DPS in CGC: effects of Quantum Statstics in Particle Production

Alex Kovner

University of Connecticut

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[with A. Rezaeian]

Before the introduction

HAPPY NEW YEAR 5778!

Motivation: Correlated production

Observation of Ridge $\rightarrow$ study of correlated particle production.

Most likely several mechanisms are at play.

Final state effects among them, but initial state effects may also be important.

Maybe those can be observed in other systems (not heavy ions/dense protons)?

In this talk effects of quantum statistics on "parton level" :

1. Double photon production. Obvious advantages and disadvantages: final state effects are not as significant but the signal is small. But really: the simplest one there is.

A. K. and A. Rezaeian, Phys.Rev. D95 (2017) no.11, 114028, e-Print: arXiv:1701.00494 [hep-ph] .

2. Double quark production. Possibly relevant for ridge (as a proxy for gluon production, but one must be careful). Much simpler color algebra is - useful to understand relevant physics. Also not tied to a specific model of the CGC state.

A.K. and A. Rezaeian, e-Print: arXiv:1707.06985 [hep-ph] .

DPS and Hybrid

"Hybrid approach": "partonic" proton scatters eikonally on color fields of the target "nucleus".

Relevant for forward scattering (projectile x values not too small). But qualitative features should be fairly universal

More than one particle is produced, so more than one parton scatters: direct connection of the CGC approach with the Double Parton Scattering (DPS).

Double Photon Production

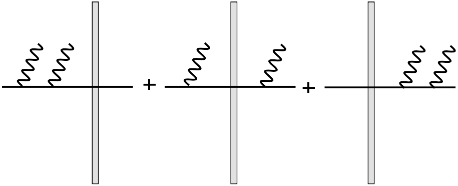

Figure: Two prompt photons amplitude from one quark in the CGC background.

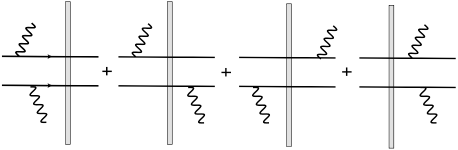

Figure: Two prompt photons amplitude from two quarks in the CGC background.

We concentrate on the process:

$$q(p_1) + q(p_2) + A \to \gamma(k_1) + \gamma(k_2) + \text{jet}(q) + \text{jet}(q') + X.$$

The proton wave function

The cross section:

$$\frac{d\,\sigma^{qq\to\gamma\gamma qq}}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \int_{q,q'} \frac{1}{4qq'^-} \quad \langle |\langle \mathrm{jet}(q), \mathrm{jet}(q'), \gamma(k_1), \gamma(k_2)|\mathrm{Proton}\rangle|^2\rangle_{\mathrm{Target}}$$

The proton Fock space wave function

$$|\mathrm{Proton}\rangle = \frac{1}{2N_c}\sum_{s_i,c_i}\int_{p_1,p_2}\frac{1}{\sqrt{4p_1^- p_2^-}}\sum_X \tilde{\mathcal{A}}(p_1,p_2,s_1,s_2,c_1,c_2;X)|p_1,s_1,c_1;p_2,s_2,c_2,X\rangle$$

X - "spectator" degrees of freedom.

Simplify (not necessary but saves some hassle) :

$$|\mathrm{Proton}\rangle \to |\mathrm{Two\ quarks}\rangle = \frac{1}{2N_c}\sum_{s_i,c_i}\int_{p_1,p_2}\frac{1}{\sqrt{4p_1^- p_2^-}}\tilde{\mathcal{P}}(p_1,p_2)|p_1,s_1,c_1;p_2,s_2,c_2\rangle$$

The amplitude

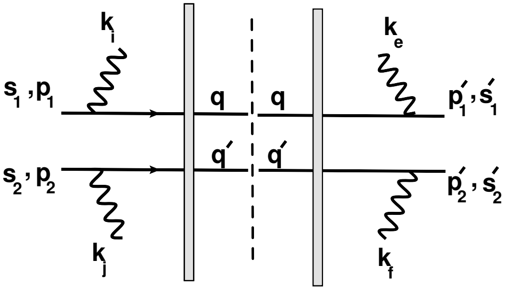

Figure: A typical diagram contributing to two prompt photons production from two quarks in the background of the CGC field

$$\langle \text{jet}(q), \text{jet}(q'), \gamma(k_1), \gamma(k_2) | \text{two quarks} \rangle = \frac{1}{2N_c} \sum_{s,s',s_1,s_2,c,c',c_1,c_2} \int\int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \tilde{\mathcal{P}}(p_1, p_2)$$

$$\Big[\langle q, s, c; k_1 | p_1, s_1, c_1 \rangle \langle q', s', c', k_2 | p_2, s_2, c_2 \rangle + \langle q, s, c; k_2 | p_1, s_1, c_1 \rangle \langle q', s', c'; k_1 | p_2, s_2, c_2 \rangle$$

$$+ \langle q', s', c'; k_1 | p_1, s_1, c_1 \rangle \langle q, s, c; k_2 | p_2, s_2, c_2 \rangle + \langle q', s', c'; k_2 | p_1, s_1, c_1 \rangle \langle q, s, c; k_1 | p_2, s_2, c_2 \rangle \Big]$$

The basics

The basic element of the calculation:

$$\langle q(q), \gamma(k_1)|q(p)\rangle = - e_q \bar{u}(q)\Big[\mathcal{F}(q; p-k_1)G_F^0(p-k_1)\not{\epsilon}(k_1) + \not{\epsilon}(k_1)G_F^0(q+k_1)\mathcal{F}(q+k_1, p)\Big]u(p),$$

with

$$\mathcal{F}(q;p) = 2\pi\delta(q^- - p^-)\gamma^- sign(p^-)\int d^2x \; U(\vec{x}) \; e^{i(\vec{q}-\vec{p})\cdot\vec{x}}$$

and U is the eikonal scattering matrix

$$U(\vec{x}) = T \exp\left(-ig^2 \int dx^- \frac{1}{\nabla^2}\rho_a(x^-, \vec{x})t^a\right)$$

To make the long story short

At large N_c:

$$d\sigma^{qq+A\to\gamma\gamma+qq} = \int_{p_1,p_2} \Big[\mathcal{T}(x_1,x_2,\vec{p}_1,\vec{p}_2,0)\; d\sigma^{q(p_1)+A\to\gamma(k_1)+q(q)} \times d\sigma^{q(p_2)+A\to\gamma(k_2)+q(q')} + d\sigma^{\text{Interference}} \Big],$$

$$\frac{d\sigma^{qq+A\to\gamma\gamma+X}}{d^3k_1 d^3k_2}\Big|_{\text{interference}} \approx \frac{e_q^4}{(2\pi)^6} k_1^- k_2^- \frac{16}{(2\pi)^2 k^8}$$
$$\times \int_{\vec{p}_1,\vec{p}_2,\xi_1,\xi_2,\vec{q},\vec{q}'} \frac{(\vec{q}'\cdot\vec{q})^2}{p_1^- p_2^-} N^{(2)}\left(\vec{q}+\vec{k}_1-\vec{p}_1\right) N^{(2)}\left(\vec{q}'+\vec{k}_2-\vec{p}_2\right) \mathcal{T}\left(x_1,x_2,\vec{p}_1,\vec{p}_2,\vec{\Delta}\right).$$

with 2GTMD

$$\mathcal{T}\left(x_1,x_2,\vec{p}_1,\vec{p}_2,\vec{\Delta}\right) = \langle Proton|\psi^{\dagger a}(x_1,\vec{p}_1)\psi^{\dagger b}(x_2,\vec{p}_2)\psi^{b}(x_2,\vec{p}_2+\vec{\Delta})\psi^{a}(x_1,\vec{p}_2-\vec{\Delta})|Proton\rangle$$

and $\vec{\Delta} = \vec{k}_1 - \vec{k}_2$.

The dipole:

$$N^{(2)}(\vec{q}) = \int_x e^{i\vec{x}\cdot\vec{q}} \frac{1}{N_c} tr[U^\dagger(x)U(0)]$$

The Hanbury-Brown, Twiss correlation

Performing q and q' integrals:

$$\frac{d\sigma^{qq+A\to\gamma\gamma+X}}{d^3k_1 d^3k_2}\Big|_{\text{interference}} \approx \frac{2e_q^4}{(2\pi)^6}\frac{k_1^- k_2^-}{s}\frac{16Q_s^4 S_{eff}^2}{(2\pi)^2 k^8}\int_{\vec{p}_i,x_i}\mathcal{T}\left(x_1,x_2,\vec{p}_1,\vec{p}_2,\vec{\Delta}\right)$$

Simple physics. For independent quarks:

$$\mathcal{T}\left(x_1,x_2,\vec{p}_1,\vec{p}_2,\vec{\Delta}\right)\approx G_{GTMD}\left(x_1,\vec{p}_1,\vec{\Delta}\right)G_{GTMD}\left(x_2,\vec{p}_2,\vec{\Delta}\right),$$

"Proton radius"

$$G_{GTMD}\left(x,\vec{p},\Delta\right) = T(x,\vec{p})e^{-\frac{1}{2}R^2|\Delta|^2}$$

And so

$$\int_{\vec{p}_i}\mathcal{T}\left(x_1,x_2,\vec{p}_1,\vec{p}_2,\vec{\Delta}\right) = f_q(x_1)f_q(x_2)e^{-R^2(\vec{k}_1-\vec{k}_2)^2},$$

This is the typical HBT behavior.

Double quark production

"Forward" production: the quarks come directly from the Proton wave function.

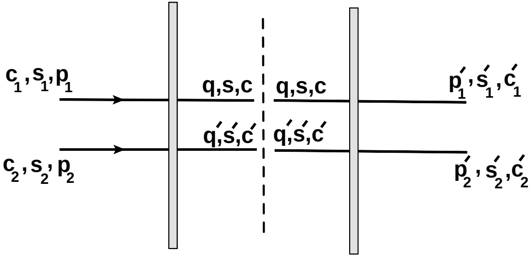

Figure: Two quark production in the background of the CGC field.

$$d\,\sigma^{p+A\to qq+X} = \frac{d^3q}{(2\pi)^3\,2q^-}\,\frac{d^3q'}{(2\pi)^3\,2q'^-}\,\langle|\langle \mathrm{jet}(q),\mathrm{jet}(q')|\mathrm{Proton}\rangle|^2\rangle_{\text{color sources}}$$

$$|\mathrm{Proton}\rangle = \sum_X \sum_{c_i,s_i} \int_{p_i} \tilde{A}(p_1,c_1,s_1;p_1,c_2,s_2;X)|p_1,c_1,s_1;p_2,c_2,s_2;X\rangle$$

2GTMDs

The cross section:

$$\mathcal{I} = 16\frac{(2\pi)^4}{4}q^- q'^-$$
$$\frac{N_c^2}{N_c^2-1}\int_{\vec{p}_1,\vec{p}_2,\vec{\Delta}}\Big\{\langle D(\vec{p}_1-\vec{q}+\vec{\Delta}/2,2\vec{\Delta})D(\vec{p}_2-\vec{q}'-\vec{\Delta}/2,-2\vec{\Delta})\rangle\Big[T^D_{qq'}(p_1,p_2,\Delta)-\frac{1}{N_c}T^E_{qq'}(p_1,p_2,\Delta)\Big]$$
$$+\ \frac{1}{N_c}\langle Q(\vec{p}_1-\vec{q}+\vec{\Delta}/2,\ \vec{p}_2-\vec{q}'-\vec{\Delta}/2,\vec{\Delta})\rangle\Big[T^E_{qq'}(p_1,p_2,\Delta)-\frac{1}{N_c}T^D_{qq'}(p_1,p_2,\Delta)\Big]\Big\}$$

with "direct" and "color exchange" 2GTMD's

$$T^D_{qq'}(p_1,p_2,\Delta)\equiv\sum_{s_i,c_i}\langle\text{Proton}|\psi^\dagger_q(\vec{p}_1,x_1,c_1,s_1)\psi^\dagger_{q'}(\vec{p}_2,x_2,c_2,s_2)\psi_{q'}(\vec{p}_2-\vec{\Delta},x_2,c_2,s_2)\psi_q(\vec{p}_1+\vec{\Delta},x_1,c_1,s_1)|\text{Proton}\rangle$$
$$T^E_{qq'}(p_1,p_2,\Delta)\equiv\sum_{s_i,c_i}\langle\text{Proton}|\psi^\dagger_q(\vec{p}_1,x_1,c_1,s_1)\psi^\dagger_{q'}(\vec{p}_2,x_2,c_2,s_2)\psi_{q'}(\vec{p}_2-\vec{\Delta},x_2,c_1,s_2)\psi_q(\vec{p}_1+\vec{\Delta},x_1,c_2,s_1)|\text{Proton}\rangle$$

The "dipole" and the "quadrupole" amplitudes:

$$D(\vec{x}_1,\vec{x}_2)\equiv\frac{1}{N_c}Tr\Big[U^\dagger(\vec{x}_1)U(\vec{x}_2)\Big],$$
$$Q(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4)\equiv\frac{1}{N_c}Tr\Big[U^\dagger(\vec{x}_1)U(\vec{x}_2)U^\dagger(\vec{x}_3)U(\vec{x}_4)\Big]$$

2GTMD's abridged - nonidentical quarks

Consider production of a u and a d quark.

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi_u^\dagger(\vec{p}_1,x_1,a,s_1)\psi_d^\dagger(\vec{p}_2,x_2,b,s_2)\psi_d(\vec{p}_2-\Delta,x_2,b,s_2)\psi_u(\vec{p}_1+\Delta,x_1,a,s_1)|\text{Proton}\rangle$$
$$=\sum_i\langle \text{Proton}|\sum_{a,s_1}\psi_u^\dagger(\vec{p}_1,x_1,a,s_1)\psi_u(\vec{p}_1+\Delta,x_1,a,s_1)|i\rangle\langle i|\sum_{b,s_2}\psi_d^\dagger(\vec{p}_2,x_2,b,s_2)\psi_d(\vec{p}_2-\Delta,x_2,b,s_2)|\text{Proton}\rangle$$

Assume a single nucleon "saturates" intermediate states:

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi_u^\dagger(\vec{p}_1,x_1,a,s_1)\psi_d^\dagger(\vec{p}_2,x_2,b,s_2)\psi_d(\vec{p}_2-\Delta,x_2,b,s_2)\psi_u(\vec{p}_1+\Delta,x_1,a,s_1)|\text{Proton}\rangle$$
$$=4N_c^2\mathcal{T}_u(x_1,0,\vec{p}_1,\Delta)\mathcal{T}_d^*(x_2,0,\vec{p}_2-\Delta,\Delta),$$

with GTMD's

$$\mathcal{T}_u(x,\eta,\vec{p},\vec{k})=\frac{1}{2N_c}\sum_{c,s}\langle \text{Proton}|\psi_u^\dagger(\vec{p},x,c,s)\psi_u(\vec{p}+\vec{k},x+\eta,c,s)|P,\vec{k},\eta\rangle$$

For "color exchange" contribution:

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi_u^\dagger(\vec{p}_1,x_1,a,s_1)\psi_d^\dagger(\vec{p}_2,x_2,b,s_2)\psi_d(\vec{p}_2-\Delta,x_2,a,s_2)\psi_u(\vec{p}_1+\Delta,x_1,b,s_1)|\text{Proton}\rangle$$
$$=\quad -2N_c^2\mathcal{M}(x_1,\eta=x_2-x_1,\vec{p}_1,\vec{p}_2-\vec{p}_1-\Delta)\mathcal{M}^*(x_1,\eta=x_2-x_1,\vec{p}_1+\Delta,\vec{p}_2-\vec{p}_1-\Delta)$$

with "off diagonal" GTMD

$$\mathcal{M}(x,\eta,\vec{p},\vec{k})=\frac{1}{2N_c}\sum_{c,s}\langle \text{Proton}|\psi_u^\dagger(\vec{p},x,c,s)\psi_d(\vec{p}+\vec{k},x+\eta,c,s)|N,\vec{k},\eta\rangle$$

Here $|N,\vec{k},\eta\rangle$ - the neutron state.

The cross section

In the single nucleon dominance approximation:

$$
\begin{aligned}
\mathcal{I}_{ud} &= 16(4N_c^2)\frac{(2\pi)^4}{4}\delta(p_1^- - q^-)\delta(p_1'^- - q^-)\delta(p_2^- - q'^-)\delta(p_2'^- - q'^-)q^- q'^- \int_{p_1,p_2,\Delta} \\
&\times \Bigg\{\Big[\langle D(\vec{p}_1 - \vec{q} + \Delta/2, 2\Delta)D(\vec{p}_2 - \vec{q}' - \Delta/2, -2\Delta)\rangle - \frac{1}{N_c^2}\langle Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta)\rangle\Big] \\
&\times \mathcal{T}_u(x_1, 0, \vec{p}_1, \Delta)\mathcal{T}_d^*(x_2, 0, \vec{p}_2, \Delta) \\
&- \frac{1}{N_c}\Big[\langle Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta)\rangle - \langle D(\vec{p}_1 - \vec{q} + \Delta/2, 2\Delta)D(\vec{p}_2 - \vec{q}' - \Delta/2, -2\Delta)\rangle\Big] \\
&\times \mathcal{M}(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1 - \Delta)\mathcal{M}^*(x_1, x_2 - x_1, \vec{p}_1 + \Delta, \vec{p}_2 - \vec{p}_1 - \Delta)\Bigg\}
\end{aligned}
$$

Identical quarks

Antisymmetry of the wave function:

$$
\begin{aligned}
T^{D}_{uu}(p_1, p_2, \Delta) &= 4N_c^2 \mathcal{T}_u(x_1, \eta = 0, \vec{p}_1, \Delta)\mathcal{T}^*_u(x_2, \eta = 0, \vec{p}_2, \Delta) \\
&- 2N_c \mathcal{T}_u(x_1, \eta = x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1 - \Delta)\mathcal{T}^*_u(x_1, \eta = x_2 - x_1, \vec{p}_1 + \Delta, \vec{p}_2 - \vec{p}_1 - \Delta),
\end{aligned}
$$

$$
\begin{aligned}
T^{E}_{uu}(p_1, p_2, \Delta) &= 4N_c \mathcal{T}_u(x_1, \eta = 0, \vec{p}_1, \Delta)\mathcal{T}^*_u(x_2, \eta = 0, \vec{p}_2, \Delta) \\
&- 2N_c^2 \mathcal{T}_u(x_1, \eta = x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1 - \Delta)\mathcal{T}^*_u(x_1, \eta = x_2 - x_1, \vec{p}_1 + \Delta, \vec{p}_2 - \vec{p}_1 - \Delta)
\end{aligned}
$$

The cross section:

$$
\begin{aligned}
&\mathcal{I}_{uu} = 16\frac{(2\pi)^4}{4}\delta(p_1^- - q^-)\delta(p_1'^- - q^-)\delta(p_2^- - q'^-)\delta(p_2'^- - q'^-)q^- q'^- \\
&\times 2N_c^2 \int_{p_1, p_2, \Delta} \Big[2\langle D(\vec{p}_1 - \vec{q} + \Delta/2, 2\Delta) D(\vec{p}_2 - \vec{q}' - \Delta/2, -2\Delta)\rangle \mathcal{T}_u(x_1, 0, \vec{p}_1, \Delta)\mathcal{T}^*_u(x_2, 0, \vec{p}_2, \Delta) \\
&- \frac{1}{N_c}\langle Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta)\rangle \mathcal{T}_u(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1 - \Delta)\mathcal{T}^*_u(x_1, x_2 - x_1, \vec{p}_1 + \Delta, \vec{p}_2 - \vec{p}_1 - \Delta)\Big]
\end{aligned}
$$

What is the physics of the different terms?

The first term is independent production.

The quadrupole term - correlated production due to quantum statistics effects. "Initial state": Pauli blocking in the incoming wave function; "Final state": HBT effect n the emission.

Quadrupole at large

Target field ensemble is color neutral on distance scales $r \sim 1/Q_s$.
Thus:

$$\lim_{|x_1-x_3|\gg 1/Q_s} Q(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) = \frac{1}{N_c}\langle U(\vec{x}_1)_{ab}U^\dagger(\vec{x}_2)_{bc}\rangle\langle U(\vec{x}_3)_{cd}U^\dagger(\vec{x}_4)_{da}\rangle = D(\vec{x}_1-\vec{x}_2)D(\vec{x}_3-\vec{x}_4)$$

And also

$$\lim_{|x_1-x_2|\gg 1/Q_s} Q(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) = \frac{1}{N_c}\langle U^\dagger(\vec{x}_4)_{da}U(\vec{x}_1)_{ab}\rangle\langle U^\dagger(\vec{x}_2)_{bc}U(\vec{x}_3)_{cd}\rangle = D(\vec{x}_1-\vec{x}_4)D(\vec{x}_2-\vec{x}_3)$$

Thus for most of configuration space where Q is not small, at leading N_c

$$Q(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) \approx D(\vec{x}_1-\vec{x}_2)D(\vec{x}_3-\vec{x}_4) + D(\vec{x}_1-\vec{x}_4)D(\vec{x}_2-\vec{x}_3)$$

First term leads to Pauli blocking, second term to quark HBT.

GTMD simplified

We need a qualitatively reasonable model for GTMD.

$$\mathcal{T}_u(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1) = T_u(x_1, \vec{p}_1) f(x_1 - x_2) \mathcal{F}(\vec{p}_1 - \vec{p}_2)$$

The form factor

$$\mathcal{F}(\vec{p}) = \frac{1}{(\vec{p}/\Lambda)^2 + 1}, \qquad \text{or} \quad \mathcal{F}(\vec{p}) = e^{-\frac{(\vec{p})^2}{\Lambda^2}}$$

where Λ^{-1} - the "quark" radius of the proton.
For "translationally invariant proton, $\Lambda \to 0$, and $\mathcal{F}(\vec{p}) \sim \delta^2(\vec{p})$

Pauli blocking

The first leading term (assuming translational invariance of the **target**)

$$Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) \approx D(\vec{x}_1 - \vec{x}_2) D(\vec{x}_3 - \vec{x}_4))$$

$$Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta) \propto S\delta^2(\Delta) D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}'),$$

Contribution to the cross section:

$$\begin{aligned} \mathcal{I}_{PB} \propto \quad & -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') |\mathcal{T}_u(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1)|^2 \\ & \approx -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') |\mathcal{T}_u(\vec{p}_1)|^2 |\delta_\Lambda(\vec{p}_2 - \vec{p}_1) \end{aligned}$$

Pauli blocking: initial configurations where two quarks have the same momentum are suppressed.

As expected, not present for non identical quarks.

Hanbury-Brown, Twiss correlation

The second term:

$$Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) \approx D(\vec{x}_1 - \vec{x}_4)D(\vec{x}_3 - \vec{x}_2))$$

$$Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta) \propto S\delta^2(\vec{p}_1 - \vec{p}_2 + \Delta + \vec{q} - \vec{q}')D(\vec{p}_1 - \vec{q})D(\vec{p}_2 - \vec{q}'),$$

The contribution to cross section:

$$\begin{aligned} \mathcal{I}_{HBT} \propto \quad & -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q})D(\vec{p}_2 - \vec{q}')\mathcal{T}_u(\vec{p}_1, \vec{q}' - \vec{q})\mathcal{T}_u^*(\vec{p}_2 + \vec{q} - \vec{q}', \vec{q}' - \vec{q}) \\ & \approx -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q})D(\vec{p}_2 - \vec{q}')\mathcal{T}_u(\vec{p}_1)\mathcal{T}_u(\vec{p}_2)\delta_\Lambda(\vec{q}' - \vec{q}) \end{aligned}$$

Typical HBT: peaked for equal momenta of final state particles.

Interestingly: there is a analogous correlated term for nonidentical particles with $\mathcal{T} \to \mathcal{M}$.

Other correlations

There are also other sources of correlations in our forlmulae. But they are suppressed by higher power of $1/N_c$:

$$\langle D(\vec{x}_1,\vec{x}_2)D(\vec{x}_3,\vec{x}_4)\rangle \approx \langle D(\vec{x}_1,\vec{x}_2)\rangle\langle D(\vec{x}_3,\vec{x}_4)\rangle + \frac{1}{N_c^2-1}\Big[\langle D(\vec{x}_1,\vec{x}_3)\rangle\langle D(\vec{x}_2,\vec{x}_4)\rangle + \langle D(\vec{x}_1,\vec{x}_4)\rangle\langle D(\vec{x}_2,\vec{x}_3)\rangle\Big]$$

Or suppressed by a power of area, i.e. $S\vec{q}^2$. This is a "classical" contribution from the region

$$|\vec{x}_1-\vec{x}_2| \sim |\vec{x}_1-\vec{x}_3| \sim |\vec{x}_1-\vec{x}_4| \sim Q_s^{-1}$$

Thus quantum statistics effects are very important!

Conclusions

Quantum staistics (quantum interference) effects are ubiquitous. For quark production they are parametrically leading source for correlations.

For gluons they are as important as "classical" effects.

Perhaps they can be observed in simpler processes. Semi inclusive DIS?

Have to think.

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