realistic evolution equation for xG_2

Nonlinear extension of the Kwiecinski-Martin-Stasto (KMS) evolution equation (below $xG_2 \equiv \mathcal{F}$):

[K. Kutak, K. Kwiecinski, Eur. Phys. J. C 29 (2003) 521]
[J. Kwiecinski, Alan D. Martin, A.M. Stasto, Phys.Rev. D56 (1997) 3991-4006]

$$\mathcal{F}\left(x, k_T^2\right) = \mathcal{F}_0\left(x, k_T^2\right) + \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_{T\,0}^2}^{\infty} \frac{dq_T^2}{q_T^2} \left\{ \frac{q_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) \theta\left(\frac{k_T^2}{z} - q_T^2\right) - k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{\left|q_T^2 - k_T^2\right|} + \frac{k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{\sqrt{4q_T^4 + k_T^4}} \right\}$$

$$+ \frac{\alpha_s}{2\pi k_T^2} \int_x^1 dz \left\{ \left(P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_{T\,0}^2}^{k_T^2} dq_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) + z P_{gq}(z)\, \Sigma\left(\frac{x}{z}, k_T^2\right) \right\}$$

$$- \frac{2\alpha_s^2}{R^2} \left\{ \left[\int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \mathcal{F}\left(x, q_T^2\right) \right]^2 + \mathcal{F}\left(x, k_T^2\right) \int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \ln\left(\frac{q_T^2}{k_T^2}\right) \mathcal{F}\left(x, q_T^2\right) \right\}$$

This equation was fitted to HERA data for proton by Kutak-Sapeta (KS).

[K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043]

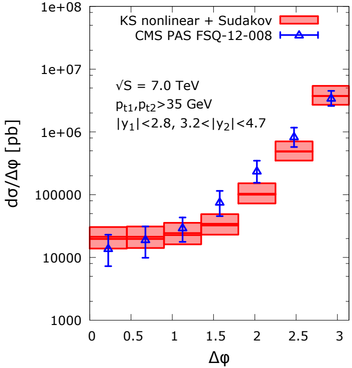
For nucleus $R_A = RA^{1/3}/\sqrt{d}$ is used so the nonlinear term is enhanced by $dA^{1/3}$.

High energy factorization (HEF): $k_T \sim P_T \gg Q_s$

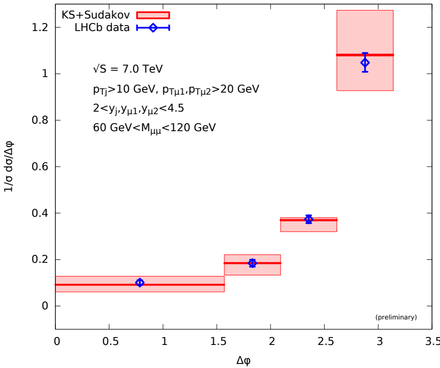
Comparison with data

[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]
[A. van Hameren, PK, K. Kutak, Phys.Rev. D92 (2015) 054007]

- central-forward dijet production



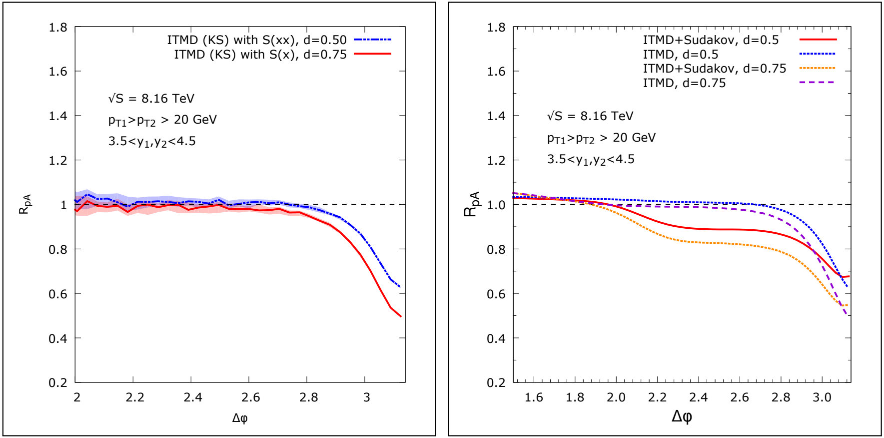
- forward Z_0+jet production



Results for dijet production in pPb at LHC

Nuclear modification ratio for azimuthal decorrelations

[A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]



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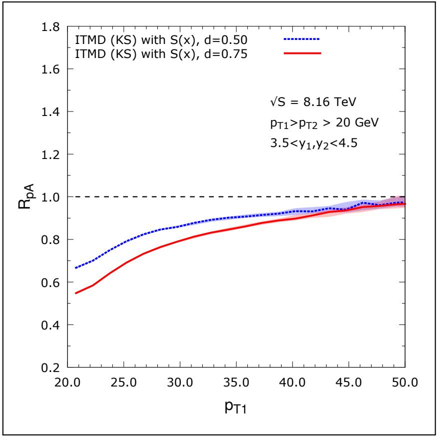
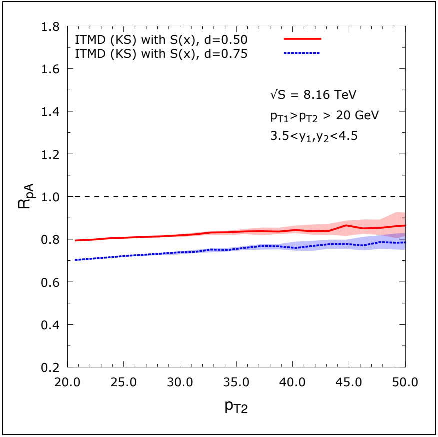




ITMD for dijets in γA

Factorization formula

[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur. Phys. J. C77 (2017) no.5, 353]

$$\frac{d\sigma_{\gamma A\to 2j+X}}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} \sim x_A G_1\left(x_A, k_T^2, \mu^2\right) \otimes K_{\gamma g^*\to q\bar{q}}\left(k_T, \mu^2\right)$$

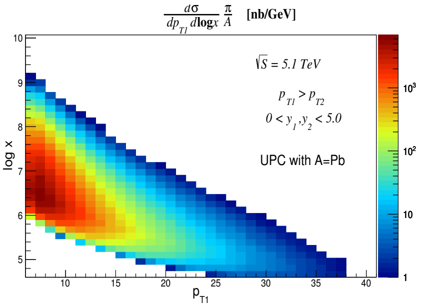
xG_1 – the Weizsacker-Williams gluon distribution
$K_{\gamma g^*\to q\bar{q}}$ – off-shell gauge invariant hard factor for the $\gamma g^* \to q\bar{q}$ process

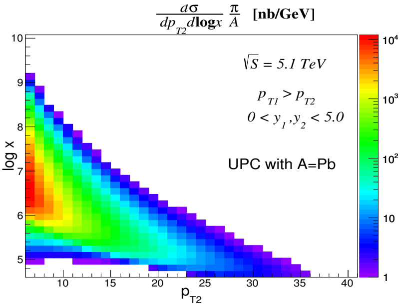
- Similar to inclusive DIS, but probes xG_1 instead of xG_2.
- For UPC one needs to convolute this with the photon flux from nucleus.
  Issue: the photon flux dies out very fast above $x_\gamma \sim 0.03$, so there is not much phase space for the asymmetric kinematics $x_A \ll x_\gamma$ which guarantees that xG_1 is probed at small x, unless we use jets with rather small p_T.

Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25$ GeV	jet algorithm: $R = 0.5$





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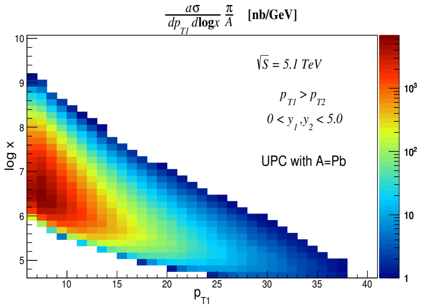

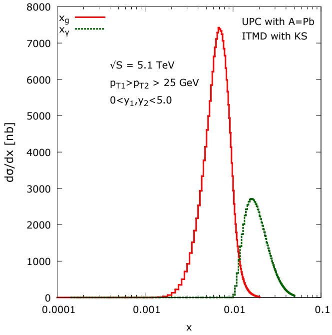

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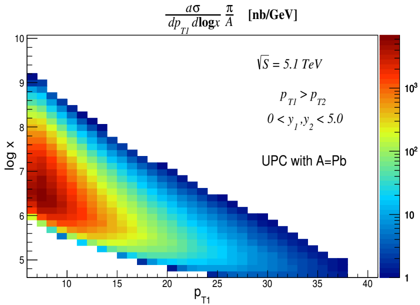

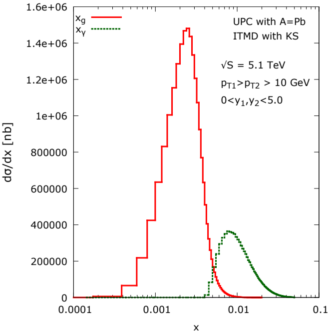

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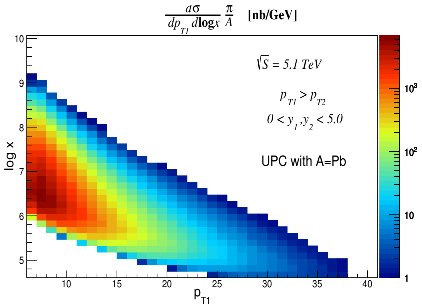

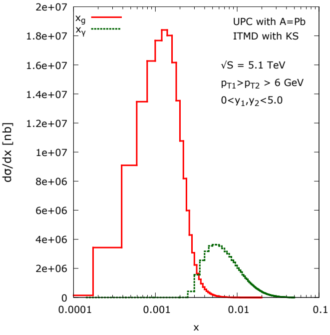

Nuclear modification ratio in UPC

Nuclear modification factor $R_{\gamma A}$

For UPC collisions we define the nuclear modification ratio as

$$R_{\gamma A} = \frac{d\sigma_{AA}^{\mathrm{UPC}}}{A d\sigma_{Ap}^{\mathrm{UPC}}}$$

where $A = \mathrm{Pb}$ and the $d\sigma_{Ap}^{\mathrm{UPC}}$ is with jets going in the nucleus direction.

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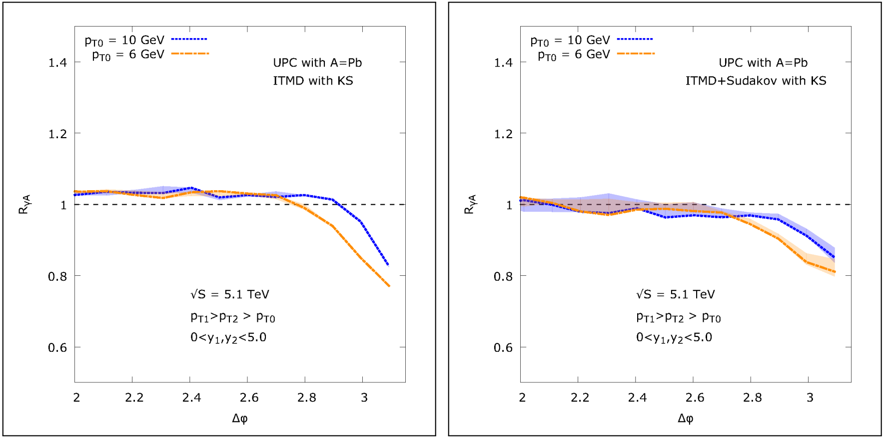
[P.K., K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur. Phys. J. C77 (2017) no.5, 353]



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