Virtual photon polarization and dilepton anisotropy in heavy-ion collisions

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Based on: E.S., A. Jaiswal, and B. Friman (to be published)

An unpolarized thermal medium produces polarized virtual photons

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UNIVERSITÄT
DARMSTADT

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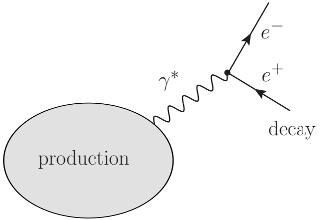
The goal



Angular distribution of the dilepton

$\Downarrow$

Information on the polarization states of the virtual photon

$\Downarrow$

Information on the production mechanism

- Early stages and onset of thermalization
  (P. Hoyer, PLB **187**, 162; E. Shuryak, arXiv:1203.1012)
- Parton anisotropic momentum distributions
  (G. Baym, T. Hatsuda, and M. Strickland, PRC **95**, 044907)

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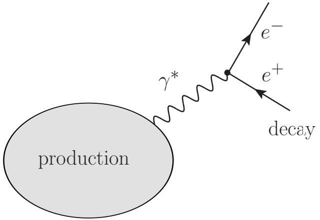

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Spin-density matrix

- **Pure state**: $|\psi\rangle = \sum_\lambda c_\lambda |\lambda\rangle$
  Expectation value of an operator $\langle O \rangle = \langle \psi | O | \psi \rangle$
- **Mixed state**: incoherent mixture of $|\psi_i\rangle$ with statistical weight a_i

$$\rho = \sum_i a_i |\psi_i\rangle\langle\psi_i| = \sum_{\lambda,\lambda'} \rho_{\lambda\lambda'} |\lambda\rangle\langle\lambda'|$$

$\rho_{\lambda\lambda'} = \sum_i a_i\, c_\lambda^{(i)} c_{\lambda'}^{(i)*}$. Expectation value: $\langle O \rangle = \mathrm{Tr}(\rho\, O)$

Example: Spin-1/2 particle (2×2 hermitian matrix):

$$\rho = \frac{1}{2}(1 + \vec{P} \cdot \vec{\sigma})$$

- Spin polarization vector: $\vec{P} = \langle \vec{\sigma} \rangle = \mathrm{Tr}(\rho\, \vec{\sigma})$

$\lvert\vec{P}\rvert = 1$	Pure state
$0 < \lvert\vec{P}\rvert < 1$	Mixed state
$\lvert\vec{P}\rvert = 0$	Completely unpolarized mixed state

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Spin-density matrix for spin-1 particles

- Three polarization states (in rest frame)

  Transverse to $\vec{q}$: $\epsilon(\pm 1) = \mp \frac{1}{\sqrt{2}} (0,\, 1,\, \pm i,\, 0)$

  Longitudinal to $\vec{q}$: $\epsilon(0) = (0,\, 0,\, 0,\, 1)$

- **Spin-density matrix**: hermitian 3×3 matrix

$$\rho = \frac{1}{3} \left[1 + \frac{3}{2} \vec{P} \cdot \vec{S} + \sqrt{\frac{3}{2}} \sum_{i,j} T_{ij} (S_i S_j + S_j S_i) \right]$$

  $\vec{S}$ are the spin-1 operators

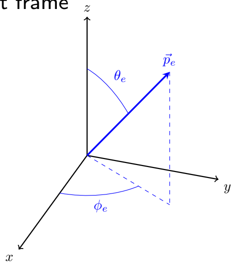
- $\mathrm{Tr}\rho = 1$ (8 parameters)
- Vector polarization: $\vec{P} = \langle \vec{S} \rangle$ (3 parameters)
- Tensor polarization: $T_{ij} = \frac{1}{2}\sqrt{\frac{3}{2}} \left(\langle S_i S_j + S_j S_i \rangle - \frac{4}{3}\delta_{ij} \right)$, $\sum_i T_{ii} = 0$

  (5 parameters)

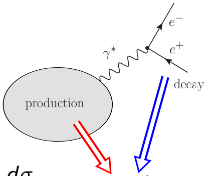
One can have no vector but tensor polarization

Lepton angular distribution

Photon rest frame



spin-1 $\rightarrow$ spin-$\frac{1}{2}$ + spin-$\frac{1}{2}$



$$\frac{d\sigma}{d\Omega_e} \propto \mathrm{Tr}(\,\rho\; O^{\mathrm{dec}}\,)$$

$$= \mathcal{N}\Big(1 + \lambda_\theta \cos^2\theta_e + \lambda_\phi \sin^2\theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos\phi_e$$

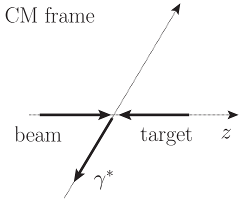
$$+ \lambda_\phi^\perp \sin^2\theta_e \sin 2\phi_e + \lambda_{\theta\phi}^\perp \sin 2\theta_e \sin\phi_e + \text{parity violating terms}\Big)$$

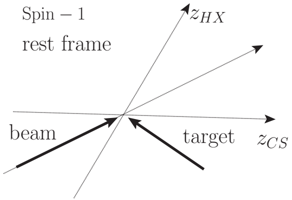
$$\lambda_\theta = \frac{\Sigma_\perp - \Sigma_\parallel}{\Sigma_\perp + \Sigma_\parallel}$$

- Transverse: $\Sigma_\perp = \rho_{-1,-1} + \rho_{+1,+1}$
  Longitudinal: $\Sigma_\parallel = 2\rho_{0,0}$
  $(\rho = \sum_{\lambda,\lambda'} \rho_{\lambda\lambda'} |\lambda\rangle\langle\lambda'|)$
- Completely **transverse** polarized: $\lambda_\theta = +1$
  Completely **longitudinal** polarized: $\lambda_\theta = -1$
- Photon polarization reflected in angular distribution

Reference frames

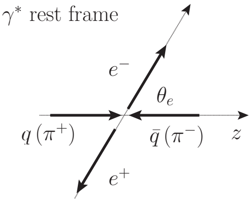
Anisotropy coefficients depend on the reference frame





- Helicity (HX): z-axis along photon momentum
- Collins-Soper (CS): z-axis along bisector between beam and target
- Different frames are related by rotation

Examples



- Drell-Yan process: $q\bar{q} \to \gamma^* \to e^+e^-$

$$\frac{d\sigma}{d\Omega_e} \sim 1 + \cos^2\theta_e$$

$\lambda_\theta = +1$. Virtual photon is completely transverse polarized along beam axis

- Pion annihilation process: $\pi^+\pi^- \to \gamma^* \to e^+e^-$

$$\frac{d\sigma}{d\Omega_e} \sim 1 - \cos^2\theta_e$$

$\lambda_\theta = -1$. Virtual photon is completely longitudinal polarized along beam axis

Virtual photon emission from a thermal medium

$$q\bar{q} \to \gamma^* \to e^+e^-$$
$$\pi^+\pi^- \to \gamma^* \to e^+e^-$$

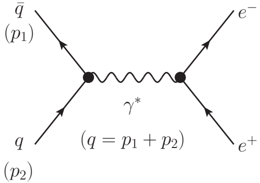
- Thermal average of initial particles momenta p through Fermi or Bose distribution
$$f(p) = \frac{1}{e^{(u\cdot p)/T} \pm 1}$$
- Fluid rest frame $u^\mu = (1,0,0,0) \Rightarrow$ Distribution is spherical symmetric

Photon momentum $\vec{q}$ breaks spherical symmetry,
but not azimuthal symmetry

- Photons are only tensor polarized
- $|\vec{q}| \to 0 \Rightarrow$ No anisotropy $\Rightarrow$ No photon polarization

Boltzmann limit

$q + \bar{q} \to \gamma^* \to e^+e^-$



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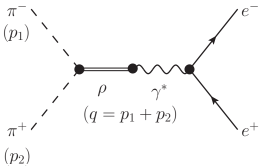


$$\int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\frac{1}{e^{(u\cdot p_1)/T} \pm 1}\frac{1}{e^{(u\cdot p_2)/T} \pm 1} \sim e^{-(u\cdot q)/T}\int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}$$

- No photon polarization independently of photon momentum

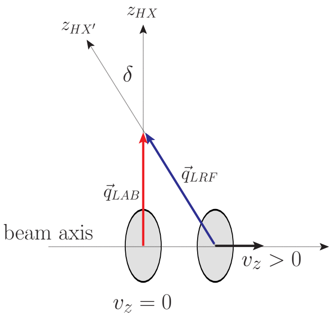
Photon polarization is due to quantum statistics!

Medium and flow

Static uniform medium

- Photon rest frame: fluid velocity $\vec{v}$ opposite to photon "direction"
- Only $\lambda_\theta \neq 0$

Longitudinal Bjorken expansion

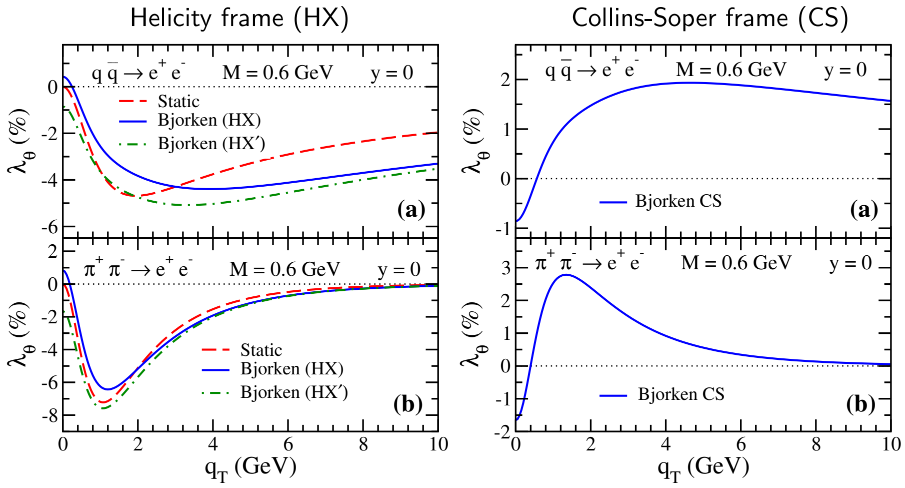


- $v_z = z/t$ along beam axis
- Photon polarized along $z_{HX'}$ defined by its momentum in local rest frame
- Rotation δ between $z_{HX'}$ and z_{HX} (Wick helicity rotation) $\Rightarrow \lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \neq 0$
- Frame invariant combination: $\lambda_\theta^{HX'} = \frac{\mathcal{N}^{HX}}{\mathcal{N}^{HX'}}(\lambda_\theta^{HX} + 3\lambda_\phi^{HX})$

Longitudinal Bjorken + Radial expansion

- All coefficients $\lambda_\theta, \lambda_\phi, \lambda_{\theta\phi}, \lambda_\phi^\perp, \lambda_{\theta\phi}^\perp \neq 0$

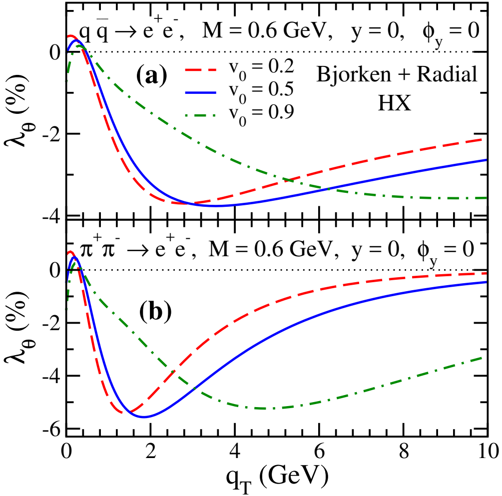
Results (static and Bjorken expansion)



- Static case: $\lambda_\theta \to 0$ for $q_T \to 0$, and for $q_T \to \infty$ (Boltzmann limit)
- λ_θ changes sign from the Helicity to Collins-Soper frame
- Experiments: average over $q_T \Rightarrow$ Cancellation in Collins-Soper

Results (Bjorken + radial expansion)

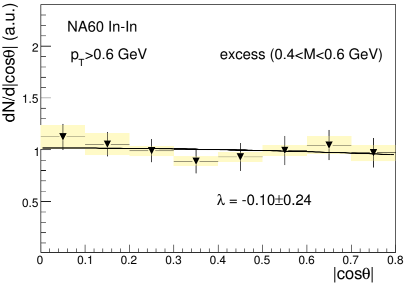
Helicity frame (HX)



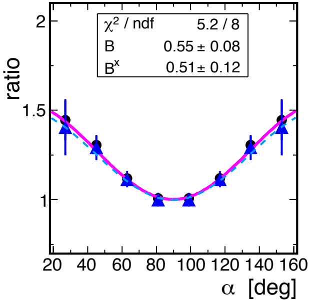
- $v_\perp = v_0 r_\perp / R_0$ (Transverse to beam axis)
- The position of the minimum shifts towards higher q_T as v_0 increases

Experimental results (NA60 and HADES)

Collins-Soper frame



Helicity frame



In-In at 158*A* GeV

(NA60 Collaboration), PRL **96**, 222301 (2009)

Ar-KCl at 1.76*A* GeV

(HADES Collaboration), PRC **84**, 014902 (2011)

- NA60: $\lambda_\theta \simeq 0$, but large error bars
- HADES: large polarization $\lambda_\theta \simeq 0.5$

Conclusions

Summary

- Anisotropy coefficients as a tool to understand heavy-ion collisions
- Virtual photons from (unpolarized) thermal sources are polarized
- Collective flow affects shape of anisotropy coefficients

Outlook

- Different elementary reactions
- Anisotropic momentum distributions ⇒ nonequilibrium
- Effect of vorticity and magnetic field (polarized medium)

BACKUP

Diagonal form of the spin-density matrix

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\frac{3}{2}P_z+\sqrt{\frac{3}{2}}T_{zz} & 0 & 0 \\ 0 & 1-\sqrt{6}T_{zz} & 0 \\ 0 & 0 & 1-\frac{3}{2}P_z+\sqrt{\frac{3}{2}}T_{zz} \end{pmatrix}$$

- In unpolarized system $P_z = 0$, but often $T_{zz} \neq 0$, i.e., no vector but tensor polarization!
- In general vector and tensor polarization axes can be different