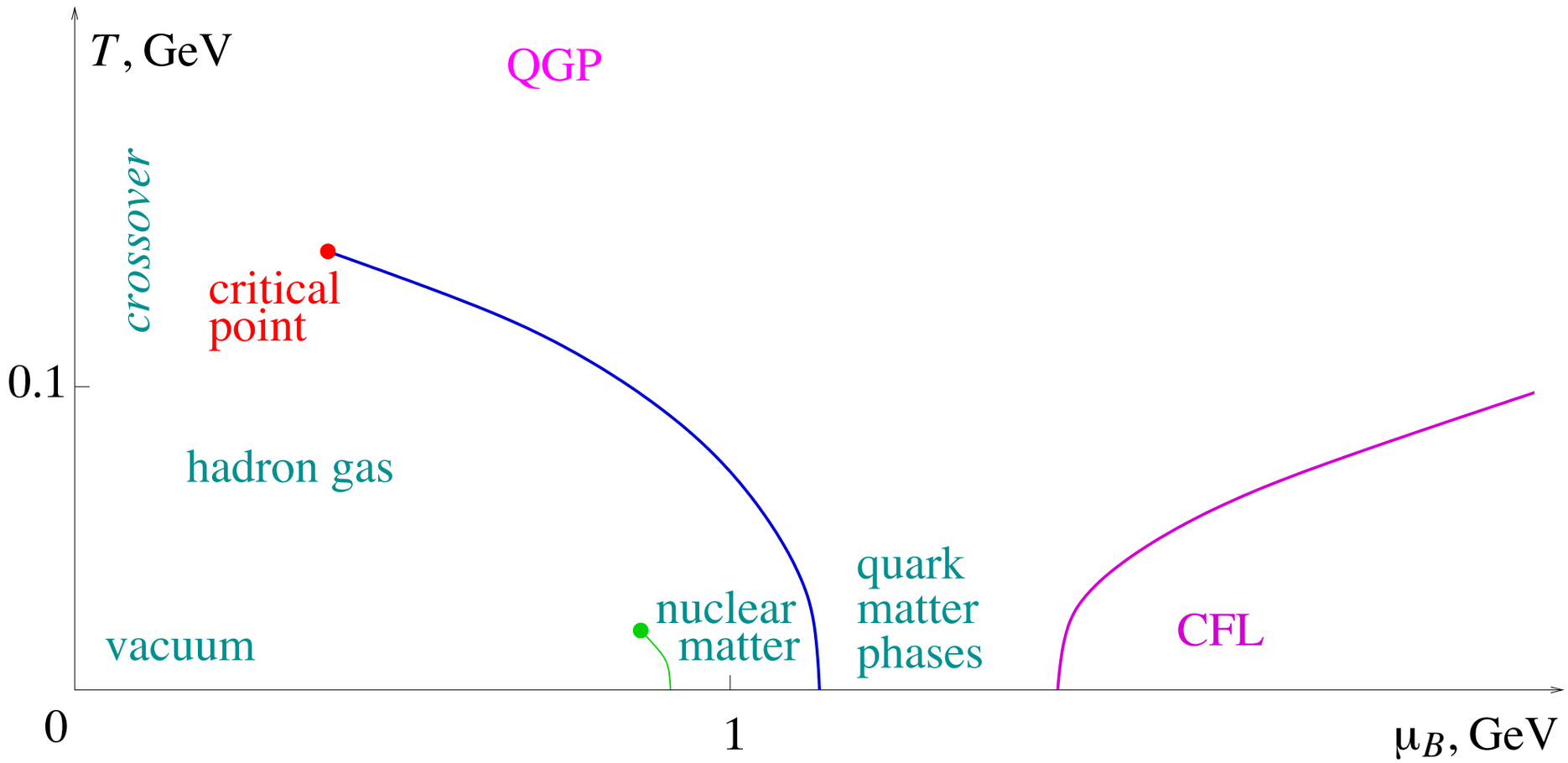
Collision Dynamics Near a Critical Point from Holography

Jorge Casalderrey-Solana

Oxford Physics

THE ROYAL SOCIETY

The QCD Phase Diagram



Taken from Stephanov hep-lat/0701002

QCD critical Point

- Phase diagram is an equilibrium property

  ➤ However it is hard to extract from lattice computations

- It may be located via experiments

  ➤ Critical behaviour in correlations of particles

  ➤ Many estimates of effects are based on (close to) equilibrium considerations

  ➤ But the dynamics of heavy ion collisions can be very far from (local) equilibrium, even if hydrodynamics work

- Can we describe collision dynamics close to a critical point?

Holography

- Gauge/gravity duality

  strongly coupled field theory in 4D ↔ weakly coupled gravity in 5D

- A powerful framework

  ➤ Access to the dynamics of strongly coupled gauge theories

  ➤ We can analyse far-from-equilibrium processes

  ➤ Allow us to simulate collision dynamics of (simplified) proyectiles

- Goal:

  Describe the collision of energetic objects in
  a theory with a critical point

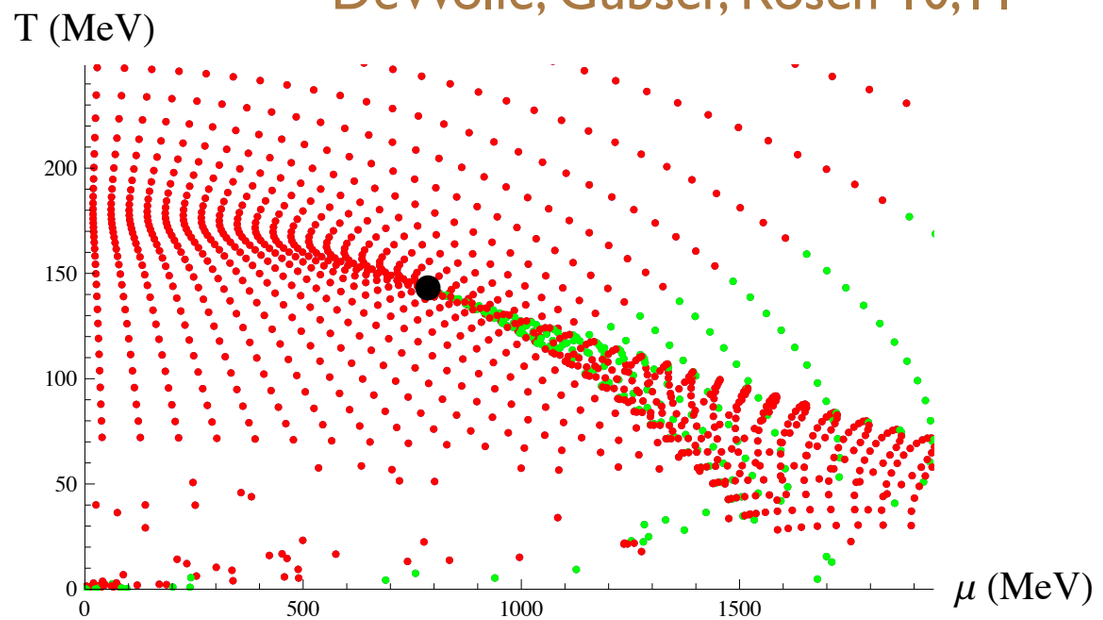
Critical Point in Holography

- Phenomenological holographic construction

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{f(\phi)}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

5D Gravity (dual to baryon charge) — Maxwell field (conformal breaking) — Scalar field

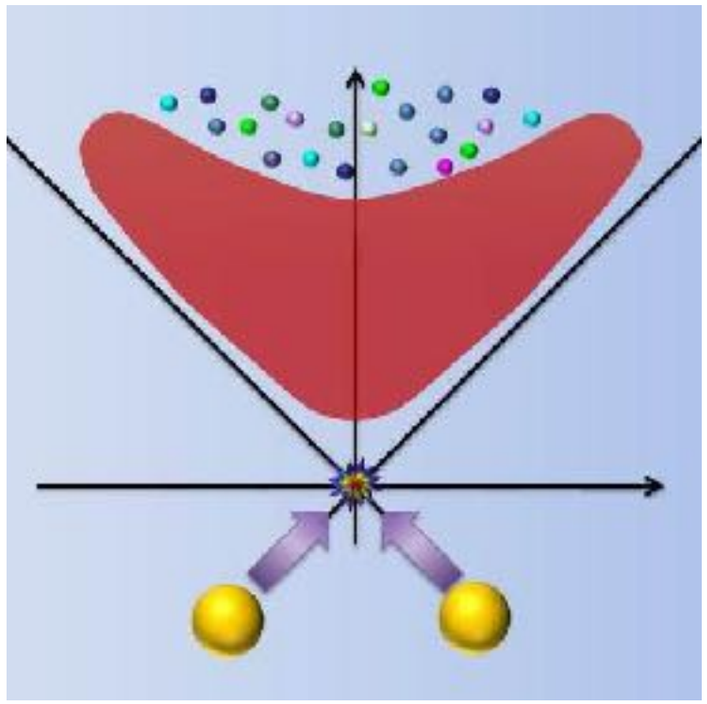
DeWolfe, Gubser, Rosen 10,11



- Holographic critical point
  - ➤ Mean field critical exponents
    - ➤ More complicated constructions go beyond mean field — Franco et al 09
  - ➤ Finite viscosities at the critical point (large N) — Natsuume & Okamure 11
- Model can be fitted to lattice data and extract equilibrium and transport properties — Rougemont et al 15, Finazzo et al 16, Rougemont et al 17, Critelli 17

Collisions at Strong Coupling

- Simulation of full collision dynamics
  - ➤ Collisions of lump of energies

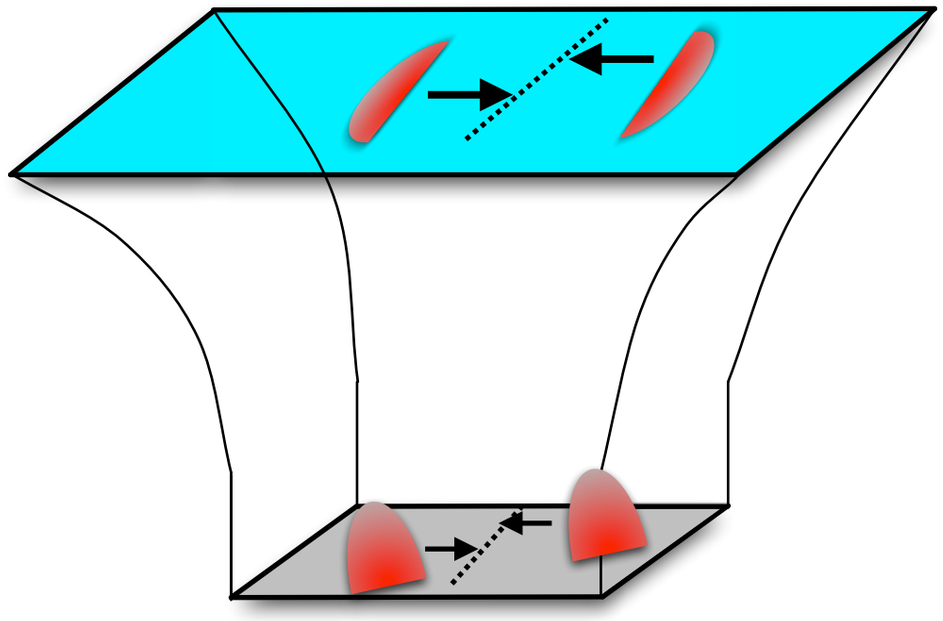


dual model

shock wave collisions

Chesler and Yaffe 11

JCS, M. Heller, D. Mateos W. van der Schee 13,14



- Fast onset of hydrodynamics
  - ➤ $t_{hydro} = 0.63 / T_{hydro}$
  - ➤ Hydrodynamics without isotropy

Chesler & Yaffe, Wu & Romatschke, Heller, Janik & Witaszczyk, Heller, Mateos, van der Schee, Trancanelli; Kurkela and Zhu

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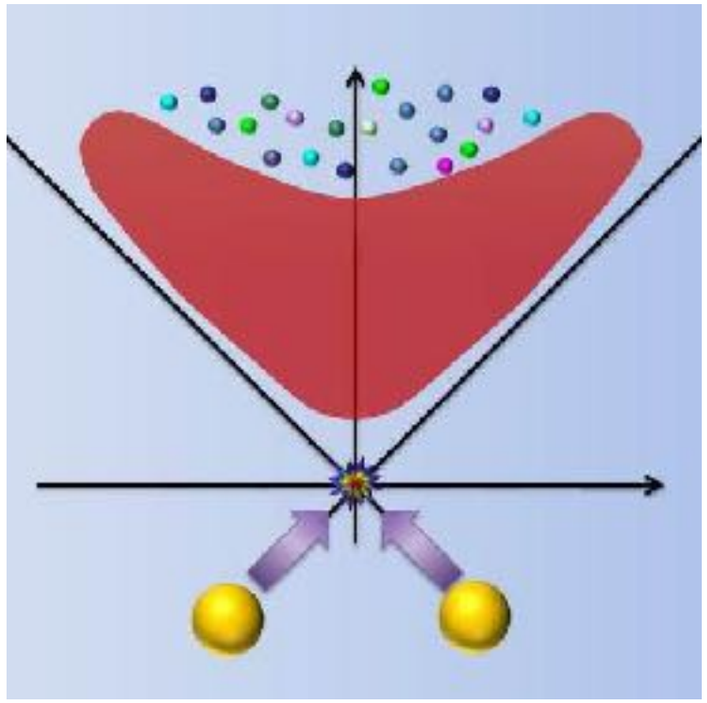

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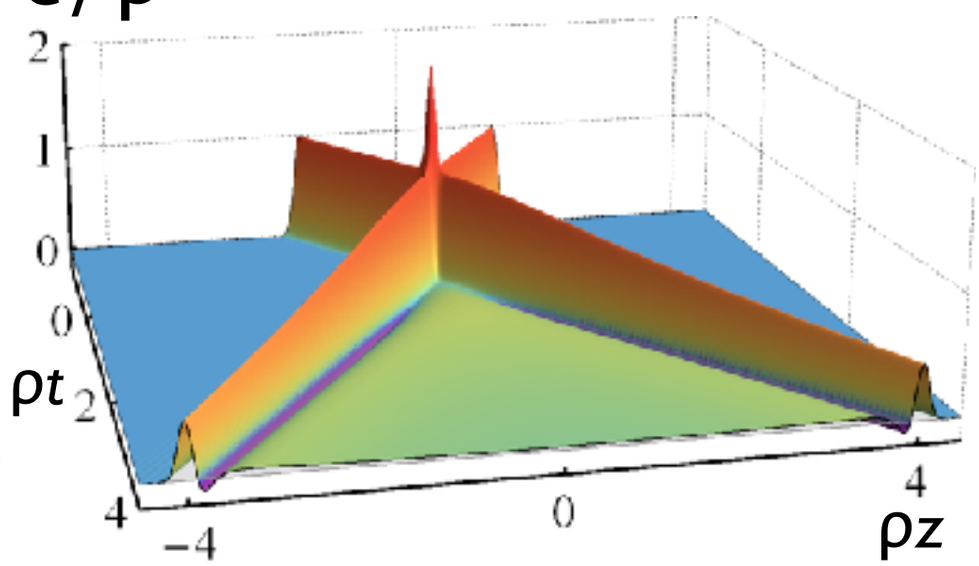

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Non-Conformal Modelling

- Einstein gravity + Scalar

$$S = \frac{2}{\kappa_5^2}\int d^5x\sqrt{-g}\left[\frac{1}{4}\mathcal{R} - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right]$$

- Phenomenological (family of) potential(s)

$$V = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

ϕ_M → parameter

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→ parameter

- Dual field theory: “mimics” a deformation of N=4 SYM with a dimension 3 operator

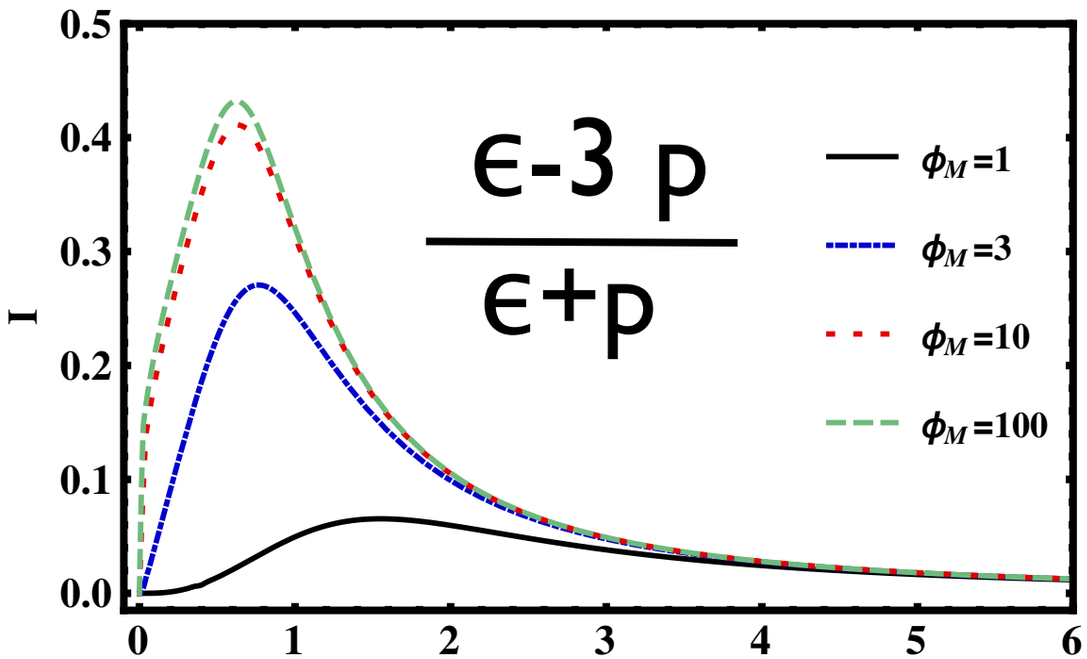
$$S_{\text{Gauge Theory}} = S_{\text{conformal}} + \int d^4x\,\Lambda\,\mathcal{O} \qquad \mathcal{O} \sim \bar{\psi}\psi + \cdots$$

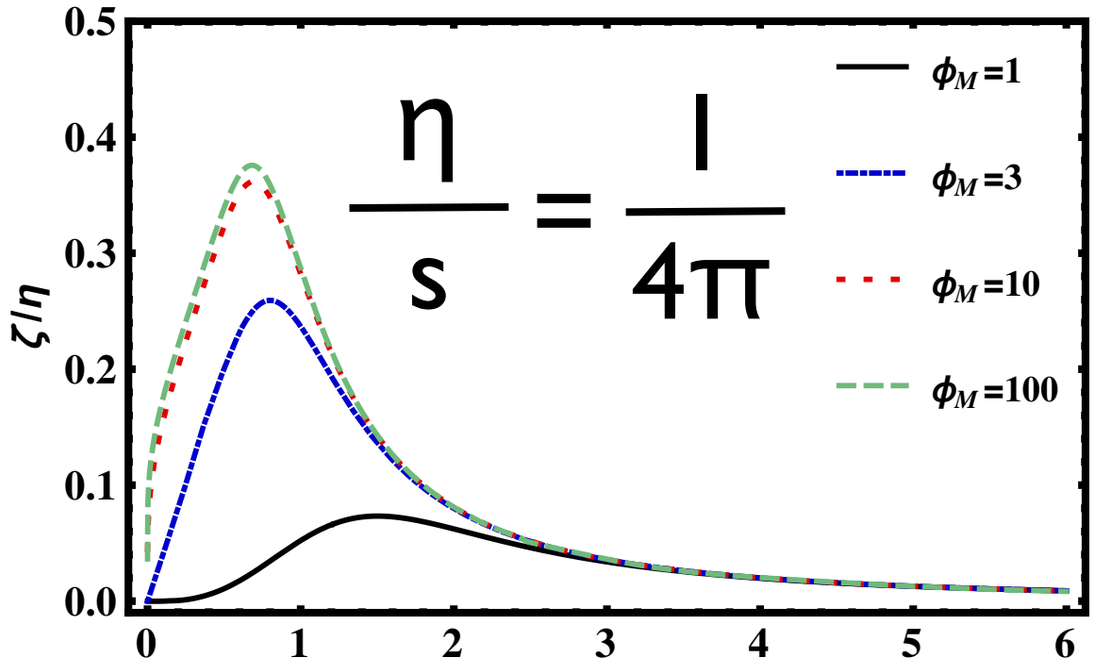
↓ “mass”

- Rich thermodynamic and transport properties

Attems, JCS, Mateos, Papadimitriou, Santos, Sopuerta, Triana, Zilhao, 16

Non-Conformal Modelling





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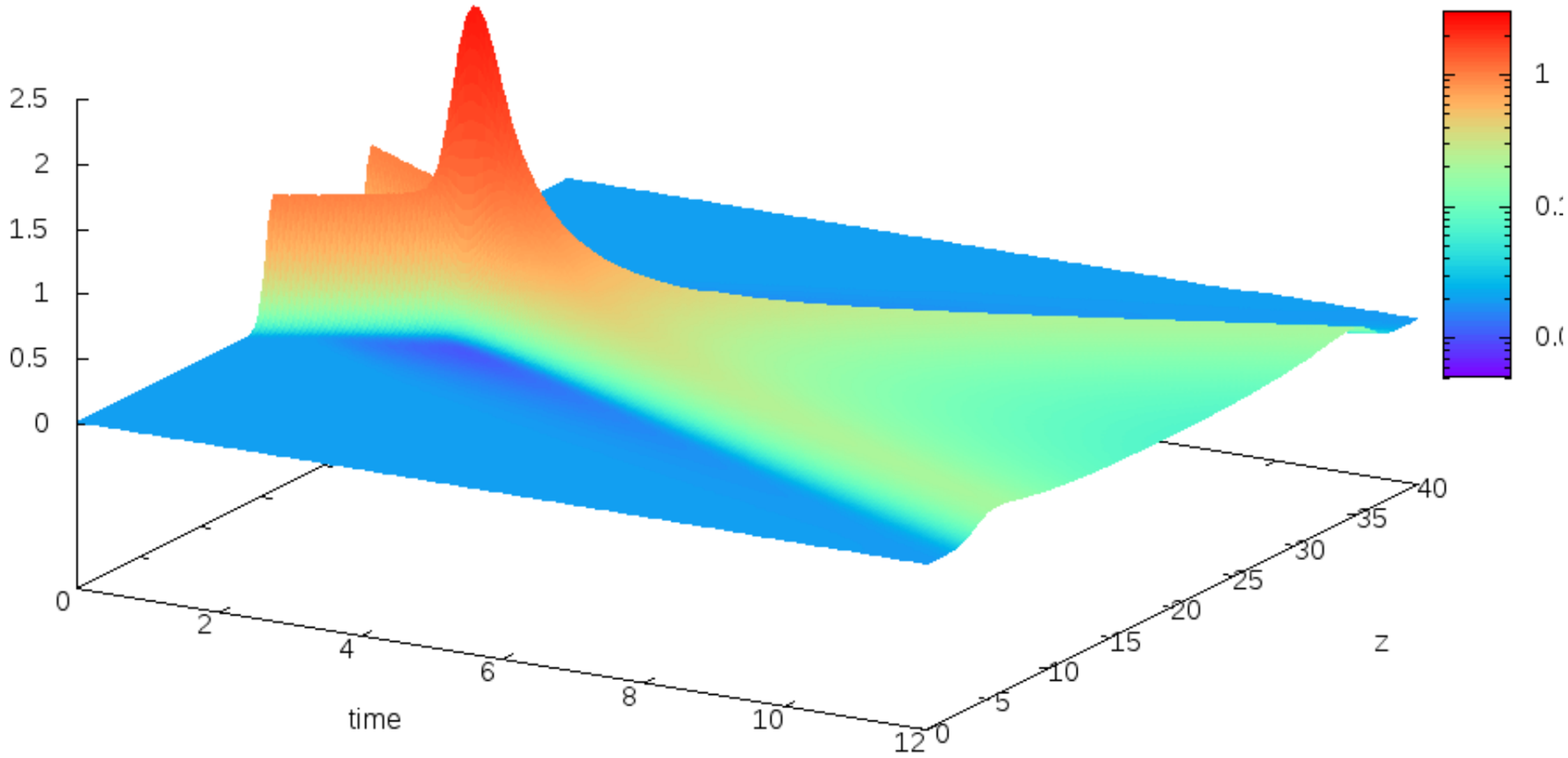
Attems, JCS, Mateos, Papadimitriou, Santos, Sopuerta, Triana, Zilhao, 16

Non-Conformal Collisions

Attems, JCS, Mateos, Santos, Sopuerta, Triana, Zilhao, 16,17



Full access to the off-equilibrium stress tensor

Hydrodynamics without E.O.S

- Does hydrodynamics describe the evolution?

  Check constitutive relationships?

$$u^\mu T^\nu_\mu \equiv \varepsilon_{\rm local} u^\nu \quad \rightarrow \quad \begin{aligned} P_L^{\rm hyd} &= P_{\rm eq} + P_\eta + P_\zeta \,, \\ P_T^{\rm hyd} &= P_{\rm eq} - \tfrac{1}{2} P_\eta + P_\zeta \end{aligned}$$

  Quickly satisfied (as in conformal models)

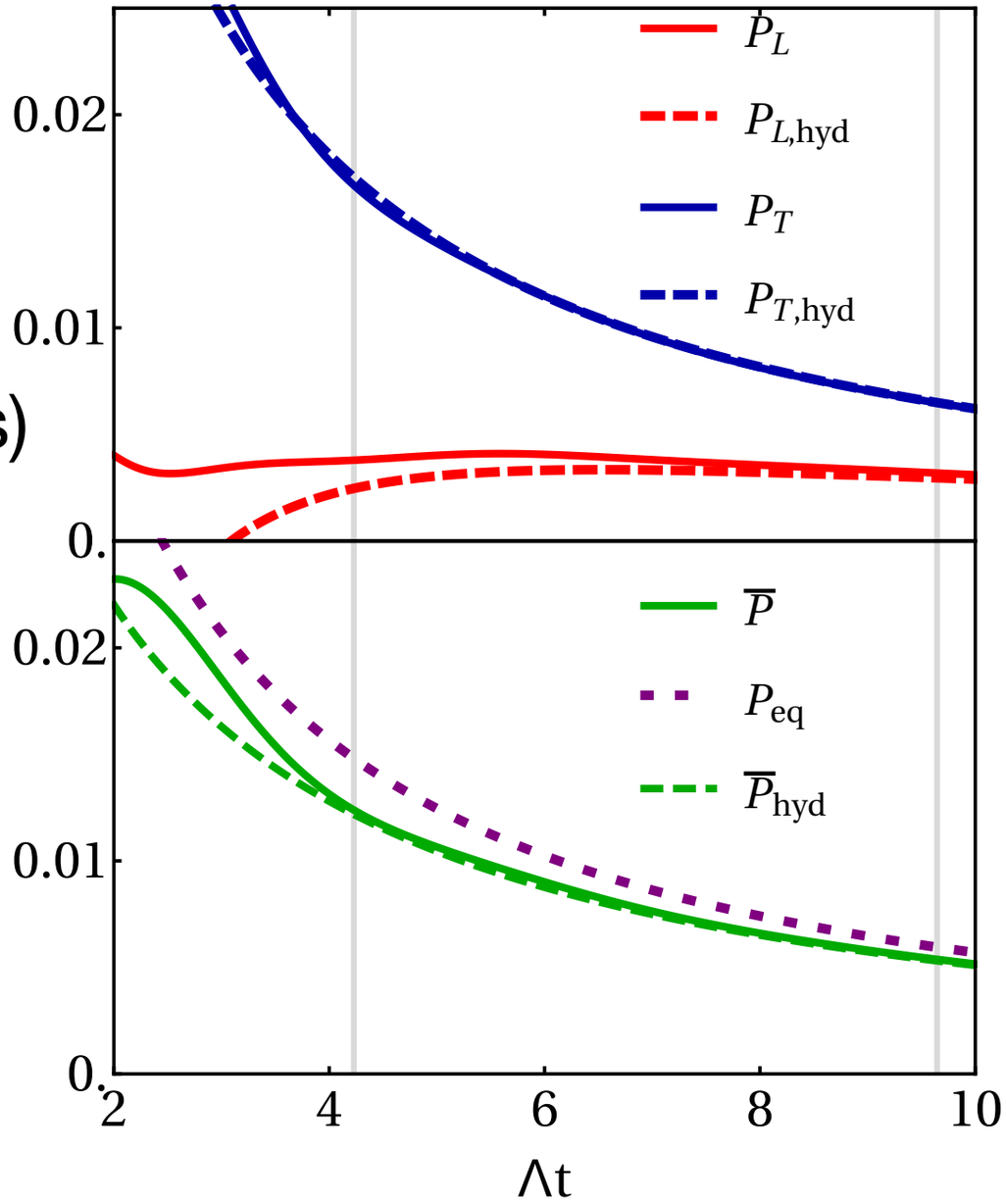
$$\frac{\left| P_{L,T} - P_{LT}^{\rm hyd} \right|}{\bar{P}} < 0.1 \qquad t_{\rm hyd} T_{\rm hyd} = 1.0$$

- Is the e.o.s satisfied?

  average pressure as a proxy of eos

$$\frac{\left| \bar{P} - \bar{P}^{\rm hyd} \right|}{\bar{P}} < 0.1 \qquad t_{\rm eos} = 2.3\, t_{\rm hyd}$$

  "eosization"



- The ordering of these times scales is not unique.

See M. Attems talk for more details

Non-Trivial Phase Structure

- Einstein gravity + Scalar

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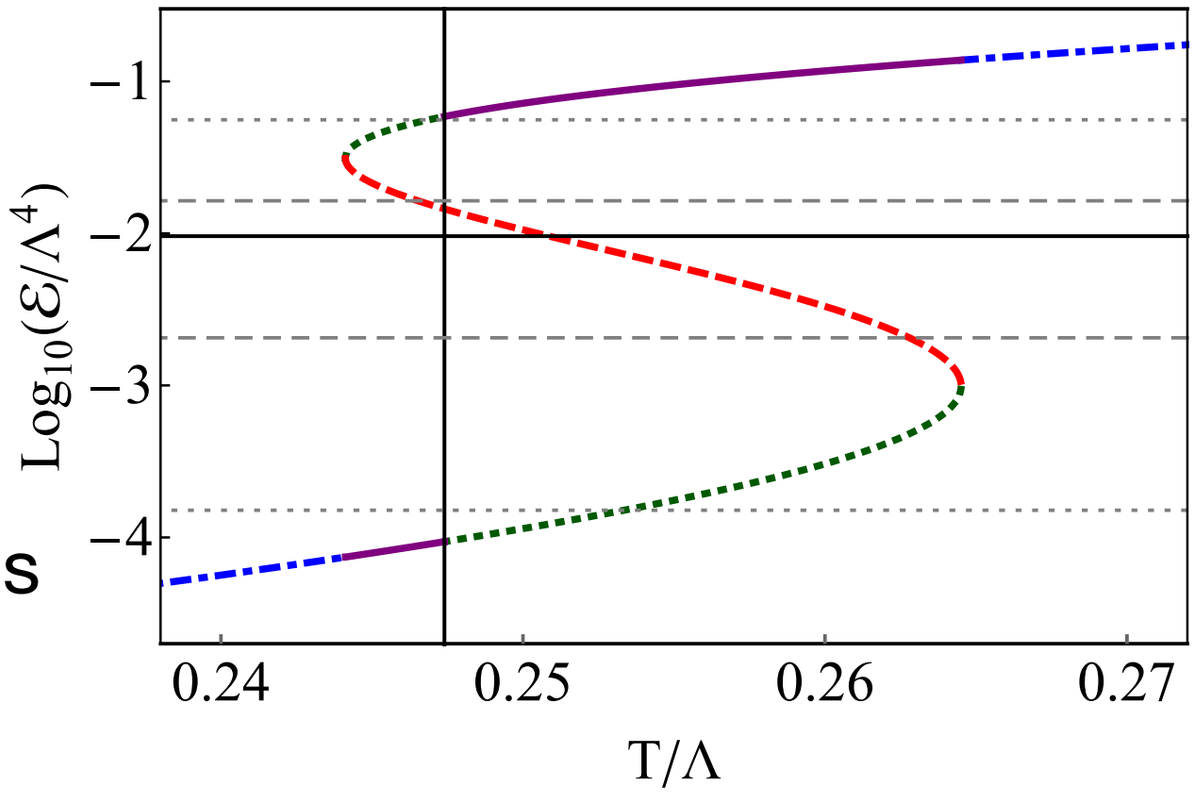
Make parameter imaginary

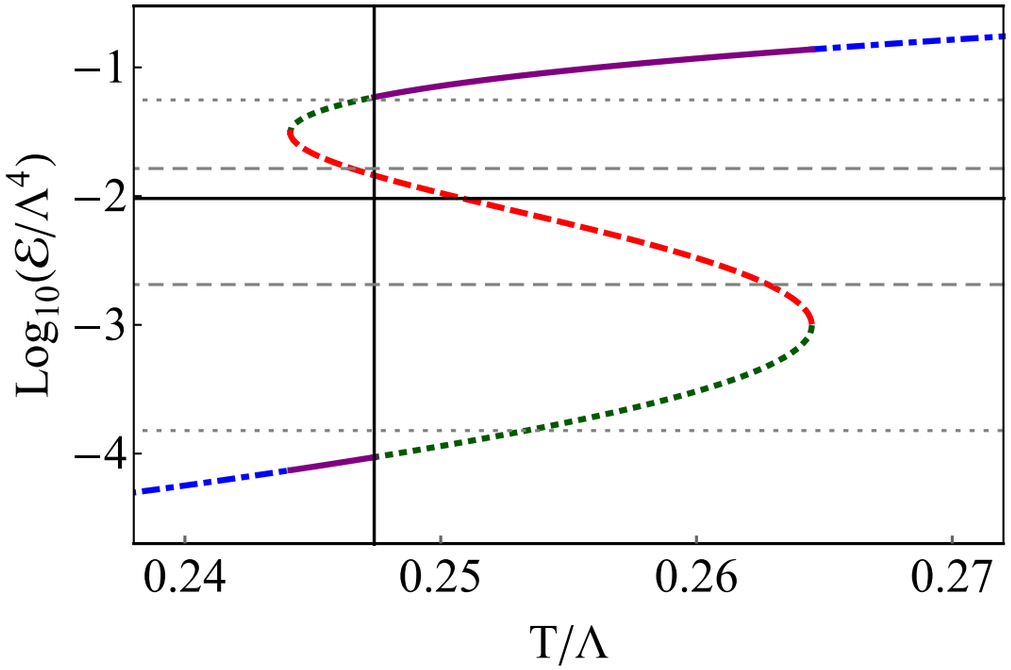
- First order Phase transition

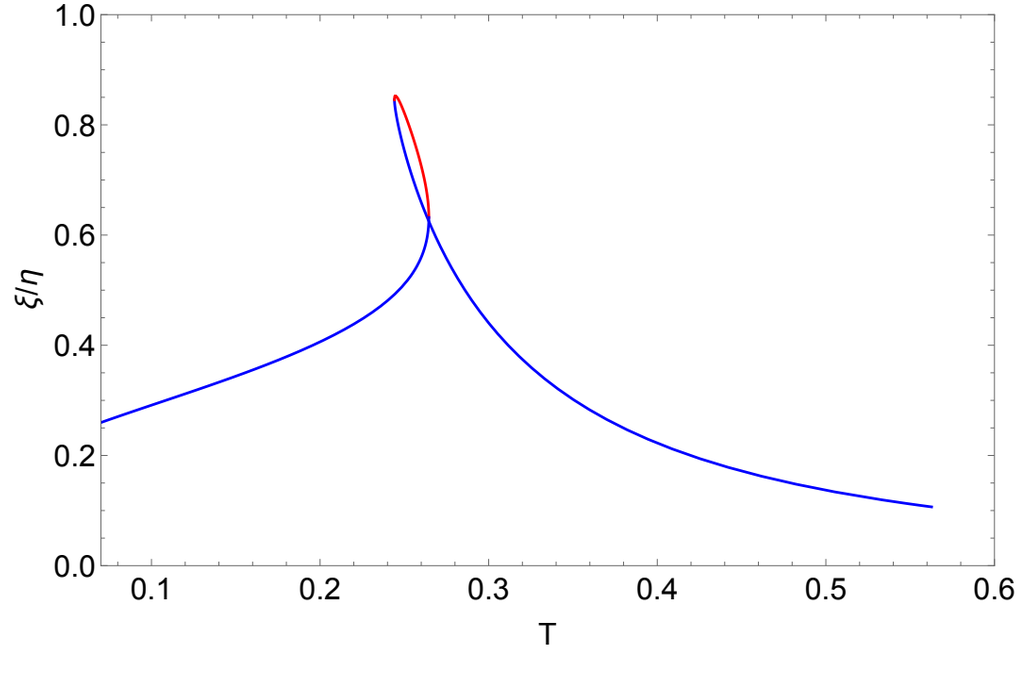
  (for certain values of ϕ_M)

- Study of homogeneous solutions



Spinodal Instability





- Thermodynamically unstable region

$$c_s^{-2} = \frac{1}{S}\frac{d\mathcal{E}}{dT} \qquad \frac{d\mathcal{E}}{dT} < 0 \to c_s^2 < 0$$

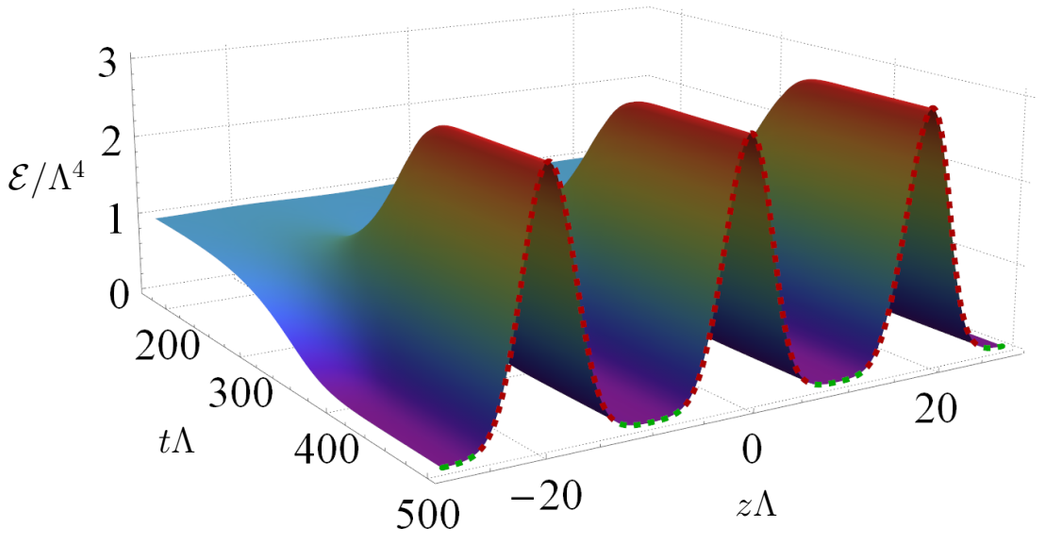
- Sound waves become unstable!

$$\Gamma(k) \simeq |c_s|\, k - \frac{1}{2T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2$$

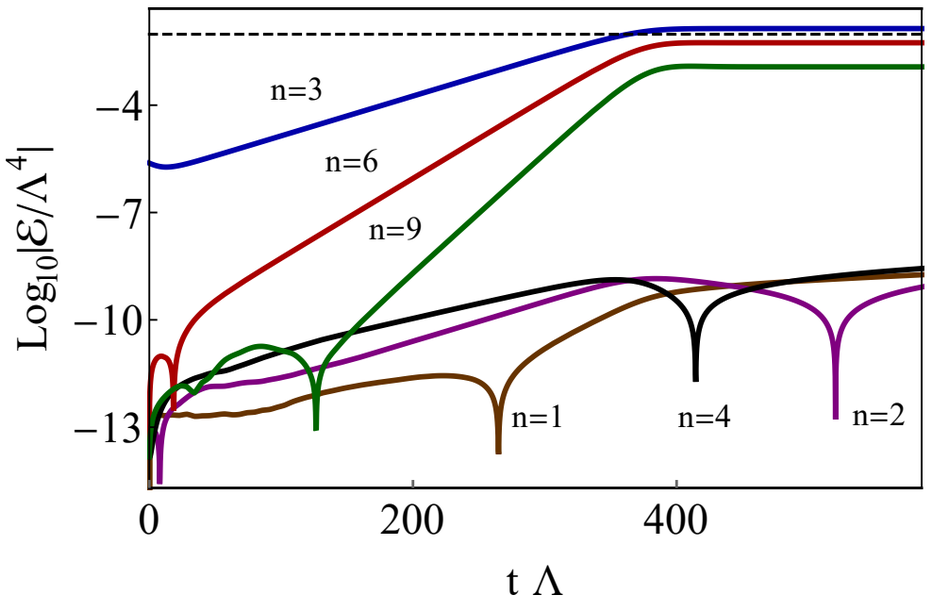
- The system wants to become inhomogeneous

see for example
Chomaz, Colonna, Randrup 03

Inhomogeneous state



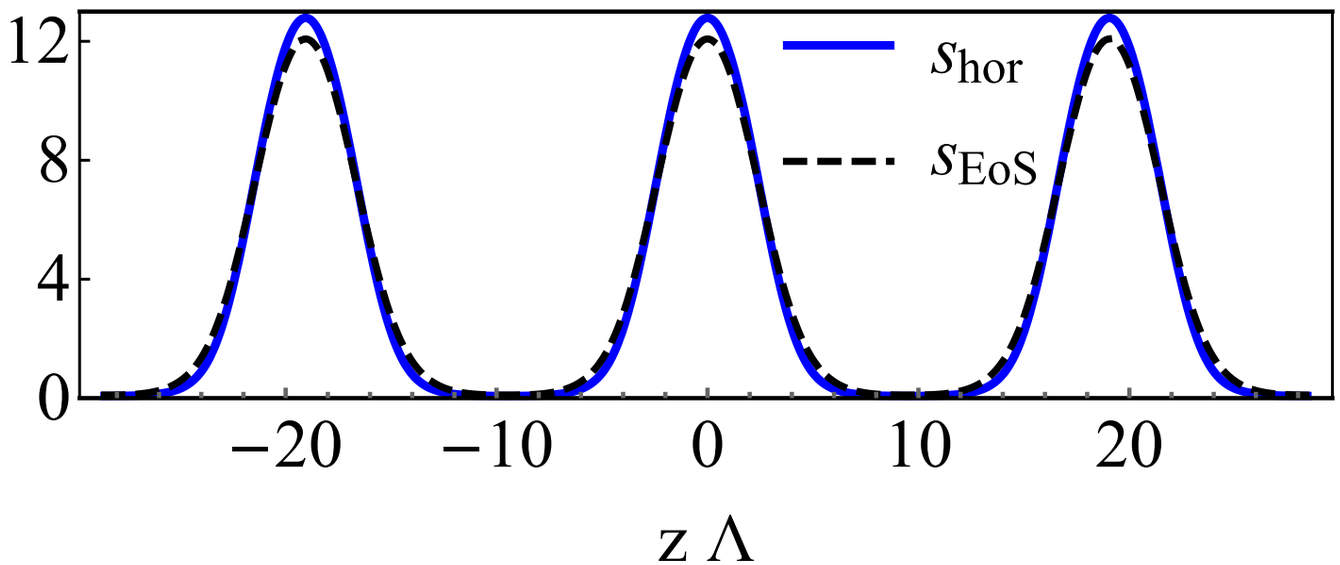
Attems, Sea, JCS, Mateos, Triana, Zilhao, 17



- Put the system is a box (cylinder) $\mapsto$ $p_z = \dfrac{2\pi}{L} n$
  - Start with the unstable solution
  - Introduce small (10^{-4}) perturbation with n=3 (lin. unstable)
- Study the evolution and end result of evolution
  - The system settles (!) into an inhomogeneous state (!)

similar observations by Janik, Jankowski, Soltanpanahi 17

Gregory-Laflamme Instability



- Evolve a simple metric ansatz

$$ds^2 = -A^2(r,z)dt^2 + \Sigma^2(r,z)\left(e^{B(z,r)}d\mathbf{x}_\perp^2 + e^{-2B(z,r)}dz^2\right) + 2dt\,(dr + F(z,r)dz)$$

- Initial state: Static & Homogeneous

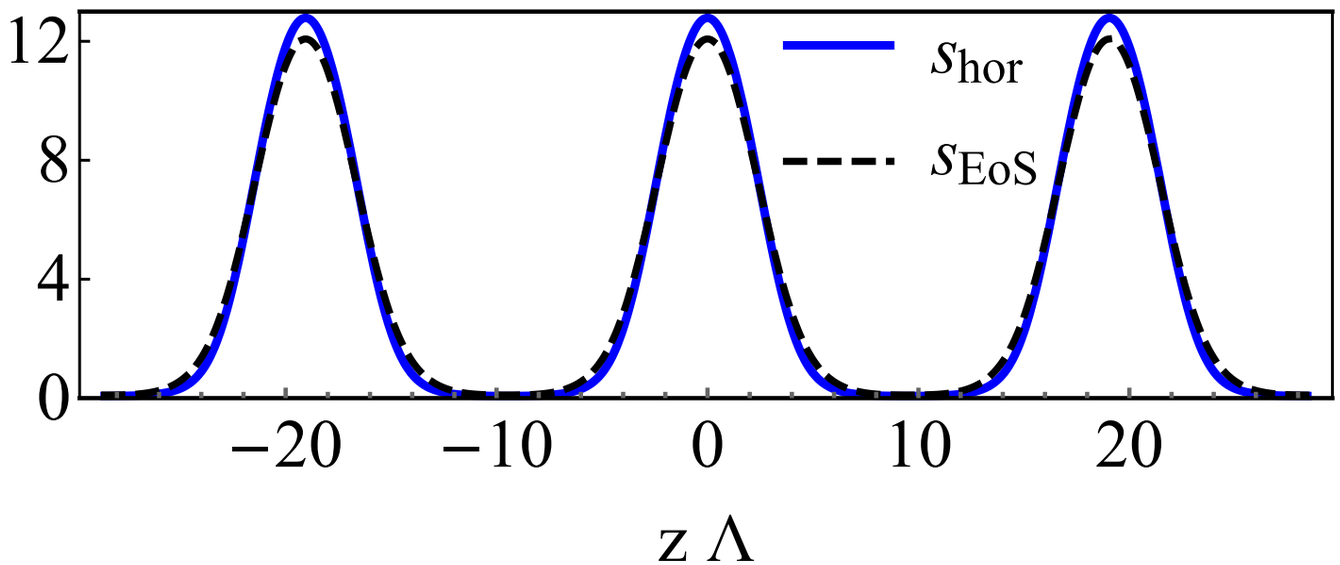
$$A(r) \quad B(r) \quad \Sigma(r) \qquad A(r_H) = 0$$

(killing) horizon

Entropy of dual theory

$$S_{\rm EOS} \propto \Sigma(r_H)^3$$

Gregory-Laflamme Instability



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(killing) horizon

Entropy of dual theory

$$S_{\rm EOS} \propto \Sigma(r_H)^3$$

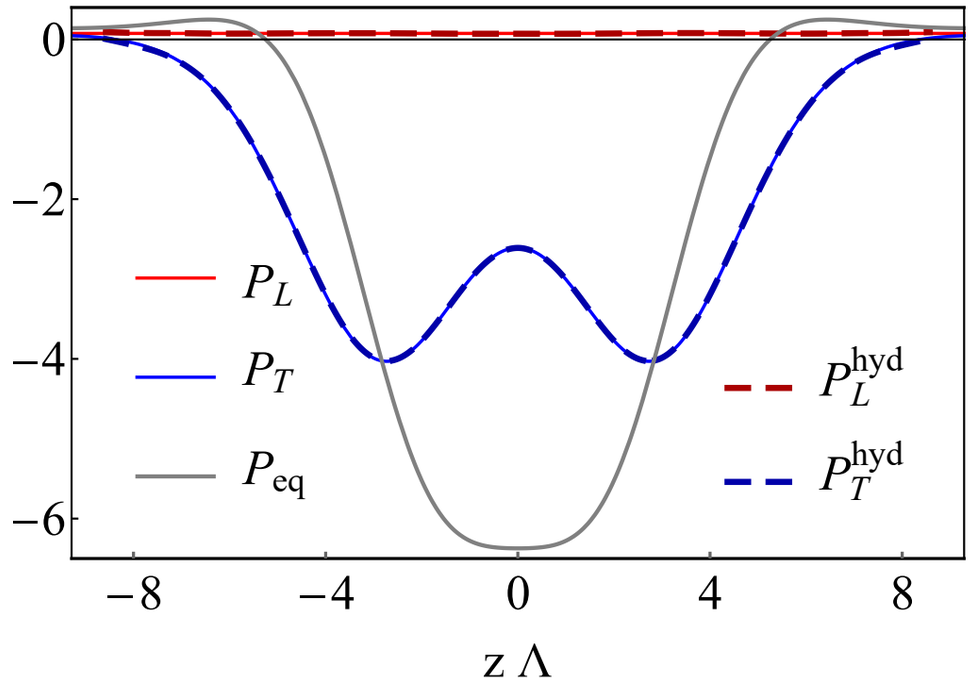
- Final state: inhomogeneous

$$A(r,z) \quad B(r,z) \quad \Sigma(r,z) \quad F(r,z) \qquad A(r_H,\ z) = 0 \qquad S \propto \Sigma(r_H,z)^3$$

(killing) horizon

Second Order HydroSatatics!

- Anisotropic state -> Anisotropic pressure



- Pressures ≠ pressure of isotropic state at local energy density.
- However, we can still find a derivative expansion!

$$P_L^{\rm hyd} = P_{\rm eq}(\mathcal{E}) + c_{\rm L}(\mathcal{E})(\partial_z\mathcal{E})^2 + f_{\rm L}(\mathcal{E})(\partial_z^2\mathcal{E})\,,$$
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(first order gradients vanish in static limit)

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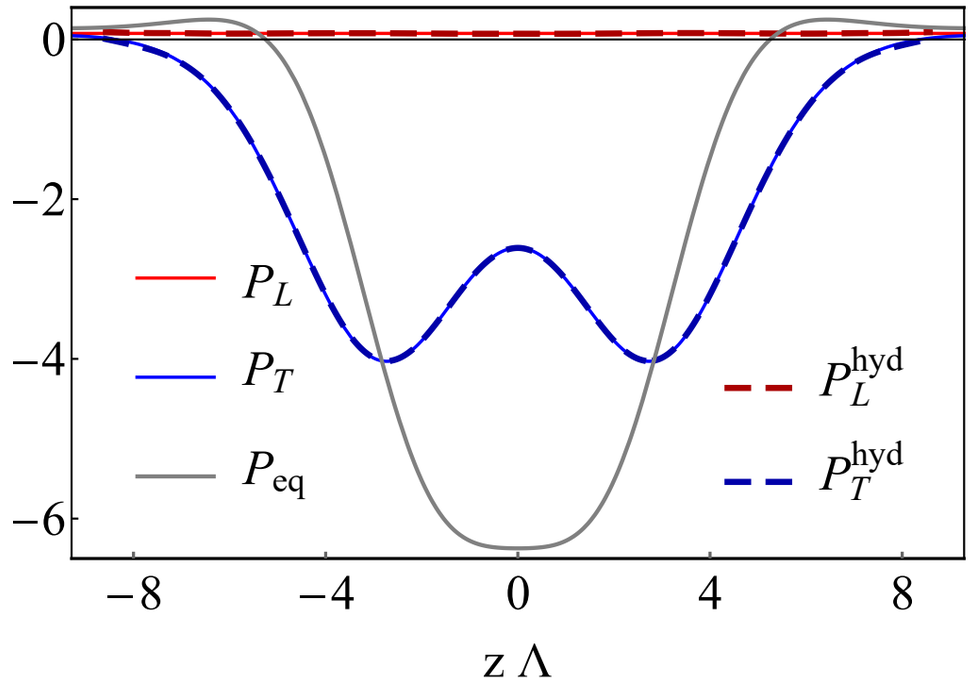

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transport coefficients

(first order gradients vanish in static limit)

- State sustained by second order gradients!

Second Order HydroSatatics!

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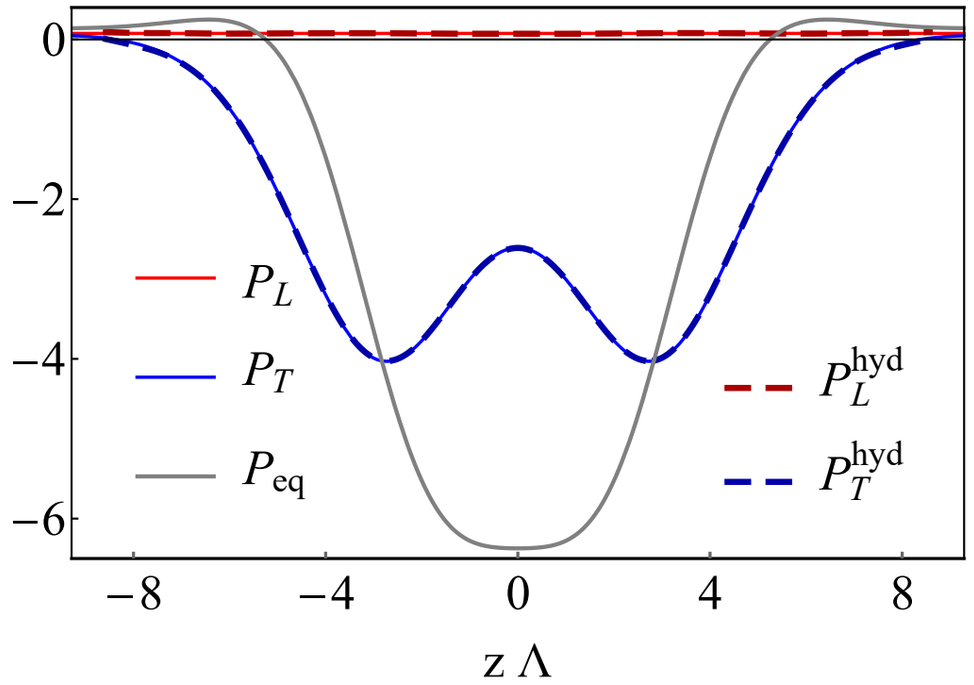

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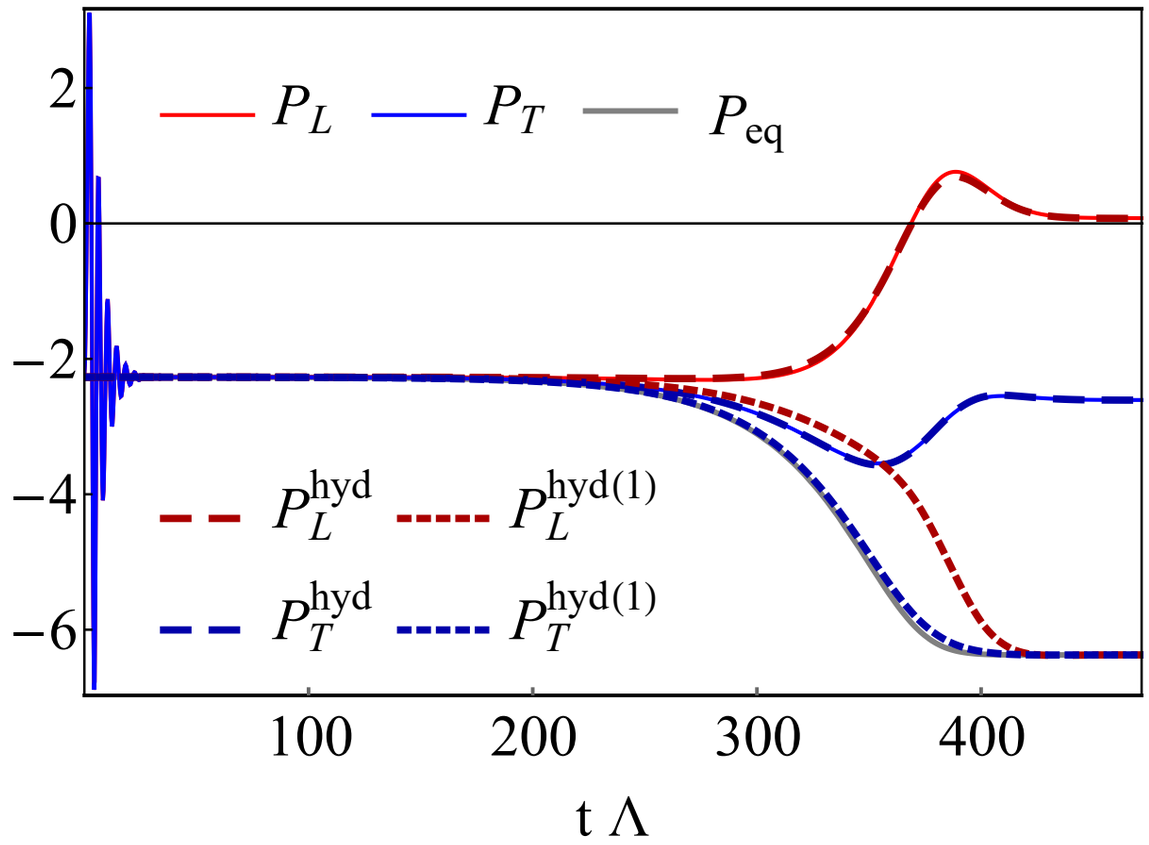
transport coefficients

(first order gradients vanish in static limit)

- Equation to determine all possible inhomogeneous solutions

Second Order Hydro Evolution

- Time evolution is also hydrodynamic!



- Onset
- Evolution
- Saturation

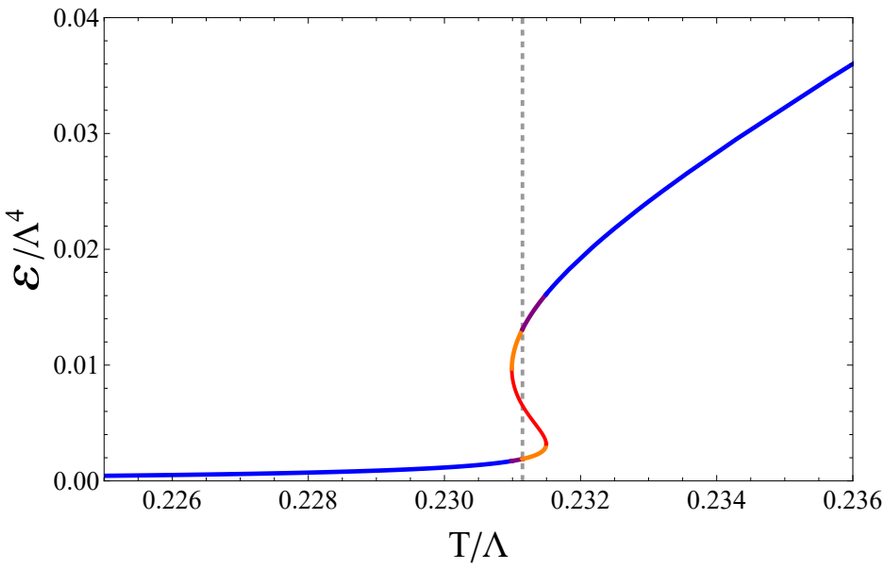
} spinodal instability

Gregory-Laflamme instability

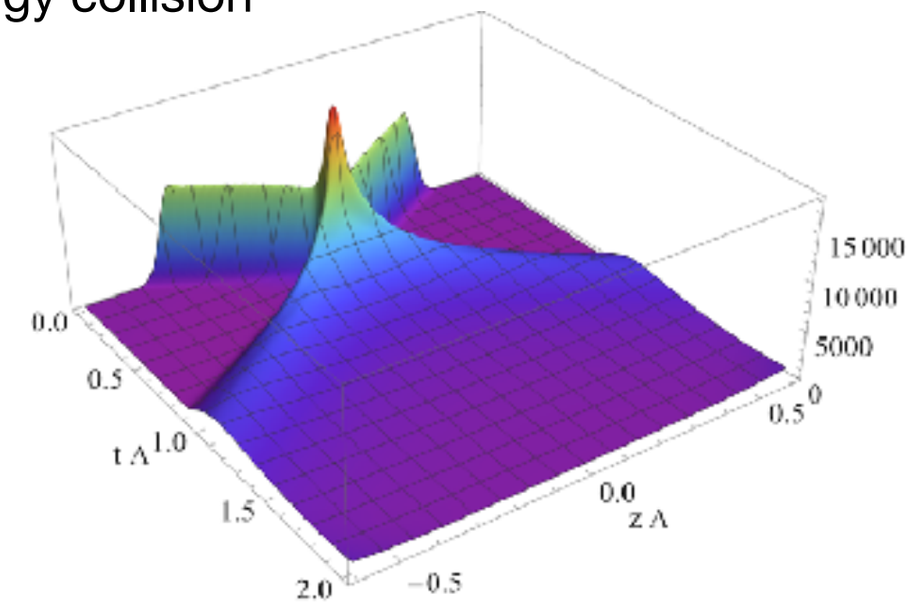
{ second order hydrodynamics

(with dominant second order gradients!)

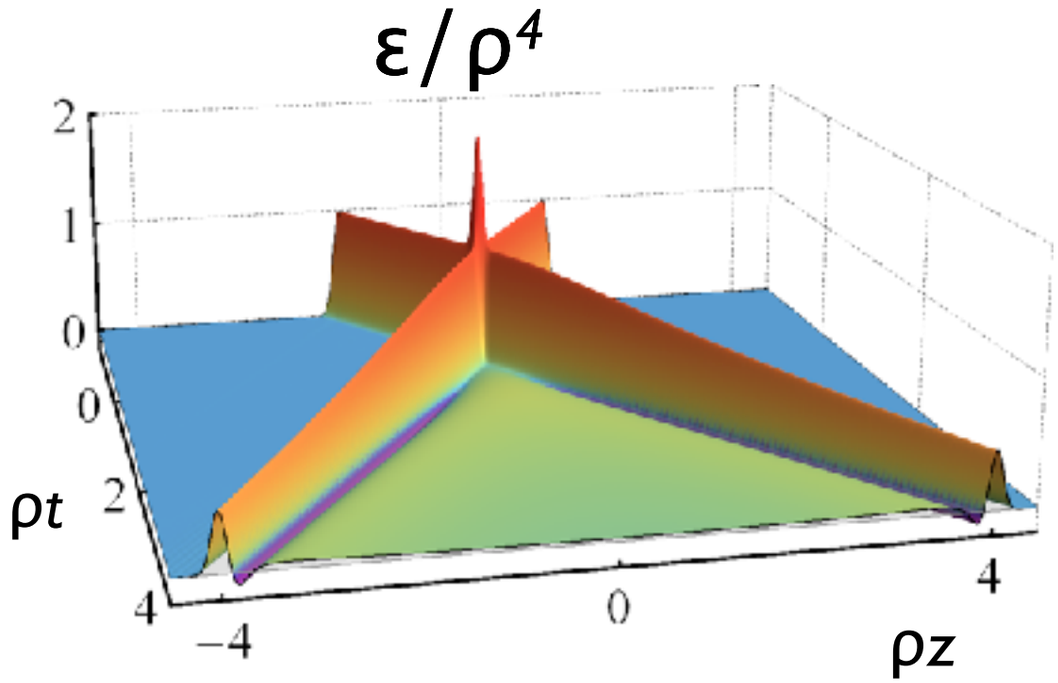
Collisions & 1st Order Transition



High energy collision



- Initial field profiles similar to conformal case



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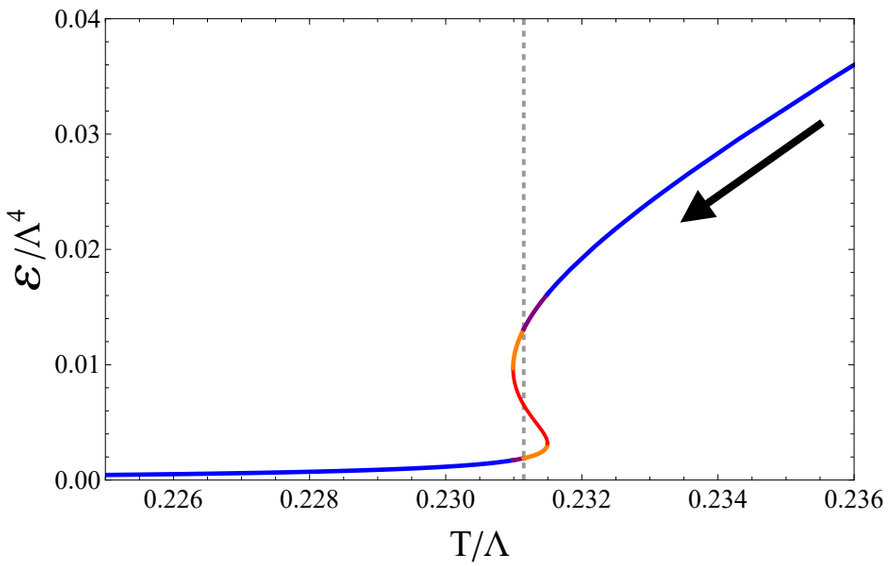

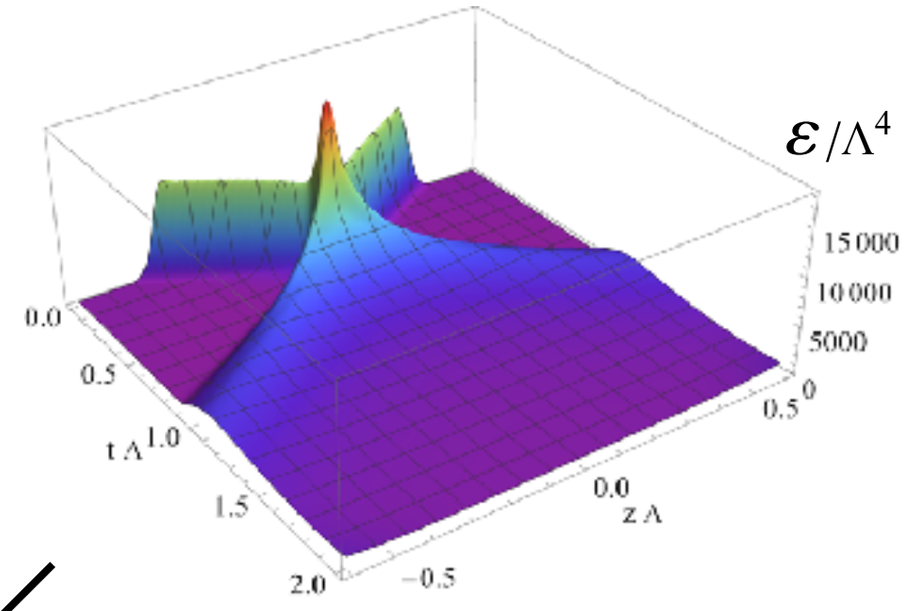

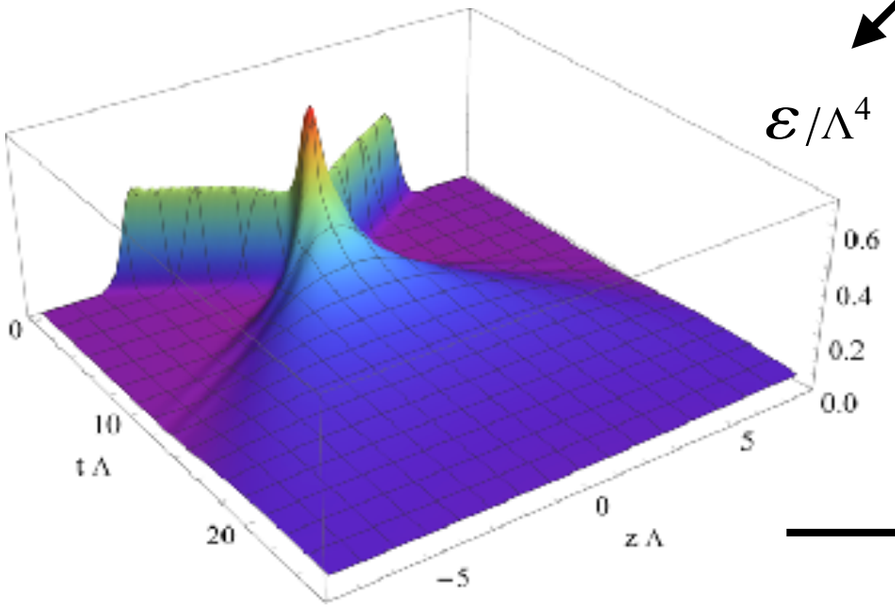

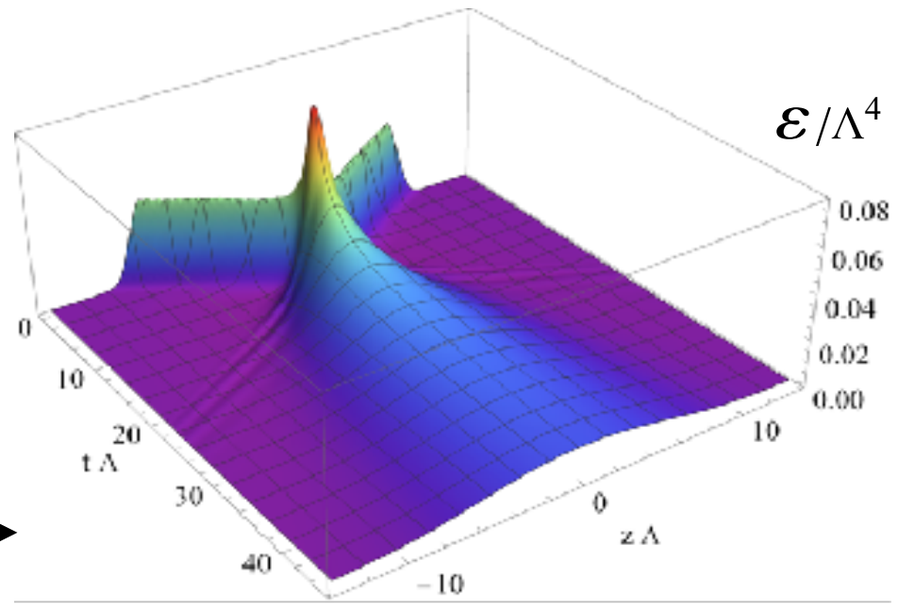

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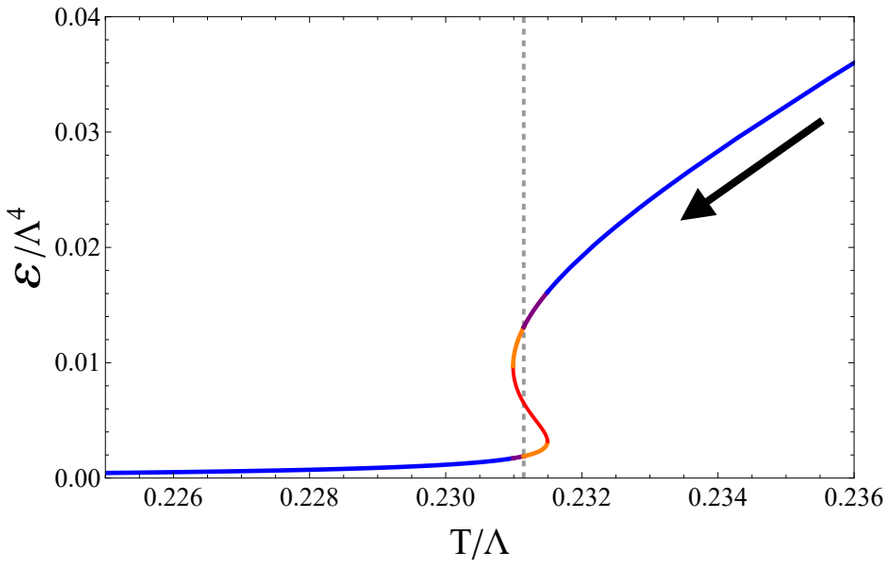
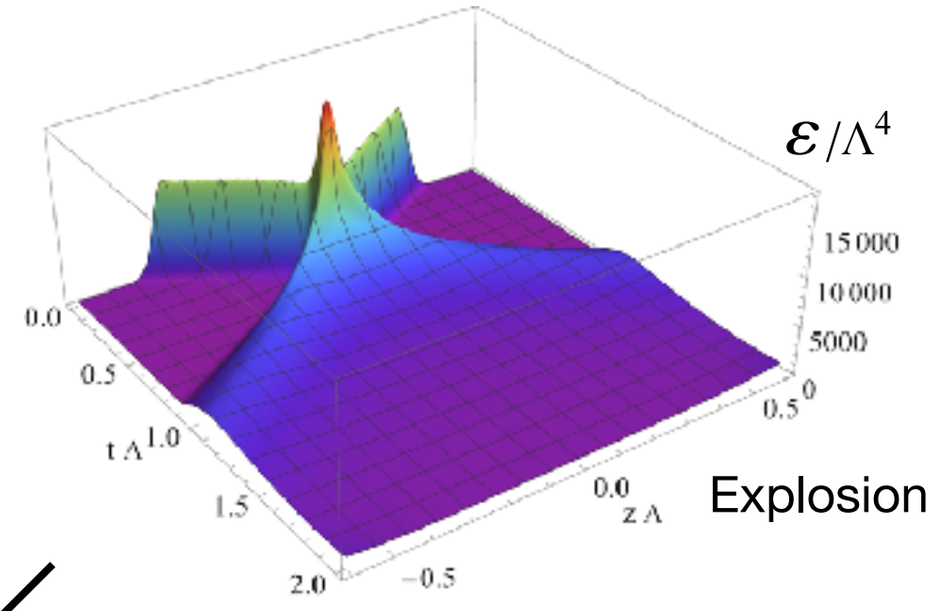
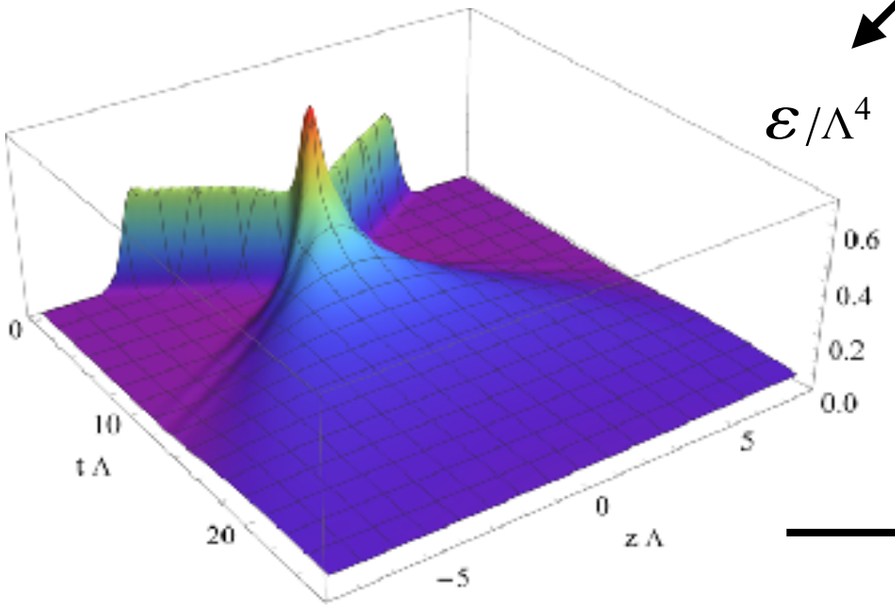
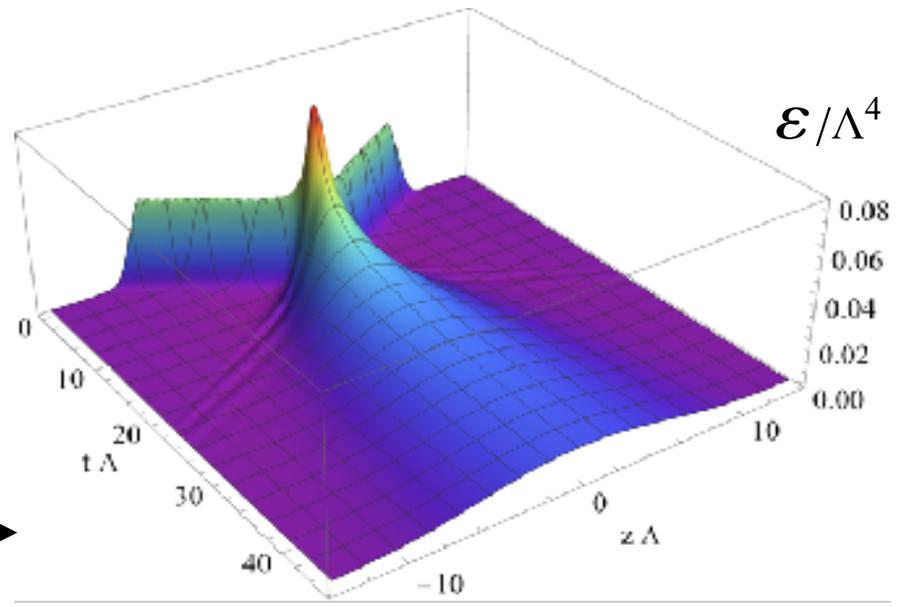




Explosion

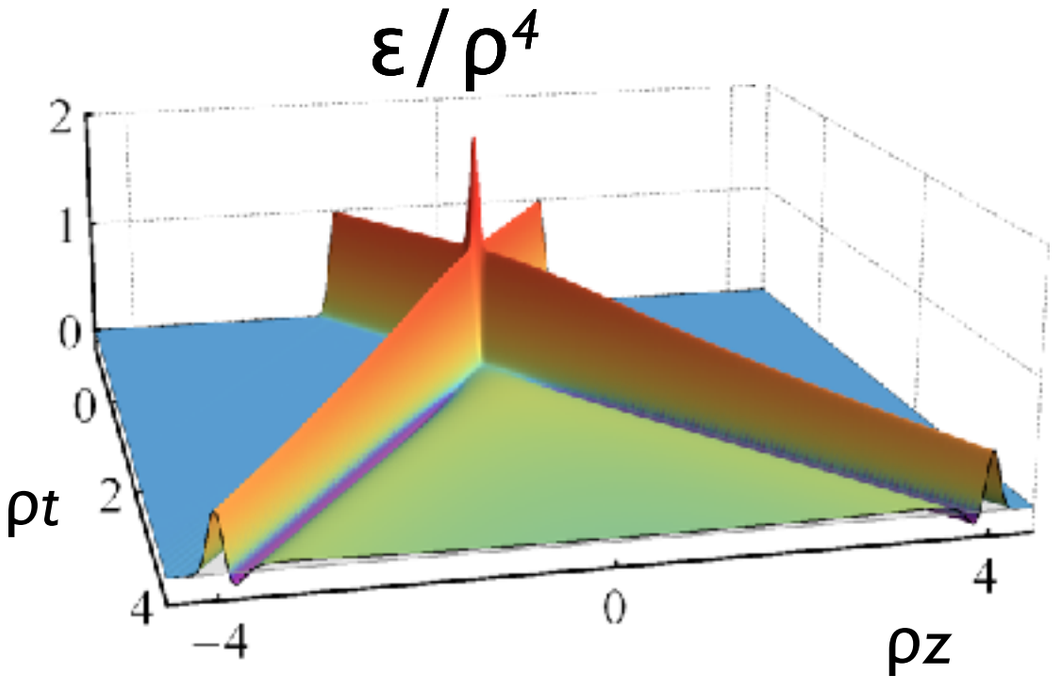




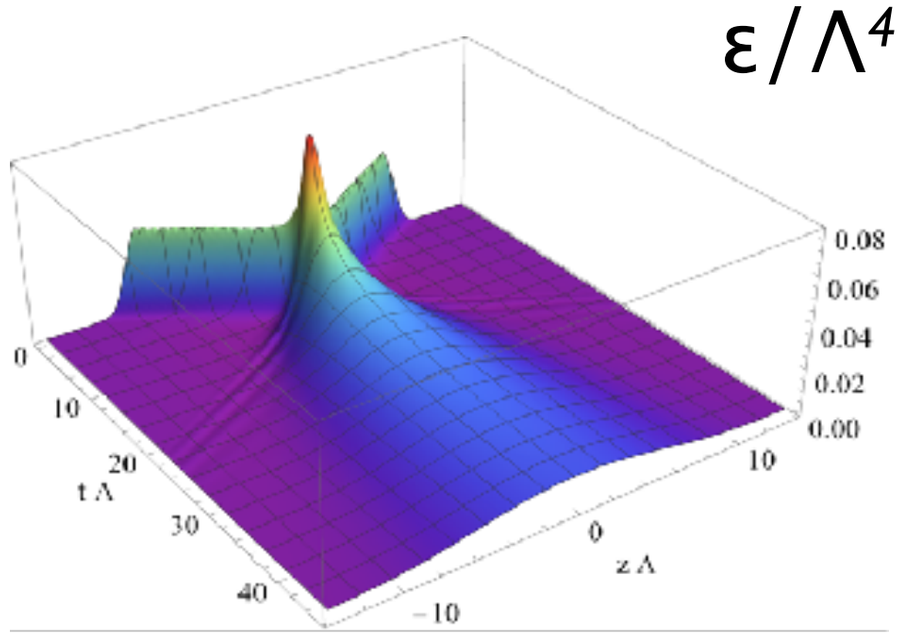
Blob formation

Collisions Through the Transition

- Conformal



- First order transition

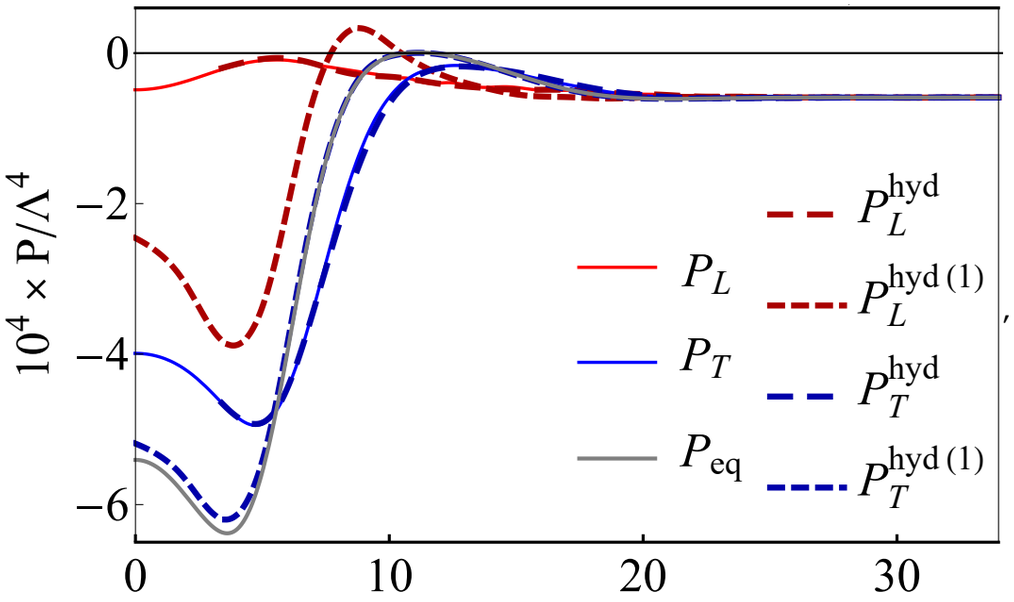


- When going through the transition, new qualitative features
  - Most of the energy gets stuck in a finite size blob
  - The evolution of the blob is extremely slow
  - Can we also describe this via hydrodynamics?

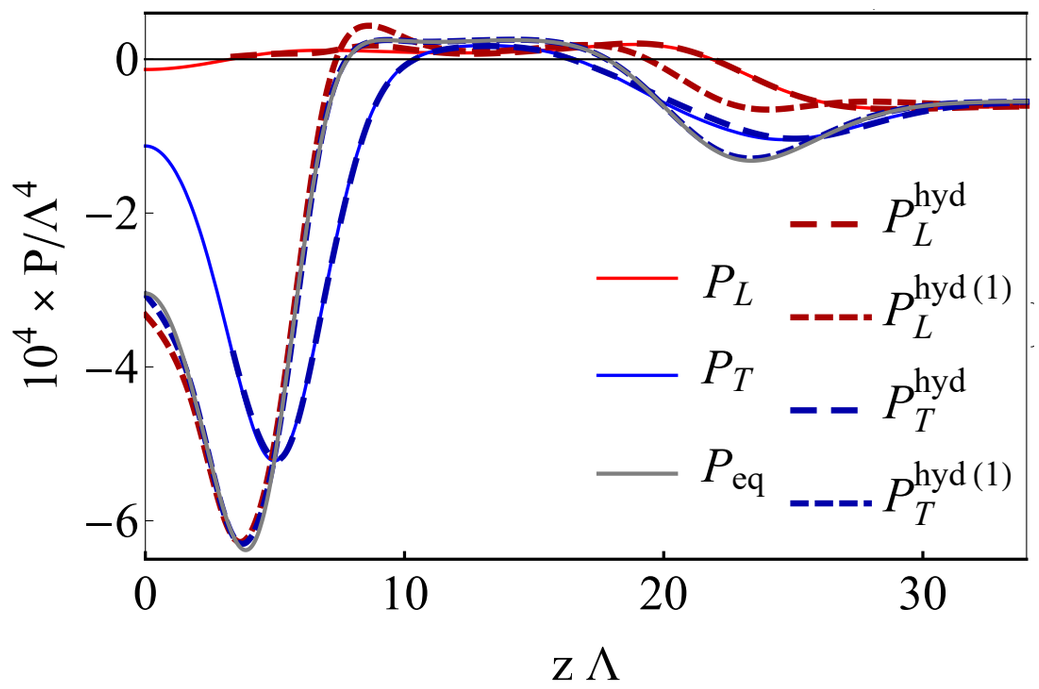
Attems, Bea, JCS, Mateos, Santos, Triana, Zilhao (in preparation)

Second Order Hydrodynamcics

t Λ=68



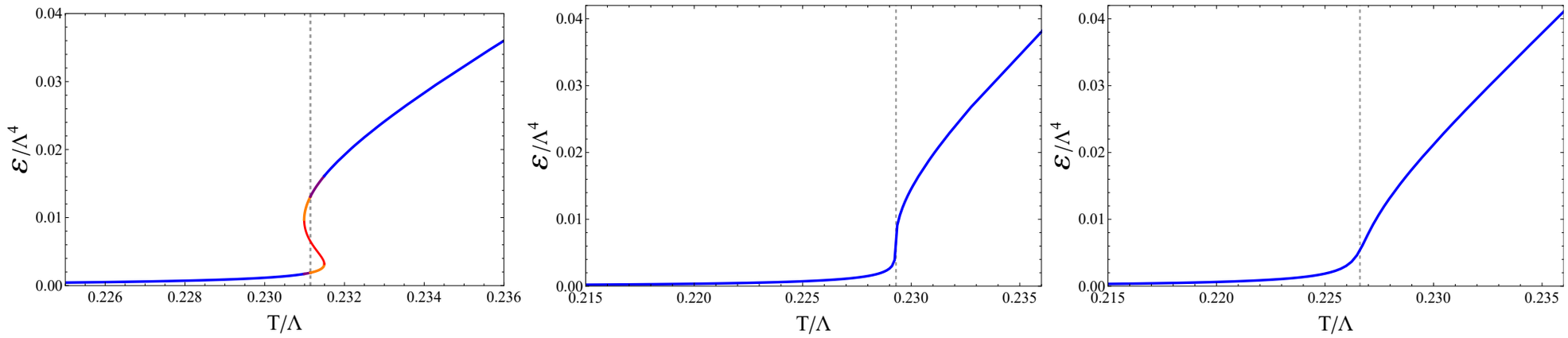
t Λ=136



- The (slow) evolution of the blob is described by second order hydrodynamics!

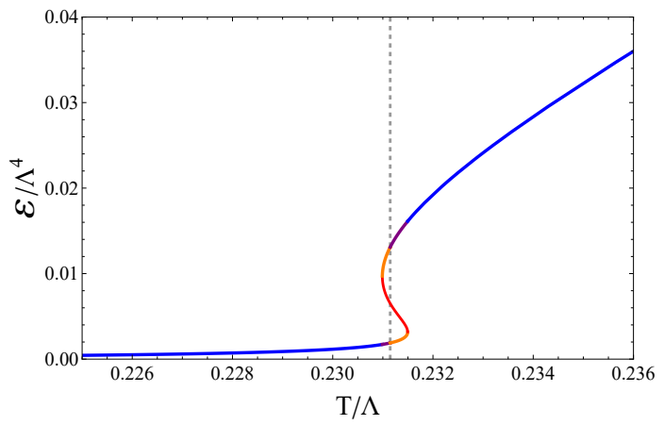
Across the Critical Point

- Different phase structures via a continuous parameter in the model

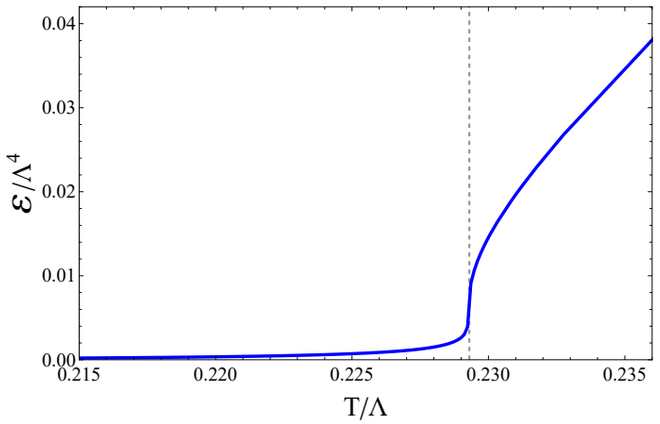


Across the Critical Point

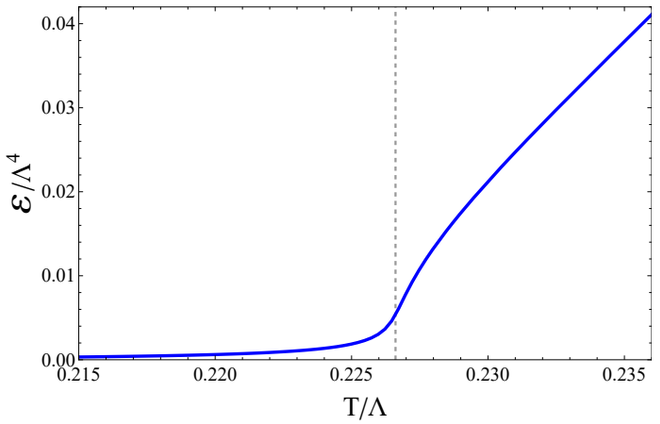
- Different phase structures via a continuous parameter in the model



1st order



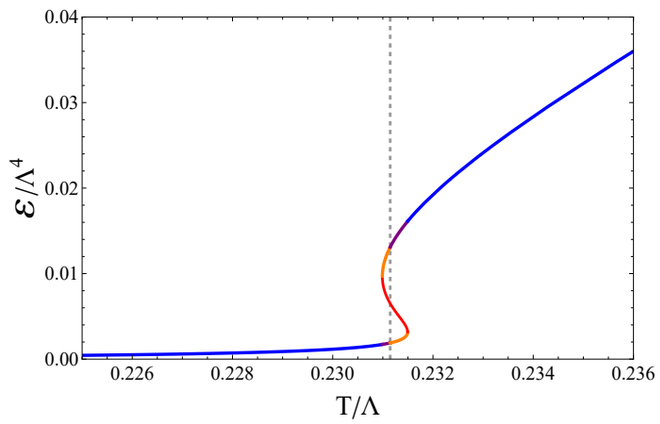
2nd order



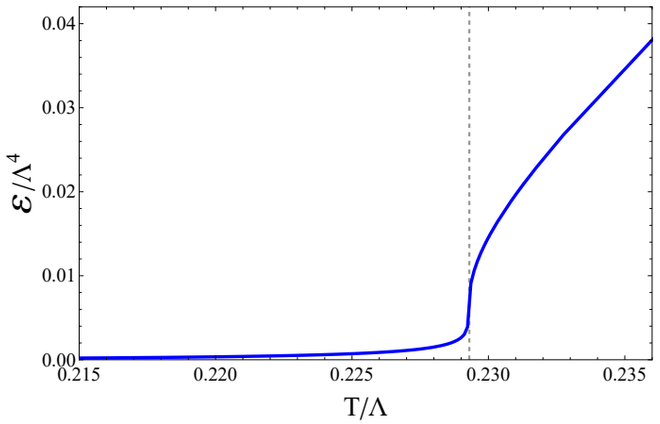
cross-over

Across the Critical Point

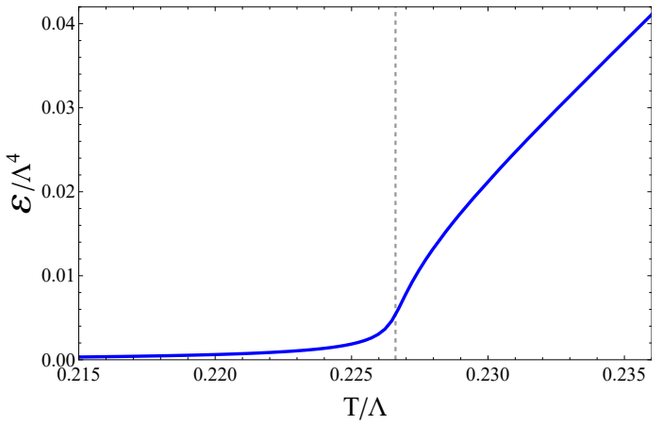
- Different phase structures via a continuous parameter in the model



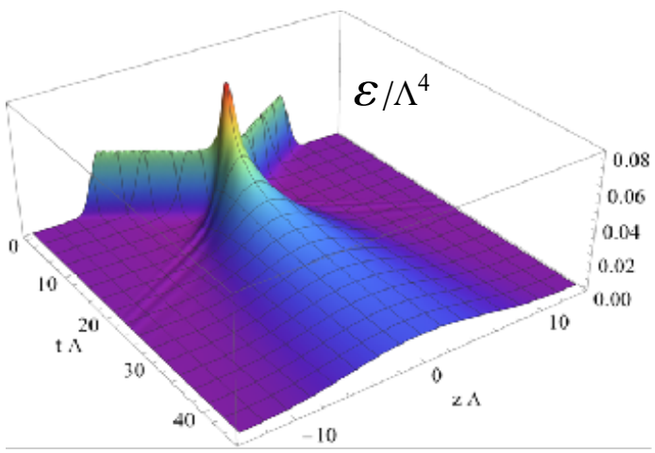
1st order

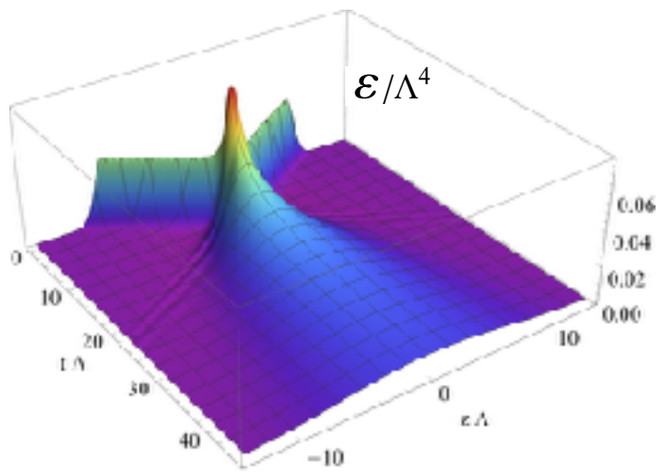


2nd order



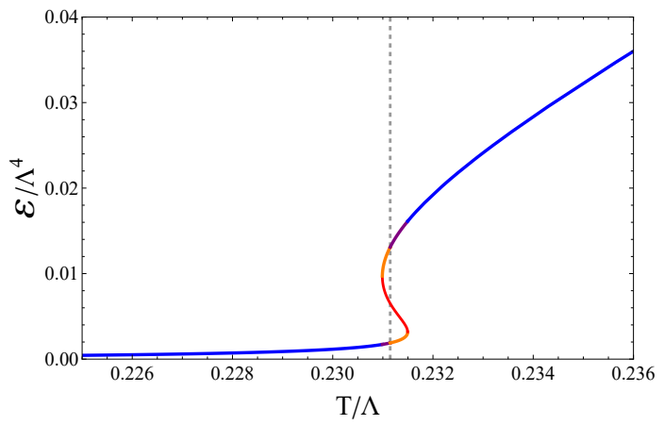
cross-over



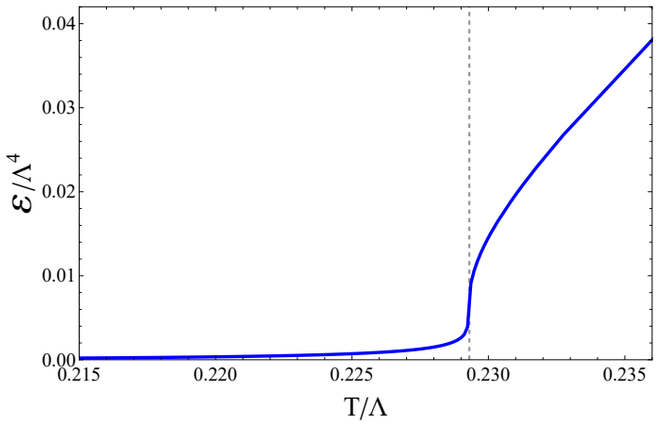


Across the Critical Point

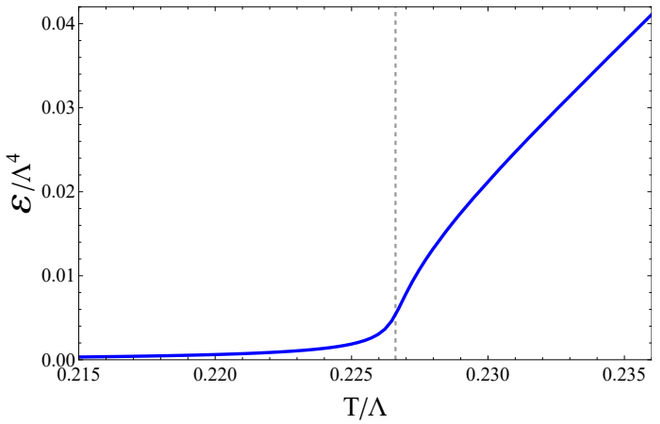
- Different phase structures via a continuous parameter in the model



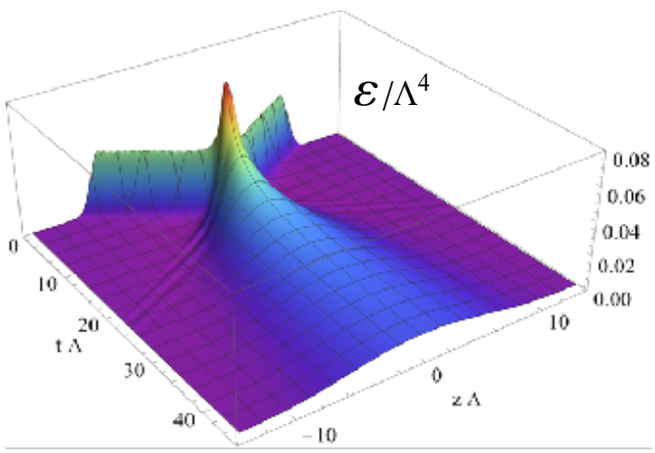
1st order

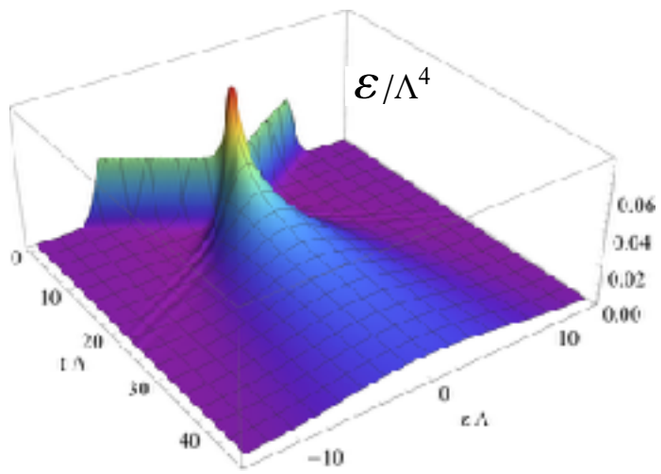


2nd order



cross-over





- No qualitative difference in the energy distribution after the collision

Attems, Bea, JCS, Mateos, Santos, Triana, Zilhao (in preparation)

Conclussions

- Lessons from holographic collisions

  ➤ Hydrodynamics works even with strong deviations from e.o.s

  ➤ Holographic analysis of thermodynamical instabilities

    ➤ Onset, evolution and saturation of instability is well described by second order hydro

    ➤ Energy distributions and flow fields near the critical point show little sensitivity to phase-structure

    ➤ Outlook: study correlations and fluctuations

Eosization

- Non-trivial VeV dynamics influence the stress tensor

$$\left\langle T^{\mu}_{\mu} \right\rangle = -\Lambda \left\langle \mathcal{O} \right\rangle$$

ward identity

equilibrium

$$P_{\mathrm{eq}}\left(\mathcal{E}\right) = \frac{1}{3}\left[\mathcal{E} - \Lambda \left\langle \mathcal{O} \right\rangle_{\mathrm{eq}}\left(\mathcal{E}\right)\right]$$

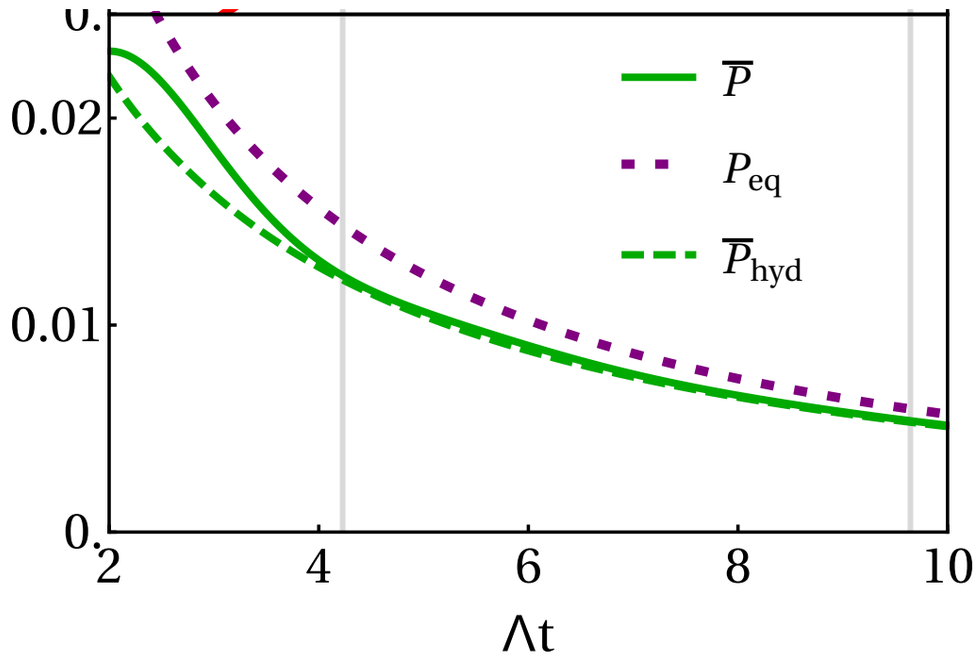
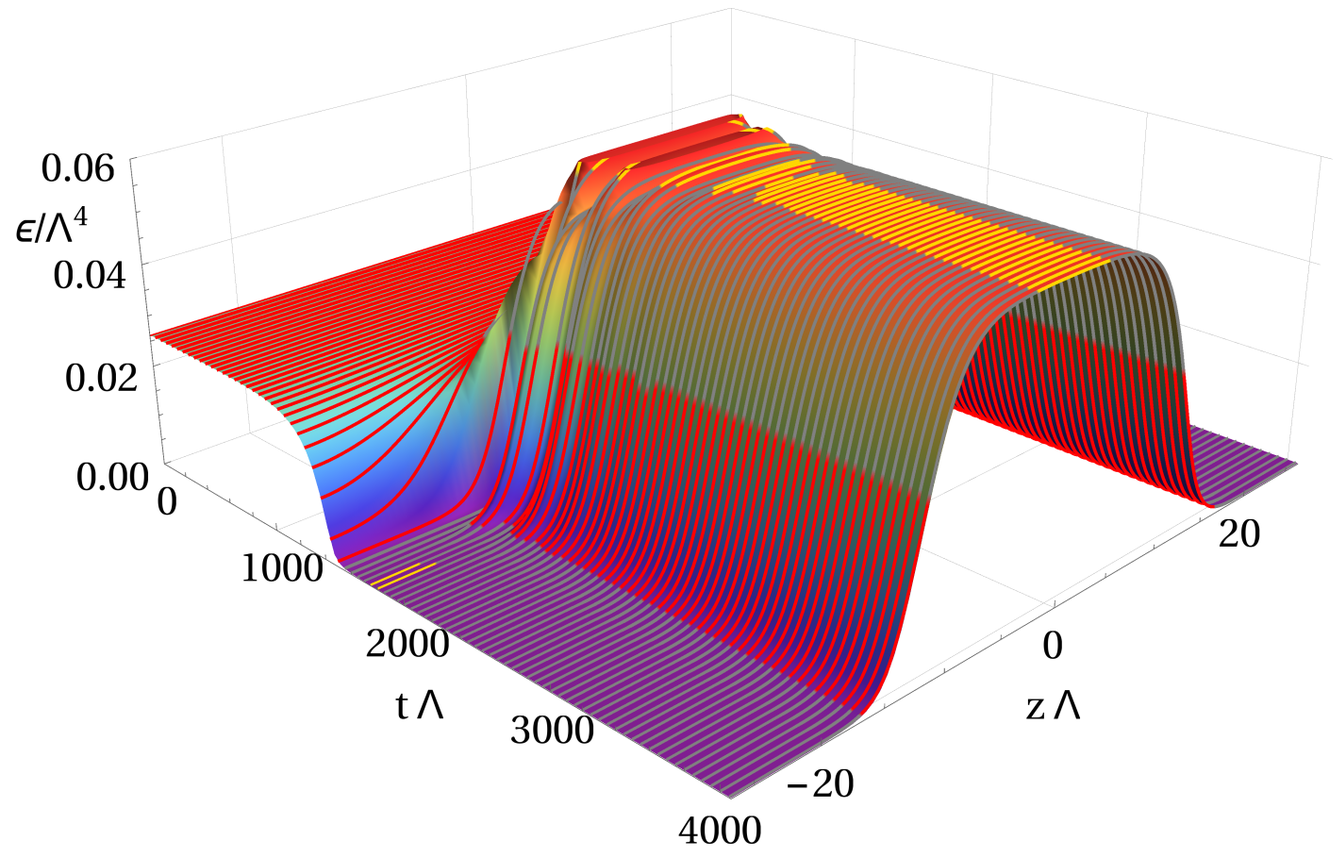
off-equilibrium

$$\bar{P} \equiv \frac{1}{3}\left[P_{\mathrm{L}} + 2P_{\mathrm{T}}\right] = \frac{1}{3}\left[\mathcal{E} - \Lambda \left\langle \mathcal{O} \right\rangle\right]$$

- The average pressure is a proxy of how well de e.o.s is satisfied

$$\frac{\left|\bar{P} - \bar{P}^{\mathrm{hyd}}\right|}{\bar{P}} < 0.1$$ “eosization”

- Large bulk corrections responsible for deviations from equilibrium!



Phase Separation