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SEIT 1386

Pre-equilibrium dynamics of the CME and anomalous transport

Niklas Mueller

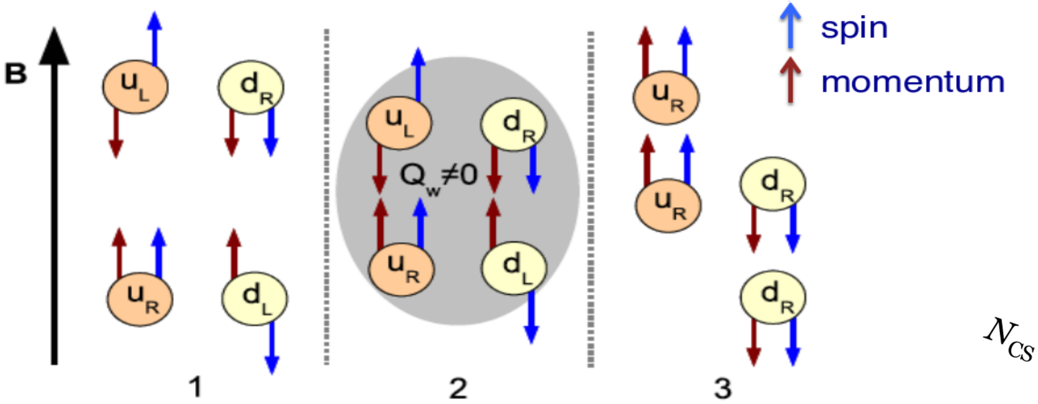
with J. Berges, M. Mace, S. Schlichting, S. Sharma, N.Tanji, R. Venugopalan

4th International Conference on the Initial Stages in High-Energy Nuclear Collisions

Cracow, Poland
Sept. 20th, 2017

Motivation

Chiral Magnetic Effect – Synonymous for a wide class of P- and CP-odd phenomena



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Nucl. Phys., A803:227–253, 2008
Fukushima, Kharzeev, Warringa
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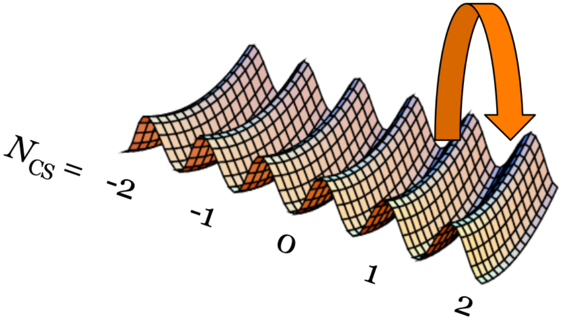

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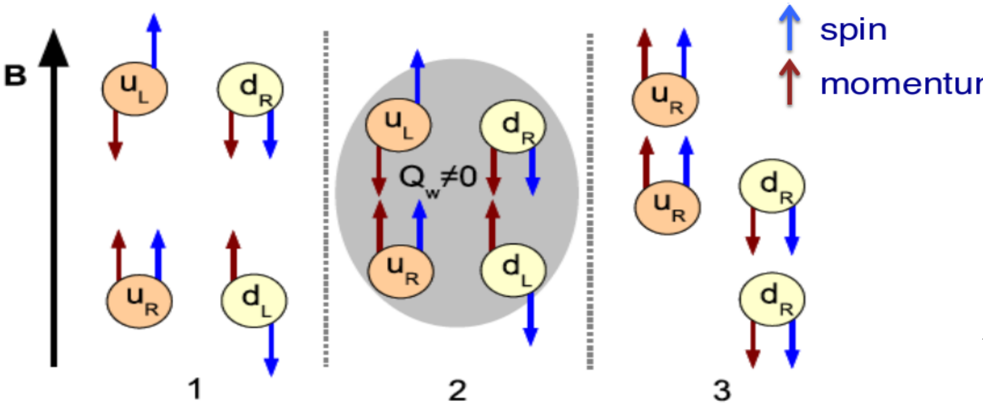

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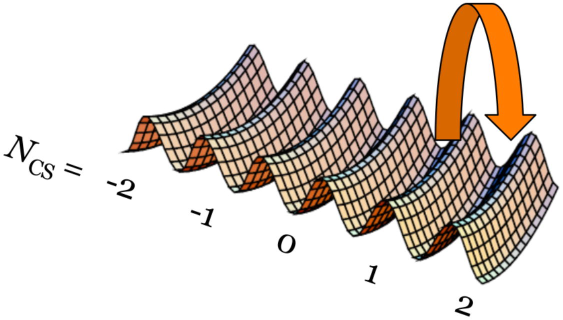
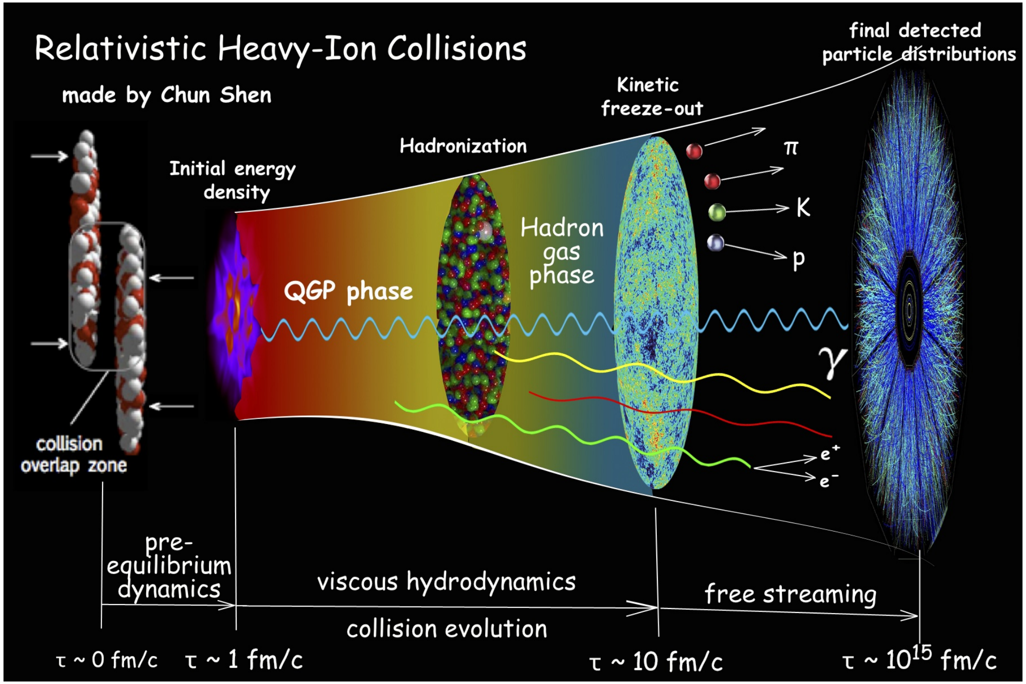


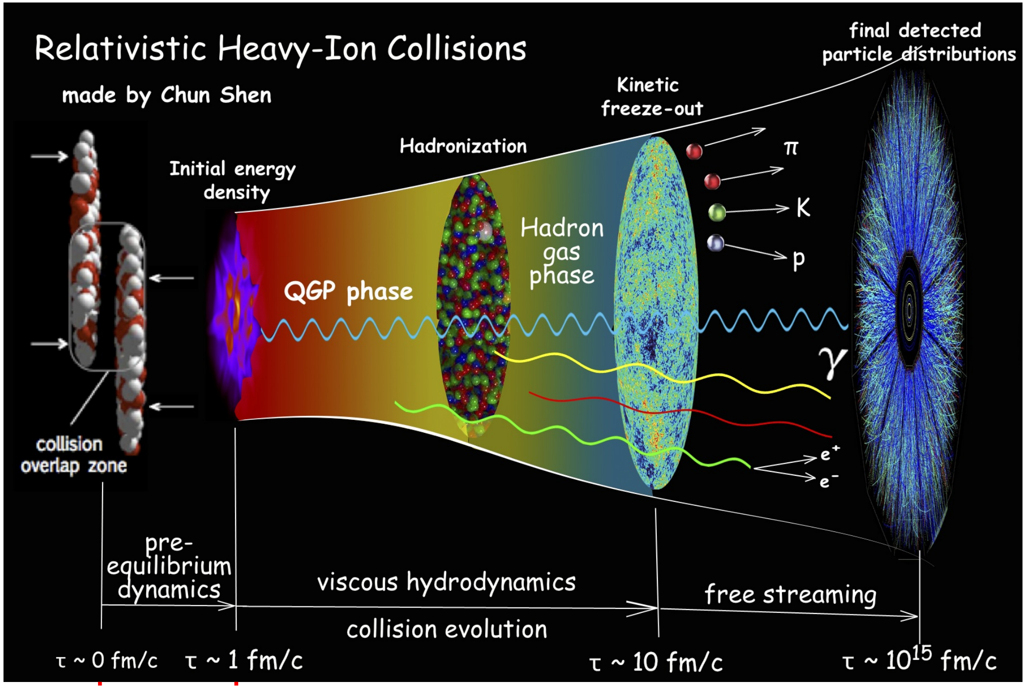
The issue in QCD experiments:

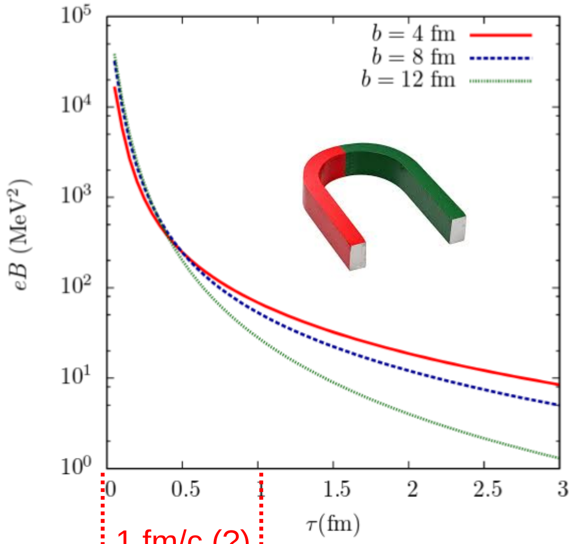
- Cannot measure P-odd effects directly
- Correlations of P-odd things are P-even, ***and so are many other things.***

Motivation


Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
QGP phase
Hadron gas phase
π
K
p
γ
e^+
e^-
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^{15} fm/c

Motivation

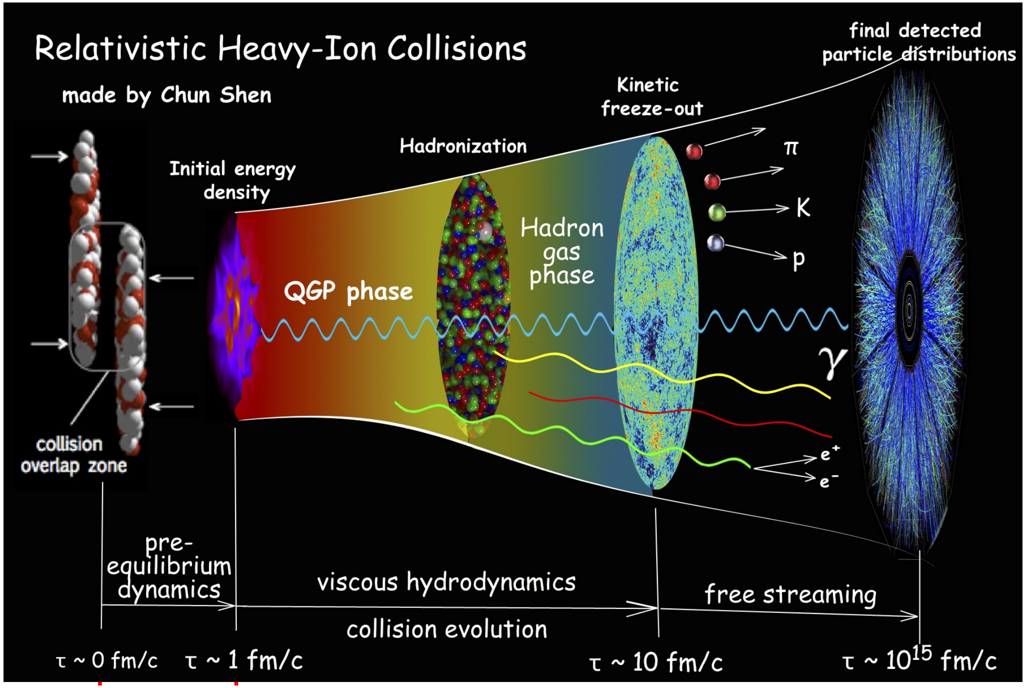


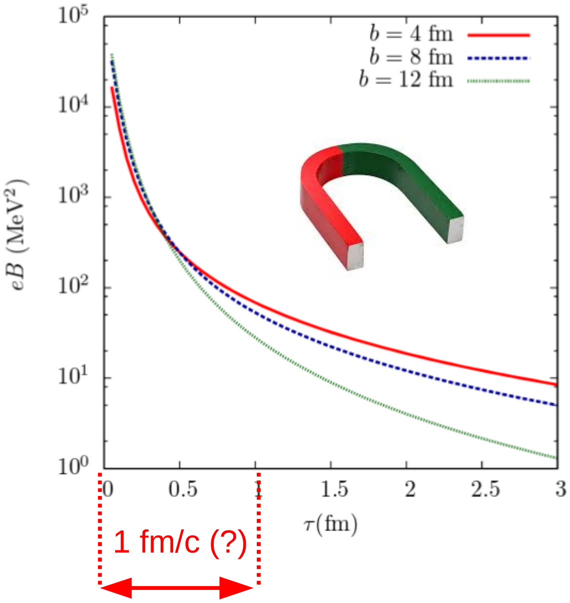


1 fm/c (?)

this talk!

Motivation





this talk!

Early times: More than just initial conditions for hydro

“no entropy production from P-odd anomalous terms”

Motivation

Correlations of charged particles

$$
\boxed{\begin{aligned}\gamma &\equiv \langle \cos(\phi_a + \phi_b - 2\Psi_{RP})\rangle = \langle \cos\Delta\phi_a \cos\Delta\phi_b\rangle - \langle \sin\Delta\phi_a \sin\Delta\phi_b\rangle \\ &= (\langle v_{1,a} v_{1,b}\rangle - \langle a_a a_b\rangle) + B_{in} - B_{out} \approx -\langle a_a a_b\rangle + B_{in} - B_{out}\,,\end{aligned}}
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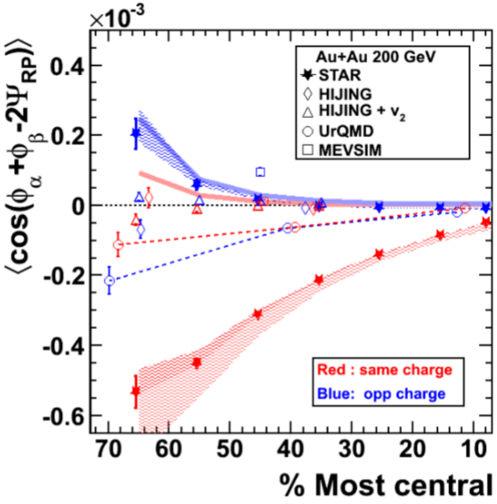
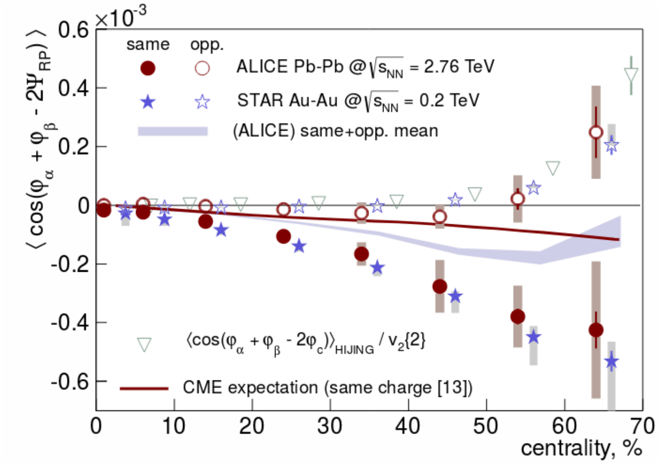
STAR PRL 103, 251601 (2009)



ALICE PRL 110 (2013) 012301



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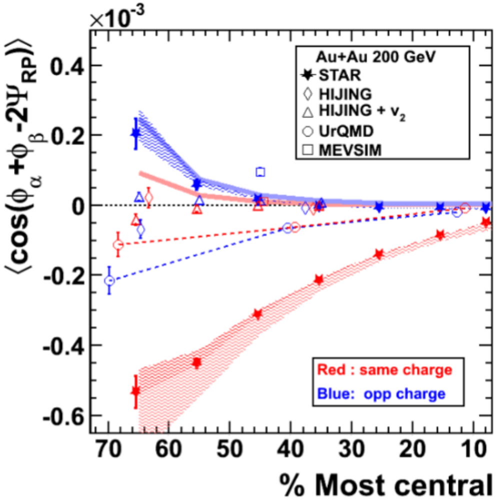

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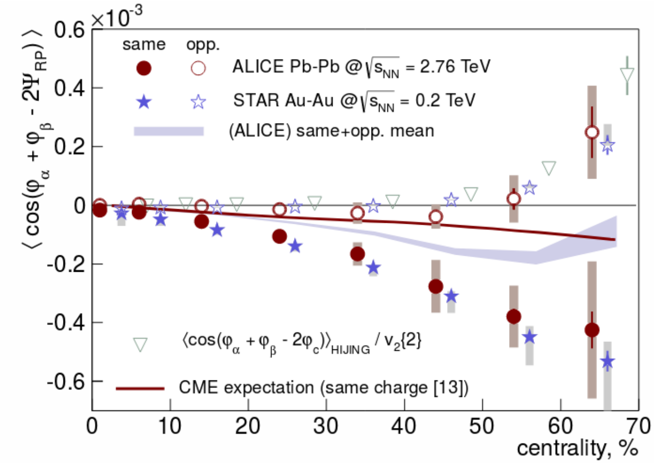
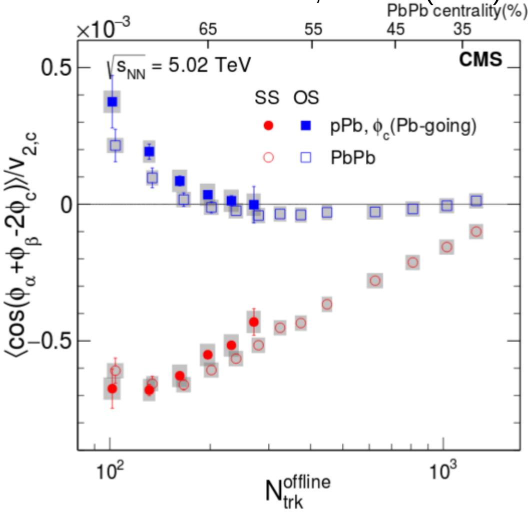


CMS PRL 118, 122301 (2017)



"...much, if not all of the original signal reported 2009 could arise from effects unrelated to the CME..."

"CME task force report", Skokov et al arxiv:1608.00982; & Kharzeev et al, Prog. Part. Nucl. Phys. 88 (2016) 1

talk by P. Tribedy

talk by V. Skokov

Motivation

Background

- **from local charge conservation at freeze-out**
- **elliptic and radial flow**

(Pratt et al, arXiv:1002.1758, Schlichting et al, PRC83 014913, Hori et al, arXiv:1208.0603, Voloshin et al, NPA 931, 992 ….)

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What to do about it?

- Beam Energy Scan II
- Isobar collisions

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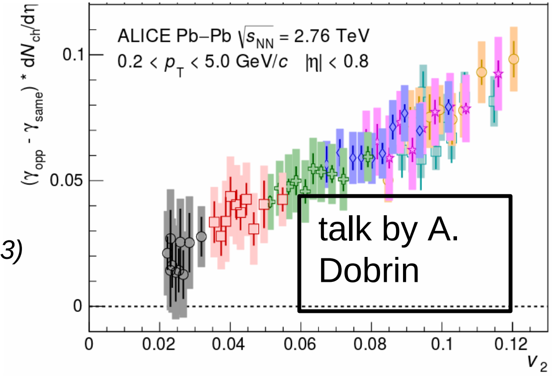
What to do about it?

- **Beam Energy Scan II**
- **Isobar collisions**
- **better and more differential observables:**

- Charge asymmetry of elliptic flow *(ALICE PRL 114, 252302 (2015); PRC 93, 044903)*
- 3-particle correlations & event shape engineering, (ALICE, arXiv:1709.04723)
- Hyperon polarization

talk by I. Karpenko



talk by A. Dobrin

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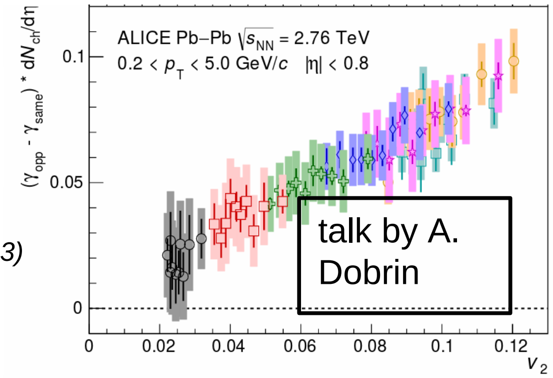

talk by A. Dobrin

Must improve the theoretical understanding !

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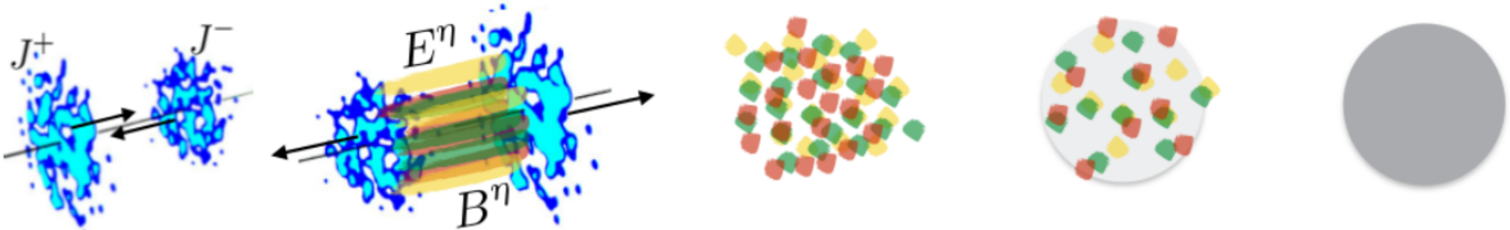
theory overview

1. Introduction



S.Schlichting

generation of anomalous axial and vector currents

strong magnetic fields present

?

transport and interactions with medium

hadronization, freeze-out

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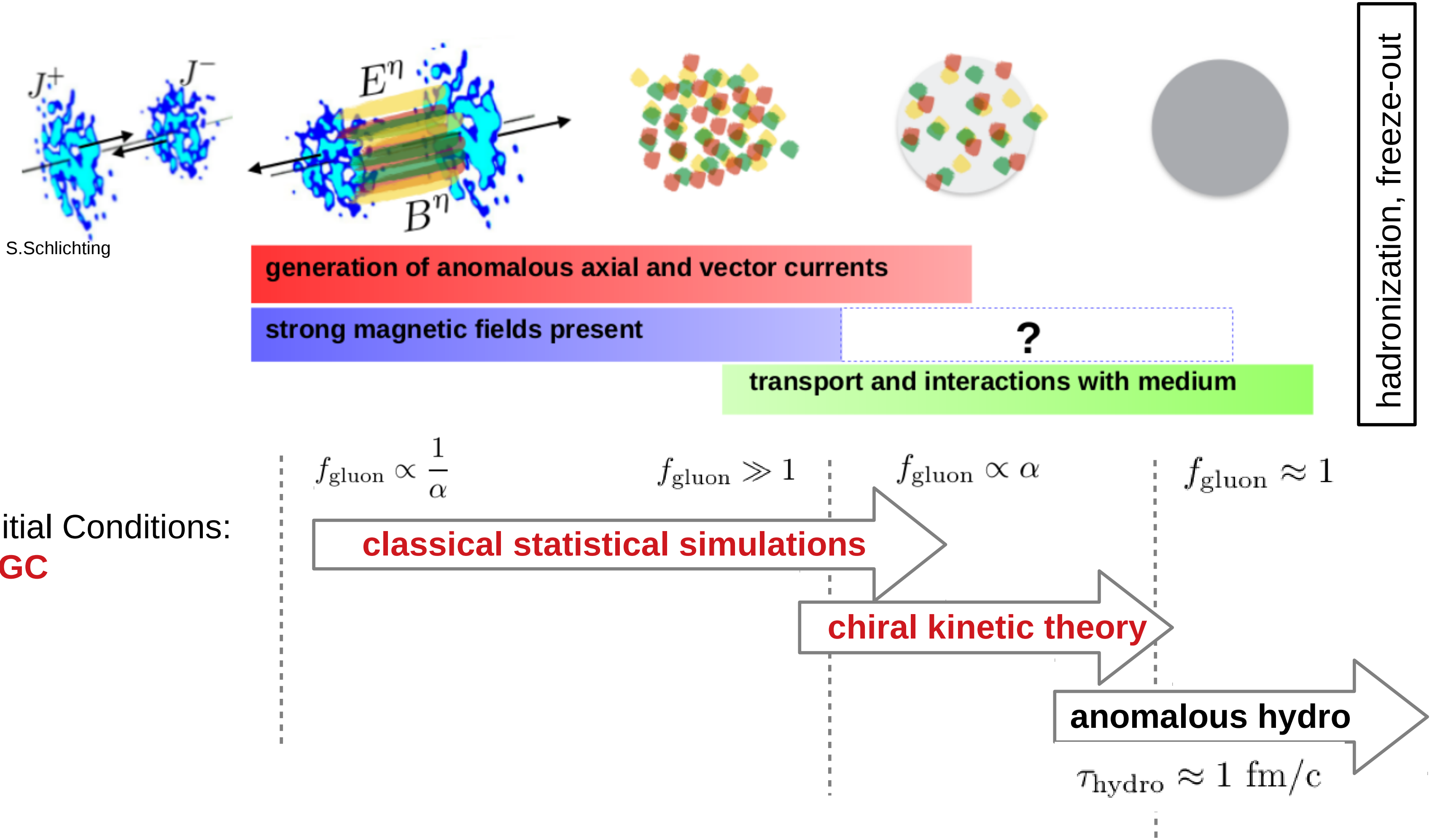

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$f_{\text{gluon}} \propto \frac{1}{\alpha}$

$f_{\text{gluon}} \gg 1$

$f_{\text{gluon}} \propto \alpha$

$f_{\text{gluon}} \approx 1$

Initial Conditions:
CGC

classical statistical simulations

chiral kinetic theory

anomalous hydro

$\tau_{\text{hydro}} \approx 1 \text{ fm/c}$

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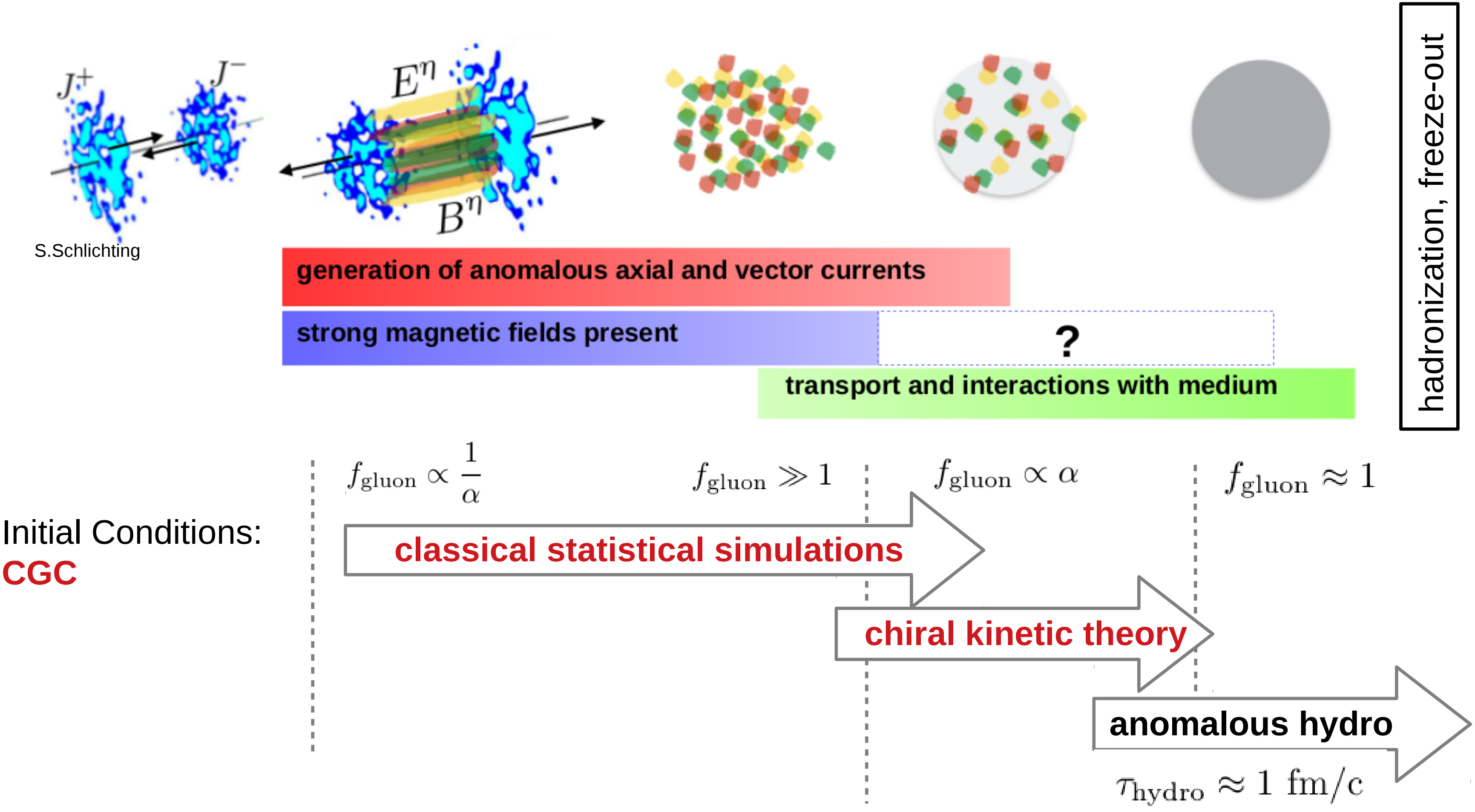

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talk by F. Gelis

talk by K. Boguslavski

talk by A. Mazeliauskas

Fermion dynamics

from Classical-statistical simulations

2. Classical-statistical simulations

Literature suggests:
"CME and related (3-lettered-acronym) effects simple and straightforward"

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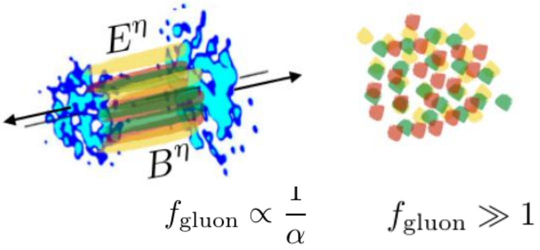
Classical-Statistical Simulations ...

"using the correspondence principle for highly occupied systems"



Berges, Boguslavski, Schlichting, Venugopalan; Kurkela, Zhu; Gelis …

many talks at this conference!

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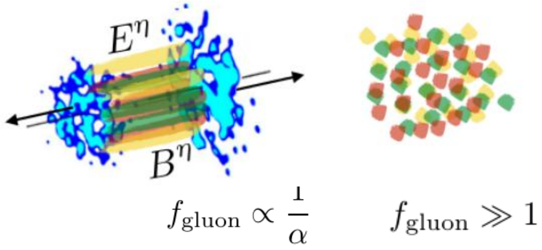


Berges, Boguslavski, Schlichting, Venugopalan; Kurkela, Zhu; Gelis …

many talks at this conference!

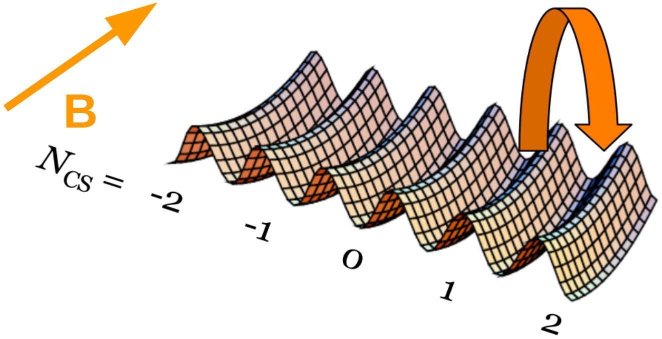
… with dynamical fermions

NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, PRD95 (2017) 036023

Groundbreaking conceptual and algorithmic progress:
Hamiltonian improvement program (a la Symanzik) in real-time

2. Classical-statistical simulations

Fermion dynamics during non-equilibrium sphaleron transition



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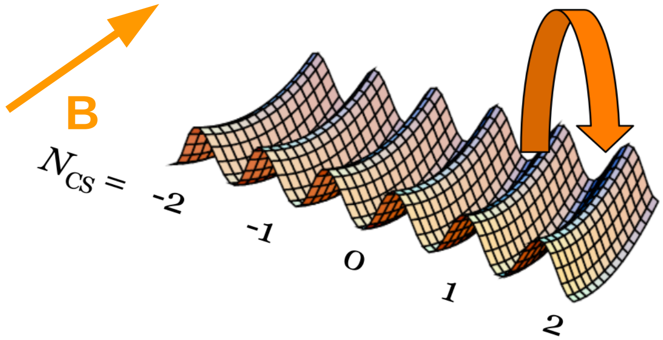
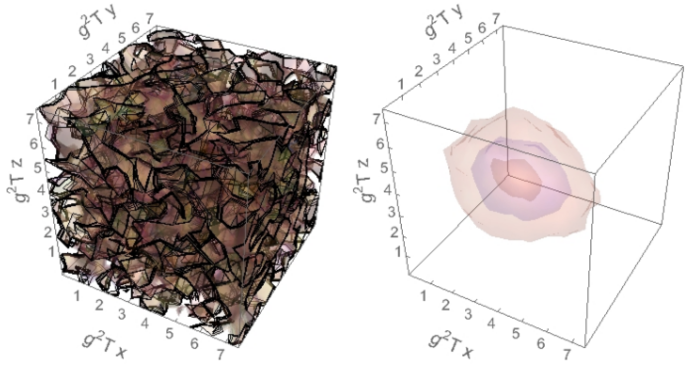


Sphalerons in the glasma

(Mace, Schlichting, Venugopalan Phys.Rev. D93 (2016) no.7, 074036



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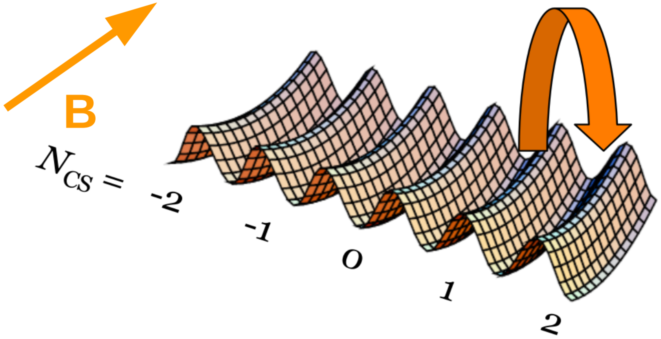
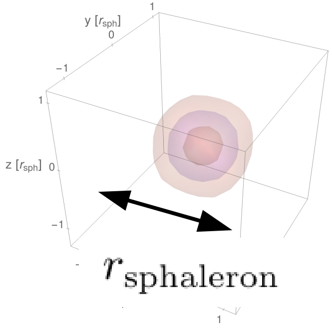


First step: a 'bare' sphaleron



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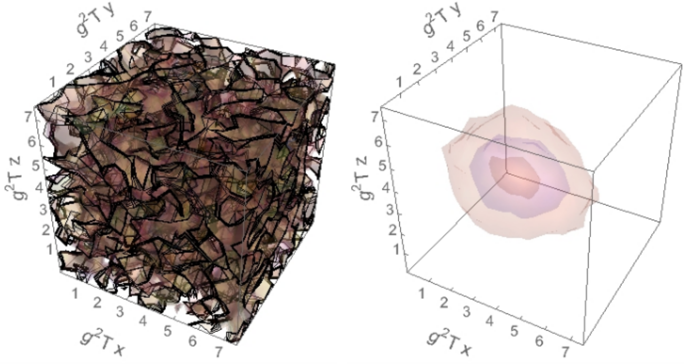
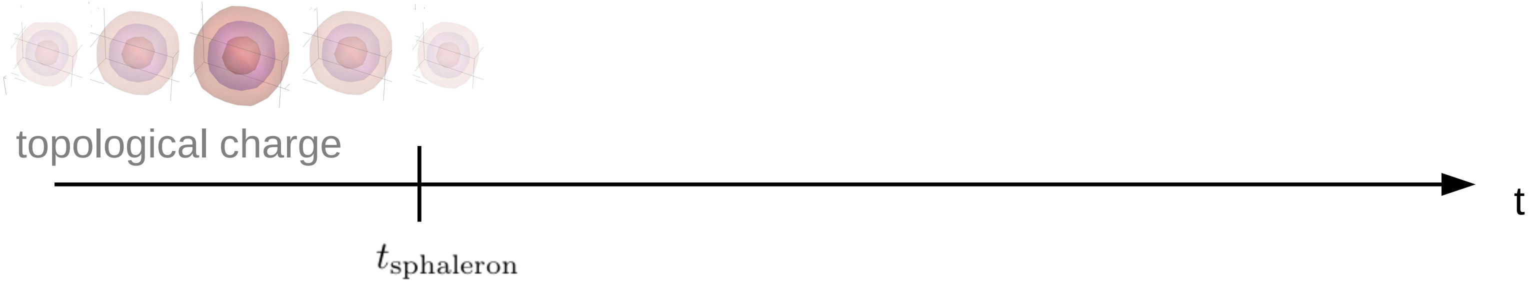


2. Classical-statistical simulations

CME, CSE and the Chiral Magnetic Wave

NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, PRD95 (2017) 036023



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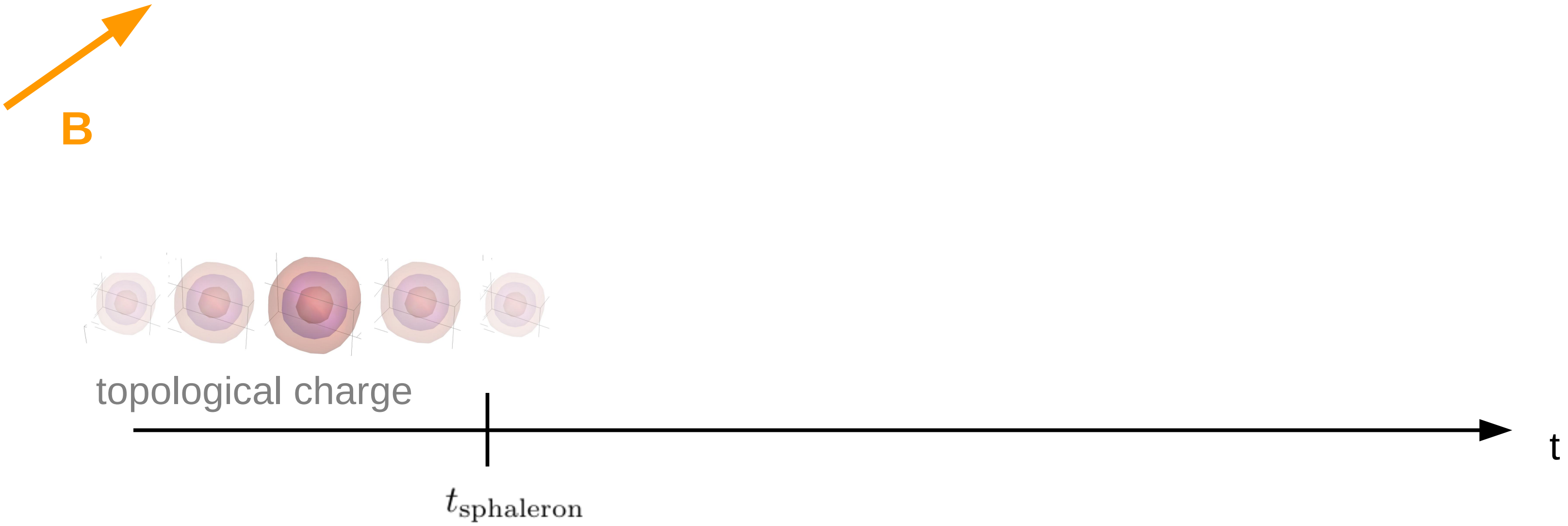

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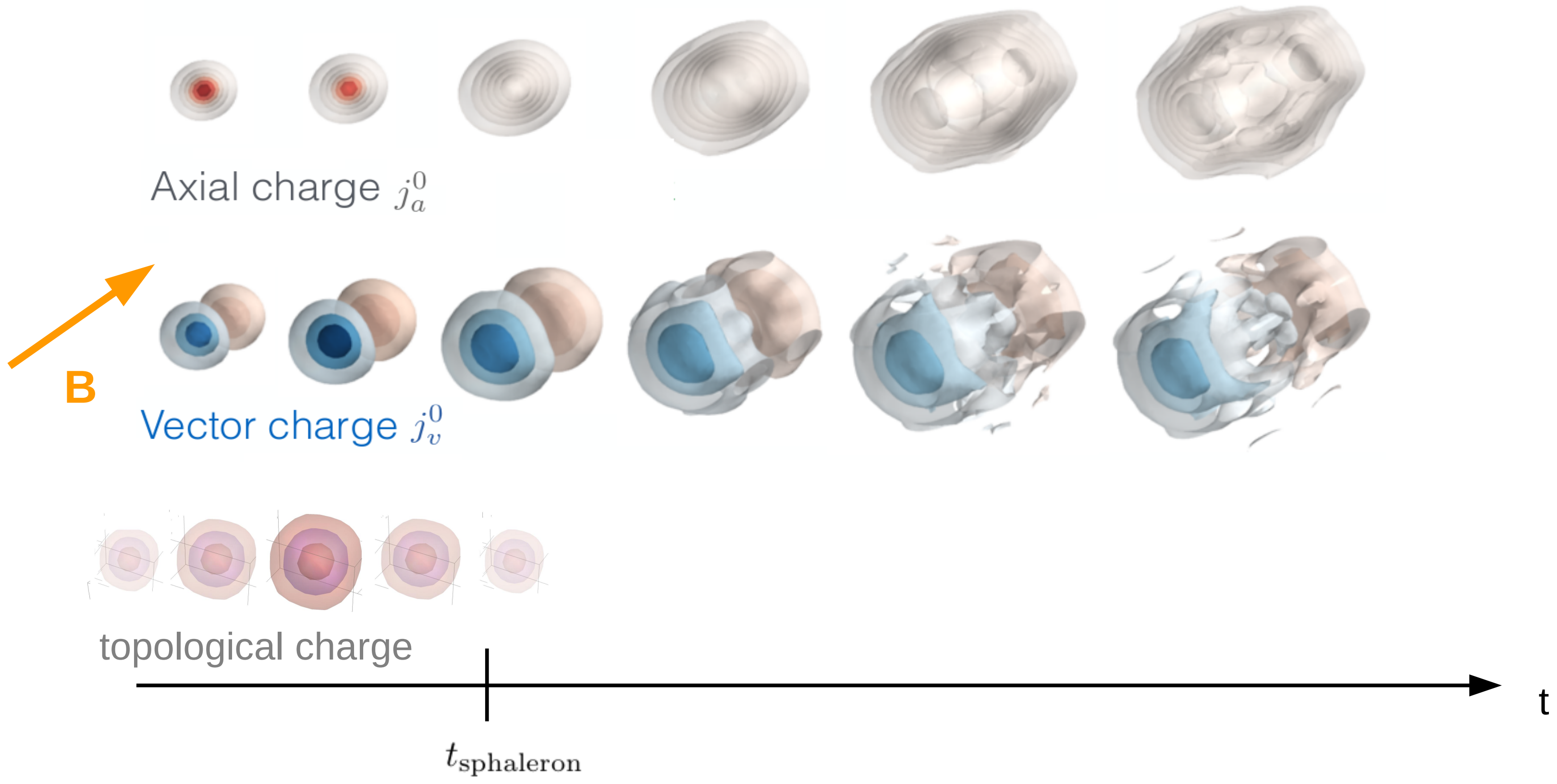
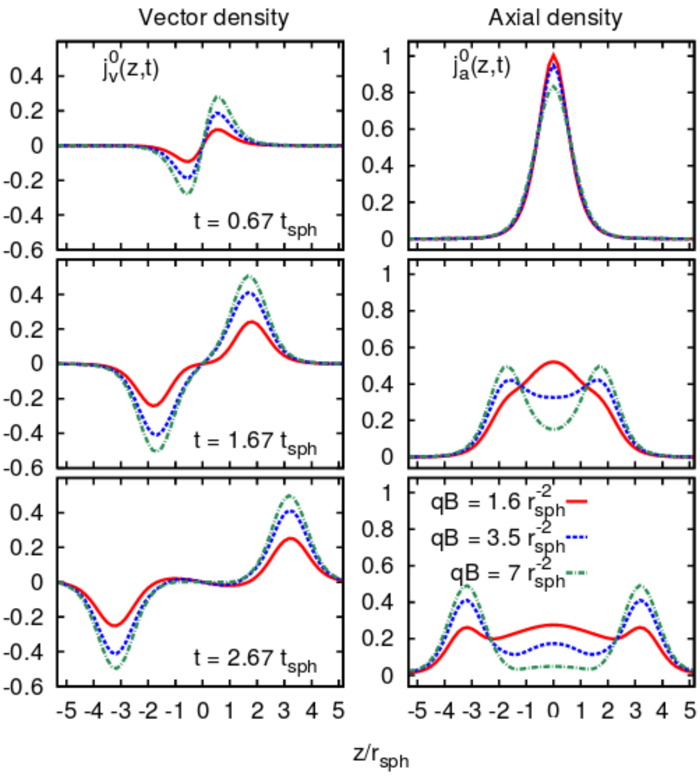


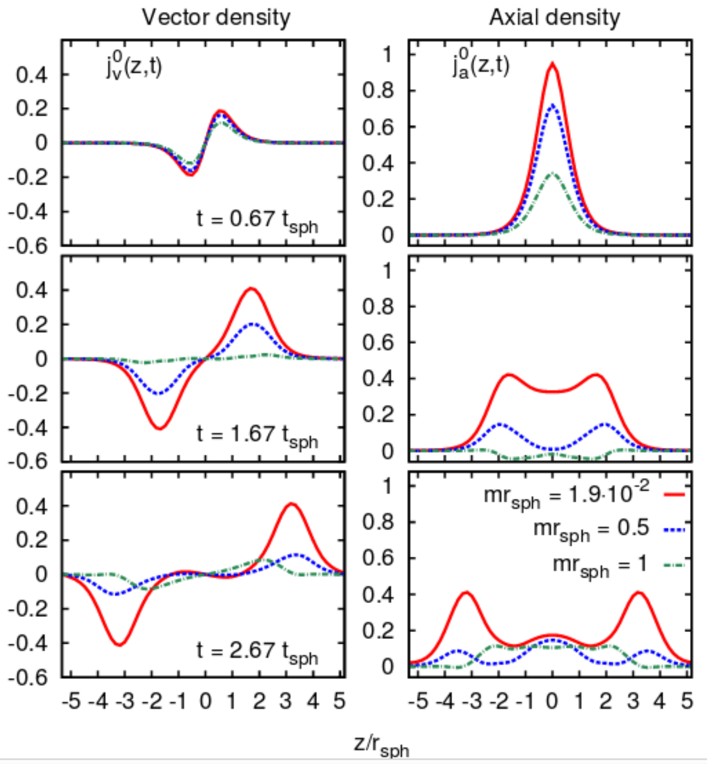
2. Classical-statistical simulations

Electric and axial charge profiles

Magnetic field dependence

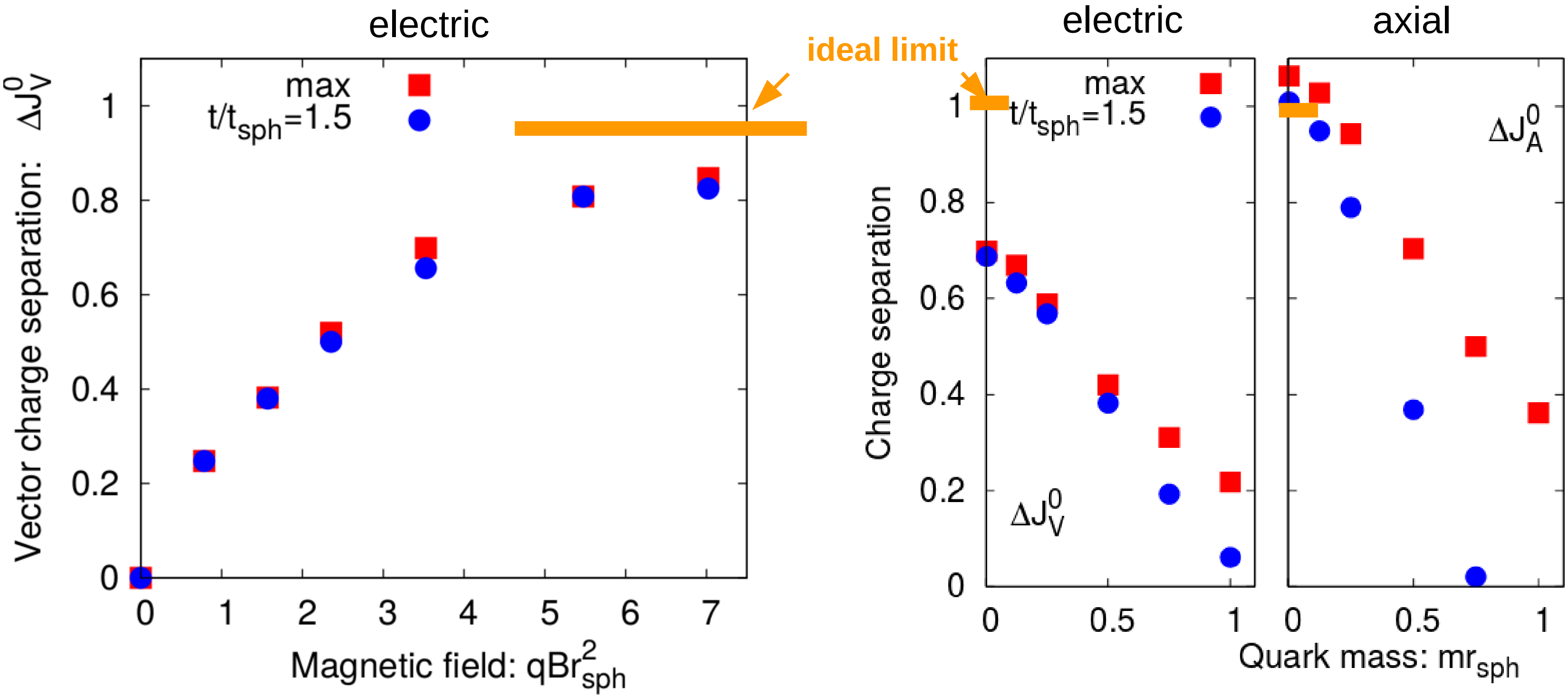


Mass dependence



2. The CME at early times

Non-equilibrium transport

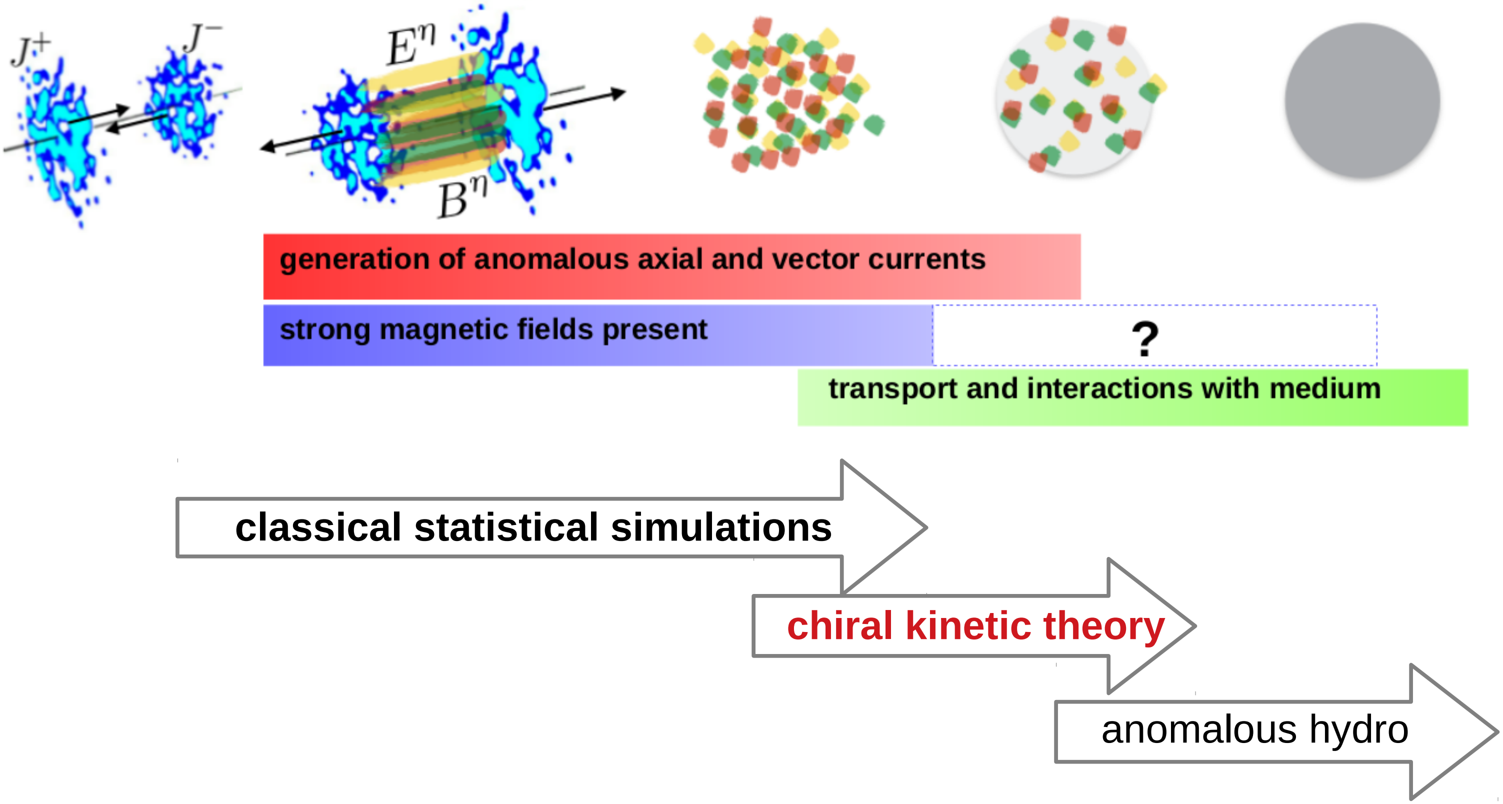


- **non-equilibrium “transport coefficients” for hydro**
- **Finite relaxation for CME and CSE vs. timescale of B-field**

Unfortunately the work is not over yet.

***Classical statistical simulations* only applicable for large occupancies**

Kinetic regime: low occupancies but still far from equilibrium



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Chiral Kinetic Theory

3. Chiral Kinetic Theory

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Anomalies and “particles”

- **Chiral Kinetic Theory with Berry curvature**
  Stephanov, Yin PRL 109 162001 (2012), Son, Yamamoto PRL 109 181602 (2012)

3. Chiral Kinetic Theory

Anomalies and “particles”

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- First solutions using relaxation time approx.
  Huang, Jiang, Shi, Liao, Zhuang, arXiv:1703.08856

- Wigner functions
  Hidaka, Pu, Yang Phys. Rev. D 95, 091901 (2017),Gao, Pu, Wang PRD 96 (2017)

- Boost Invariance
  Ebihara, Fukushima, Pu PRD 96 (2017)

- Lorentz covariance a big issue – “side jumps”
  Chen, Son, Stephanov, Yee, Yin PRL 113 (2014), Chen, Son, Stephanov

3. Chiral Kinetic Theory

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Open issues

- Topology of Berry's phase and the anomaly *(c.f. Fujikawa 2005)*
- Lorentz covariance, “side jumps”
- scattering terms, time scales etc.

3. Chiral Kinetic Theory

Quantum kinetic theory from microscopic approach

World-lines in quantum field theory

NM, R. Venugopalan, PRD96 (2017), 016023 , arXiv:1701.03331 [hep-ph]

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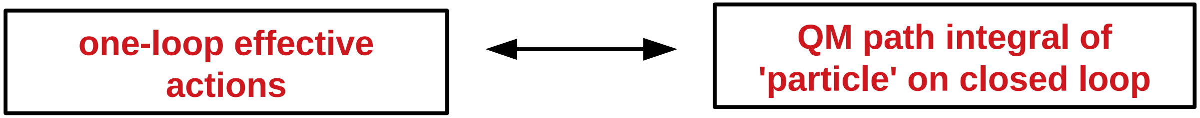

Bern, Kosower, Polyakov; Strassler 1992,

- Dirac Algebra
- anomalies from fermion measure

- Grassmanian degrees of freedom
- anomalies from Grassmanian zero modes

3. Chiral Kinetic Theory

Quantum kinetic theory from microscopic approach

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one-loop effective actions ⟷ **QM path integral of 'particle' on closed loop**

Bern, Kosower, Polyakov; Strassler 1992,

- Dirac Algebra
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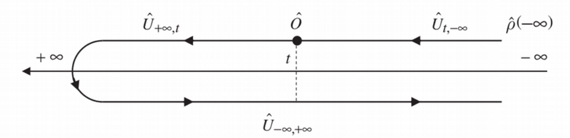
Some consequences

- **Anomaly and Berry monopole not related.**
- **Lorentz covariance – no assumptions needed (adiabaticity, chemical potential)**
- **Saddle point = point particle limit of QFT**
- **Anomaly microscopically included in kinetic approximation**

3. Chiral Kinetic Theory

CKT theory from world lines (in prep. with R. Venugopalan, Y. Yin)



$$f(x^\mu, P^\mu) \to f(x^\mu, P^\mu, \psi^\mu, \psi_5)$$

- **Schwinger-Keldysh representation of colored world lines**
  Jalilan-Marian, Jeon, Venugopalan, Wirstam, Phys.Rev. D62 (2000) 045020
- **Grassmanian phase space: spin and color**
  Berezin, Marinov, Annals Phys. 104 (1977) 336
- **dielectric properties and scattering from world lines**

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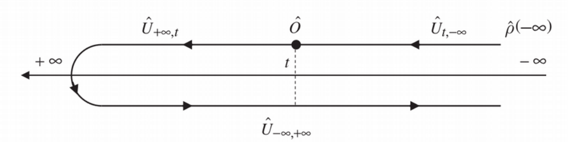
  Jalilan-Marian, Jeon, Venugopalan, Wirstam, Phys.Rev. D62 (2000) 045020

- **Grassmanian phase space: spin and color**
  Berezin, Marinov, Annals Phys. 104 (1977) 336

- **dielectric properties and scattering from world lines**



$$f(x^\mu, P^\mu) \to f(x^\mu, P^\mu, \psi^\mu, \psi_5)$$

Next: anomalous hydrodynamics *(Son, Surowka, PRL 103, 191601; Landsteiner, Megias, Pena-Benitez; Sadofyev, Isachenkov; Kharzeev, Yee...)*

- **hydrodynamic approximation from world lines, constrain transport coefficients**

- **typical time scales for scattering, sphaleron transitions**

- **initial conditions from simulations**

4. Conclusions

Theory must give quantitative predictions for anomalous effects

I have presented some recent advances here

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- **Powerful approaches exist and have been used for gluonic dynamics**
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- **A possible closer look at the hydro paradigm?**

- **Weak vs. strong coupling?**

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- **A possible closer look at the hydro paradigm?**

- **Weak vs. strong coupling?**

Experiment: **more differential and detailed observables,**
more flow coefficients / n-particle correlations of charged particles!

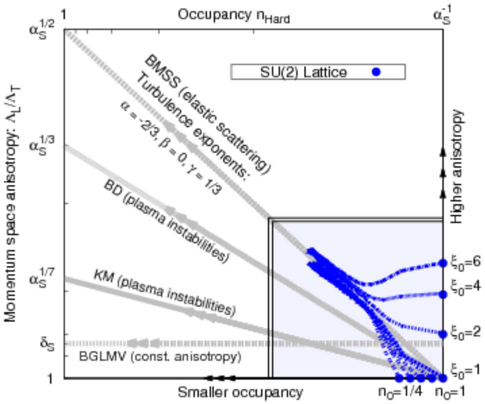
- **Variation of magnetic field and nothing else – isobar runs**

Thanks for your attention!

Backup

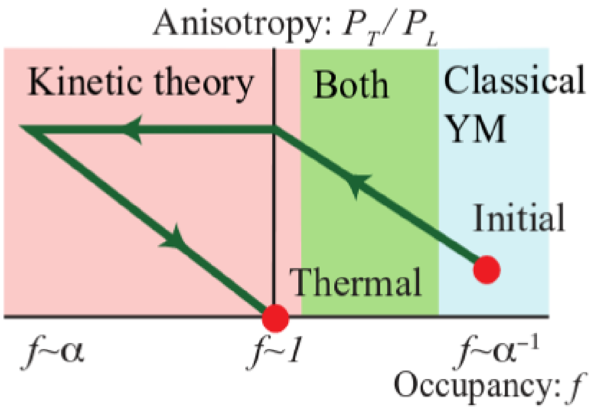
A. Thermalization and Isotropization

“If you believe weak coupling, everything points to the BMSS scenario”



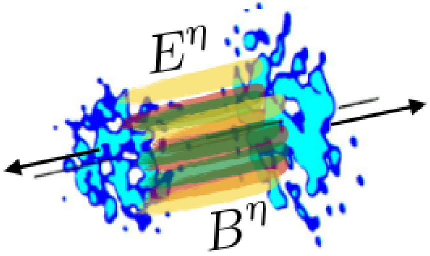
Universal attractor in a highly occupied non-Abelian plasma -
Berges et al. Phys.Rev. D89 (2014) no.11, 114007

A. Kurkela / Nuclear Physics A 00 (2016) 1–8

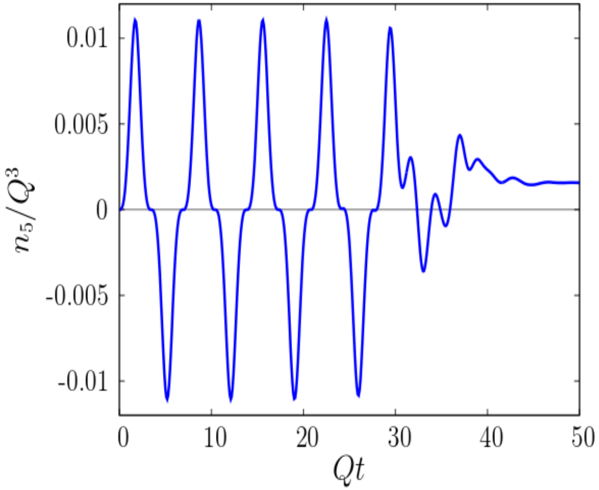


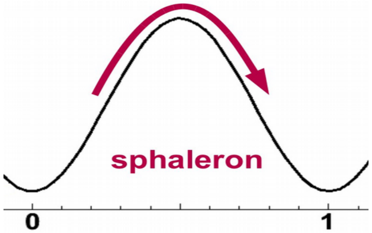
Backup

B. Simulations

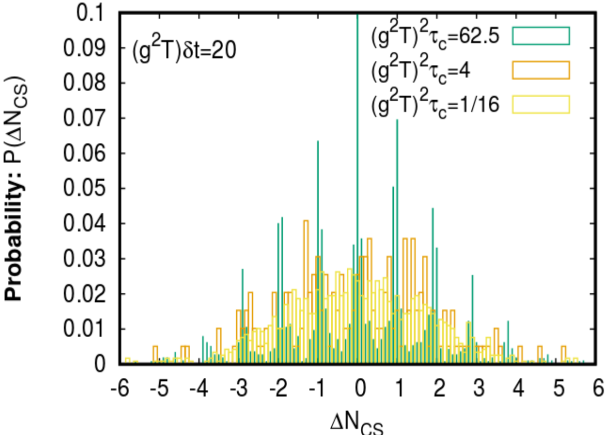


- **Color flux-tubes naturally provide large coherent** $\mathbf{E}^a \cdot \mathbf{B}^a$ (topological charge via field strength, *see e.g. N. Tanji, NM, J. Berges, Phys.Rev. D93 074507, arxiv:1603.03331*)





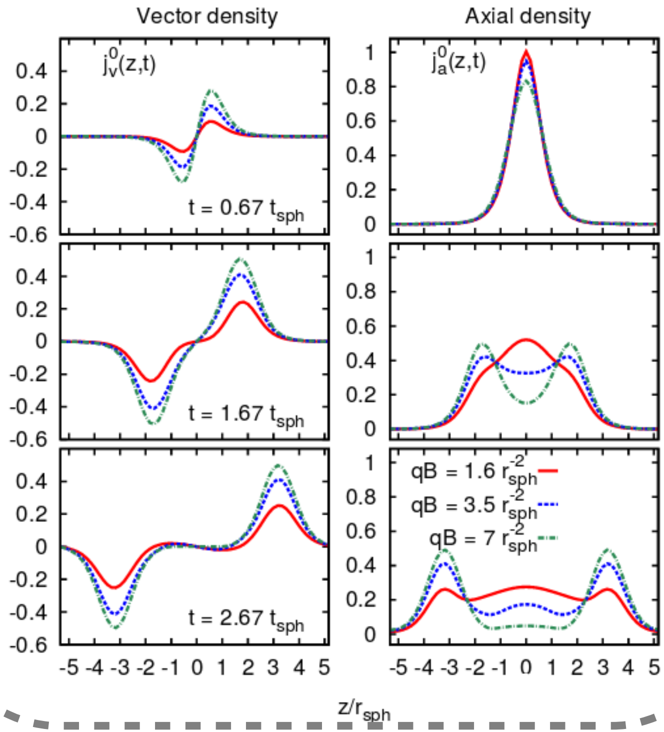
- **Sphaleron transition rates far from equilibrium can be huge!** *see M.Mace, S.Schlichting, R.Venugopalan, Phys.Rev. D93, 074036, arxiv: 1601.07342*



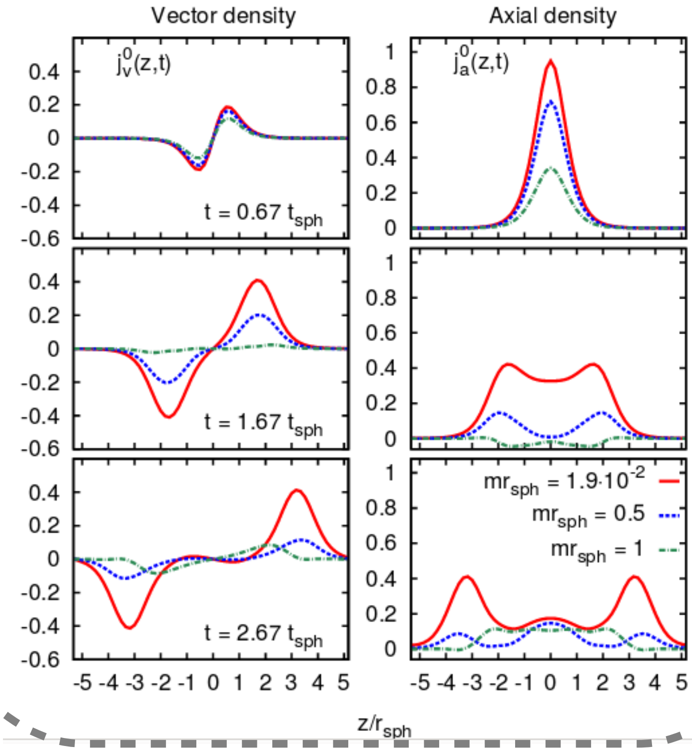
Backup

B. Simulations

Magnetic field



Mass

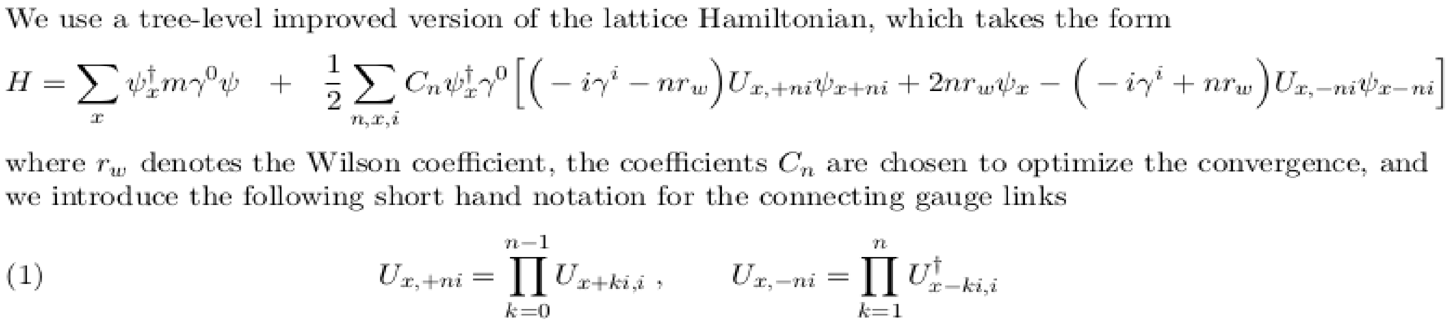


Backup

B. Simulations

Fermions: Exact description via **modefunctions**
up to 24x24x64 lattices

Improved operators (NM, S.Schlichting, S.Sharma, **arXiv:1606.00342, arXiv:1612.02477**)



We use a tree-level improved version of the lattice Hamiltonian, which takes the form

$$H = \sum_x \psi_x^\dagger m \gamma^0 \psi \quad + \quad \frac{1}{2} \sum_{n,x,i} C_n \psi_x^\dagger \gamma^0 \Big[\Big(- i\gamma^i - n r_w \Big) U_{x,+ni} \psi_{x+ni} + 2 n r_w \psi_x - \Big(- i\gamma^i + n r_w \Big) U_{x,-ni} \psi_{x-ni} \Big]$$

where r_w denotes the Wilson coefficient, the coefficients C_n are chosen to optimize the convergence, and we introduce the following short hand notation for the connecting gauge links

$$U_{x,+ni} = \prod_{k=0}^{n-1} U_{x+ki,i} \,, \qquad U_{x,-ni} = \prod_{k=1}^{n} U^\dagger_{x-ki,i} \tag{1}$$

Wilson-averaging (M.Mace, NM, S.Schlichting, S.Sharma, *arXiv:1612.02477*)

→ improvement of chiral properties
→ extremely important for larger fermion masses
→ average fermionic observables over Wilson parameters with opposite sign
→ leading order errors in the anomaly equation cancel

Backup

C. Worldlines

Always a challenge, as natural to formulate in first quantization (point-particles), whereas most of our modern concepts (anomalies etc.) are in second quantization (fields)

Berry connection and chiral kinetic theory

Son and Yamamoto, Stephanov and Yin (2012)

Weyl equation: $(\boldsymbol{\sigma} \cdot \mathbf{p}) u_{\mathbf{p}} = \pm |\mathbf{p}| u_{\mathbf{p}}$

Effective theory in the **adiabatic limit**, describing excitations near the fermion surface:

$$\boxed{S = \int dt [p^i \dot{x}^i + A^i(x)\dot{x}^i - \mathcal{A}^i(p)\dot{p}^i - \epsilon_{\mathbf{p}}(x) - A^0(x)]}$$

Berry phase: $i\boldsymbol{\mathcal{A}}_{\mathbf{p}} \equiv u_{\mathbf{p}}^{\dagger} \boldsymbol{\nabla}_{\mathbf{p}} u_{\mathbf{p}}$

Berry monopole: $\boldsymbol{\Omega}_{\mathbf{p}} \equiv \boldsymbol{\nabla}_{\mathbf{p}} \times \boldsymbol{\mathcal{A}}_{\mathbf{p}} = \pm \frac{\hat{\mathbf{p}}}{2|\mathbf{p}|^2}$

Geometric only in the adiabatic limit

Claim: accounts for the dynamics of the anomaly by some continuity arguments (incompressibility of phase space)

Backup

C. Worldlines

some rather old stuff (Schwinger, Feynman) … re-discovered many times (Strassler) …
and some pioneers of the modern interpretation from Heidelberg (Schmidt, Schubert)! …

one-loop effective actions

⟷

quantum mechanical particle, quantized on closed loop (world line)

$$Z_{S^1} = \int \frac{d^Dp}{(2\pi)^D}\left(-\int_0^\infty \frac{dT}{T}\, e^{-(p^2+m^2)T}\right) = \int \frac{d^Dp}{(2\pi)^D}\ln(p^2+m^2) = \Gamma^{QFT}_{eff}$$

$$\longleftarrow$$

$$\int_P \mathcal{D}x\, e^{-\int_0^1 d\tau\, \frac{1}{4T}\dot{x}^2} = \mathrm{Tr}\, e^{-\hat{p}^2 T} = \int \frac{d^Dp}{(2\pi)^D} e^{-p^2 T}$$

1st ingredient:

integral-representation of the logarithm

Backup

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C. Worldlines

Matter + Gauge fields:

$$\Gamma[A] = -\log\left[\det(-D^2)\right] \equiv -\mathrm{Tr}\left(\log(-D^2)\right) = \int_0^\infty \frac{dT}{T}\mathrm{Tr}\,\exp(-TD^2)$$

$$= \int_0^\infty \frac{dT}{T}\mathcal{N}\int \mathcal{D}x\,\mathcal{P}\exp\left[-\int_0^T d\tau\,\left(\frac{1}{2\varepsilon}\dot{x}^2 + igA[x(\tau)]\cdot\dot{x}\right)\right]$$

Fermions *(do not expect a complete derivation here!)*

anomalies live here

$$S[A,B] = \int d^4x\,\bar{\psi}\,(i\partial\!\!\!/ + A\!\!\!/ + \gamma_5 B\!\!\!/)\,\psi \longrightarrow W[A,B] = W_R + i\,W_I$$

real part:

$$\boxed{W_R = -\frac{1}{8}\log\,\det\left(\tilde{\Sigma}^2\right) \equiv -\frac{1}{8}\mathrm{Tr}\,\log\left(\tilde{\Sigma}^2\right)}$$

$$\Sigma^2 = (p-\mathcal{A})^2\,\mathbf{I}_8 + \frac{i}{2}\Gamma_\mu\Gamma_\nu F_{\mu\nu}[\mathcal{A}]$$

$$F_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu - [\mathcal{A}_\mu,\mathcal{A}_\nu]$$

$$\mathcal{A} = \begin{pmatrix} A+B & 0 \\ 0 & A-B \end{pmatrix}$$

2nd ingredient:

representation of the trace over Gamma matrices via <u>fermionic</u> coherent states

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \gamma_\mu & 0 \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}, \quad \Gamma_6 = \begin{pmatrix} 0 & i\mathbb{I}_4 \\ -i\mathbb{I}_4 & 0 \end{pmatrix}$$

$$\Gamma_7 = -i\prod^{6}\Gamma_A = \begin{pmatrix} \mathbb{I}_4 & 0 \\ 0 & -\mathbb{I}_4 \end{pmatrix} \qquad \{\Gamma_7,\Gamma_A\} = 0$$

$$a_r^\pm = \frac{1}{2}(\Gamma_r \pm i\Gamma_{r+3})$$

$$\langle\theta|a_r^- = \langle\theta|\theta_r \qquad a_r^-|\theta\rangle = \theta_r|\theta\rangle \qquad \langle\bar{\theta}|a_r^+ = \langle\bar{\theta}|\bar{\theta}_r \qquad a_r^+|\bar{\theta}\rangle = \bar{\theta}_r|\bar{\theta}\rangle$$

The origin of Berry's phase ... **Backup**

Adiabatic (and non-relativistic) approximation of the world-line path integral gives

$$\boxed{W_R} \longrightarrow \int \mathcal{D}x\mathcal{D}p \, \exp\left(i\int dt\left[\dot{\mathbf{x}}\cdot\mathbf{p} - \tilde{H}\right]\right)$$

$$\tilde{H} = mc^2 + \frac{(\boldsymbol{p}-\boldsymbol{A}/c)^2}{2m} + A^0(x) + \boxed{\dot{\mathbf{p}}\cdot\boldsymbol{\mathcal{A}}(\boldsymbol{p})}$$

$$\boxed{\boldsymbol{\mathcal{A}}(\boldsymbol{p}) \equiv -i\langle\psi^+(\boldsymbol{p})|\boldsymbol{\nabla}_p|\psi^+(\boldsymbol{p})\rangle}$$

Berry's phase

It is pretty clear what the role of Berry's phase is:
→ spin transport along the world-line of the particle
→ related to defining the 'moving frame' for the spin variables
Most importantly: **It is not robust!** ***(Part of the dynamics away from adiabatic approx.)***

… we got it from the real part of the effective action …
… the anomaly stems from the imaginary part (more soon) …