The Search for Gluon Saturation in pA Collisions and at the Future Electron Ion Collider

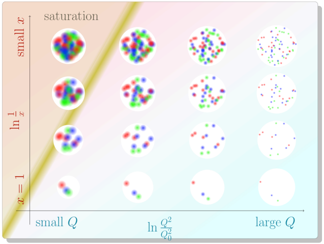
Bo-Wen Xiao

Institute of Particle Physics, Central China Normal University

Initial Stages 2017
Polish Academy of Arts and Sciences
September 18-22 2017, Cracow, Poland

Saturation Physics (Color Glass Condensate)

QCD matter at extremely high gluon density



$$\sigma_0 \sum_{i=0}^{\infty} \alpha_s^i \left(L^i + C^{(i)}\right)$$ ideal QCD expansion

$$\sigma_0 \sum_{i=0}^{n-1} \alpha_s^i L^i \quad \Big| \quad \sigma_0 \sum_{i=0}^{n-1} \alpha_s^i C^{(i)}$$ $\Leftarrow$ pQCD

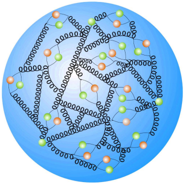
$$\sigma_0 \sum_{i=n}^{\infty} \alpha_s^i L^i \quad \Big| \quad \sigma_0 \sum_{i=n}^{\infty} \alpha_s^i C^{(i)}$$

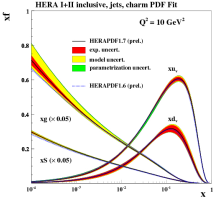
$\Uparrow$ resummation

$\nwarrow$ negligible

- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK equation) $\Rightarrow$ saturation in gluon distributions.
- From QCD expansion point of view, various types of resummations often is vital to get reliable results for a given physical processes.
- Core ingredients: Multiple interactions (tree) + Small-x (high energy) evolution (loop, Resummation of the $\alpha_s \ln \frac{1}{x}$).
- Introduce $Q_s(x)$ to separate the saturated dense regime from the dilute regime.
- Gluons at small-x carry typical transverse momentum of order $Q_s(x)$. (Cf. Collinear pdf)

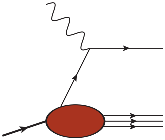
HERA (Hadron Elektron Ring Anlage)

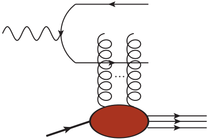




- $e^{\pm}p$ collisions at $\sqrt{s} = 318\,\text{GeV}$ (1992-2007);
  - Partons in the low-x region is dominated by rapid growing gluons.
  - Hint of gluon saturation at low-x region.
    Geometric Scaling [Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]

Dual Descriptions of Deep Inelastic Scattering


Bjorken frame


Dipole frame

Bjorken frame

$$F_2(x,Q^2) = \sum_q e_q^2 x \left[f_q(x,Q^2) + f_{\bar{q}}(x,Q^2) \right].$$

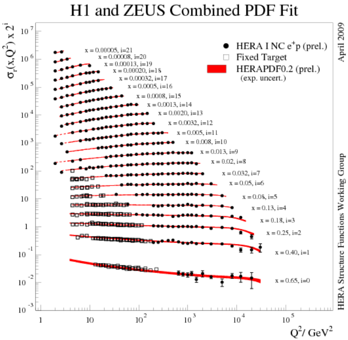
Dipole frame [A. Mueller, 01; Parton Saturation-An Overview]

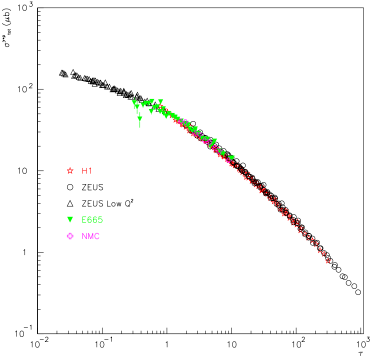
$$F_2(x,Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\rm em}} S_\perp \int_0^1 {\rm d}z \int {\rm d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

- Bjorken: the partonic picture of a hadron is manifest. Saturation shows up as a limit on the occupation number of quarks and gluons.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

Geometrical Scaling in DIS

[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]





- Define $Q_s^2(x) = (x_0/x)^\lambda \mathrm{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$. All low-x data of $\sigma_{tot}^{\gamma^* p}$ is function of a single variable $\tau = Q^2/Q_s^2$.
- This scaling can be naturally explained in small-x formalism.

A Tale of Two Gluon Distributions

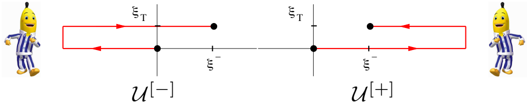
In terms of operators (TMD def. [Bomhof, Mulders and Pijlman, 06]), two gauge invariant gluon definitions: [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution: conventional gluon distributions

$$xG_{\rm WW}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG_{\rm DP}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



- Modified Universality for Gluon Distributions:

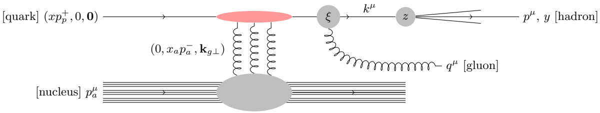
	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
$xG_{\rm WW}$	×	×	√	×	√
$xG_{\rm DP}$	√	√	×	√	√

× ⇒ Do Not Appear. √ ⇒ Apppear.

- Measurements in pA collisions and at the EIC are tightly connected with complementary physics missions.

Forward rapidity single hadron productions in pA collisions

Dilute-Dense factorizations: large x proton or γ^* $\rightarrow$ as dilute probe:

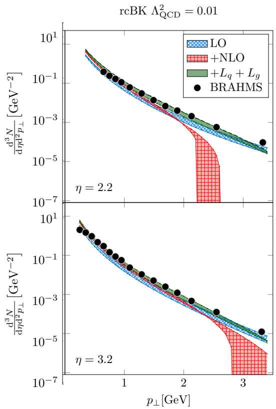


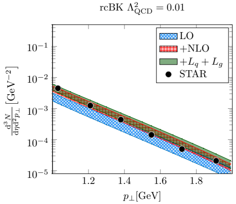
$$x_p = \frac{k_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{dilute}$$
$$x_A = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{dense}$$

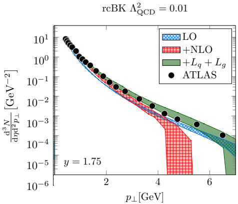
- LO [Dumitru, Jalilian-Marian, 02]: probing $xG_{\mathrm{DP}}(x, k_\perp)$ at small-x.
- NLO Cutoff[Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11]
- NLO Complete NLO in DR: [Chirilli, BX and Yuan, 12].
  1. 1. soft, collinear to the target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$. Subtraction scheme is not Unique. See Iancu and Lappi's talk.
  2. 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs. $\overline{MS}$ scheme.
  3. 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs, $\overline{MS}$ scheme.
  4. The importance of subtraction: systematic resummation of large logarithms. ($\alpha_s \ln 1/x_g$ or $\alpha_s \ln M_F^2/\mu^2$), which allows us to have $\mathcal{H} \sim \mathcal{O}(\alpha_s)$.
- Numerical implementations:
  - Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]
  - NLO pdf and FF and running coupling.
  - NLO hard factors. Partially by [Albacete, Dumitru, Fujii, Nara, 12]
  - rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Recent progress: [Kovner, Lublinsky, and Mulian, 13; Caron-Huot, Herranen, 16].

Numerical implementation of the NLO result

SOLO results [Stasto, Xiao, Zaslavsky, 13; Watanabe, Xiao, Yuan, Zaslavsky, 15]



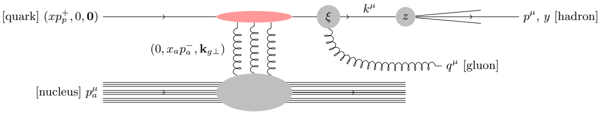




- Agree with RHIC and LHC data in low $p_\perp$ region where pQCD does not apply.
- SOLO (1.0 and 2.0) break down in the large $p_\perp$ region.
- Towards a more complete framework. [Iancu and Lappi's talk, Tues] (Different scheme?) [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller and Triantafyllopoulos, 16]
- Another idea: threshold resummation! The resummation of plus-functions or $\bar{\alpha}_s \ln(1 - x_p) < 0$.

Threshold resummation in the saturation formalism

Dilute-Dense factorizations: large x proton or γ^* $\rightarrow$ as dilute probe:



$$x_p = \frac{k_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{dilute}$$

$$x_A = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{dense}$$

- Threshold resummation is the resummation of plus-functions. In single forward hadron production, [Stasto, Zaslavsky, 16]

$$\int_{x_p}^{1} \frac{d\xi}{(1-\xi)_+} f(\xi) \sim f(1) \ln(1-x_p)$$

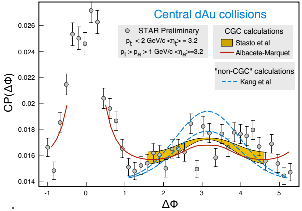
- It is also the resummation of logarithm $\bar{\alpha}_s \ln(1-x_p) < 0$. For example: let $X = \bar{\alpha}_s \ln(1-x_p)$, $e^X = 1 + X + \frac{1}{2}X^2 + \cdots$
- Typical feature of asymmetric forward pA collisions.
- Mellin transform is the technique used to perform resummation.

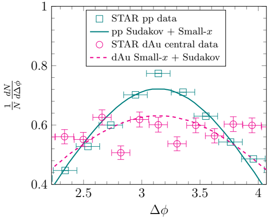
$$\int_0^1 d\tau \tau^{N-1} \int_\tau^1 \frac{d\xi}{\xi} \mathcal{P}(\xi) q\left(\frac{\tau}{\xi}, \mu\right) = \int_0^1 d\xi \xi^{N-1} \mathcal{P}(\xi) \int_0^1 dx x^{N-1} q(x, \mu) = P_N q_N,$$

- $\frac{1}{(1-\xi)_+} \rightarrow P_N \sim\simeq -\ln N$. ($\tau \rightarrow 1 \Leftrightarrow N \rightarrow \infty$.)

Dihadron correlations in *dAu* collisions

Forward dihadron correlation in Dilute-Dense factorizations as a probe to saturation.

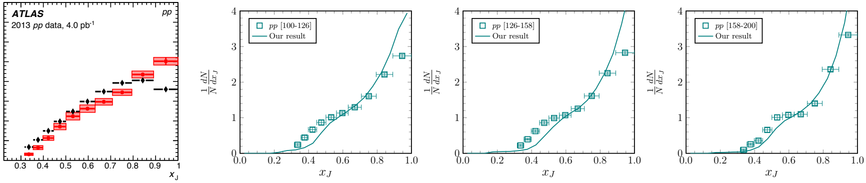




- Physics predicted by [C. Marquet, 09]. Important hint of gluon saturation.
- Further calculated in [Marquet, Albacete, 10; Stasto, BX, Yuan, 11]
- Interpretation: de-correlation due to interaction with low-x gluonic matter.
- Sudakov resummation $\alpha_s \ln^2 \frac{P_\perp^2}{q_\perp^2}$ in dijet processes. [Mueller, BX, Yuan, 13; K. Kutak, *et al*, 15, 16; Also see P. Kotko's talk]
- More sophisticated and robust theoretical computation, and more precise experimental *pAu* data will be released soon.[Marquet *et al*; Stasto *et al*]

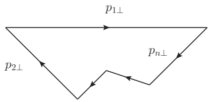
Dijet asymmetry at the LHC

Fully corrected dijet asymmetry $x_J \equiv \frac{p_{2\perp}}{p_{1\perp}} = \frac{1-A_J}{1+A_J}$ data from ATLAS, 1706.09363.

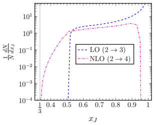


Sudakov resummation improved pQCD approach: [Chen, Qin, Wei, Xiao, Zhang, 16]

- pQCD expansion up to $2 \to n$ process is bounded $x_J \geq \frac{1}{n-1}$ (or $A_J \leq \frac{n-2}{n}$).

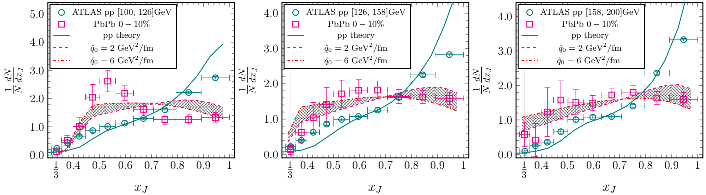


$\Rightarrow \quad (n-1)p_{2\perp} \geq p_{1\perp}$



- Sudakov resummation for the back-to-back dijet configurations when $x_J \sim 1$.

Dijet asymmetries as a probe of QGP



New methods to probe transport coefficient $\hat{q}$: [Chen, Qin, Wei, Xiao, Zhang, 16]

- Use BDMPS energy loss distribution for medium induced soft gluon emissions

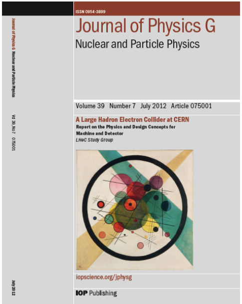
$$D(\epsilon) = \sqrt{\frac{\alpha^2\omega_c}{2\epsilon}} \exp\left(-\frac{\pi\alpha^2\omega_c}{2\epsilon}\right), \text{ with } \quad \omega_c \equiv \int dL\hat{q}L \quad \text{and} \quad \alpha \equiv \frac{2\alpha_s C_R}{\pi}.$$

- Calculation in medium is embedded in OSU $2+1$ d viscous hydro. [Z. Qiu, C. Shen, and U. Hein, 11]
- Dijet asymmetries gives $\hat{q}_0 \sim 2 - 6\text{GeV}^2/\text{fm}$ at the LHC. ($T = 481\text{MeV}$)
- Assuming T^3 scaling, this roughly agrees with the original BDMPS estimate of $\hat{q}_0 = 0.5\text{GeV}^2/\text{fm}$ at $T = 250\text{MeV}$.
- Global fit from dihadron correlations ⇒ quark jet $\hat{q}_0 \sim 4\text{GeV}^2/\text{fm}$ at RHIC.

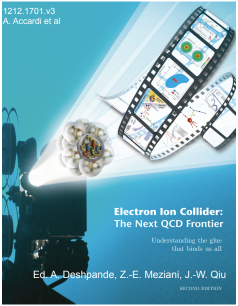
Into the future

Electron Ion Collider (LHeC)

arXiv:1108.1713



arXiv:1206.2913

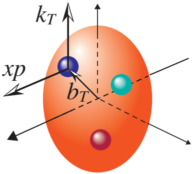


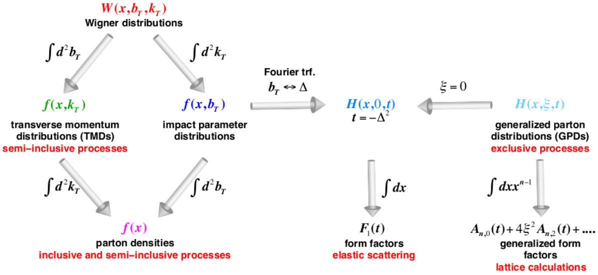
arXiv:1212.1701

- A lot of interesting physics. [Nestor Armesto Perez and Thomas Ullrich's talk]
- EIC will be a fantastic stereoscopic "camera" with extremely high resolution, which allows us to visualise protons and nuclei in a multi-dimensional fashion. (Theorist's version of EIC)

3D Tomography of Proton

Wigner distributions ingeniously encode all quantum information of how partons are distributed inside hadrons. [Ji, 03; Belitsky, Ji, Yuan, 03]





- Small-x gluon distributions $\Leftrightarrow$ gluon Wigner distributions? [Ji, 03]

Can we measure Wigner distributions?

VOLUME 70, NUMBER 9 PHYSICAL REVIEW LETTERS 1 MARCH 1993

Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Raymer
Department of Physics and Chemical Physics Institute, University of Oregon, Eugene, Oregon 97403

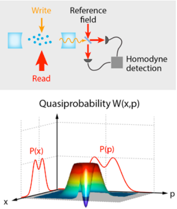
A. Faridani
Department of Mathematics, Oregon State University, Corvallis, Oregon 97331
(Received 16 November 1992)

PRL **116**, 130402 (2016) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2016

Wigner Distribution of Twisted Photons

Mohammad Mirhosseini,[1,*] Omar S. Magaña-Loaiza,[1] Changchen Chen,[1] Seyed Mohammad Hashemi Rafsanjani,[1] and Robert W. Boyd[1,2]
[1]*The Institute of Optics, University of Rochester, Rochester, New York 14627, USA*
[2]*Department of Physics and Max Planck Centre for Extreme and Quantum Photonics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada*
(Received 4 December 2015; published 1 April 2016)

We present the first experimental characterization of the azimuthal Wigner distribution of a photon. Our protocol fully characterizes the transverse structure of a photon in conjugate bases of orbital angular momentum (OAM) and azimuthal angle. We provide a test of our protocol by characterizing pure superpositions and incoherent mixtures of OAM modes in a seven-dimensional space. The time required for performing measurements in our scheme scales only linearly with the dimension size of the state under investigation. This time scaling makes our technique suitable for quantum information applications involving a large number of OAM states.



- Small-x gluon distributions $\Leftrightarrow$ gluon Wigner distributions? [Ji, 03]
- Yes, we can measure the small-x gluon Wigner distribution
- Impact on the spin side of EIC: gluon OAM [Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Yuan, Zhao, 16, Bhatttacharya, Metz, Zhou, 17]

The exact connection between dipole amplitude and Wigner distribution

[Hatta, Xiao, Yuan, 16] Def. of gluon Wigner distribution:

$$xW_g^T(x, \vec{q}_\perp; \vec{b}_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \times \left\langle P + \frac{\Delta_\perp}{2} \left| F^{+i}\left(\vec{b}_\perp + \frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp - \frac{\xi}{2}\right) \right| P - \frac{\Delta_\perp}{2} \right\rangle ,$$

Def. of GTMD [Meissner, Metz and Schlegel, 09]

$$xG(x, q_\perp, \Delta_\perp) \equiv \int d^2 b_\perp e^{-i\Delta\cdot b_\perp} xW_g^T(x, \vec{q}_\perp; \vec{b}_\perp).$$

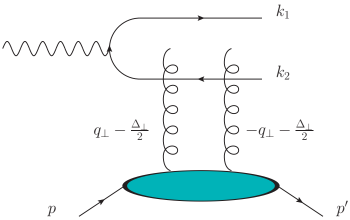
- With one choice of gauge link (dipole like) and $b_\perp = \frac{1}{2}(R_\perp + R'_\perp)$, we demonstrate

$$xG_{\rm DP}(x, q_\perp, \Delta_\perp) = \frac{2N_c}{\alpha_s} \int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp\cdot(R_\perp - R'_\perp) + i\frac{\Delta_\perp}{2}\cdot(R_\perp + R'_\perp)} \times \left(\nabla_{R_\perp} \cdot \nabla_{R'_\perp}\right) \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(R_\perp) U^\dagger(R'_\perp)\right]\right\rangle_x .$$

- This provides the $3D$ quasiprobabilistic information $x, b_\perp, k_\perp$ of small-x gluon.

Probing 3D Tomography of Proton at small-x

Diffractive back-to-back dijet productions in DIS [Hatta, Xiao, Yuan, 16]



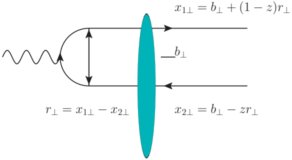
- Cross-Sections are positive-definite, although Wigner distributions may not be.
- Elliptic Wigner distribution: angular correlation between $b_\perp$ ($\Delta_\perp$) and $q_\perp$.

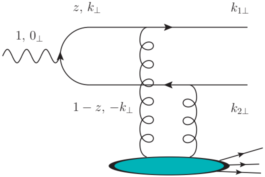
$$F_x(q_\perp, \Delta_\perp) = F_0(|q_\perp|, |\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) F_\epsilon(|q_\perp|, |\Delta_\perp|) + \cdots$$

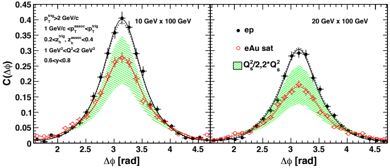
$$\text{with} \quad xG \equiv \frac{2N_c}{\alpha_s}\left(q_\perp^2 - \frac{\Delta_\perp^2}{4}\right) F_x.$$

- WW Wigner (WWW) distribution can be also defined.
- Similar measurement may be possible in ultra-peripheral diffractive AA collisions. [Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 17]

Gluon TMD and Dijet production in DIS



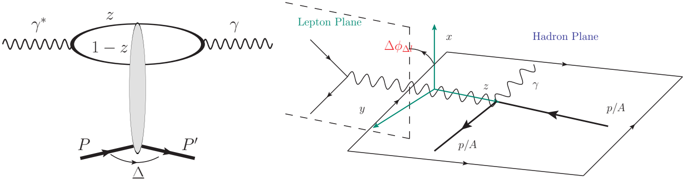




- Back-to-back correlation $C(\Delta\phi)$: [Dominguez, Marquet, Xiao and Yuan, 11; Zheng, Aschenauer, Lee and BX, 14]
- Unique golden measurement for the Weizsäcker Williams gluon distributions.
- Also depends on the linearly polarized WW gluon distribution [Metz, Zhou, 11]
- Due to linearly polarized gluon distribution, there could be the analog of elliptic flow v_2 in DIS as well. [Dumitru, Lappi, Skokov, 15]

DVCS and gluon GPD at small-x

Deeply Virtual Compton Scattering [Hatta, Xiao, Yuan, 17] in the Breit frame

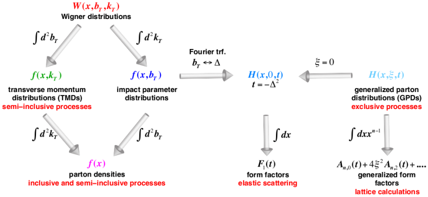


- The relation between gluon GPDs and dipole gluon Wigner distributions

$$\frac{1}{P^+}\int \frac{d\zeta^-}{2\pi} e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$= \frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp) + \frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j - \frac{\delta^{ij}\Delta_\perp^2}{2}\right) + \cdots,$$
$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0\,, \quad xE_{Tg}(x,\Delta_\perp) = \frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

- The helicity-flip gluon GPD xE_{Tg} probed directly by measuring $\cos 2\phi_{\Delta l}$ correlation.
- Vector meson productions at NLO. [Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon, 16]

Summary



- Rich physics in dilute-dense factorization formalism. (Multiple scattering, small-x resummation $\alpha_s \ln 1/x_g$, collinear logarithms $\alpha_s \ln \frac{Q^2}{\mu^2}$, Sudakov resummation $\alpha_s \ln^2 \frac{P_\perp^2}{q_\perp^2}$ and threshold resummation $\alpha_s \ln(1-x_p)$, etc.)
- Reliable higher order calculations and robust predictions[Balitsky, Chirilli, Beuf, Iancu, Lappi, Ducloue, Paatelainen, Boussarie, etc.], as well as new ideas are emerging.
- EIC will be a superb "stereoscopic camera", which allows us to depict 3D the internal structure of protons and heavy nuclei with unprecedented precision and significantly advance our knowledge of hadron structure.
- Complementary studies in pA collisions and the future EIC can give us the opportunity to discover the gluon saturation phenomenon.