Momentum anisotropy in the quark-gluon plasma

Michael Strickland
Kent State University

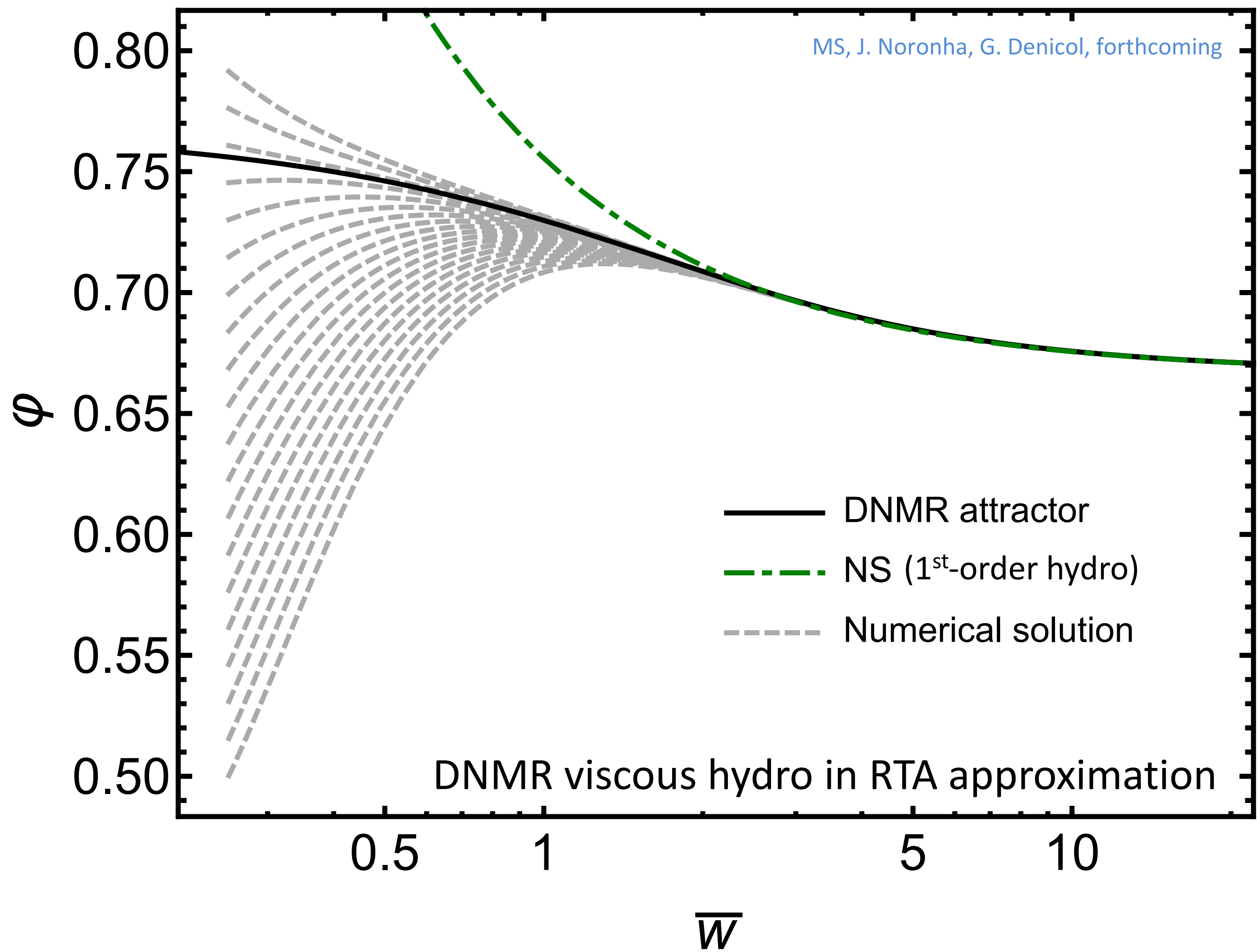
4^{th} International Conference on the
Initial Stages in High-Energy Nuclear Collisions
Krakow, Poland
September 18, 2017

Introduction

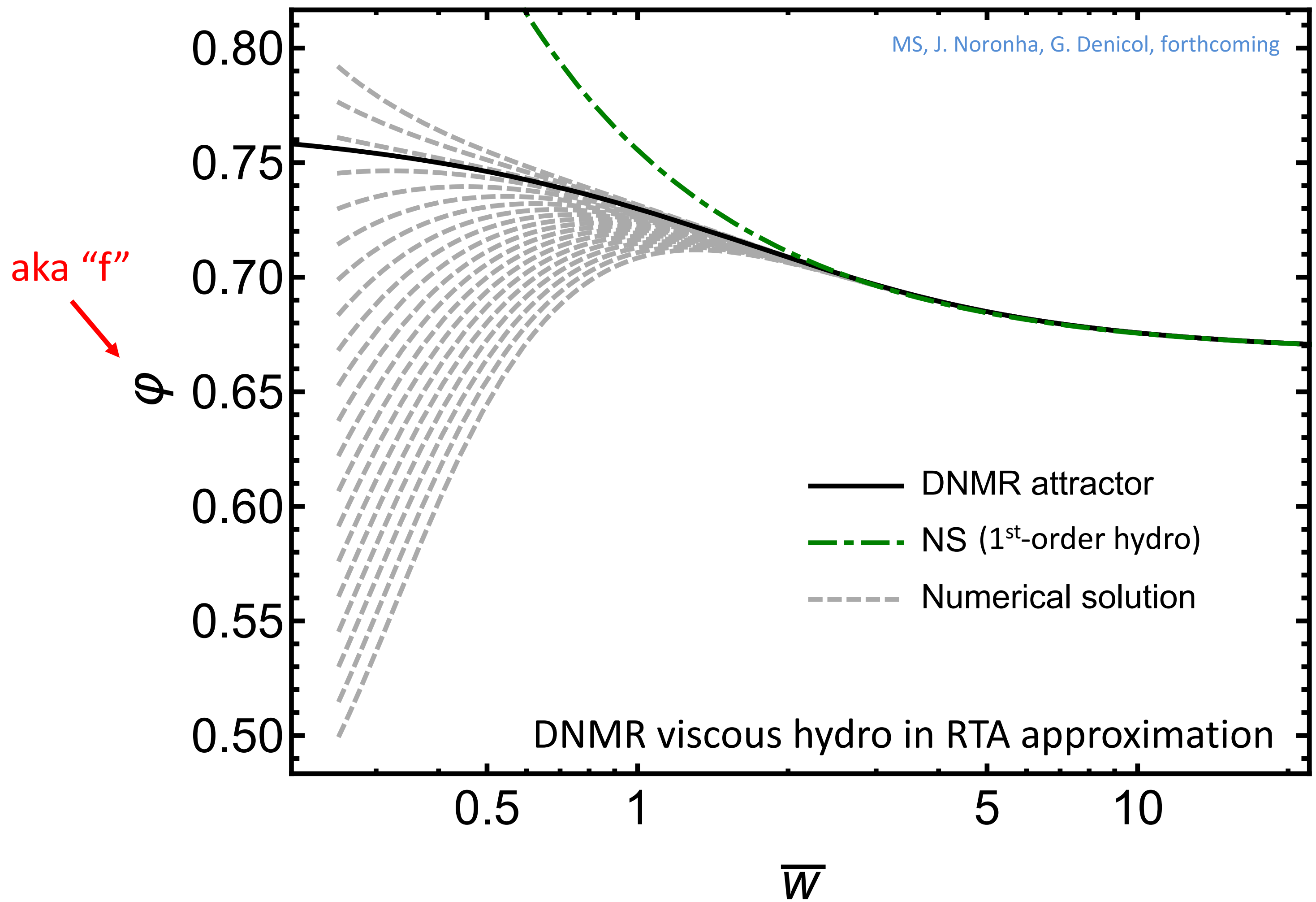
- Many disparate approaches find that, for the first few fm/c, the center of the QGP is highly momentum-space anisotropic (AdS-CFT models, kinetic theory with/without plasma instabilities, 3d CGC, …).
- The length of time over which the system is highly momentum-space anisotropic increases as one approaches the "edges" of the system (large r or spatial rapidity, ς).
- Today, I would like to discuss this using an idea which was born here in Krakow namely the **non-equilibrium dynamical attractor**.
[see e.g. Heller and Spalinski, Phys. Rev. Lett. 115 (7), 072501 (2015)]

The non-equilibrium attractor

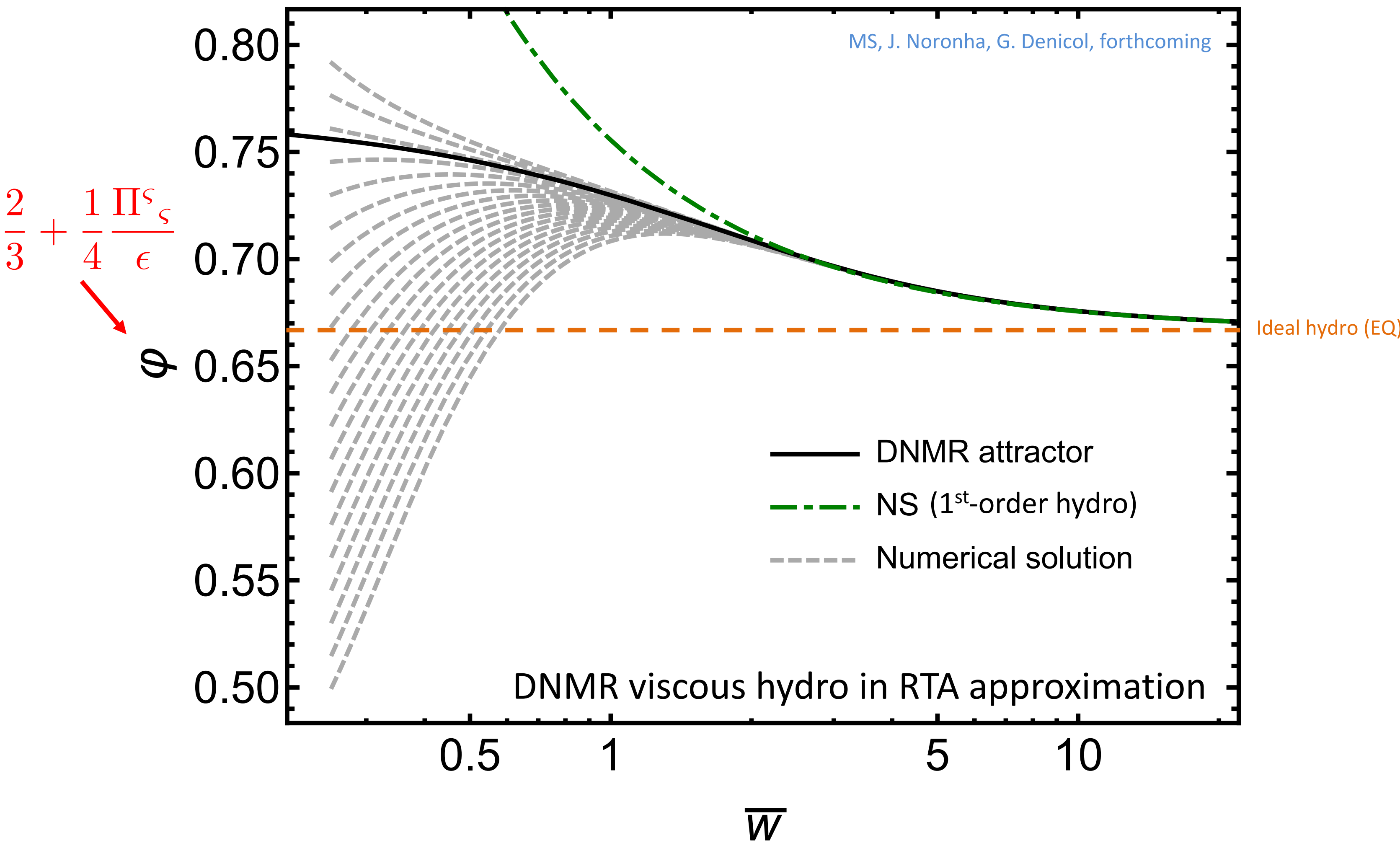
The attractor concept – 0+1d



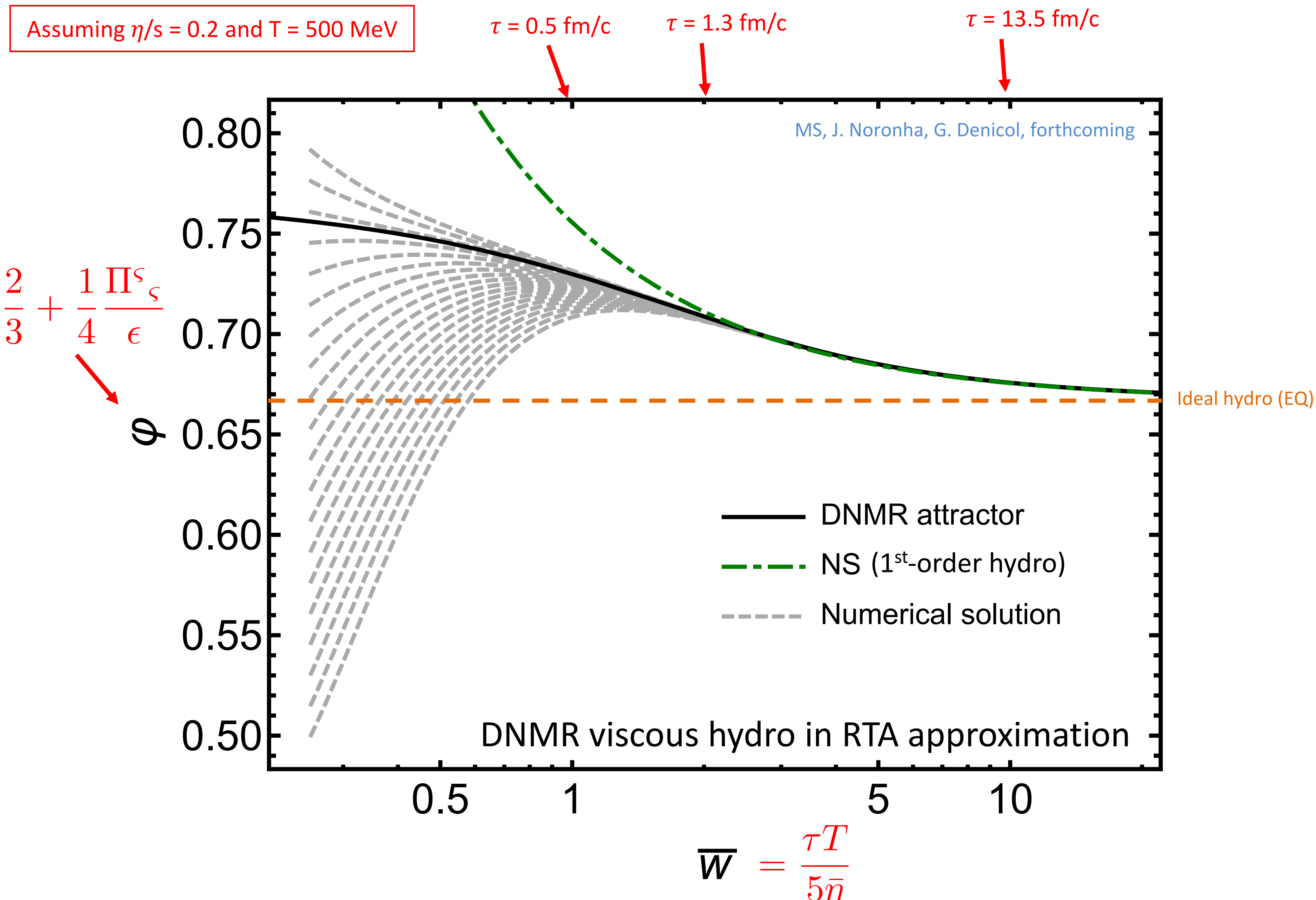
The attractor concept – 0+1d



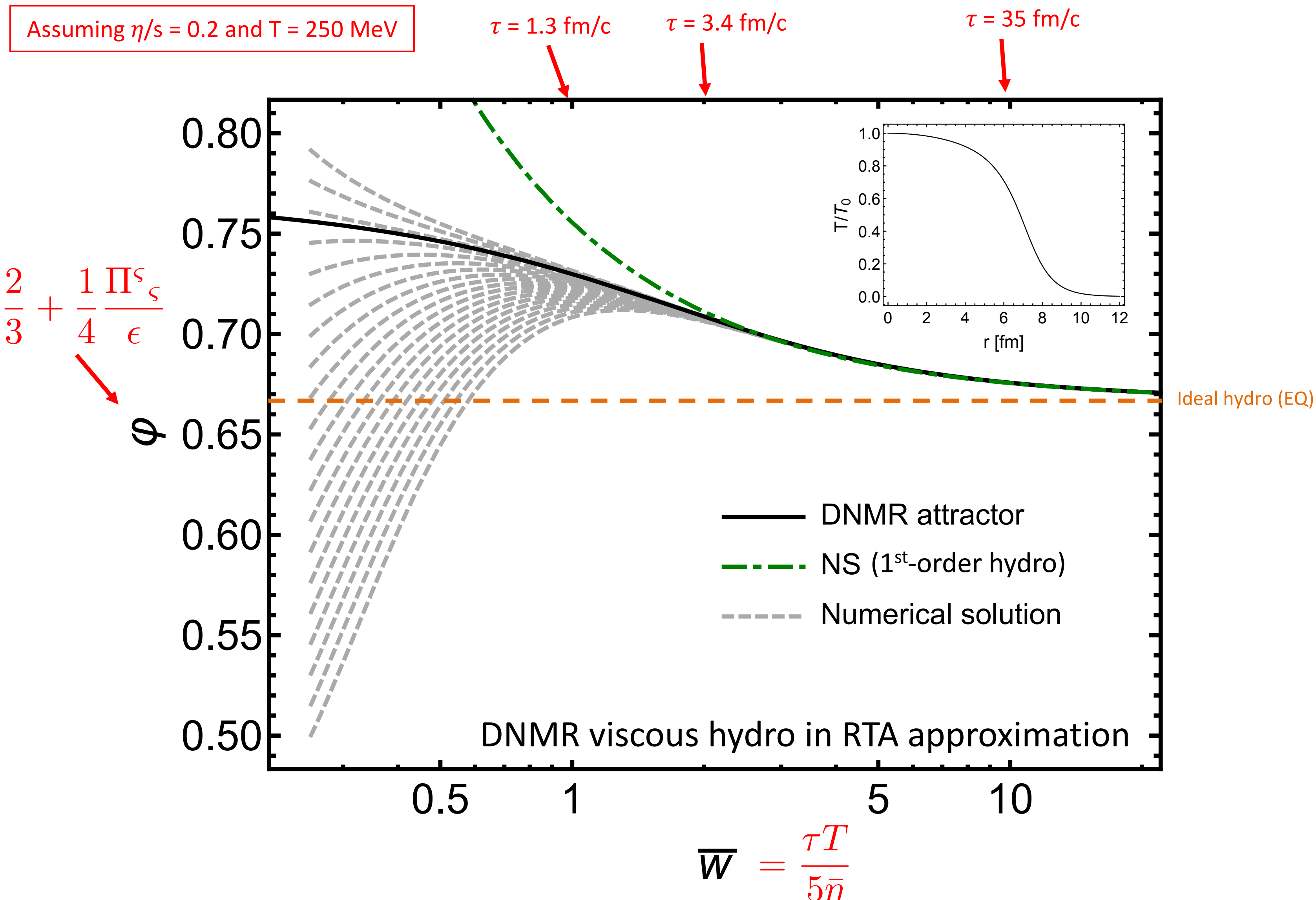
The attractor concept – 0+1d



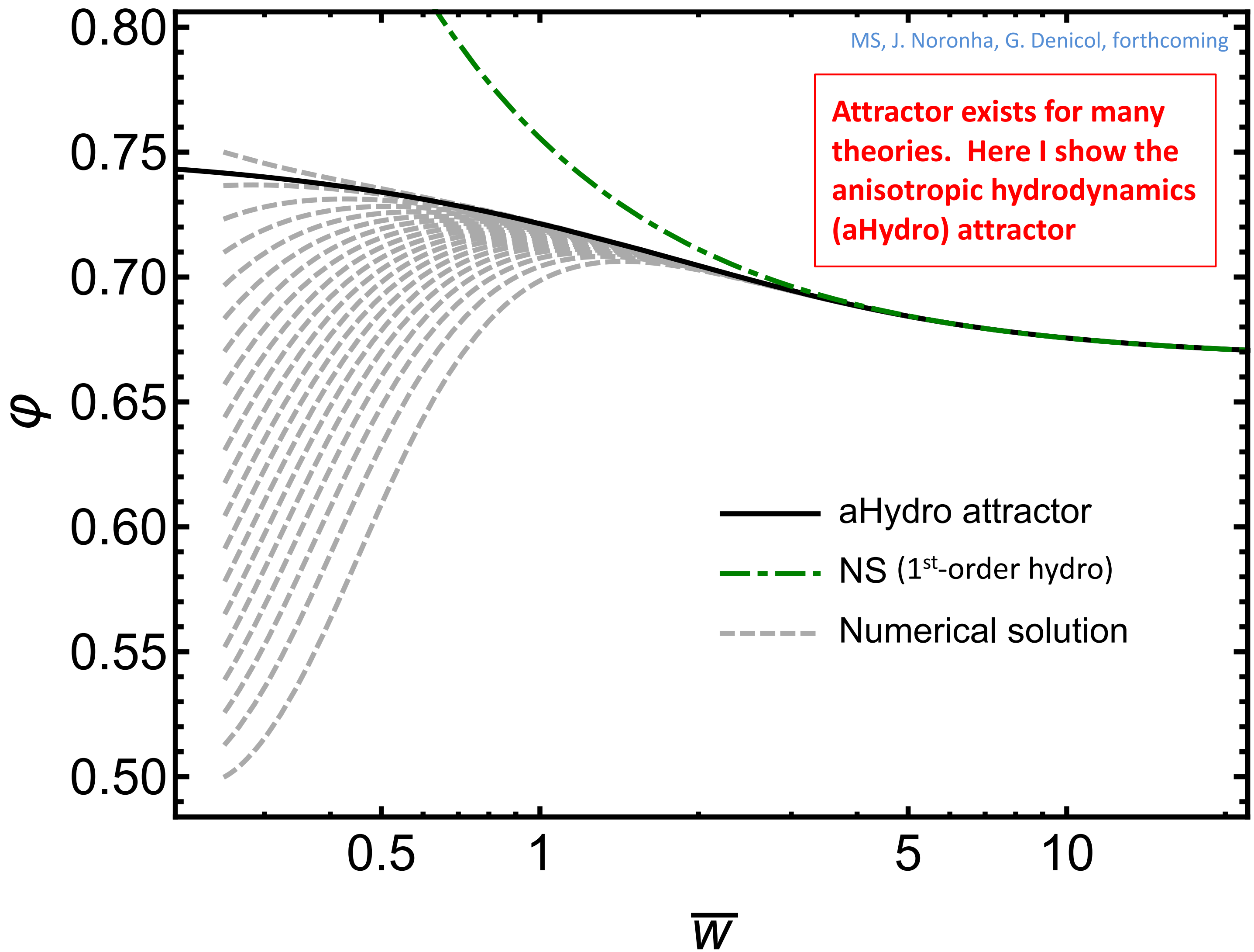
The attractor concept – 0+1d



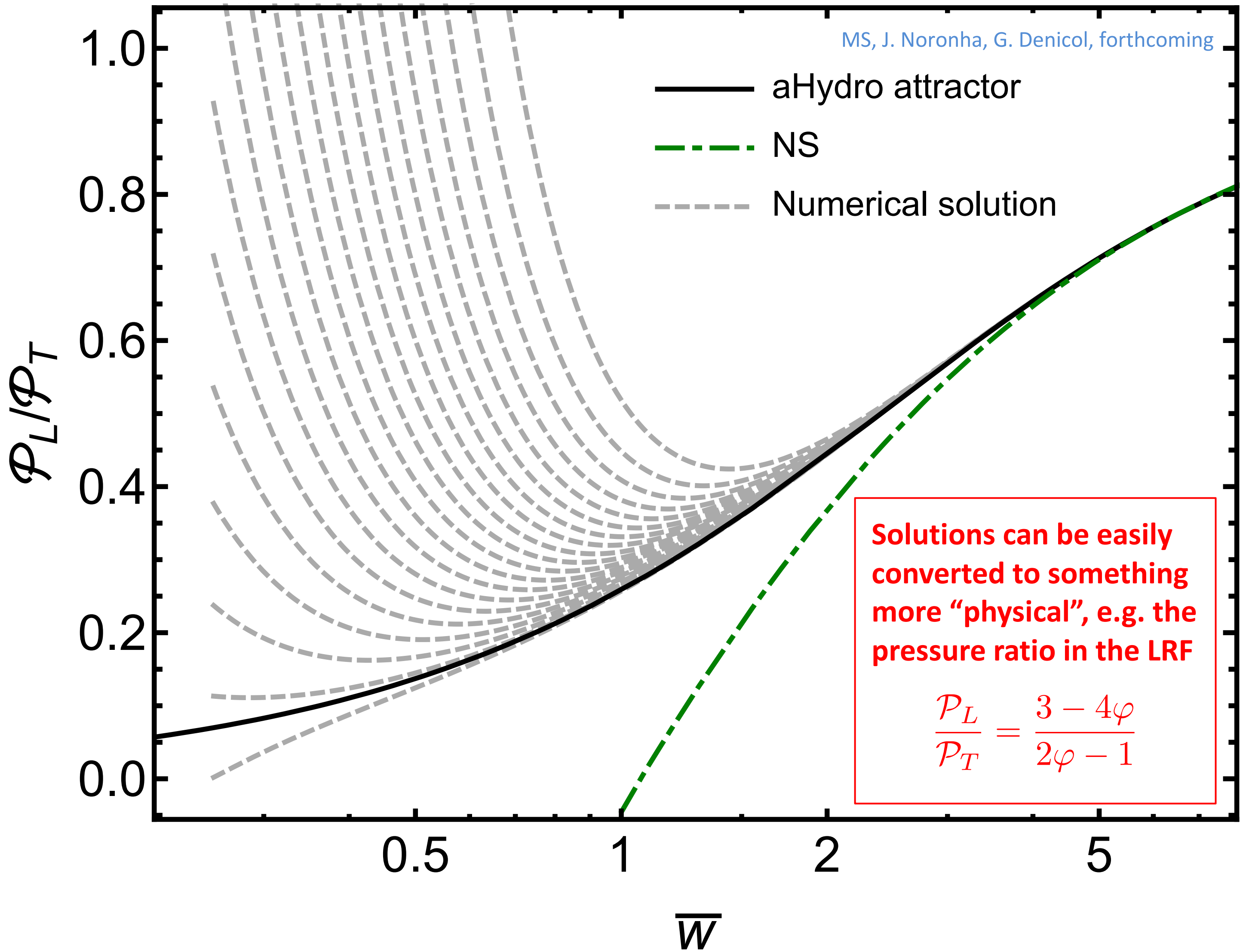
The attractor concept – 0+1d



The attractor concept – 0+1d



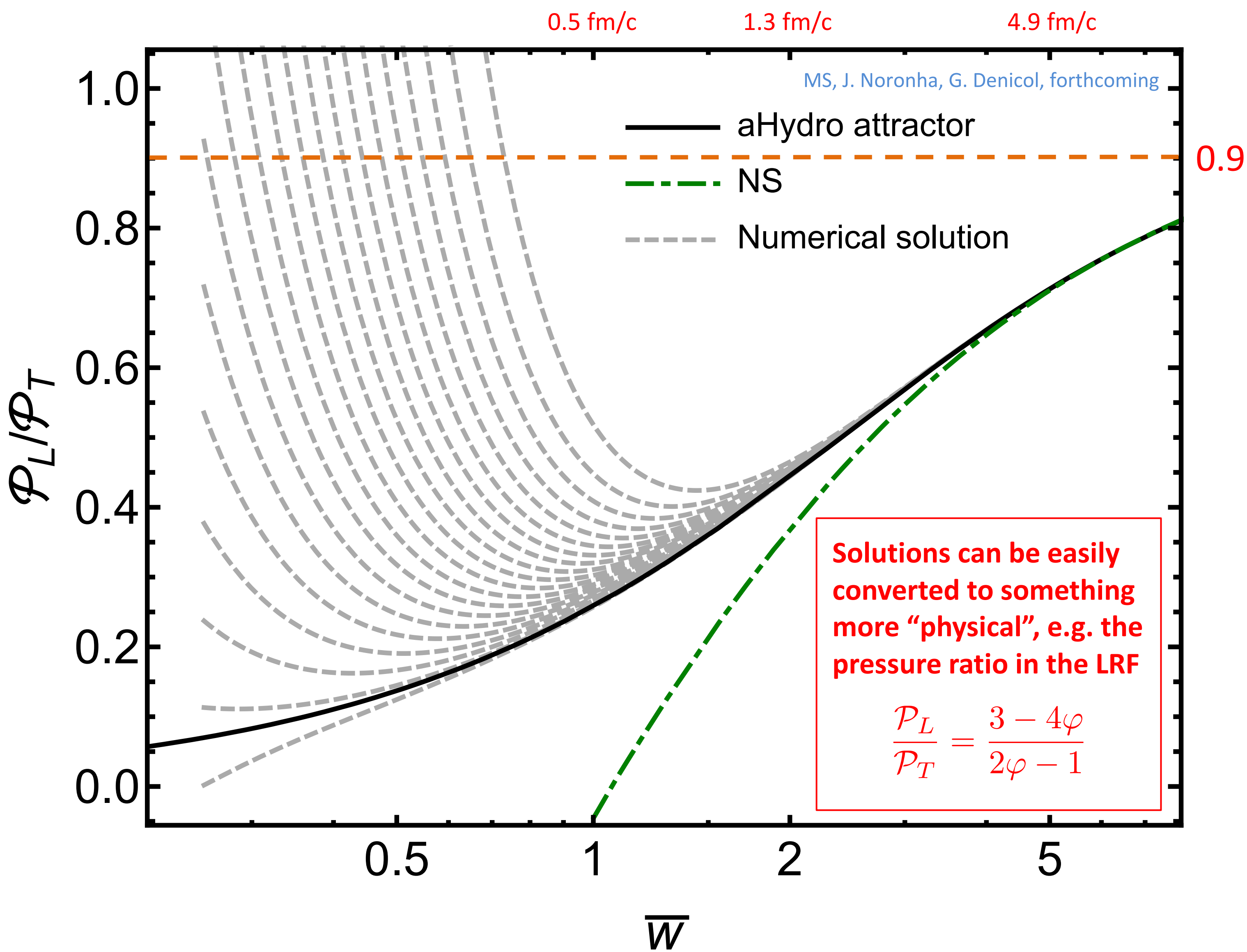
The attractor concept – 0+1d



Solutions can be easily converted to something more "physical", e.g. the pressure ratio in the LRF

$$\frac{\mathcal{P}_L}{\mathcal{P}_T} = \frac{3-4\varphi}{2\varphi-1}$$

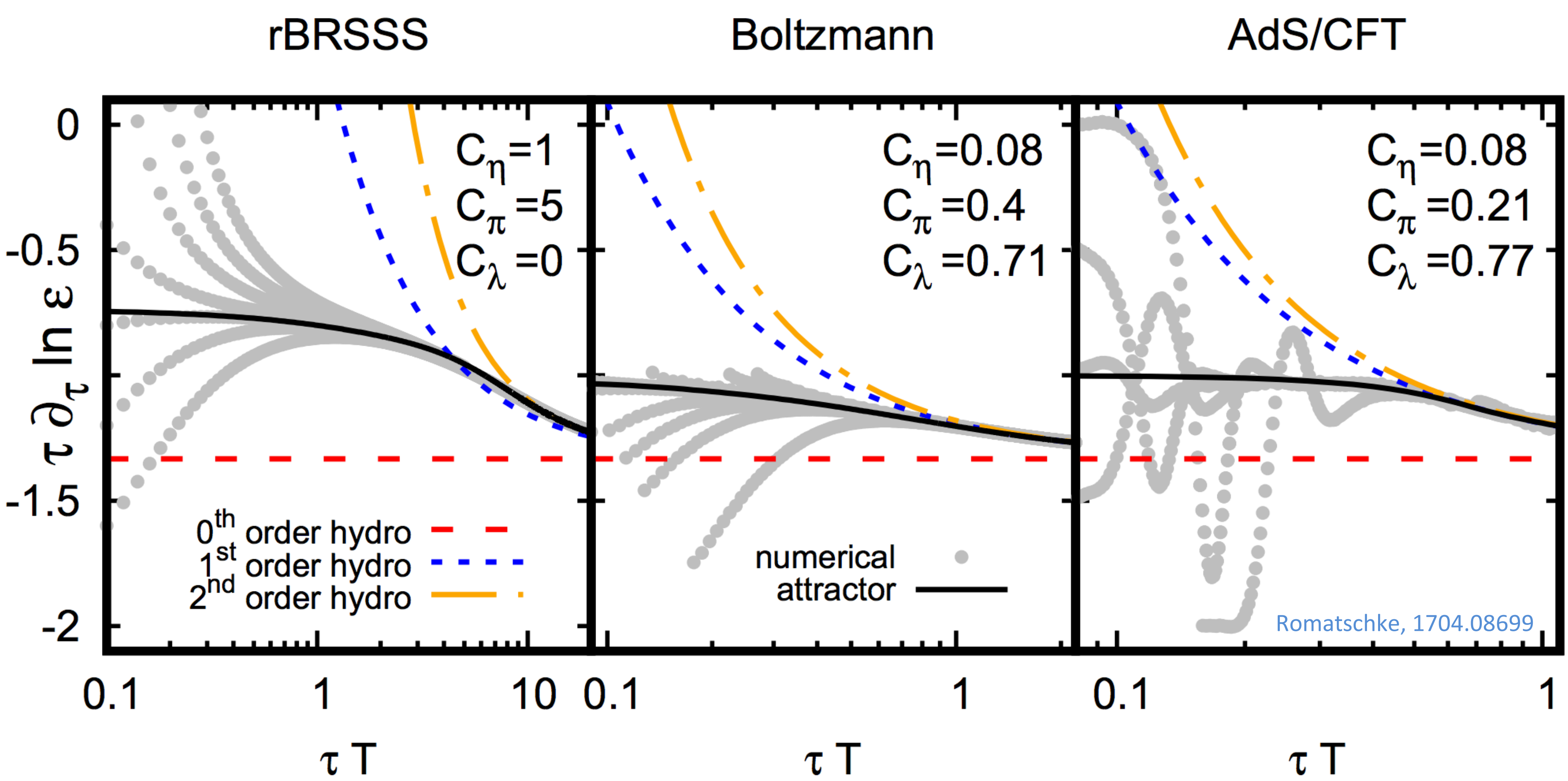
The attractor concept – 0+1d



Solutions can be easily converted to something more "physical", e.g. the pressure ratio in the LRF

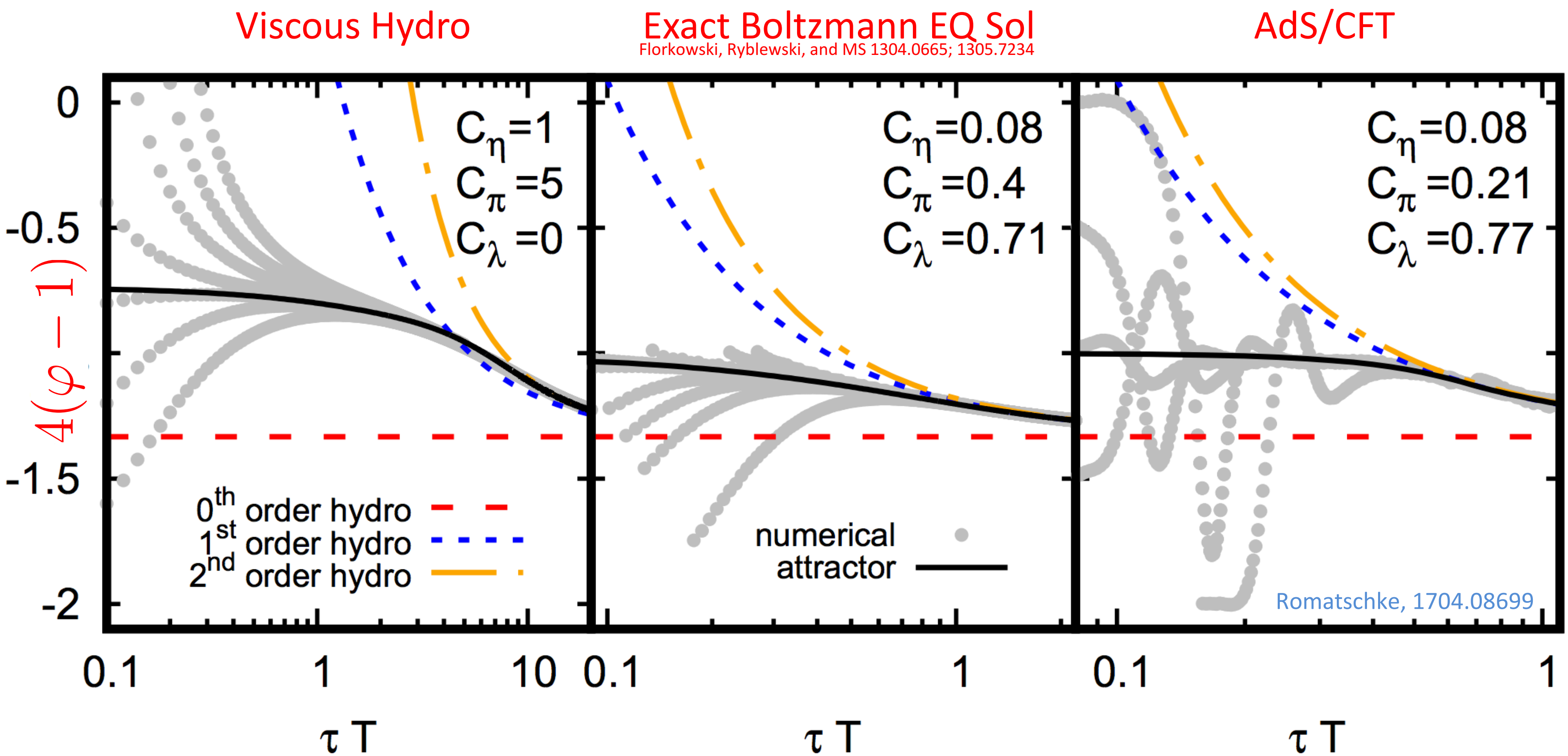
$$\frac{\mathcal{P}_L}{\mathcal{P}_T} = \frac{3-4\varphi}{2\varphi-1}$$

Attractor exists in many theories



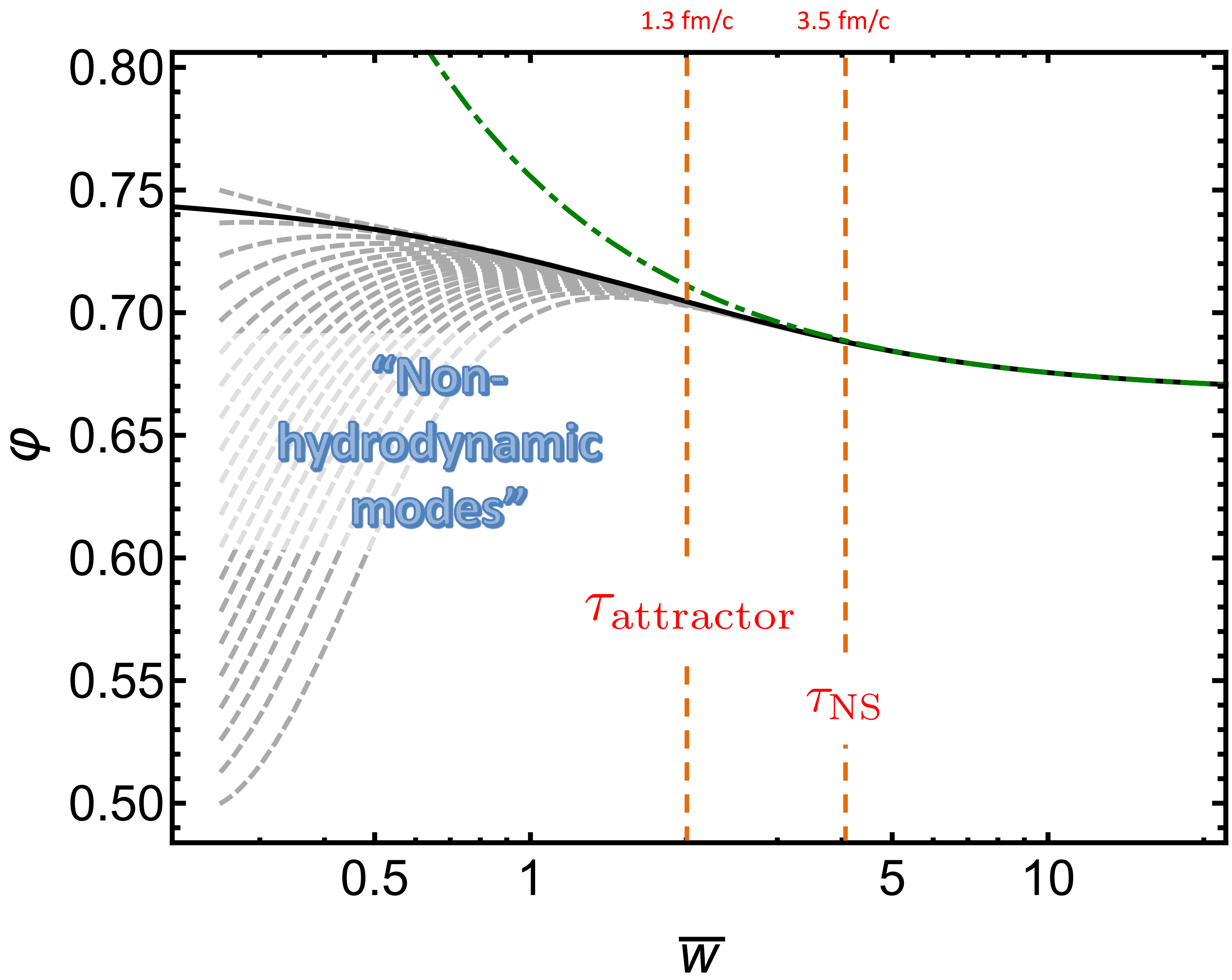
Romatschke, 1704.08699; see also Keegan et al, 1512.05347

Attractor exists in many theories



Romatschke, 1704.08699; see also Keegan et al, 1512.05347

The attractor concept – 0+1d



How does one obtain the attractor?

- Let's look at hydrodynamics-like theories for simplicity (e.g. MIS, DNMR, aHydro, etc.)
- Start with the 0+1 d energy conservation equation

$$\tau\dot{\epsilon} = -\frac{4}{3}\epsilon + \Pi \qquad \Pi = \Pi^{\varsigma}{}_{\varsigma}$$

- Change variables to

$$w = \tau T \qquad \varphi(w) \equiv \tau\frac{\dot{w}}{w} = 1 + \frac{\tau}{4}\partial_\tau \log\epsilon$$

$$w\varphi\frac{\partial\varphi}{\partial w} = -\frac{8}{3} + \frac{20}{3}\varphi - 4\varphi^2 + \frac{\tau}{4}\frac{\dot{\Pi}}{\epsilon}$$

How does one obtain the attractor?

- Need the evolution equation for the viscous correction.
- To linear order in the shear correction (e.g. MIS, DNMR) one has

$$\dot{\Pi} = \frac{4\eta}{3\tau\tau_\pi} - \beta_{\pi\pi}\frac{\Pi}{\tau} - \frac{\Pi}{\tau_\pi}$$

For DNMR in RTA $\beta_{\pi\pi} = \frac{38}{21}$

- Plugging this into the energy-momentum conservation equation gives

$$\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0$$

$$\overline{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\overline{\eta}} \qquad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

How does one solve for the attractor?

$$\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0$$

$$\overline{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\overline{\eta}} \qquad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

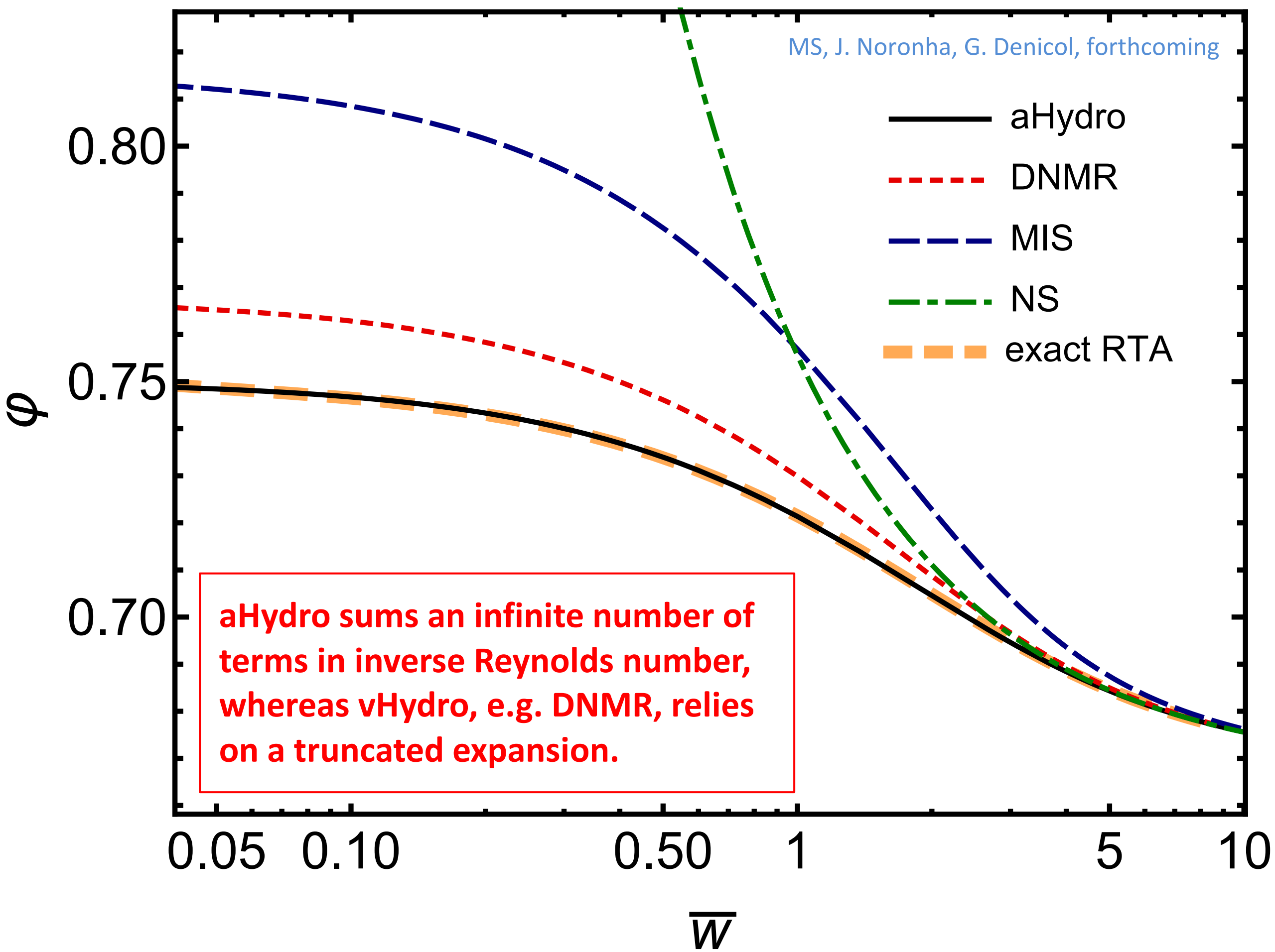
- First try to approximate using "slow-roll" approx ($\varphi' = 0$)
- From this, we can read off the boundary condition as $w \to 0$

$$\lim_{\overline{w}\to 0} \varphi(\overline{w}) = \frac{1}{24}\left(-3\beta_{\pi\pi} + \sqrt{64c_{\eta/\pi} + (3\beta_{\pi\pi} - 4)^2} + 20\right)$$

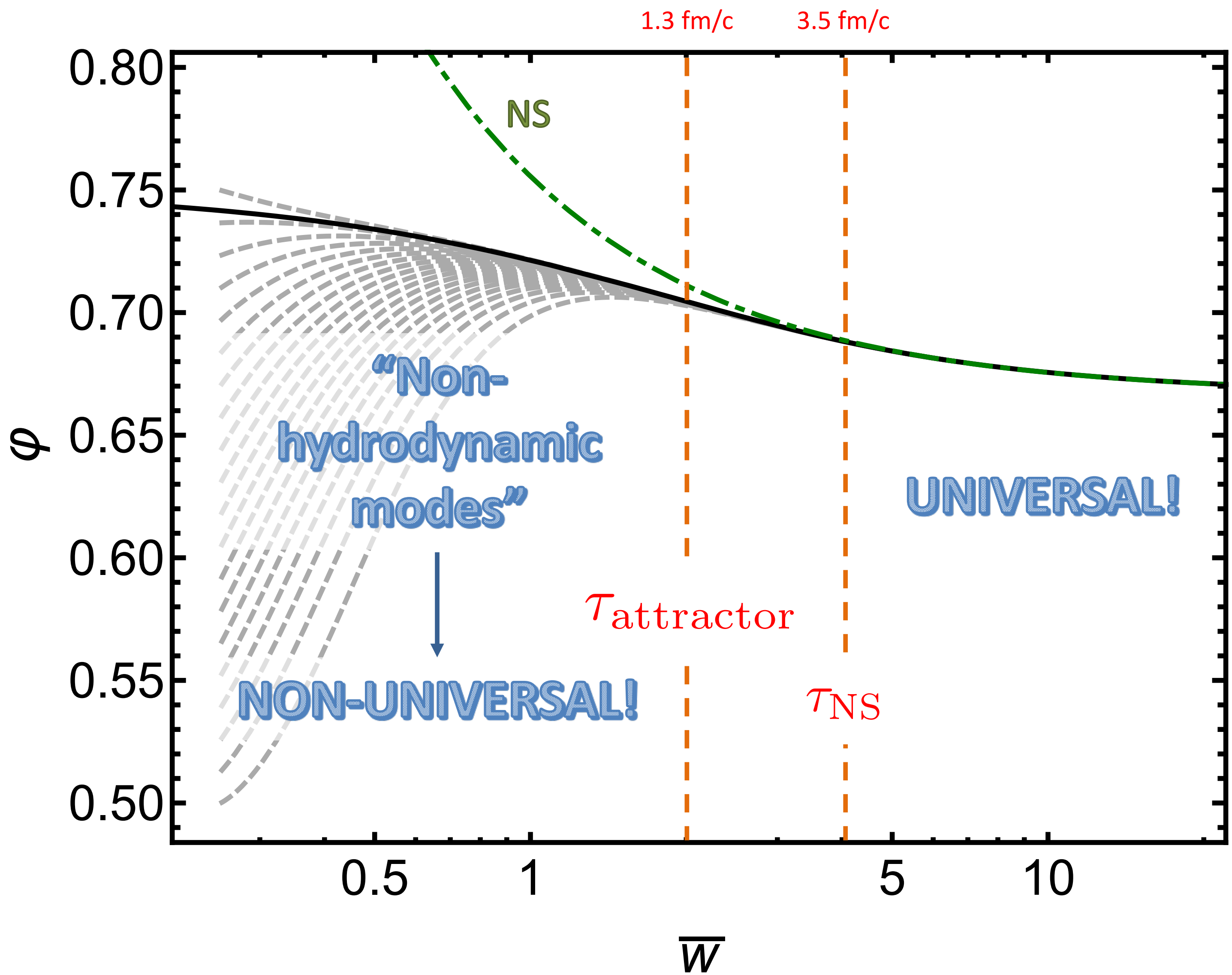
- Then numerically solve the ODE at the top of the slide

Results

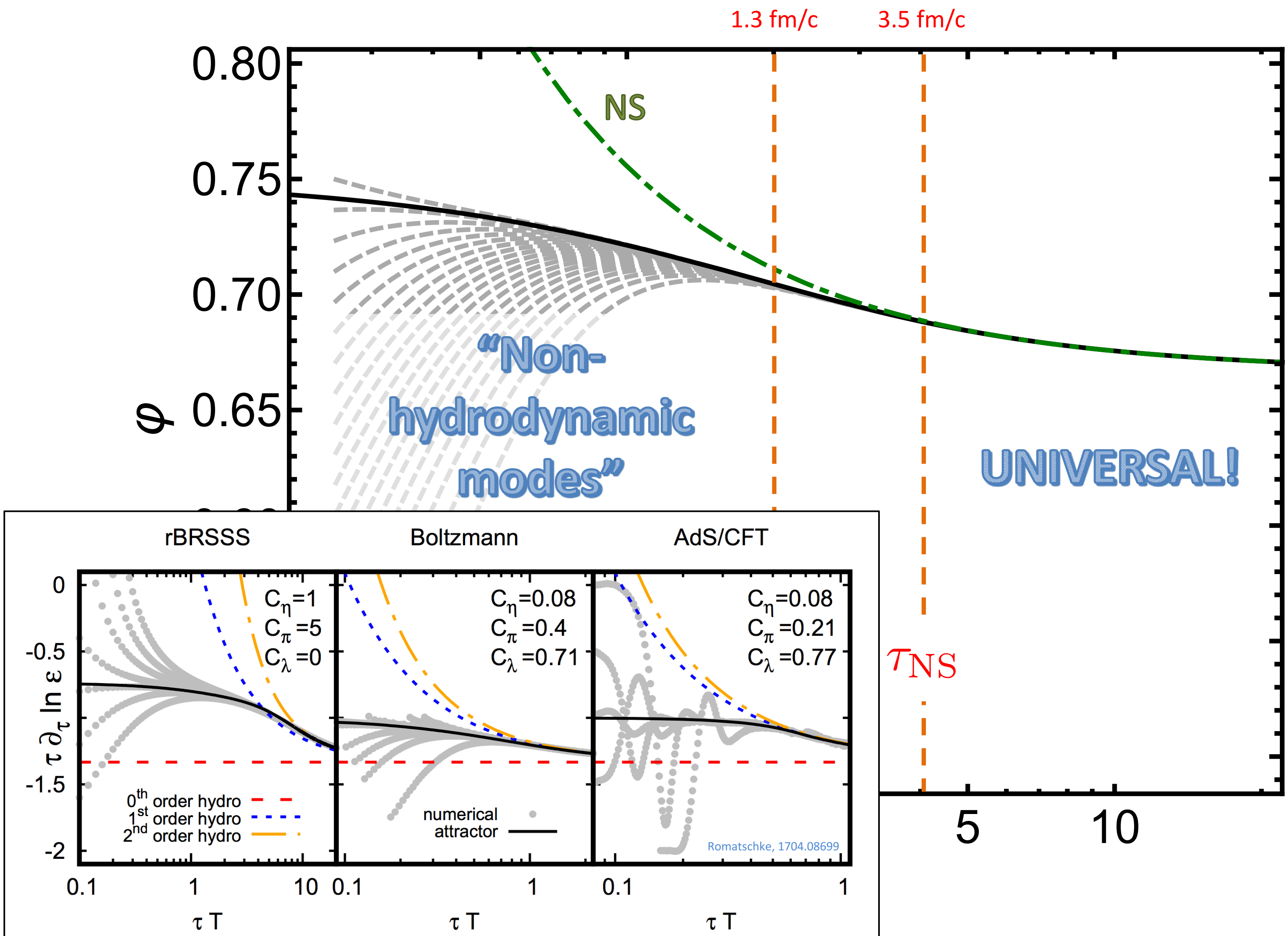
- Can extract the attractor for MIS, DNMR, aHydro, and the exact solution in this manner



The attractor concept – 0+1d



The attractor concept – 0+1d



Univerisality and non-universality

- Universality → All microscopic theories are equally good starting points; pick the one that's easiest to deal with
- Non-universality → We have to pick which microscopic theory we think best describes the system's dynamics
- I would argue that some sort of pQCD/kinetic theory inspired microscopic theory makes the most sense at early times (particles + CGC) and in the dilute regions (hadronic transport) which are precisely where "non-universal" physics pops up.
- And, since kinetic-theory based models also share the "universal properties" of hydrodynamics at later times, they will also work well in this region.

aHydro application

Generalized aHydro formalism

In generalized aHydro, we use kinetic theory and assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{“Bulk”}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

→ 11 DOFs

See e.g.
- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Implementing the equation of state

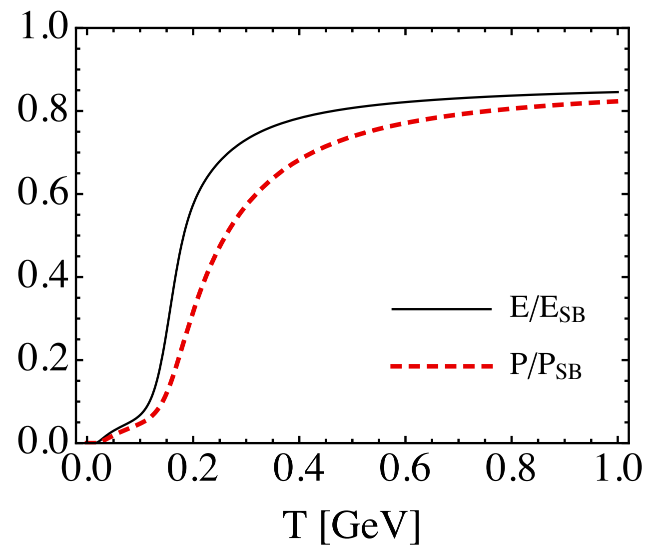
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

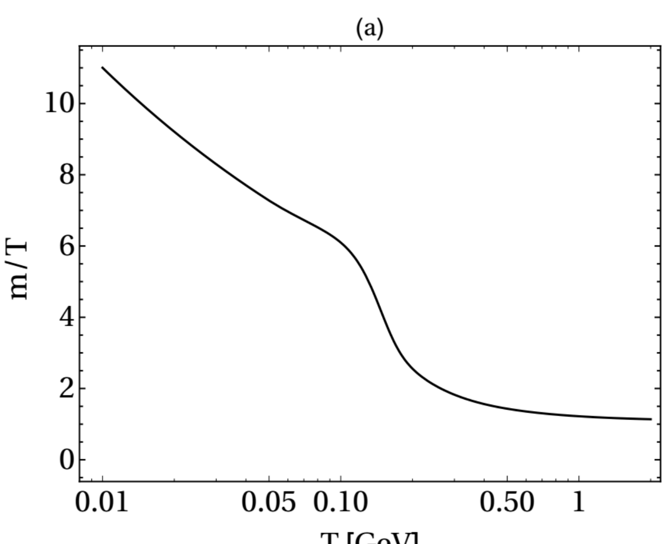
Quasiparticle Method

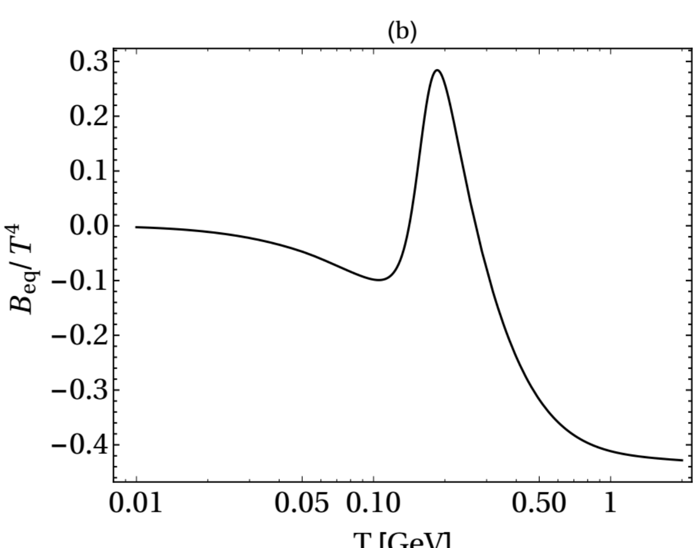
$$T^{\mu\nu} = T^{\mu\nu}_{\text{kinetic}} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$







Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entropy density ratio.

$$\frac{\eta}{\tau_{\rm eq}} = \frac{1}{T} I_{3,2}(\hat{m}_{\rm eq})$$

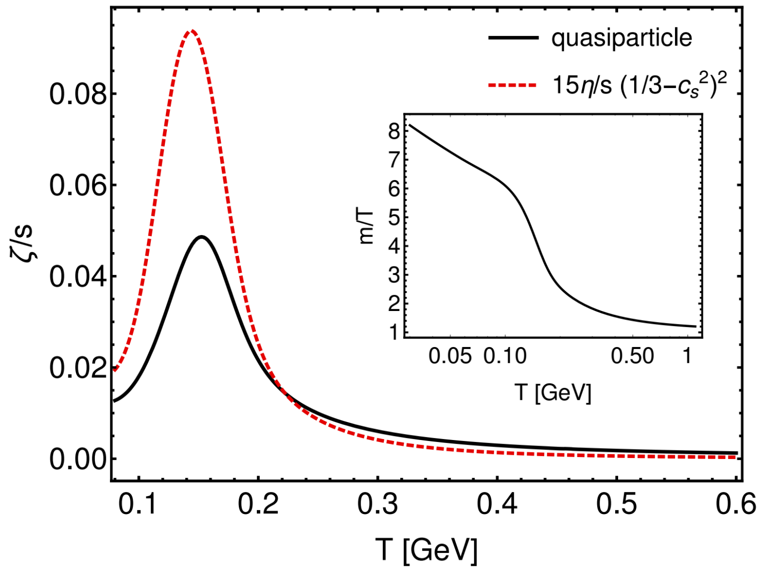
Bulk viscosity

$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,2} - c_s^2(\mathcal{E}+\mathcal{P}) + T\hat{m}^3 \frac{dm}{dT} I_{1,1}$$

$$I_{3,2}(x) = \frac{N_{\rm dof} T^5 x^5}{30\pi^2}\left[\frac{1}{16}\Big(K_5(x) - 7K_3(x) + 22K_1(x)\Big) - K_{i,1}(x)\right],$$

$$K_{i,1}(x) = \frac{\pi}{2}\left[1 - xK_0(x)\mathcal{S}_{-1}(x) - xK_1(x)\mathcal{S}_0(x)\right],$$

$$I_{1,1} = \frac{g\, m^3}{6\pi^2}\left[\frac{1}{4}(K_3 - 5K_1) + K_{i,1}\right]$$



Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\text{kinetic}} + Bg^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entropy density ratio.

$$\frac{\eta}{\tau_{\text{eq}}} = \frac{1}{T} I_{3,2}(\hat{m}_{\text{eq}})$$

Bulk viscosi

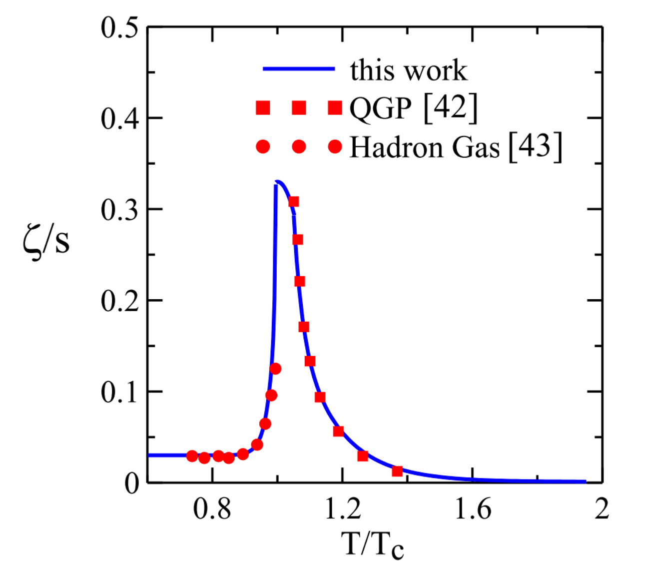
$$\frac{\zeta}{\tau_{\text{eq}}} = \frac{5}{3T} I_{3,}$$

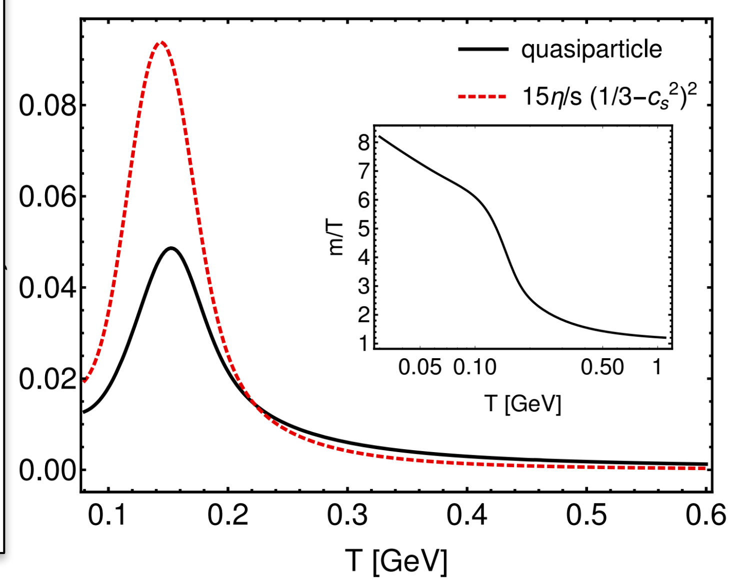
$I_{3,2}(x) = \frac{N_{\text{dof}}}{3(}$

$K_{i,1}(x) = \frac{\pi}{2}\Big[1$

$I_{1,1} = \frac{g\, m}{6\pi^2}$

Ryu et al, PRL 115 (2015) no.13, 132301





Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101

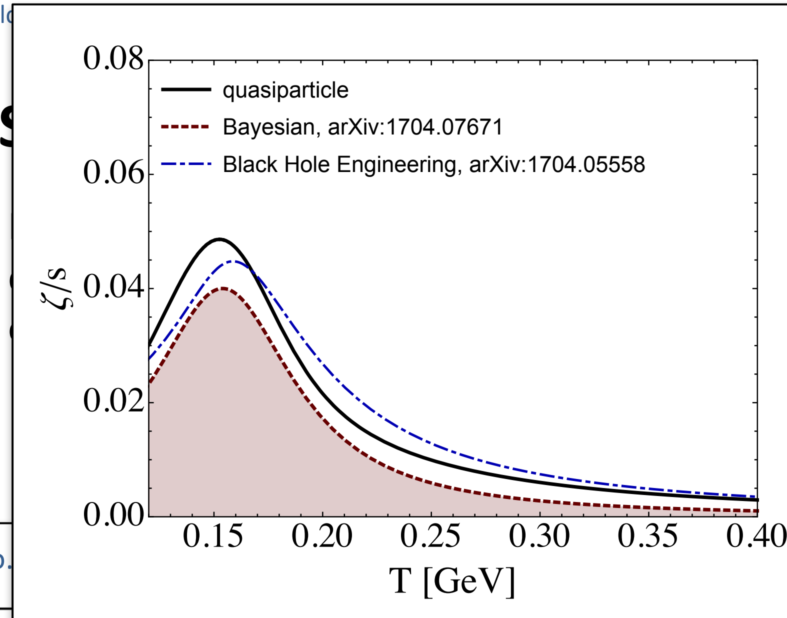
M. Al[...] [...]91

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$



Bulk viscosi

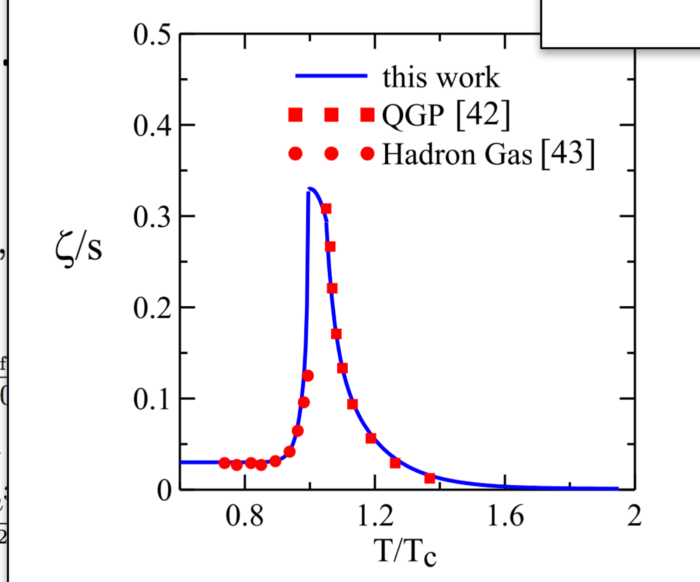
$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,}$$

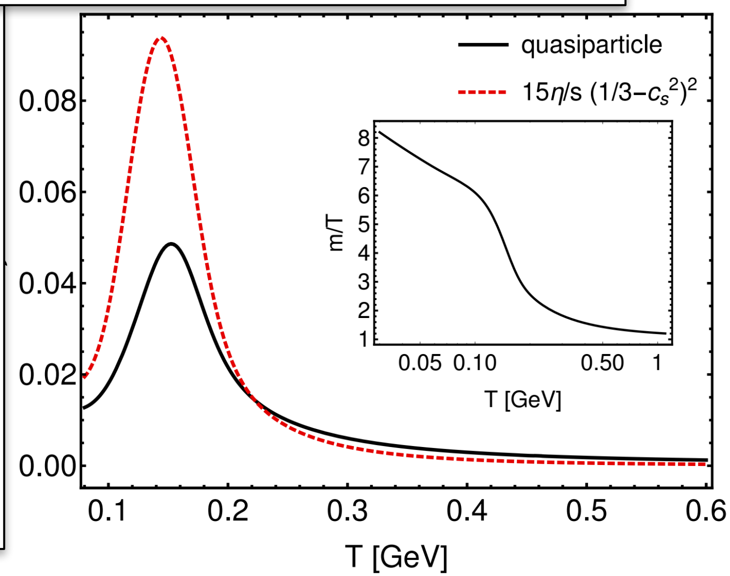
$I_{3,2}(x) = \frac{N_{\rm dof}}{30}$

$K_{i,1}(x) = \frac{\pi}{2}\Big[1$

$I_{1,1} = \frac{g\, m}{6\pi^2}$

Ryu et al, PRL 115 (2015) no.





Anisotropic Cooper-Frye Freezeout

M. Alqahtani, M. Nopoush, and MS, 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

- Use same generalized-RS form for "anisotropic freeze-out" at LO
- Form includes both shear and bulk corrections to the distribution function
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\rm eff,FO}$ = 130 MeV

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Phi\Delta^{\mu\nu}$$

isotropic | anisotropy tensor | bulk correction

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3}\int f_i(x,p)\, p^\mu d\Sigma_\mu\,,$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq})\frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p\cdot u/T) + a] \qquad a = \text{-}1,\ +1,\text{ or } 0$$

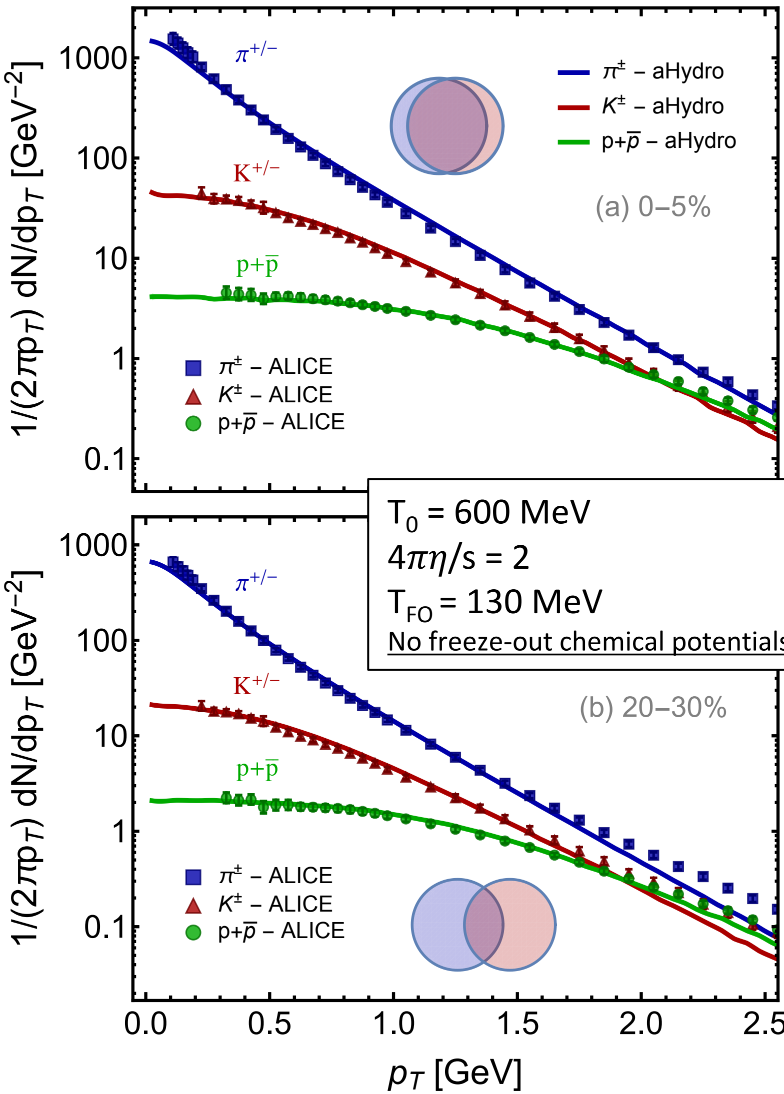
- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes worse when including the bulk viscous correction**.

The phenomenological setup

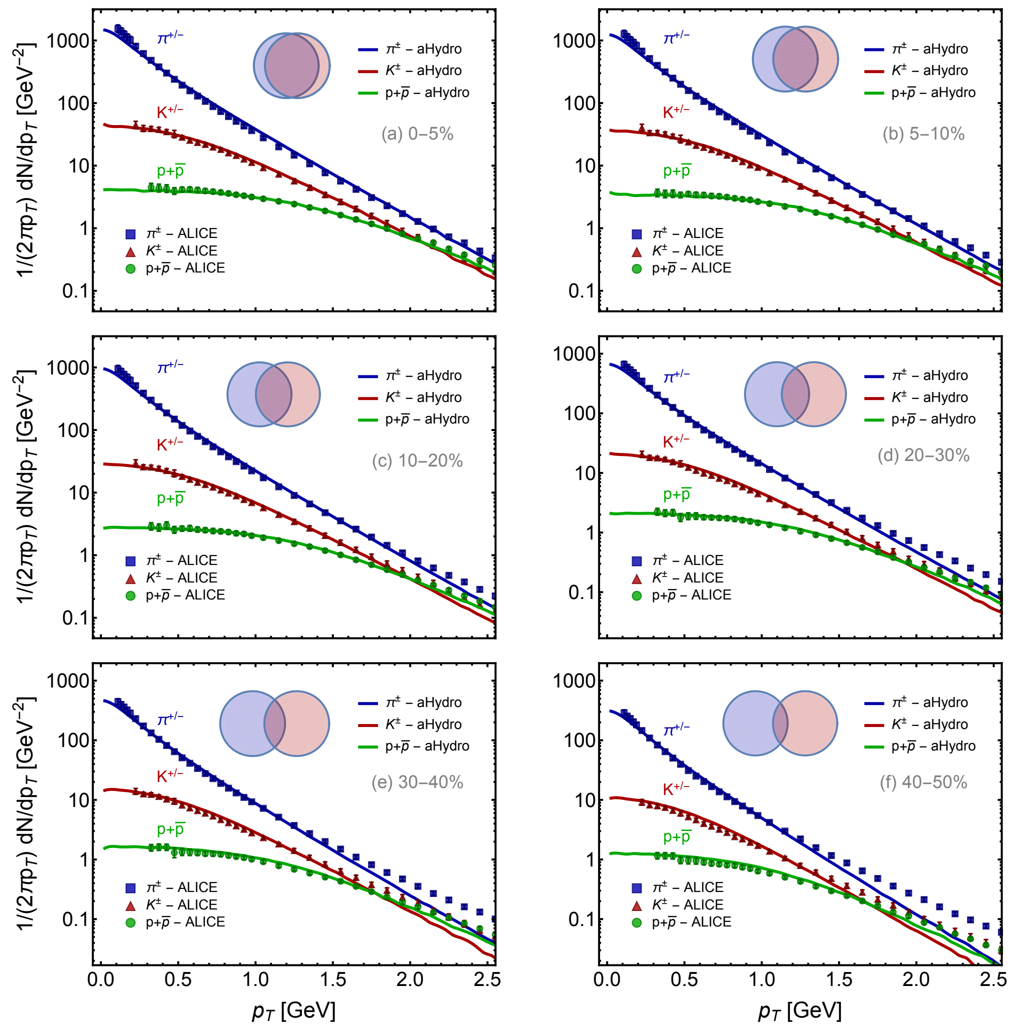
- Keep it simple → three anisotropy parameters (diagonal) + smooth Glauber initial conditions
- Mixture of wounded nucleon and binary collision profiles with a binary mixing fraction of 0.15 (empirically suggested from prior viscous hydro studies)
- In the rapidity direction, we use a rapidity profile with a "tilted" central plateau and Gaussian "wings"
- We take the system to be initially isotropic in momentum space
- We then run the code and extract the freeze-out hypersurface
- The primordial particle production is then Monte-Carlo sampled using the Therminator 2 [Chojnacki, Kisiel, Florkowski, and Broniowski, arXiv:1102.0273]
- Therminator also takes care of all resonance feed downs
- All data shown are from the **ALICE collaboration 2.76 TeV/nucleon**

Identified particle spectra

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



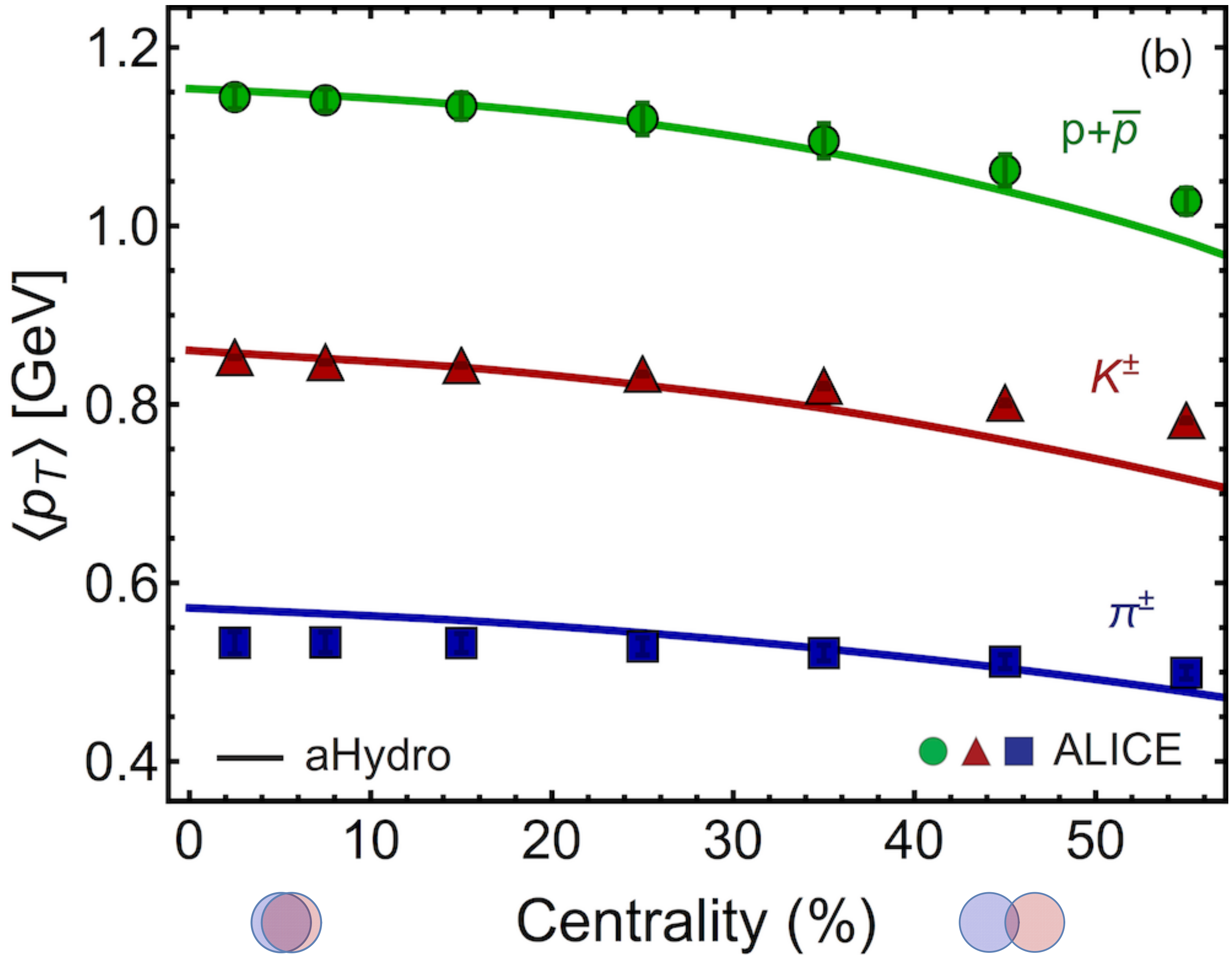
T_0 = 600 MeV
$4\pi\eta/s = 2$
T_{FO} = 130 MeV
No freeze-out chemical potentials



Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

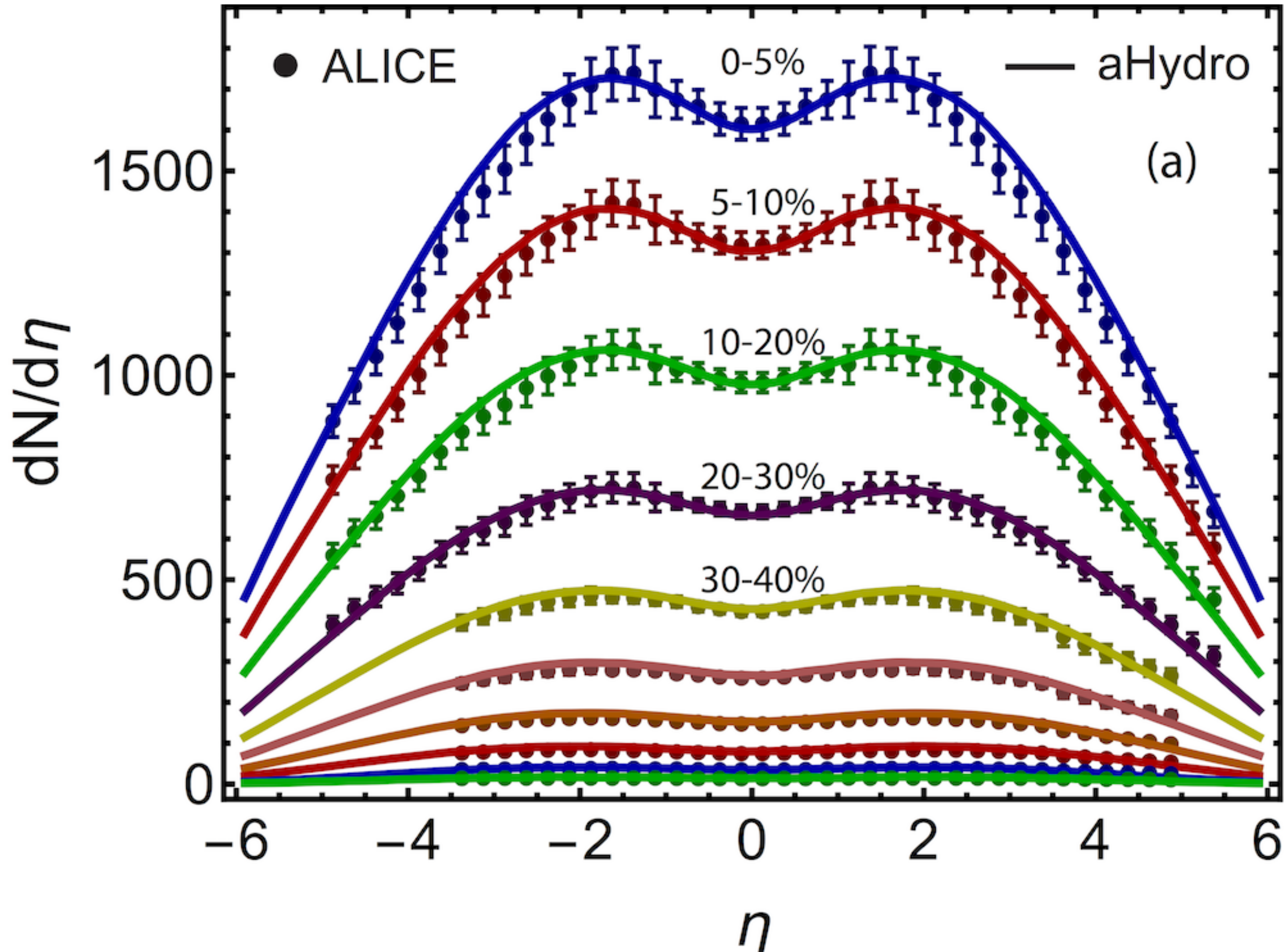
Identified particle average p_T

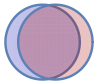
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

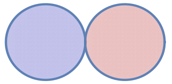


Charged particle multiplicity

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

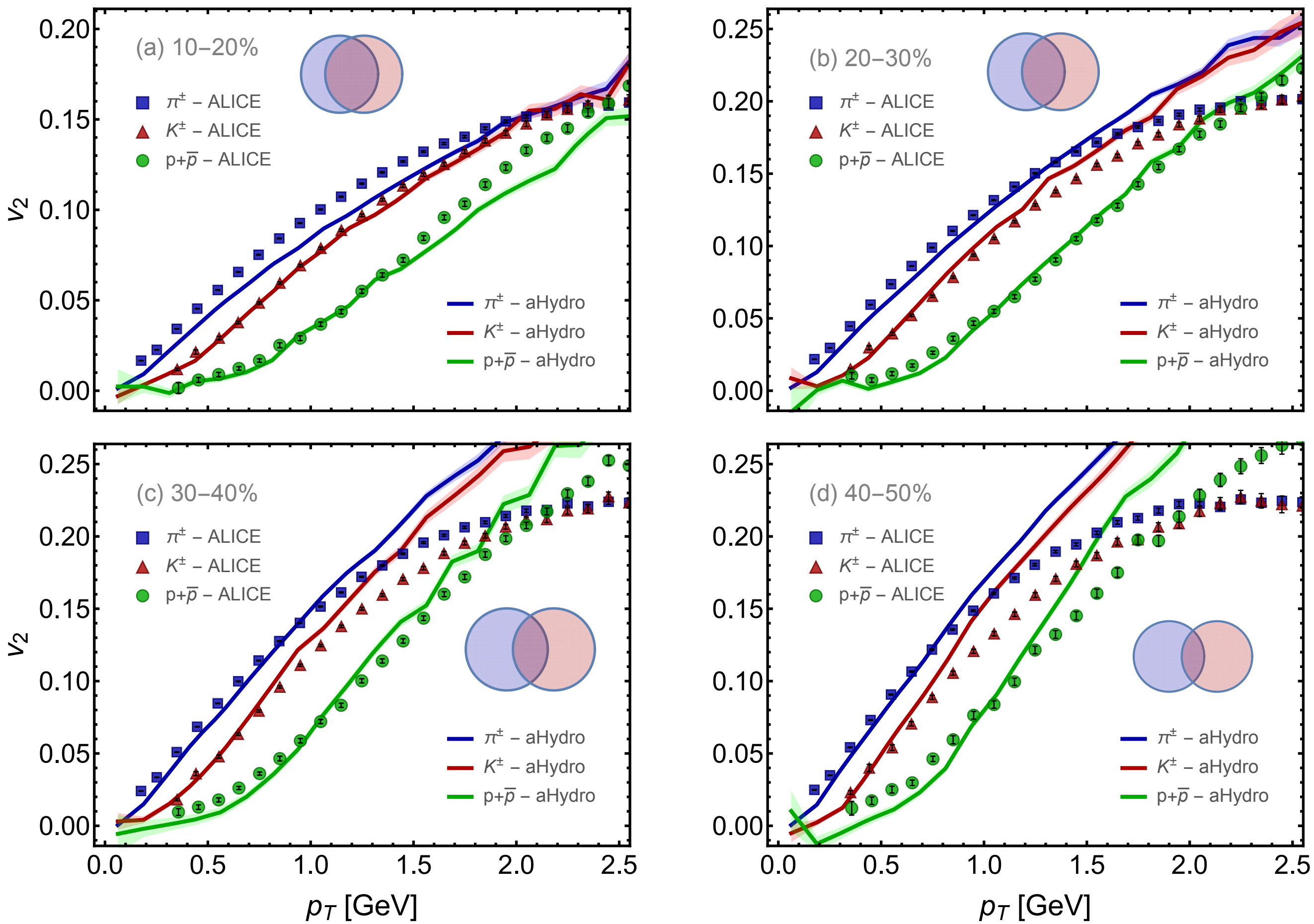






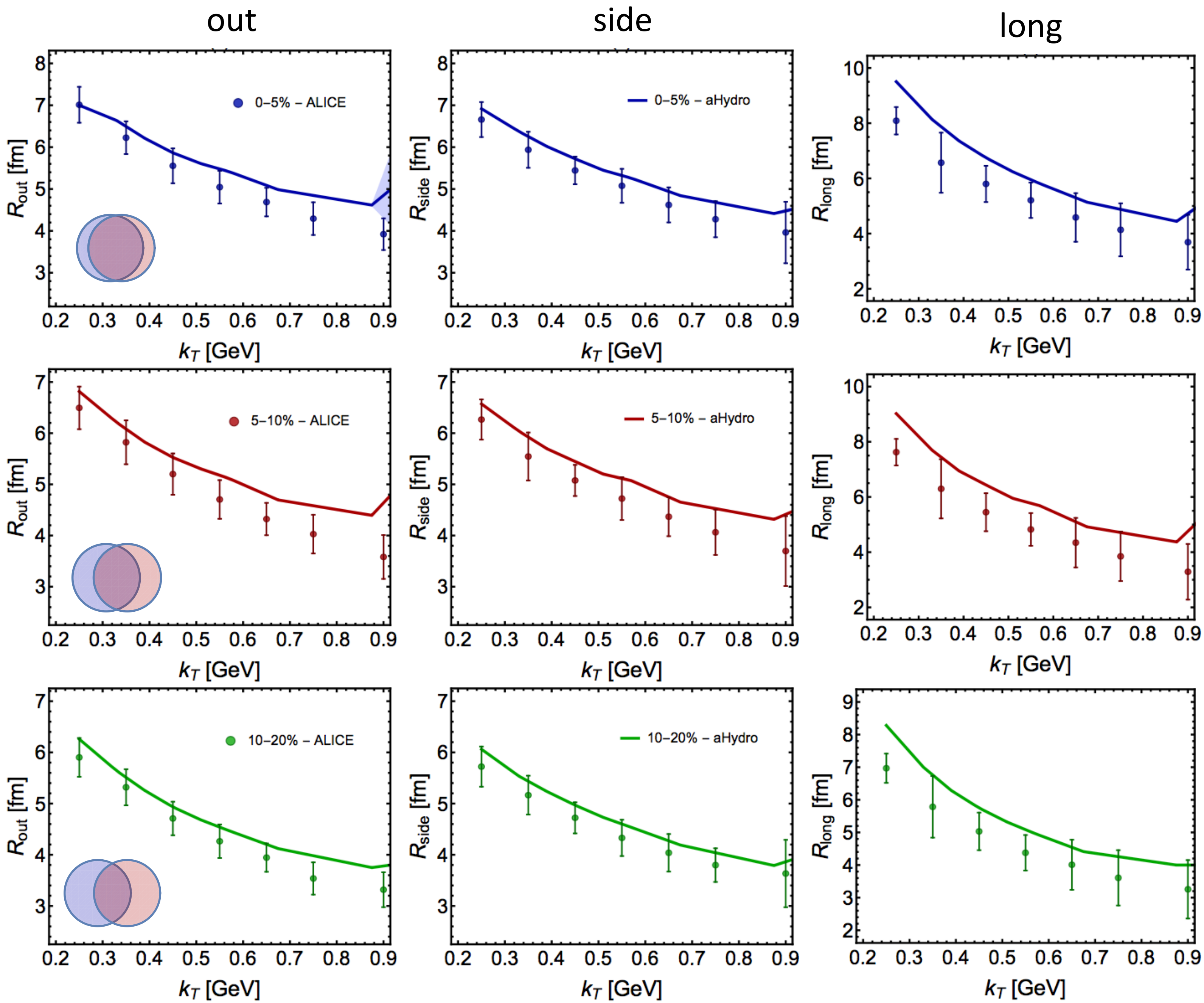
Elliptic flow

- Quite good description of identified particle elliptic flow as well
- Central collisions → need to include fluctuating init. Conditions!

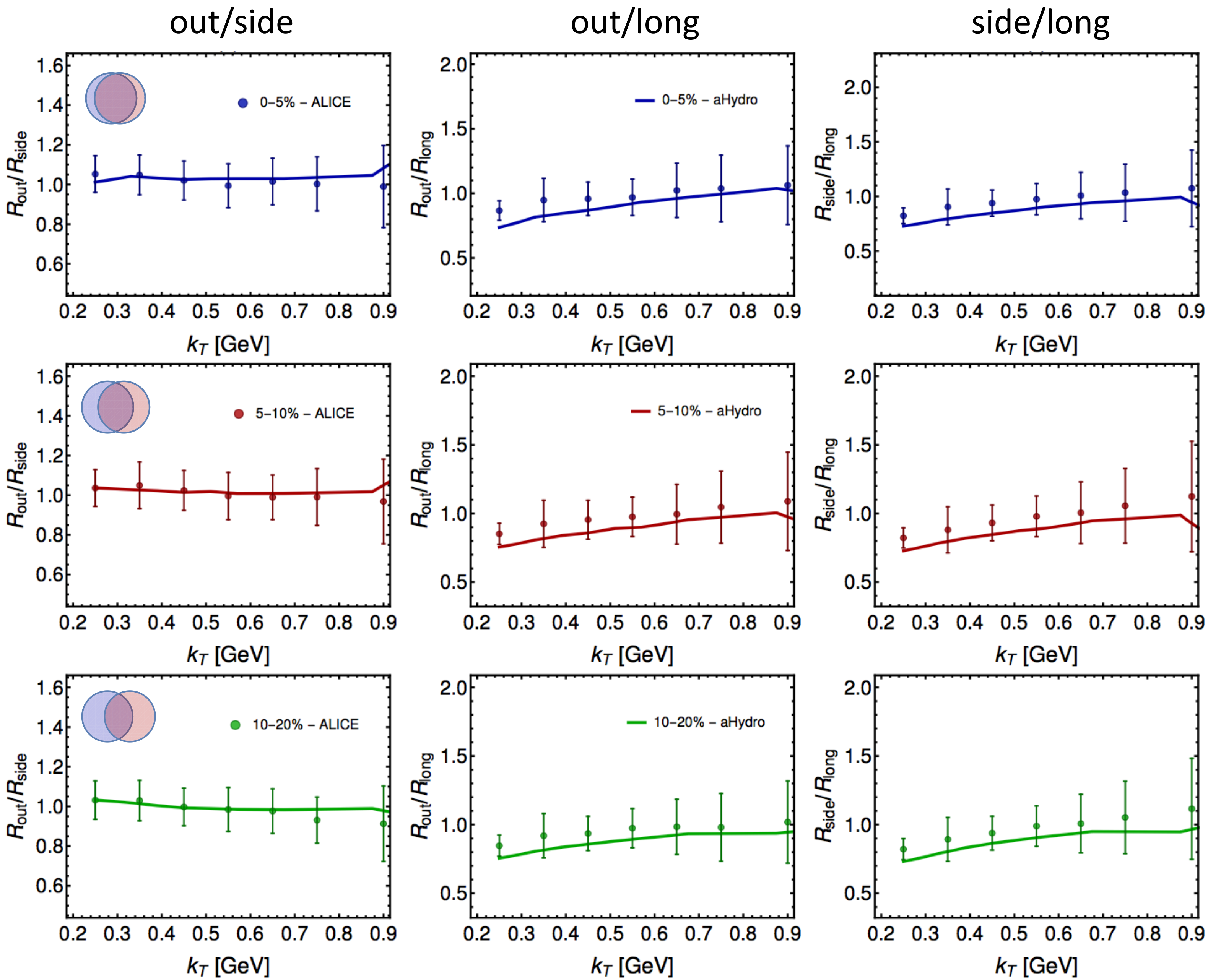


Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

HBT Radii


out
side
long
0–5% – ALICE
0–5% – aHydro
5–10% – ALICE
5–10% – aHydro
10–20% – ALICE
10–20% – aHydro
R_{out} [fm]
R_{side} [fm]
R_{long} [fm]
k_T [GeV]

HBT Radii Ratios



Conclusions and Outlook

- The non-equilibrium attractor is momentum-space anisotropic → **hydronization instead of thermalization**
- Hydrodynamics can describe such non-equilibrium dynamics (see Paul's talk and others later this week); however, some formalisms are better than others as we leave the magical land of near-equilibrium
- aHydro RTA attractor agrees best with the exact RTA attractor.
- aHydro builds upon prior advances in relativistic hydrodynamics in an attempt to **create an even more quantitatively reliable model of QGP evolution**.
- Now have a 3+1d "ellipsoidal" aHydro code with realistic EoS, anisotropic freeze-out, and fluctuating initial conditions. **Our preliminary fits to experimental data using smooth Glauber initial conditions look quite nice.**
- There is also work ongoing to incorporate momentum-space anisotropies into photon production, dilepton production, quarkonia suppression, etc. Much work to be done still…