Sub-Eikonal Corrections to the Gluon Fields of a Heavy Nucleus

Douglas Wertepny

Universidade de Santiago de Compostela

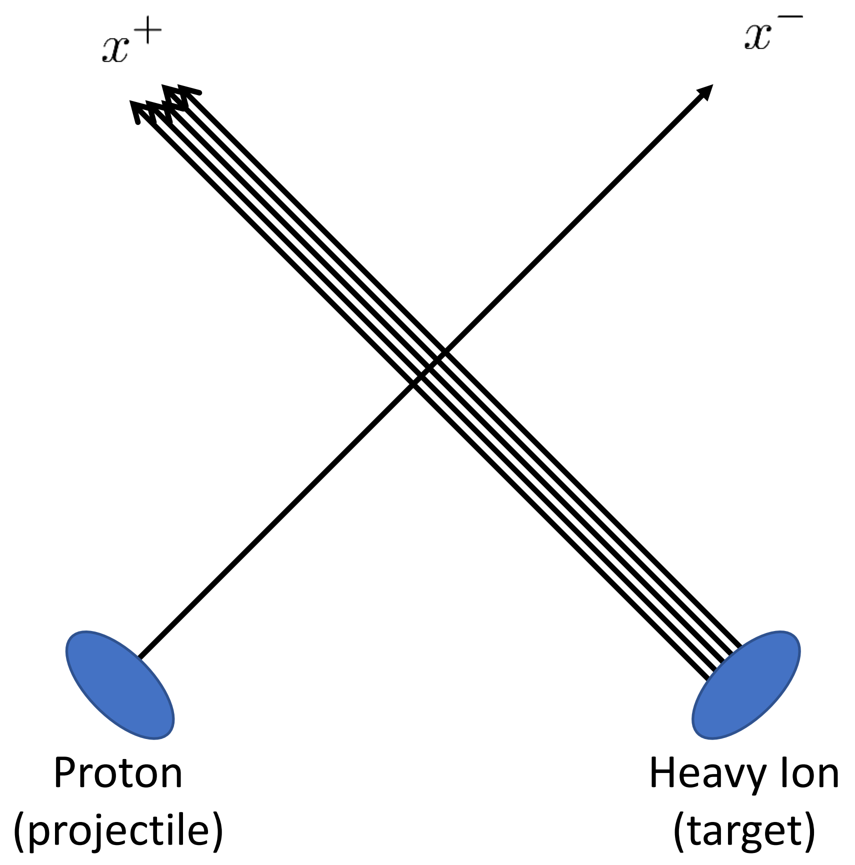
IS 2017, Krakow

Outline

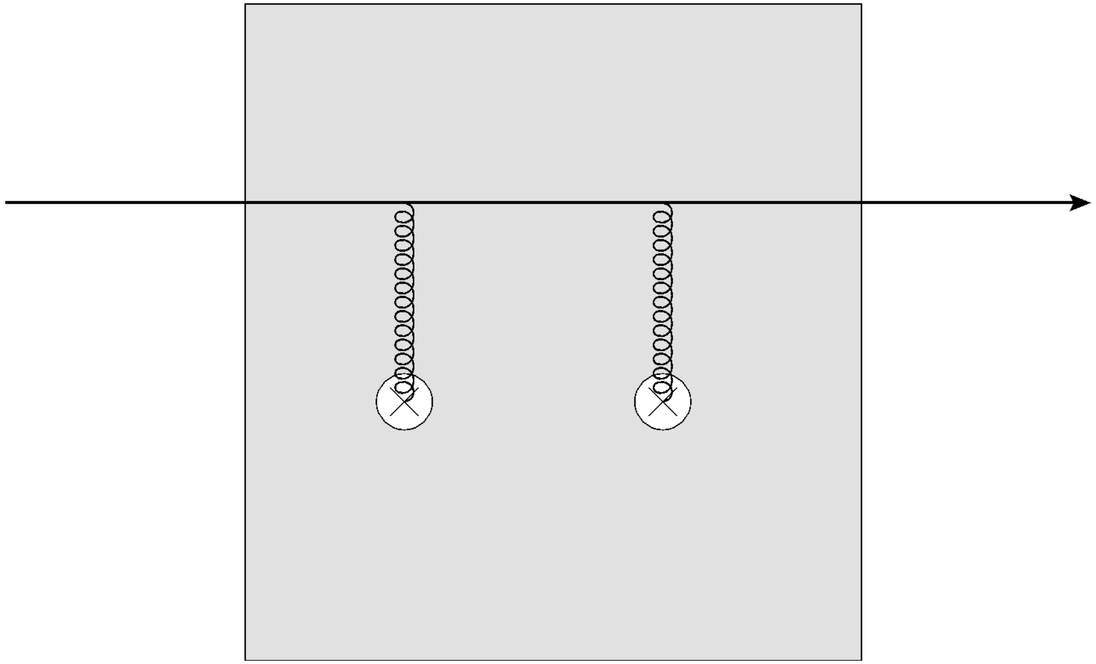
- Overview of the correction.
- Details of the formalism.
- Applying this to the Weizsäcker-Williams distribution
- Conclusions and Outlook

Overview of the Kinematics

- Example: Proton Heavy-Ion collision.
- Travelling at high momenta along the light-cone.
- Proton travels in the x^- direction.
- Heavy-nucleus travels in the x^+.
- Use the light cone gauge $A^-=0$.
- View this as the proton interacts with the gluon fields produced by the charges in the nucleus.

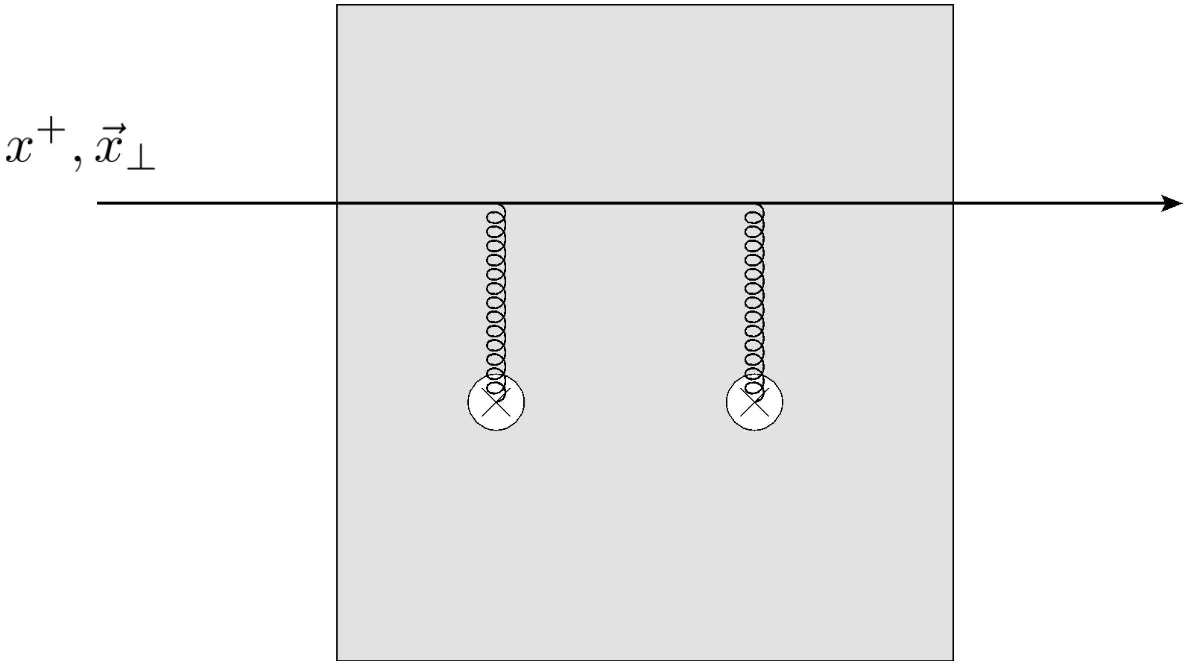


Particle Passing through a QCD Medium



- We have a particle passing though a QCD medium.
- The medium is created by the heavy-ion
- Examples: Jets, particle emission from a heavy-ion collision.
- The particle is traveling with a large momentum through a series of gluon fields.

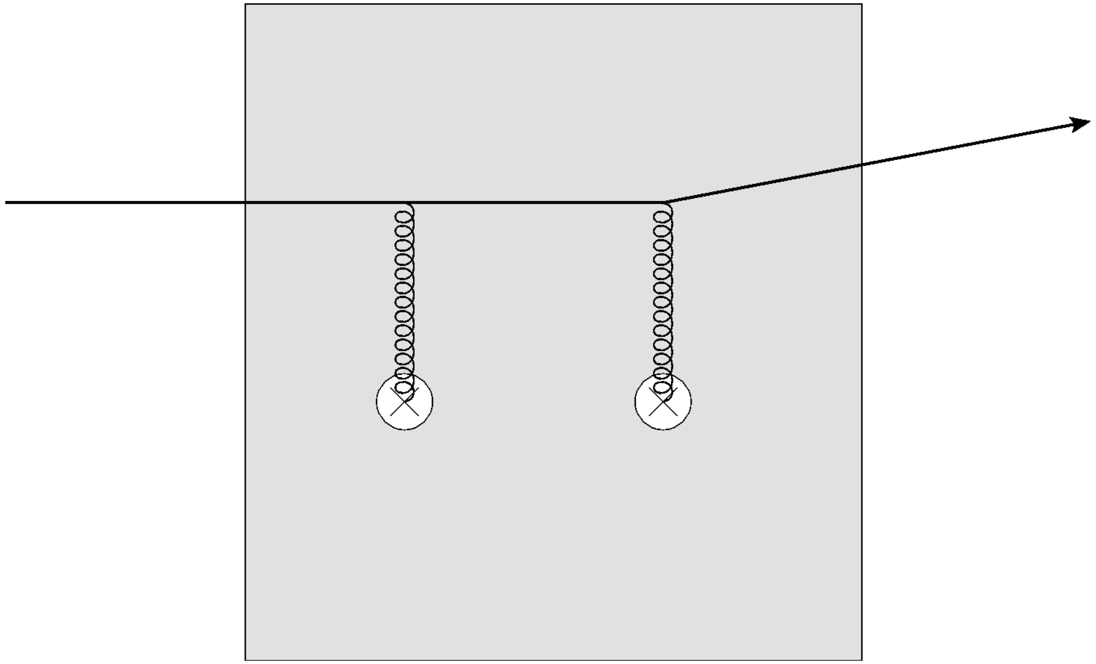
Particle Passing through a QCD Medium



- To leading order the particle does not recoil off the fields produced by the medium.
- Only introduces a color rotation.
- For a quark this is represented by a fundamental Wilson line:

$$V_{\vec{x}_\perp} = \mathcal{P} \exp\left(ig \int dx'^- t^a A^a_-(x^+, x'^-, \vec{x}_\perp) \right)$$

Particle Passing through a QCD Medium

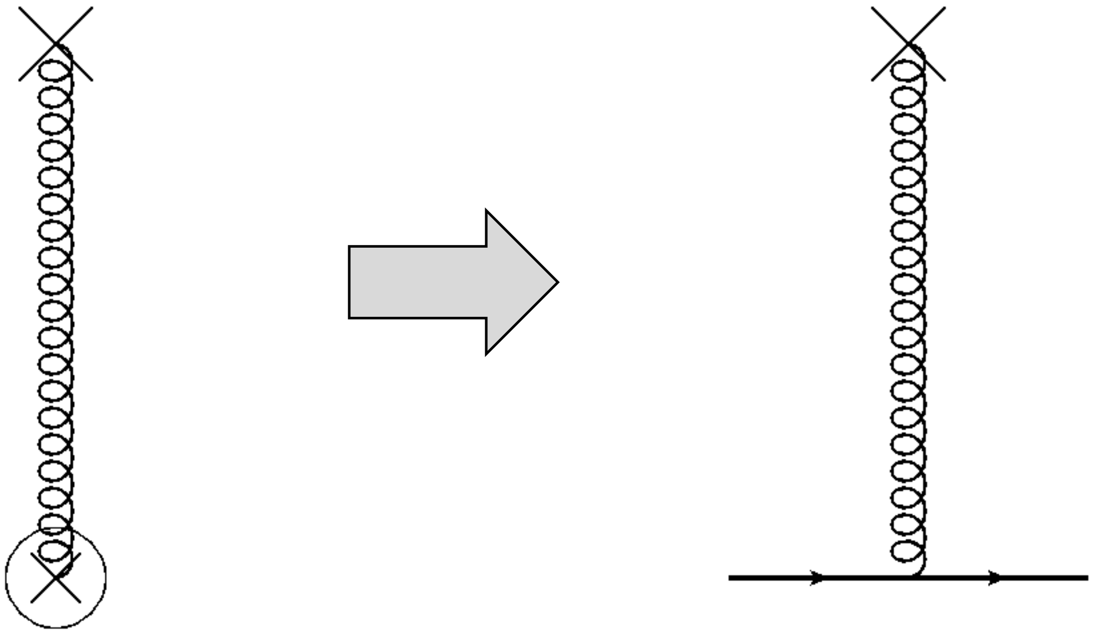


- To consider higher-order effects one often allows the particle to recoil off the fields produced by the medium.
- These corrections are proportional to the length of the medium.

Particle Passing through a QCD Medium

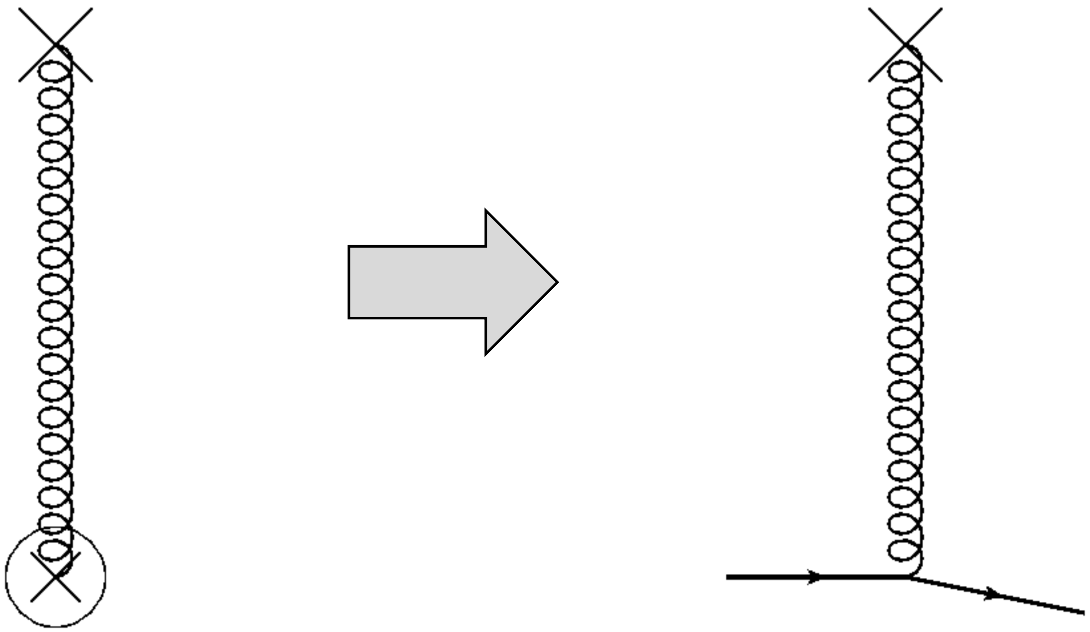
- This allows for the particle's momentum to change as it passes through the medium.
- Important for various phenomenon such as Jets and energy loss calculations.
- Notice that we did not include the effect this particle has on the medium, the back-reaction.
- How does the back-reaction effect the particle?

Modelling the Back-Reaction



- The fields originating from the medium are often modeled as classical gluon fields.
- The off-shell gluon emitted by an eikonal quark.
- A classical current that does not lose energy.
- Breaks energy conservation.

Modelling the Back-Reaction

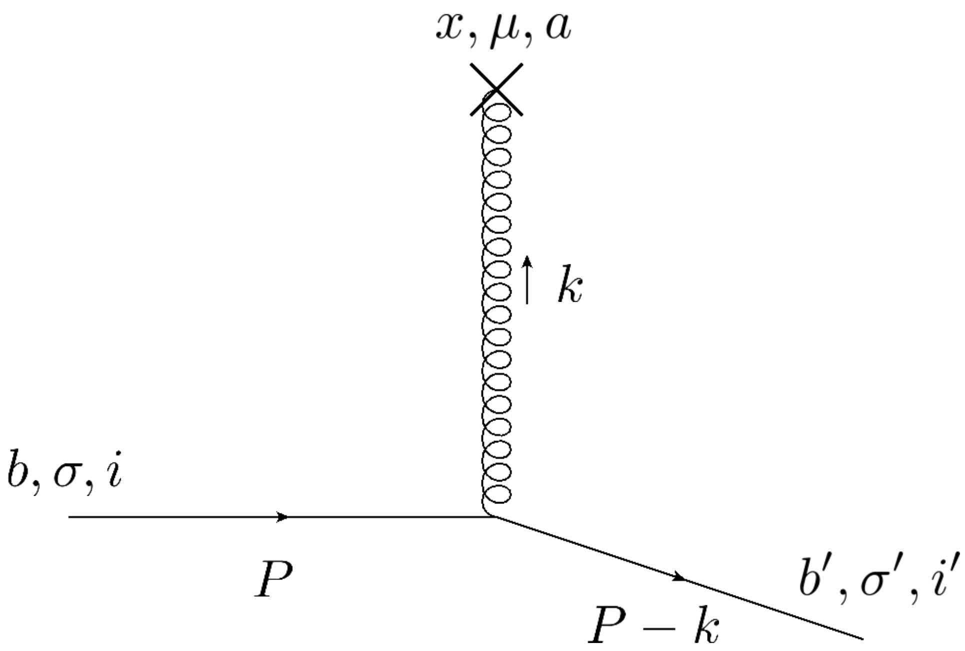


- Here we break this classical approximation.
- The source quark is allowed to recoil due to the emission of the gluon (back-reaction).
- Allows us to look at energy conserving corrections.

Goal of the Calculation

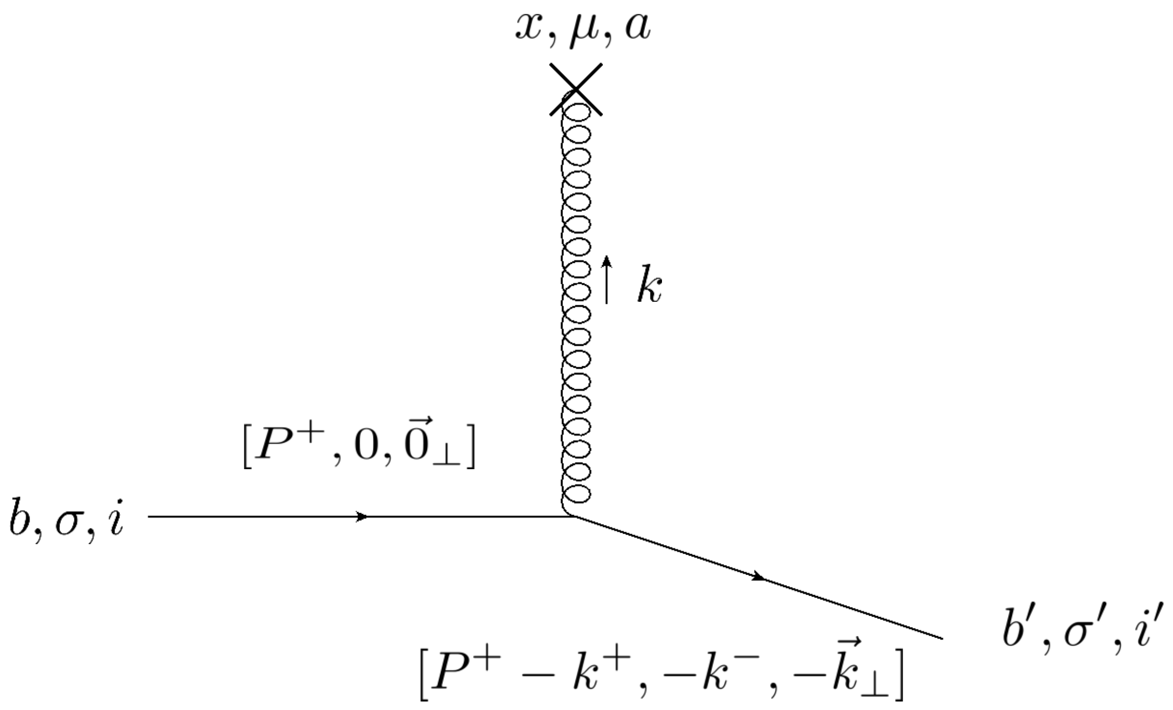
- We use this concept to model corrections to the gluon fields in the QCD medium.
- Then apply this to the Weizsäcker-Williams distribution.
- Allows us to see how the backreaction starts to effect physical processes.
- We look at the lowest order correction, ends up being proportional to the length of the medium, inversely proportional to the energy of the sources.
- Back-Reaction is just as important as the recoil of the passing particle

Field Emission - General



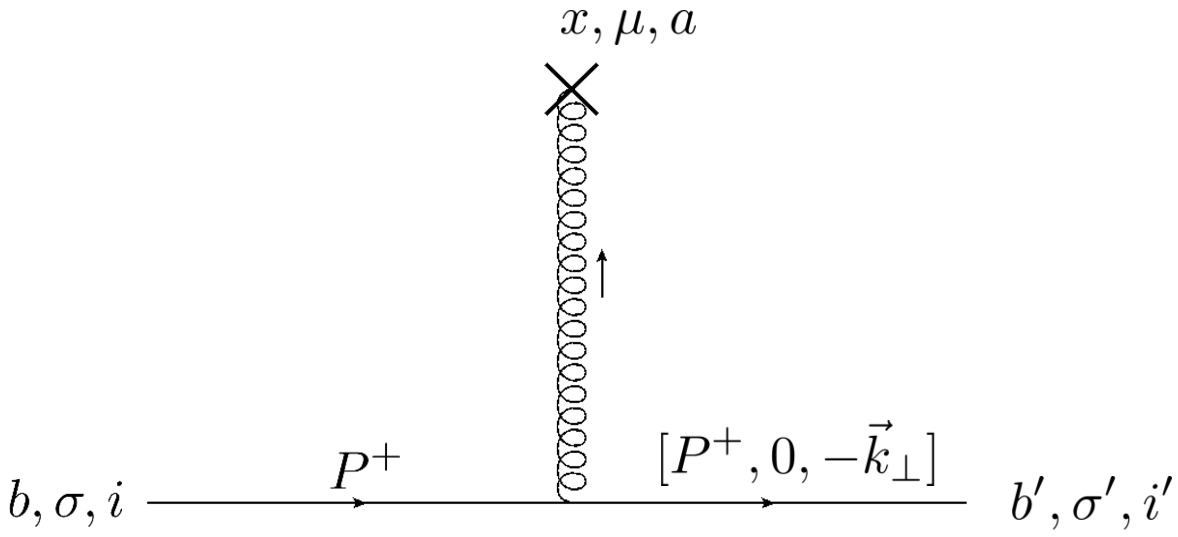
- Source quark emits an off-shell gluon.
- Allow for the source quark to recoil.
- Important Properties:
  - Mass dependence
  - Spin dependence
  - Depends on the transverse momentum of the quark
- Complex formula, major simplifications if assumptions are made.

Field Emission – Translational Invariance



- Here we assume the target nucleus is translationally invariant and the quark is massless.
- Allows us to set the transverse momentum of the source quark to zero.
- Major simplifications

Field Emission – Classical Limit



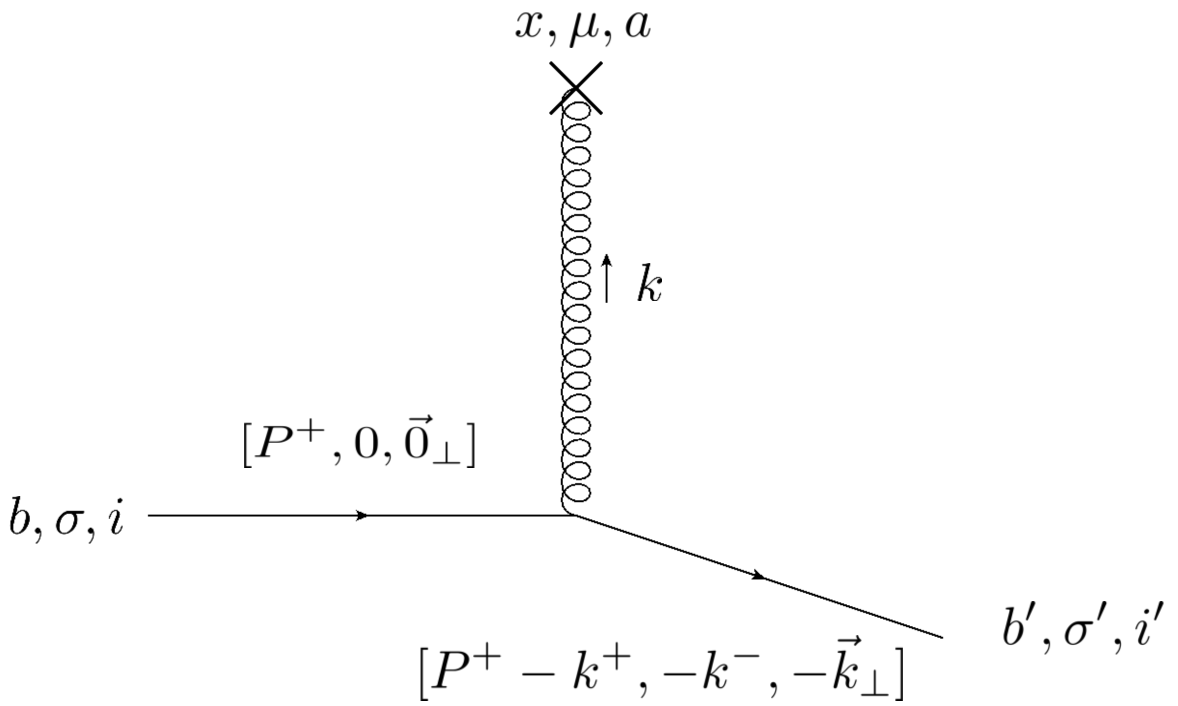
- Source quark has infinite energy (large P^+).
- Have to average over all possible source quark positions.

$$A^a_\mu(x)\Big|_{q,LO} = -\,gt^a\delta_{\sigma,\sigma'}\sqrt{T_A}\int db^-\psi(b^-)\int\frac{dP^+}{2\pi}\frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}e^{-iP^+(b'^- - b^-) - ik^+(x^- - b^-) + i\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{b}'_\perp)}\frac{1}{k^2_\perp}\eta_\mu$$

$$A^a_\mu(x)\Big|_{q,LO} = -\,gt^a\delta_{\sigma,\sigma'}\sqrt{T_A}\psi(b'^-)\delta(x^+ - b'^+)\ln\left(\frac{1}{|\vec{x} - \vec{b}'|_\perp\Lambda}\right)$$

$$\eta\cdot x = x^-$$

Field Emission – Sub-eikonal Correction

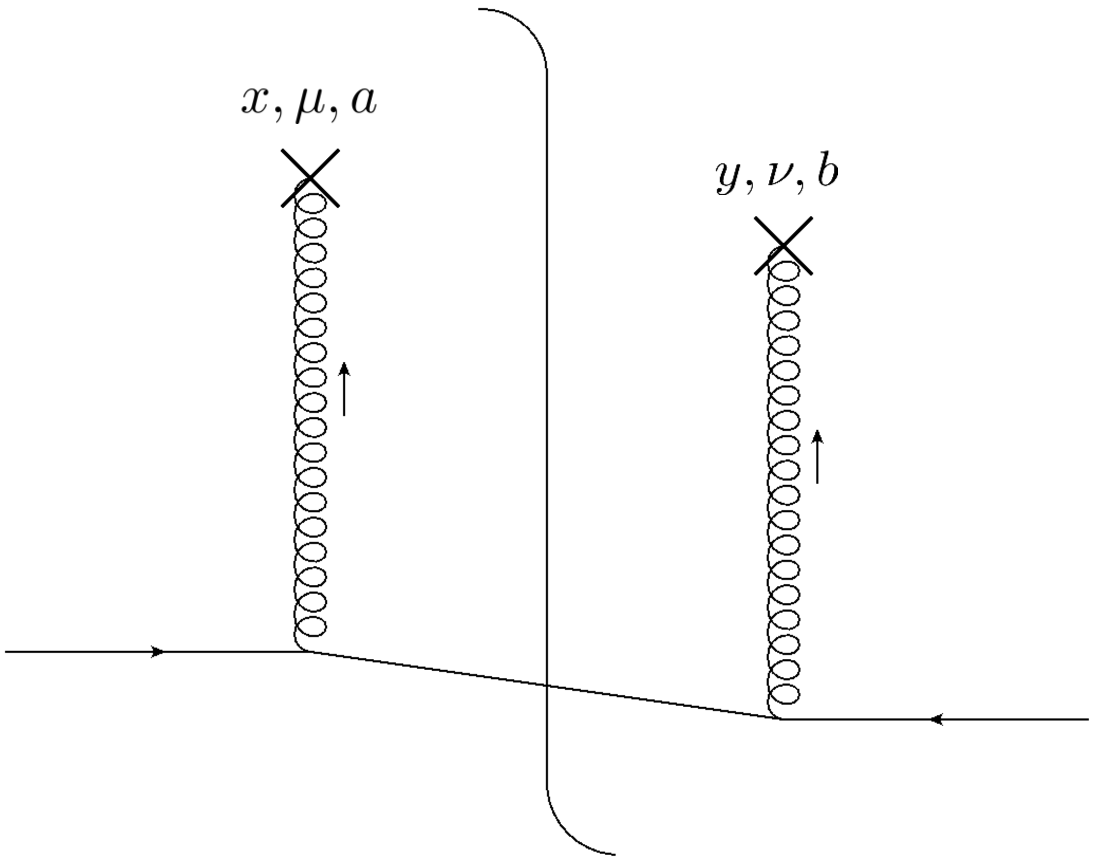


- Expand in terms of powers of the momentum of the source quark.
- Have transverse component.
- Notice that the power count varies between the contributions.

$$A^a_\mu(x)\Big|_q \approx -gt^a\delta_{\sigma,\sigma'}\sqrt{T_A}\int db^-\psi(b^-)\int\frac{dP^+}{2\pi}\frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\Theta\left(P^+-k^+\right)e^{-iP^+(b'^- - b^-)-ik^+(x^- - b^-)+i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{b}'_\perp)}$$

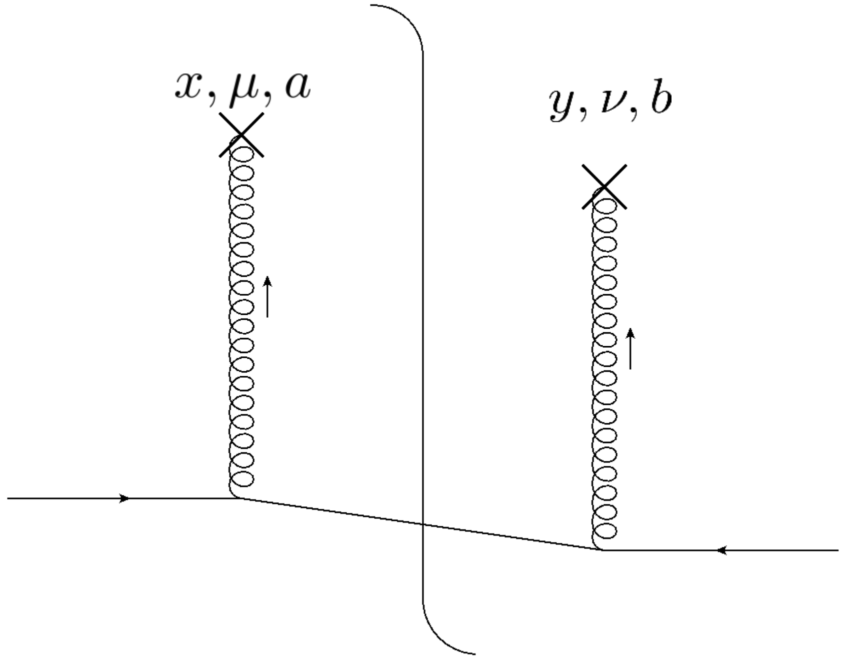
$$\times\frac{1}{k_\perp^2}\left(\left(1+\frac{k^+}{2P^+}\right)\eta_\mu-\frac{1}{2P^+}\left(\vec{g}_{\mu i,\perp}-i\sigma\epsilon_{\mu i}\right)\vec{k}_{i,\perp}\right)$$

Gaussian Averaging



- In a physical process the quark must emit at least two gluons.
- The emission of more gluon is suppressed by α_s
- This object is what is used when calculating observables.
- Here we assume $x^+ = y^+$
- This condition is used when calculating the Weizsäcker-Williams distribution.

Gaussian Averaging – Classical Limit

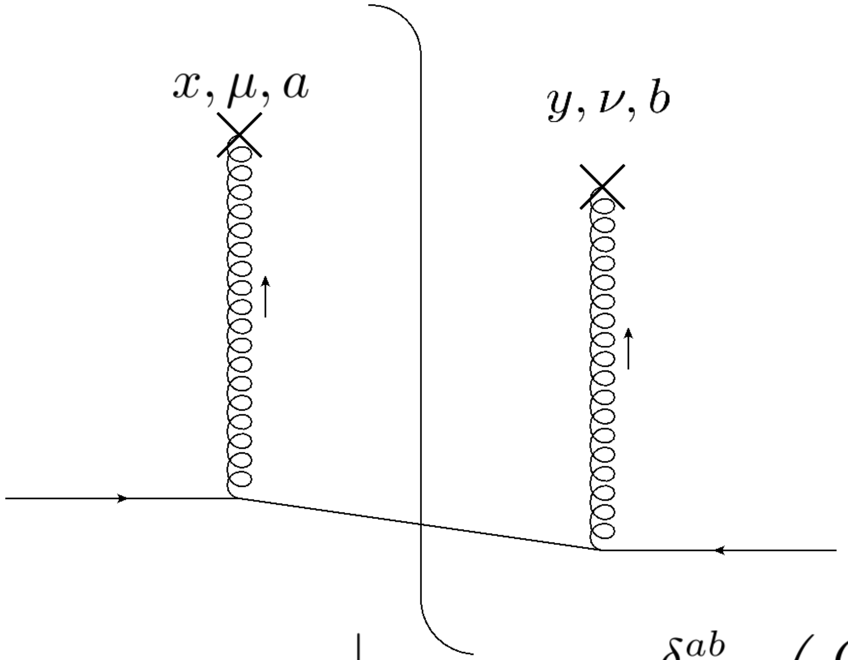


- To find the classical we summed and averaged over the spins and the colors of the quark.
- This is known as the MV model.

$$\left\langle A^a_\mu(x)\mathcal{A}^b_\nu(y)\right\rangle\Big|_{LO} = \frac{\delta^{ab}}{N_C\, g^2}\eta_\mu\eta_\nu\left(\frac{Q_s^2}{2\Lambda^2} - \Gamma(|\vec{x}-\vec{y}|_\perp)\right)\int db^-|\psi(b^-)|^2\delta(x^- - b^-)\delta(y^- - b^-)$$

$$\Gamma(r_\perp) = \frac{1}{4}Q_s^2 r_\perp^2 \ln\left(\frac{1}{r_\perp\Lambda}\right)$$

Gaussian Averaging – Sub-Eikonal Corrections



- This has sub-eikonal corrections.

$$\psi_1(b^-) = \int \frac{dP^+}{2\pi} \frac{1}{P^+} \phi(P^+) e^{-iP^+ b^-}$$

$$\left\langle A^a_-(x) \mathcal{A}^b_-(y) \right\rangle \Big|_{LO+NLO} = \frac{\delta^{ab}}{N_C\, g^2} \left(\frac{Q_s^2}{2\Lambda^2} - \Gamma(|\vec{x} - \vec{y}|_\perp) \right)$$
$$\int db^- \left(\left(|\psi(b^-)|^2 - \frac{1}{2} \partial_{b^-}\, Re \left[i\psi_1(b^-)\psi^*(b^-) \right] \right) \right.$$
$$\left. + \frac{i}{2} Re \left[\psi_1(b^-)\psi^*(b^-) \right] \left(-\partial_{x^-} + \partial_{y^-} \right) \right) \delta(x^- - b^-) \delta(y^- - b^-)$$

Gaussian Averaging - Sub-Eikonal Corrections

- The sub-eikonal correction also allows for the production of transverse fields and thus we have the following objects as well.

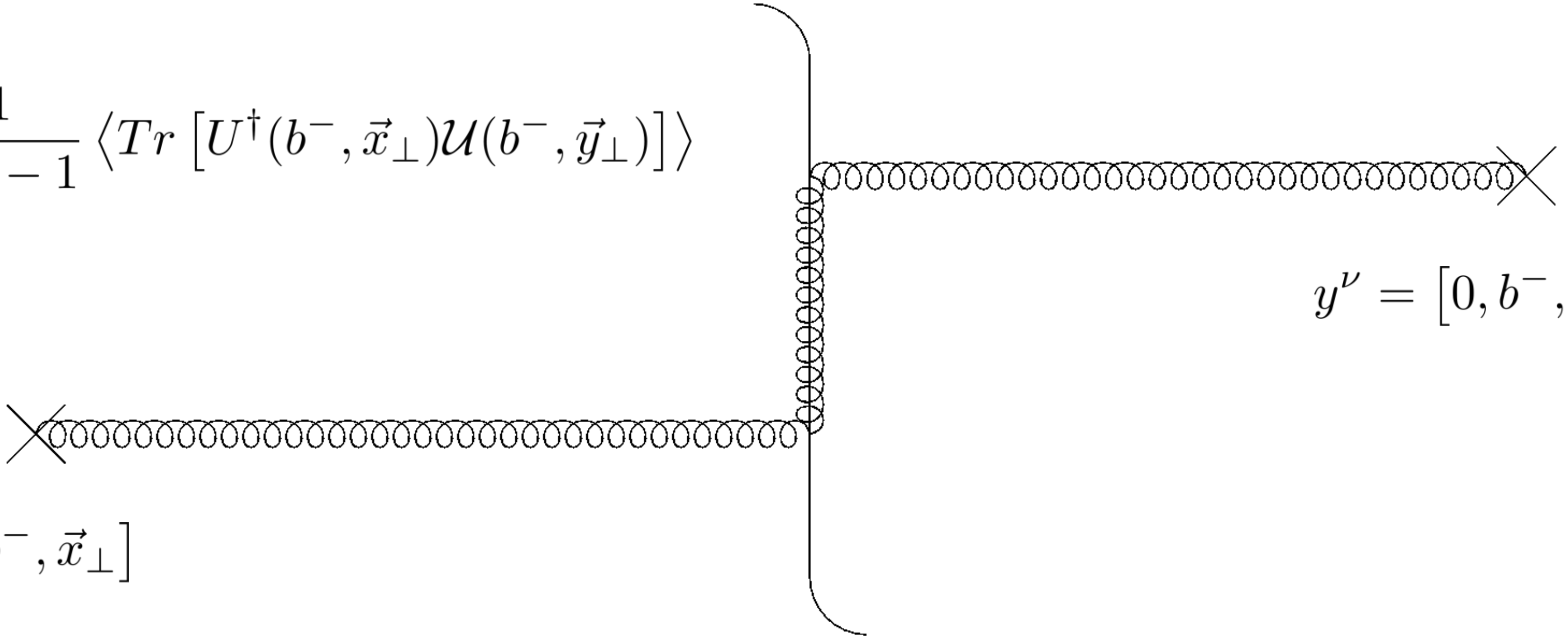
$$\left\langle A^a_-(x)\vec{\mathcal{A}}^b_{i,\perp}(y)\right\rangle\Big|_{NLO} = \frac{\delta^{ab}}{N_C\, g^2}\left(-\vec{\nabla}_{i,\vec{x}_\perp}\Gamma(|\vec{x}-\vec{y}|_\perp)\right)$$
$$\int db^-\left(Re\left[i\,\psi_1(b^-)\psi^*(b^-)\right] + i\,Re\left[\psi_1(b^-)\psi^*(b^-)\right]\right)\frac{1}{2}\delta(x^- - b^-)\delta(y^- - b^-)$$

$$\left\langle \vec{A}^a_{i,\perp}(x)\mathcal{A}^b_-(y)\right\rangle\Big|_{NLO} = \frac{\delta^{ab}}{N_C\, g^2}\left(-\vec{\nabla}_{i,\vec{y}_\perp}\Gamma(|\vec{x}-\vec{y}|_\perp)\right)$$
$$\int db^-\left(Re\left[i\,\psi_1(b^-)\psi^*(b^-)\right] - i\,Re\left[\psi_1(b^-)\psi^*(b^-)\right]\right)\frac{1}{2}\delta(x^- - b^-)\delta(y^- - b^-)$$

Gluon Dipole

- Now we use the previous results to analyze the gluon dipole.
- Modify the usual definition so that we consider that gluon fields could be on either side of the cut.

$$S_G(b^-, |\vec{x} - \vec{y}|_\perp) = \frac{1}{N_C^2 - 1} \left\langle Tr \left[U^\dagger(b^-, \vec{x}_\perp) \mathcal{U}(b^-, \vec{y}_\perp) \right] \right\rangle$$



$$y^\nu = \left[0, b^-, \vec{y}_\perp\right]$$

$$x^\mu = \left[0, b^-, \vec{x}_\perp\right]$$

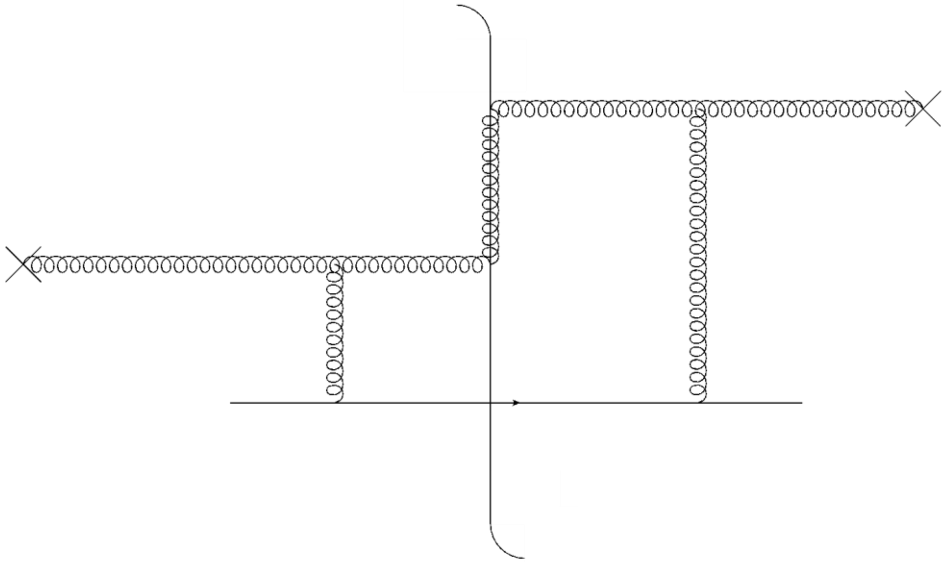
Gluon Dipole - Expansion

- To get an idea of how this is analyzed we expand the Wislon lines out

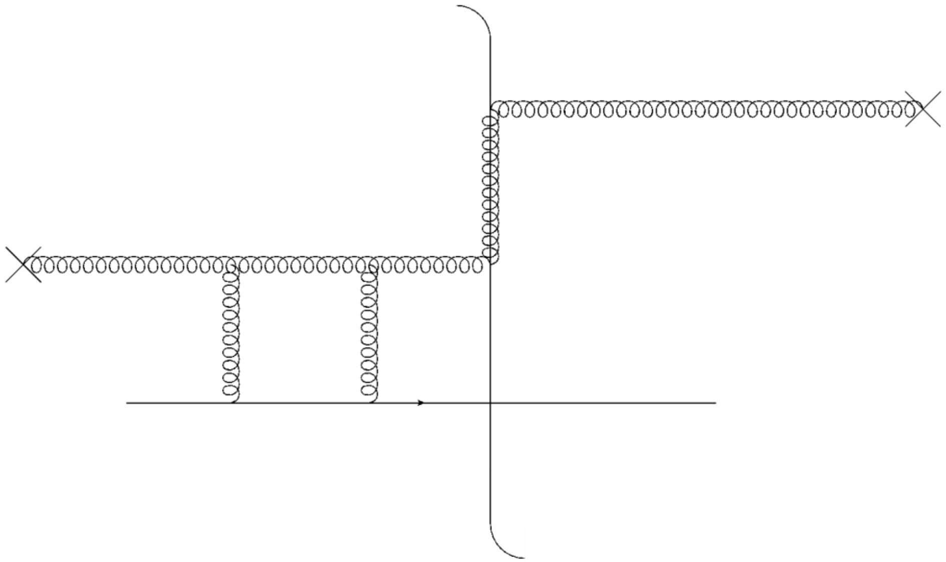
$$
\begin{aligned}
S_G(b^-, |\vec{x}-\vec{y}|_\perp) =& 1 + \frac{N_C\, g^2}{N_C^2-1}\delta^{ab}\left(\int_{b^-}^{\infty} dx^- \int_{b^-}^{\infty} dy^- \left\langle A_-^a(x^-,\vec{x}_\perp)\mathcal{A}_-^b(y^-,\vec{y}_\perp)\right\rangle\right.\\
&- \int_{b^-}^{\infty} dx_1^- \int_{b^-}^{x_1^-} dx_2^- \left\langle A_-^a(x_1^-,\vec{x}_\perp)A_-^b(x_2^-,\vec{x}_\perp)\right\rangle\\
&\left.- \int_{b^-}^{\infty} dy_1^- \int_{b^-}^{x_1^-} dy_2^- \left\langle \mathcal{A}_-^a(y_1^-,\vec{y}_\perp)\mathcal{A}_-^b(y_2^-,\vec{y}_\perp)\right\rangle\right) + ...
\end{aligned}
$$

Types of Scatterings

- We have two basic types of scattering



$$\left\langle A_-^a(x^-,\vec{x}_\perp)A_-^b(y^-,\vec{y}_\perp)\right\rangle$$



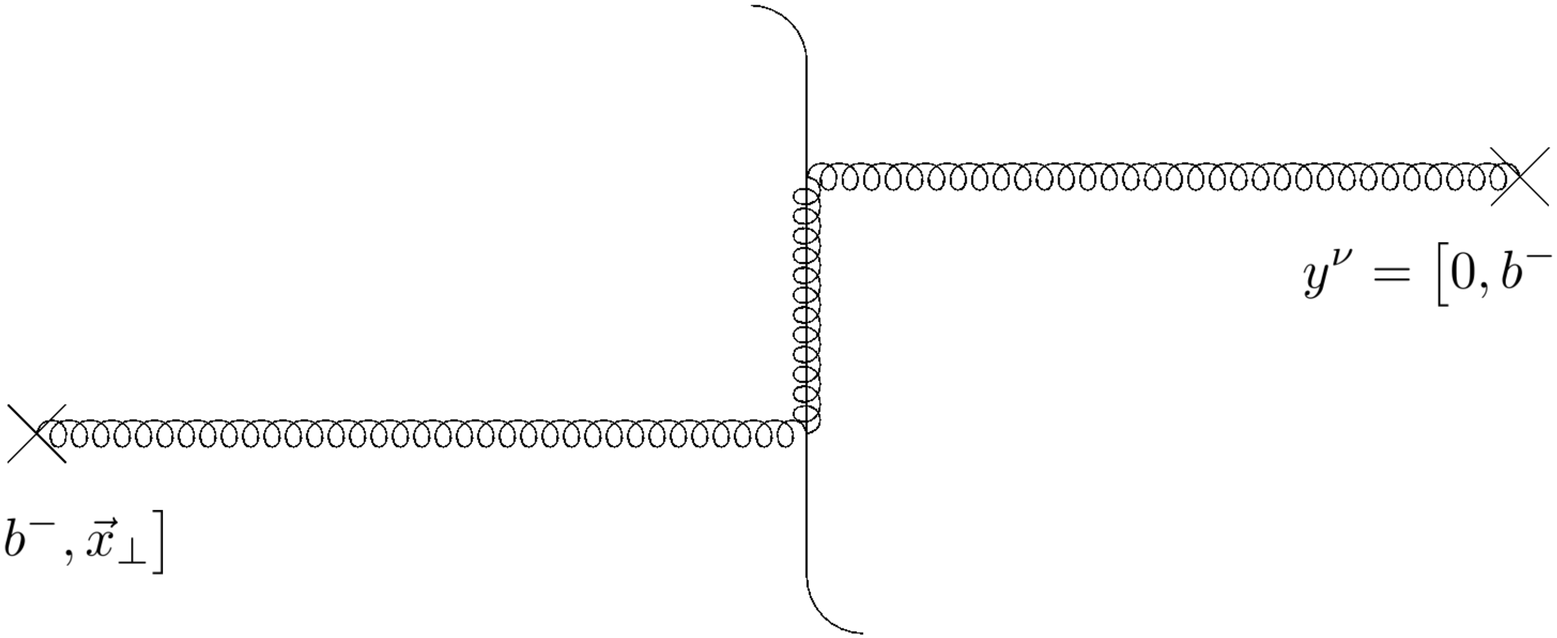
$$\left\langle A_-^a(x_1^-,\vec{x}_\perp)A_-^b(x_2^-,\vec{x}_\perp)\right\rangle$$

- In the classical case, the diagram to the right just cancels out the divergences in the diagram to the left. Assume that remains true.

Gluon Dipole – Final Result

- It can be show that one gets the final result.

$$S_G(b^-, |\vec{x}-\vec{y}|_\perp) = \exp\left(-\Gamma(|\vec{x}-\vec{y}|_\perp)\int_{b^-}^{\infty} dz^- \left(|\psi(z^-)|^2 - \frac{1}{2}\partial_{z^-} Re[i\psi_1(z^-)\psi^*(z^-)]\right)\right)$$



Gluon Dipole – Limits

- In the classical limit the medium is just a delta function (here we choose it to be at $\tau^- = 0$).
- The gluon dipole just becomes

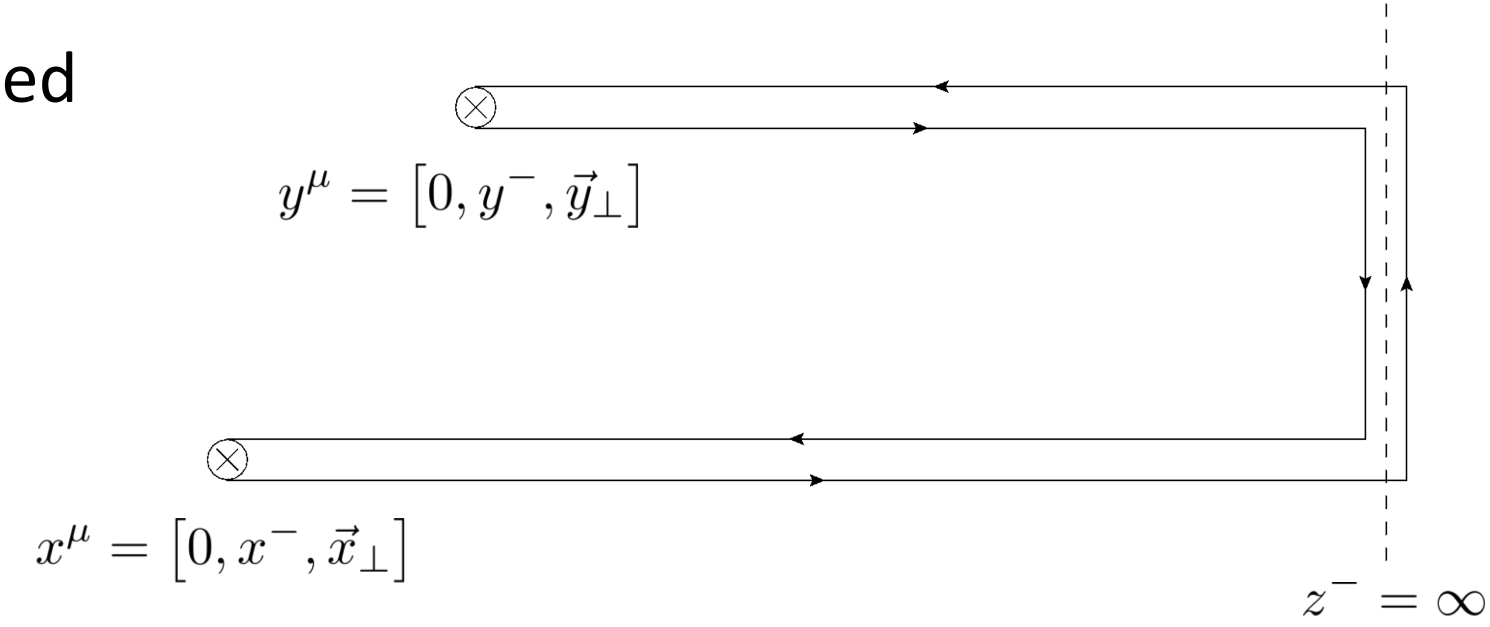
$$S_G(b^-, |\vec{x}-\vec{y}|_\perp) = \exp\left(-\Gamma(|\vec{x}-\vec{y}|_\perp)\Theta(-b^-)\right)$$

- Now if we take b^- to $-\infty$ we get the usual result.

$$S_G(|\vec{x}-\vec{y}|_\perp) = \exp\left(-\Gamma(|\vec{x}-\vec{y}|_\perp)\right)$$

Weizsäcker-Williams Distribution

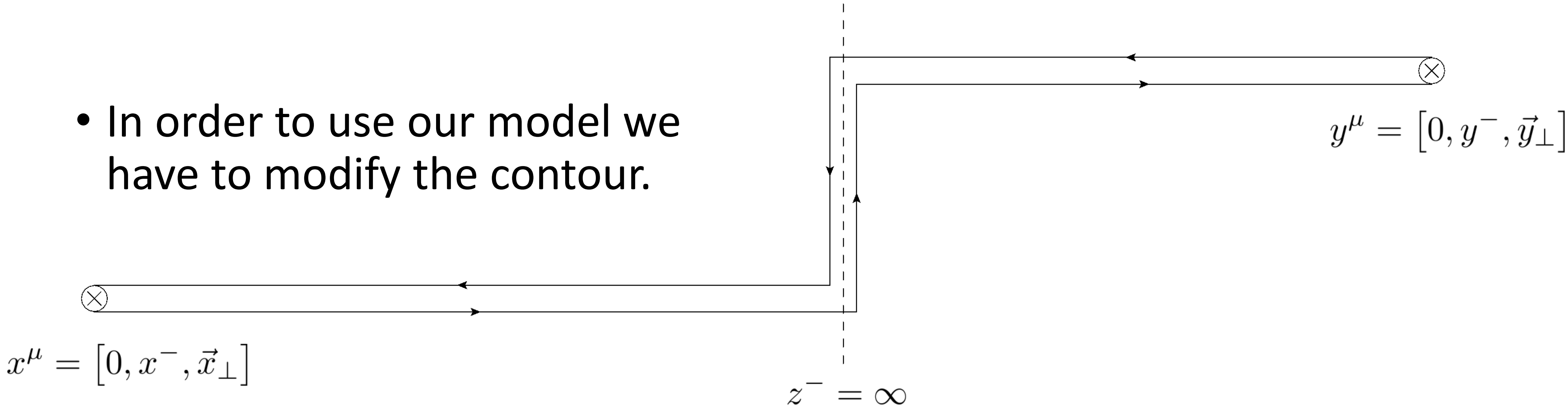
- Now lets apply the described model to the WW distribution.
- It is defined as such



$$\phi^{WW}(x, k_\perp^2) = \frac{(k^+)^2}{2\pi^2} \int dx^- d^2x_\perp dy^- d^2y_\perp e^{-ik^+(x^- - y^-) + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{y}_\perp)} \int_{x^-}^{\infty} dx'^- \int_{y^-}^{\infty} dy'^-$$
$$\times \langle A| \, tr \left[S(x', \vec{x}_\perp) F_{-i}(x', \vec{x}_\perp) S^{-1}(x', \vec{x}_\perp) S(y', \vec{y}_\perp) F_{-i}(y', \vec{y}_\perp) S^{-1}(y', \vec{y}_\perp) \right] |A\rangle$$

Weizsacker-Williams Distribution

- In order to use our model we have to modify the contour.



$$\phi^{WW}(x, k_\perp^2) = \frac{(k^+)^2}{2\pi^2} \int dx^- d^2x_\perp dy^- d^2y_\perp e^{-ik^+(x^- - y^-) + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{y}_\perp)} \int_{x^-}^{\infty} dx'^- \int_{y^-}^{\infty} dy'^-$$
$$\times \sum_X \left\langle A \left| \left[S(x', \vec{x}_\perp) F_{-k}(x', \vec{x}_\perp) S^{-1}(x', \vec{x}_\perp) \right]^{ij} \right| X \right\rangle$$
$$\left\langle X \left| \left[S(y', \vec{y}_\perp) F_{-k}(y', \vec{y}_\perp) S^{-1}(y', \vec{y}_\perp) \right]^{ji} \right| A \right\rangle$$

Weizsäcker-Williams Distribution

- Using the model outline in this talk we arrive at:

$$\phi^{WW}_{NLO}(x,k_\perp^2) = \frac{1}{(2\pi)^3}\frac{C_F}{\alpha_s}\int db^2\, dr^2\, e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\nabla_\perp^2\Gamma(r_\perp)\right)$$
$$\times\int db^-\left(|\psi(b^-)|^2 - \frac{1}{2}\partial_{b^-}Re[i\psi_1(b^-)\psi^*(b^-)] + 2k^+Re[\psi_1(b^-)\psi^*(b^-)]\right)S_G(b^-,|\vec{x}-\vec{y}|_\perp)$$

- While this looks complicated this actually can be written in a much more intuitive form.

Final Result

- The final result can be written as

$$\phi^{WW}_{NLO}(x,k_\perp^2) = \frac{1}{(2\pi)^3}\frac{C_F}{\alpha_s}\int db^2\, dr^2\, e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\frac{1}{r_\perp^2}\left(N(r_\perp) + x\left\langle\frac{P_N^+}{P^+}\right\rangle\Gamma(r_\perp)(1+S(r_\perp))\right)$$

- Where

$$\left\langle\frac{P_N^+}{P^+}\right\rangle \sim A^{\frac{1}{3}} \qquad \Gamma(r_\perp) = \frac{1}{4}Q_s^2 r_\perp^2 \ln\left(\frac{1}{r_\perp\Lambda}\right) \qquad N(r_\perp) = 1 - S(r_\perp) \qquad S(r_\perp) = e^{-\Gamma(r_\perp)}$$

- We can see that including back-reaction gives a contribution that is enhanced by the size of the nucleus.

Conclusions and Future Directions

- We examined the effect including back-reaction had on the gluon fields produced by the quark sources.
- Specifically used this to model the first order back-reaction correction to the Weizsäcker-Williams distribution.
- Found that the correction is enhanced by the length of the medium.

- In the future we need to examine these corrections in the context of some physical process.
- This will help gain a greater understand of exactly what these corrections imply.