Initial stages: from theory to practice

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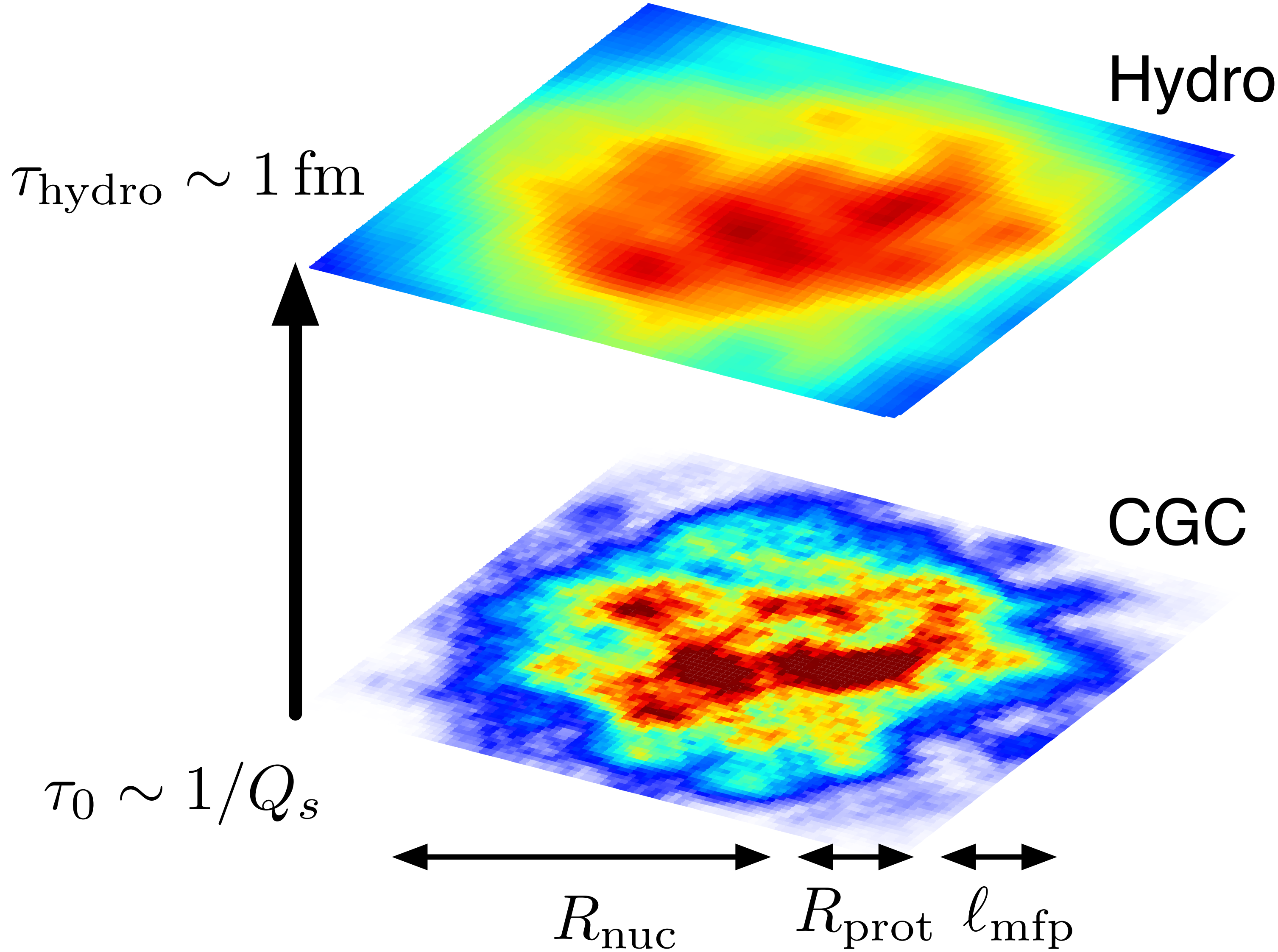
1. L. Keegan, A. Kurkela, A. Mazeliauskas, DT, JHEP (2016)
2. A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, DT, 65 pages, <u>almost done</u>
3. See talk by Aleksas Mazeliauskas at this conference

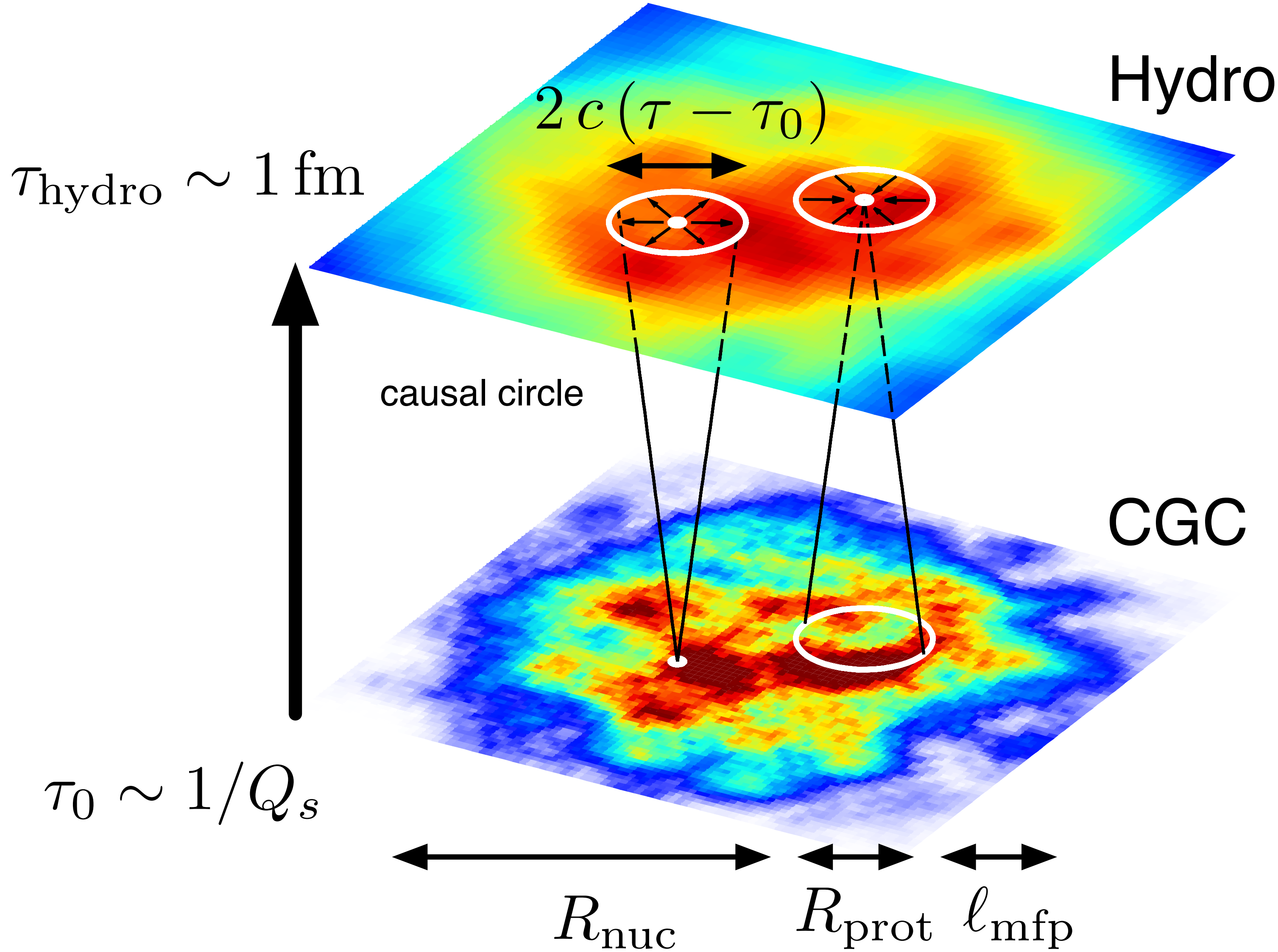
Mapping the CGC fluctuating initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics with approximations:

$$R_{\text{nuc}} \gg R_{\text{prot}} \sim \ell_{\text{mfp}} \gg 1/Q_s$$

Mapping the CGC fluctuating initial conditions to hydro

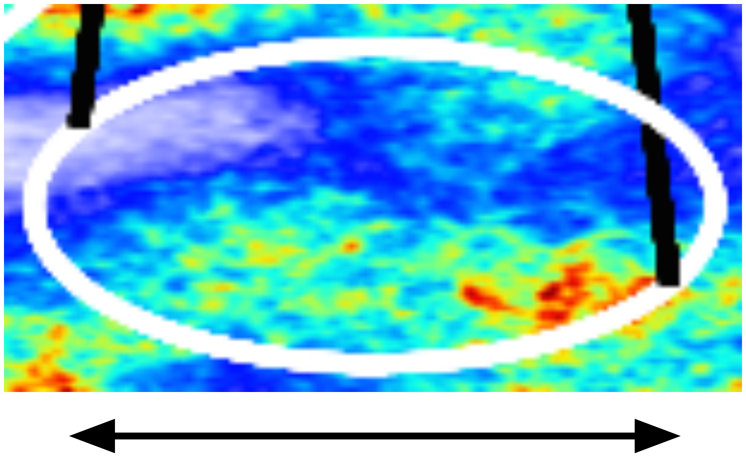


Causality limits the equilibration dynamics within a causal circle

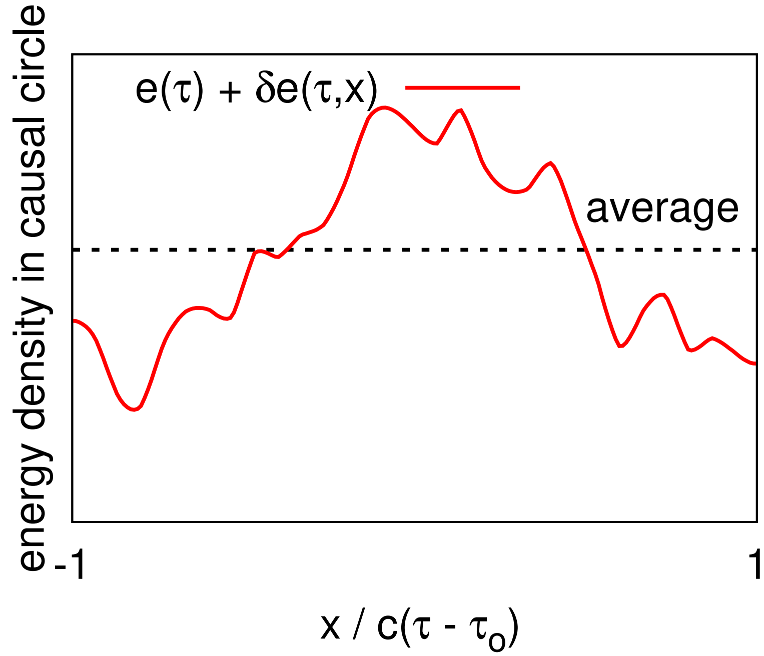
$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \sim c\tau_{\rm hydro} \gg 1/Q_s$$

An approximation scheme for the equilibration dynamics:

look in causal circle



$$2\,c\,(\tau - \tau_0)$$



1. Determine the evolution of the average (homogeneous) background

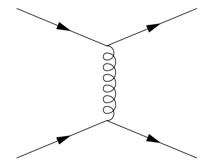
Bottom-Up Thermalization!

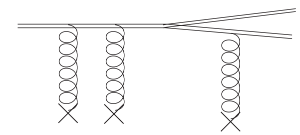
2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}'\, G(\boldsymbol{x} - \boldsymbol{x}'; \tau, \tau_o) \underbrace{\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

How to compute the background and perturbations:

$$\partial_\tau f + \frac{\mathbf{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]} \quad ,$$





Gluon distribution function for background and perturbations

$$f = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp,\mathbf{p}} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}}_{\text{transverse perturbations}} \quad .$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp\cdot\boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\mathbf{p}} = -\delta\mathcal{C}[\bar{f},\delta f] \qquad \text{perturbation}$$

We will discuss the background and perturbations separately

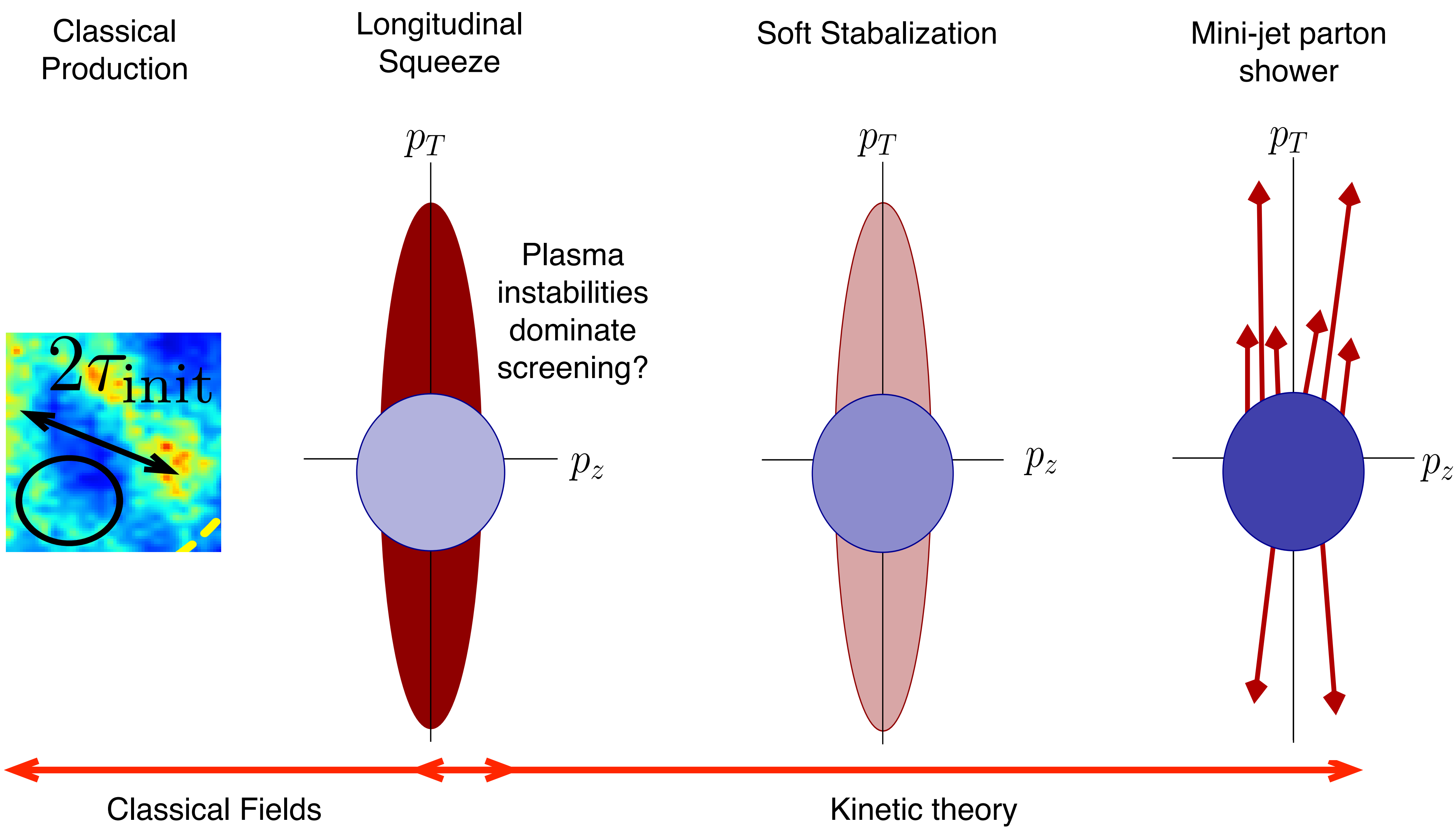
Outline

I. Evolution of the backgound: “bottom-up” thermalization

II. Evolution of the perturbations

The background and “bottom-up” thermalization

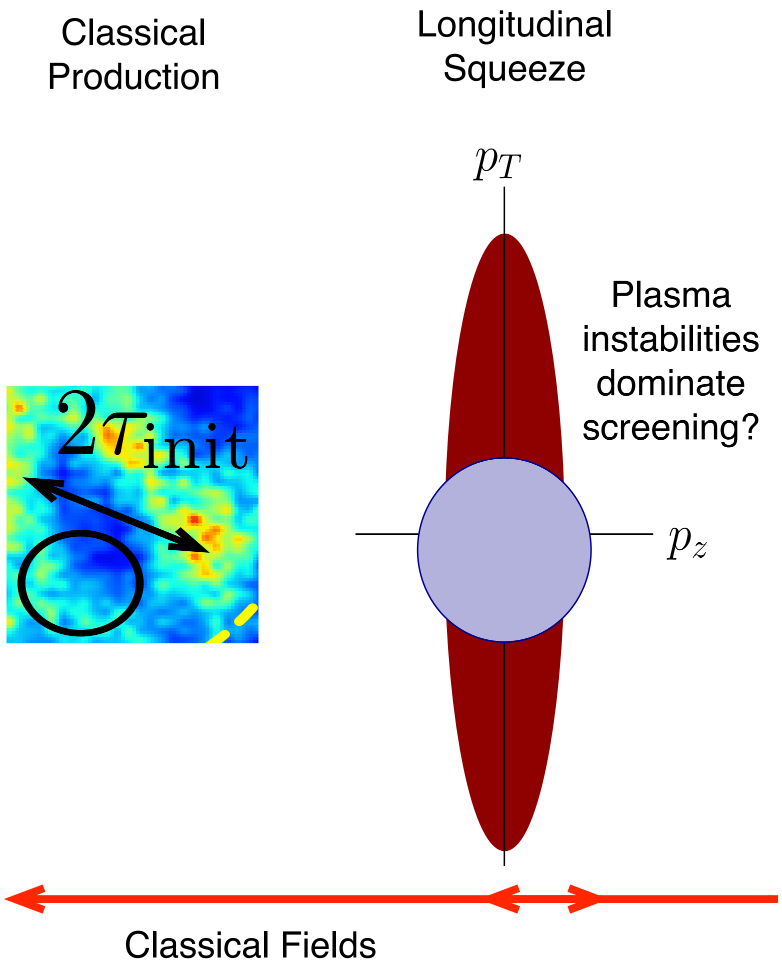
Baier, Mueller, Schiff, Son



Reach a thermal state in $\tau_{\rm hydro} \sim 1/(\alpha_s^{13/5} Q_s)$

The classical phase of “bottom-up” redo:

see talk by Boguslavski and Gelis



Progress on the first phase of “bottom-up”

(Berges, Boguslavski, Schlichting, Venugopalan, Mace . . .)

- Scaling solution for classical phase space dist:

$$f(\tau, p_z, p_T) = \frac{(Q\tau)^{1/3}}{\tau} f_o(p_z (Q\tau)^{1/3}\,, p_T)$$

  ⋆ But, finite coupling may limit its utility (Gelis)

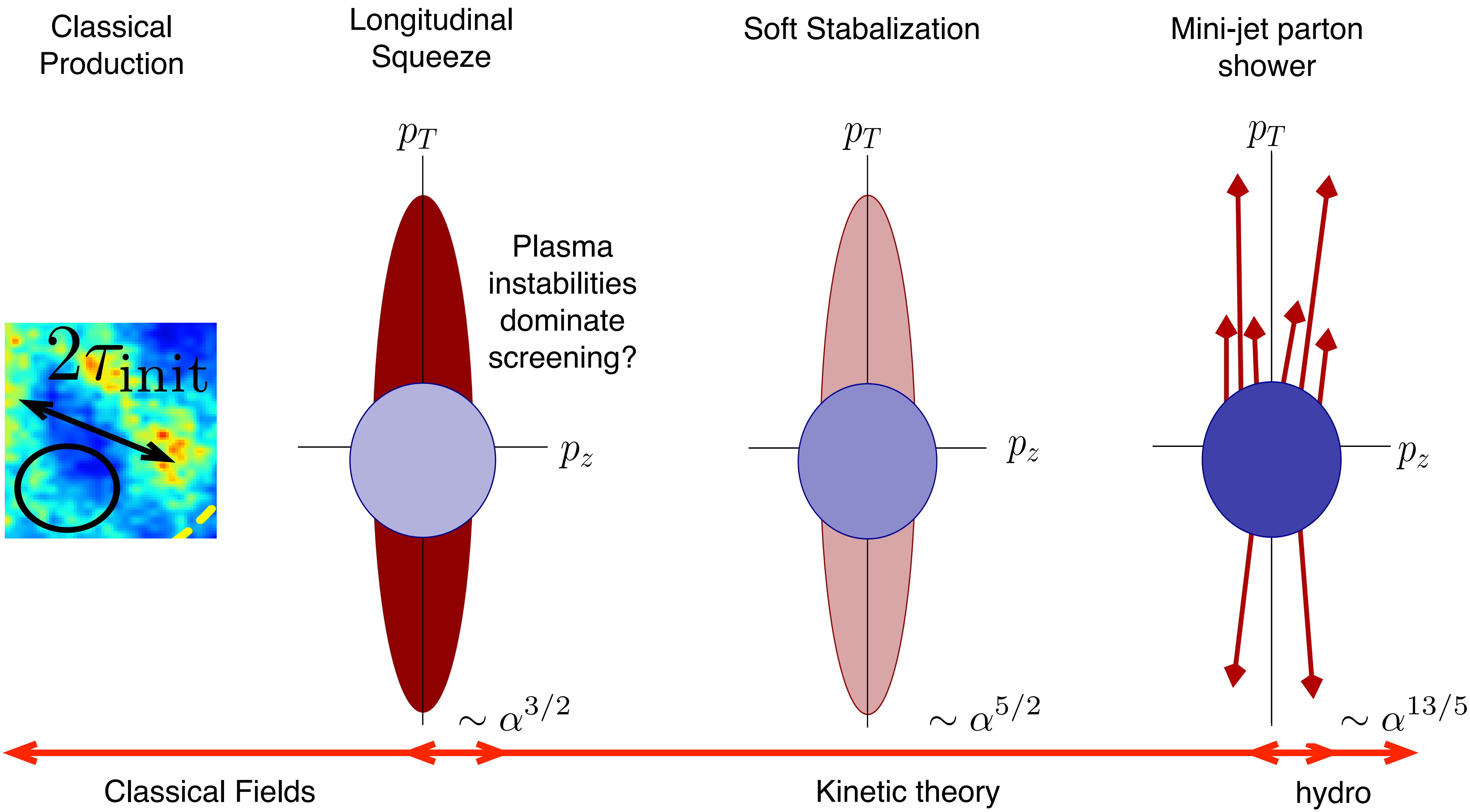
- See the hard, electric and magnetic scales emerge

- Instabilities do not seem to play a significant role

After the first classical phase, we should continue with kinetics!

The background and “bottom-up” thermalization

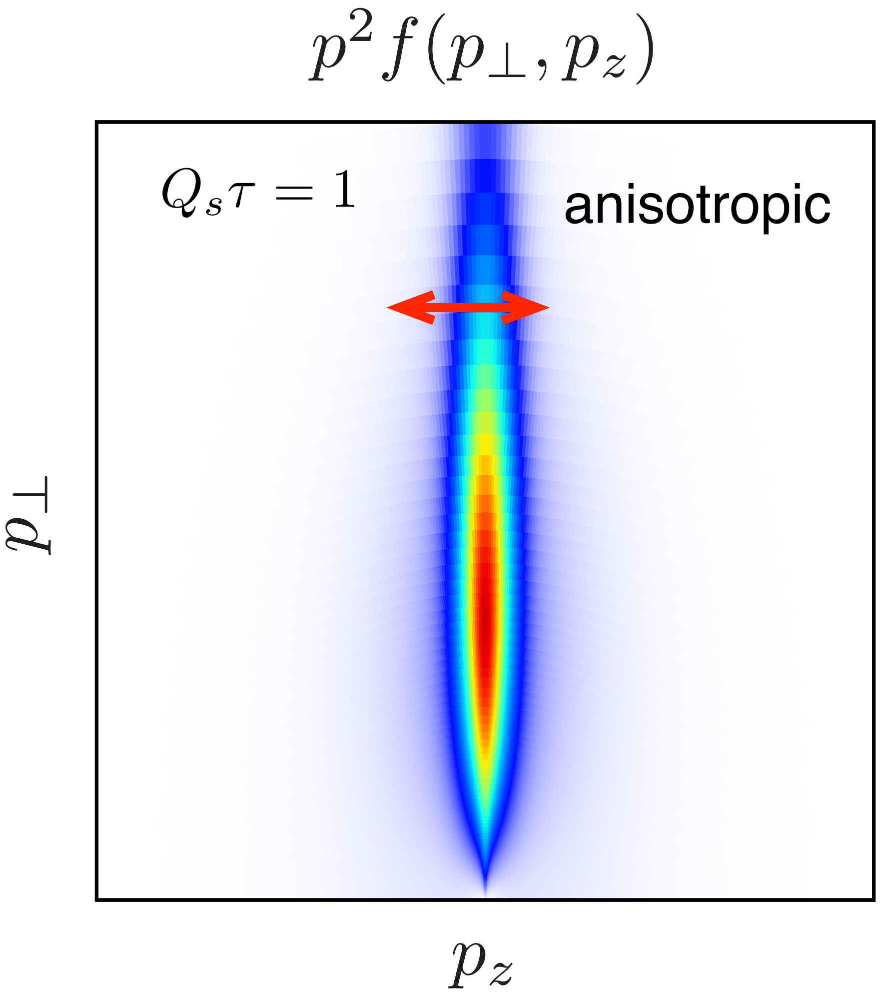
Baier, Mueller, Schiff, Son



Do these regimes qualitatively exist for realistic couplings?

A numerical realization of bottom-up

- Builds upon the first numerical realization — Kurkela, Zhu PRL (2015)



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling constant of $\alpha_s = 0.3$

$$\underbrace{\lambda \equiv 4\pi\alpha_s N_c = 10}_{\text{theorists version of } \alpha_s = 0.3}$$

corresponding to

$$\frac{\eta}{s} = 0.6 = \frac{7.5}{4\pi}$$

We see “Bottom-Up” in the computer code.

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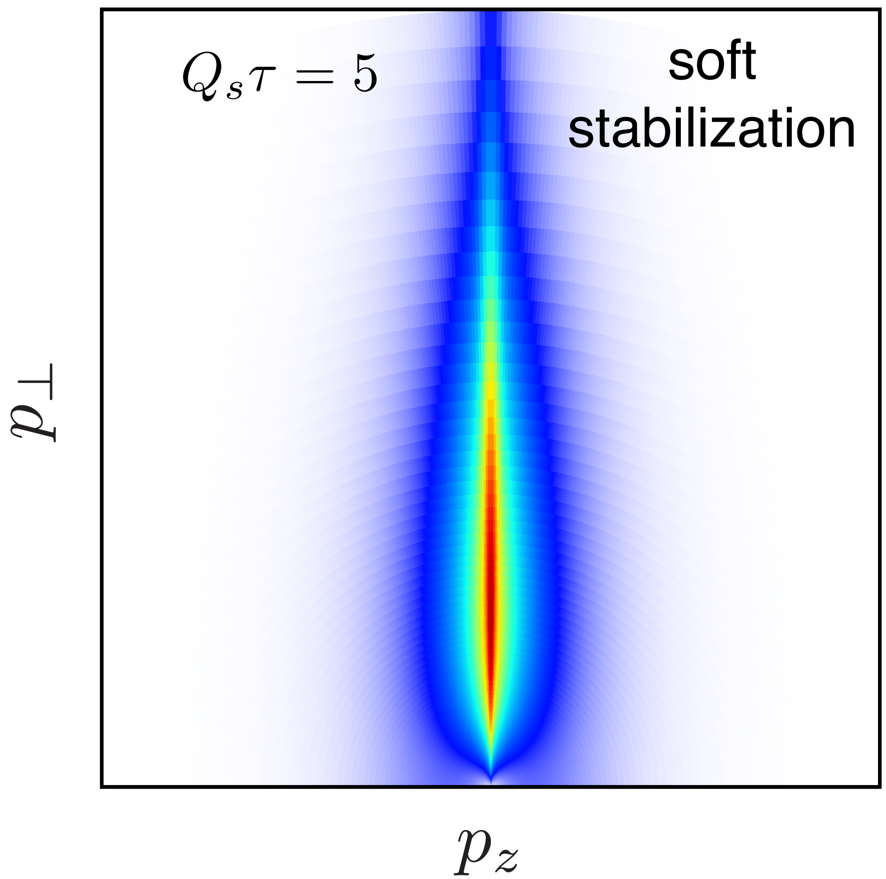
$$p^2 f(p_\perp, p_z)$$



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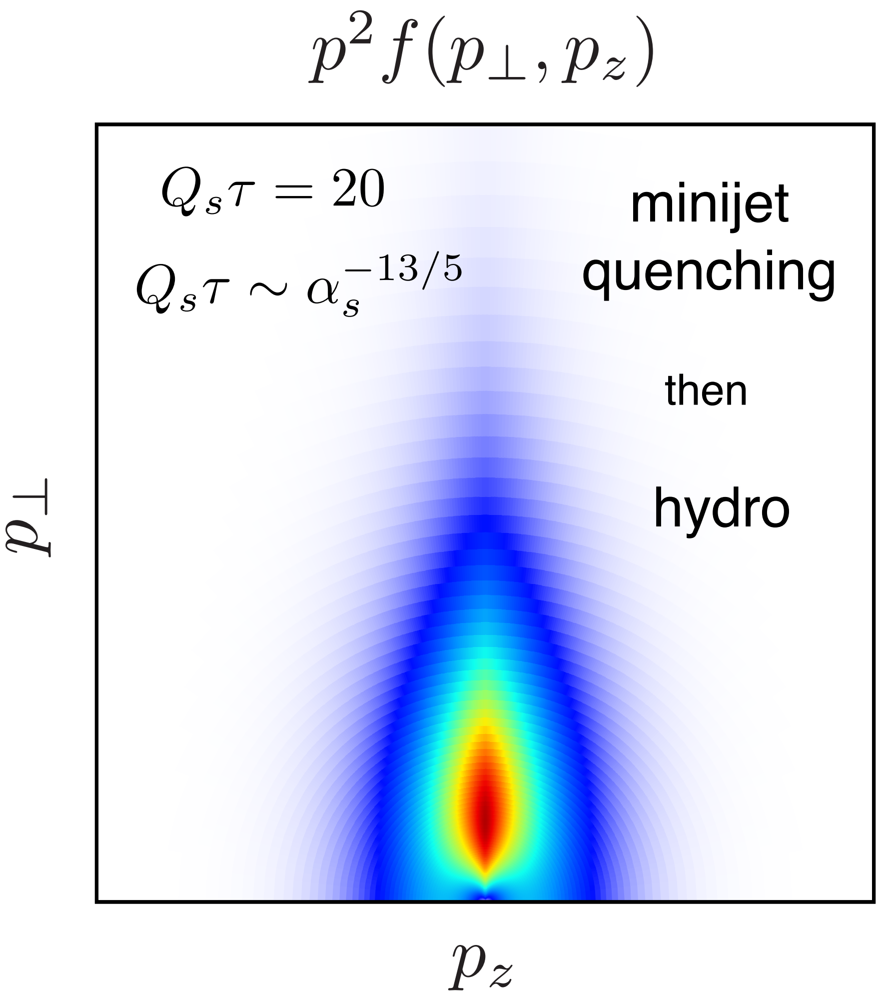

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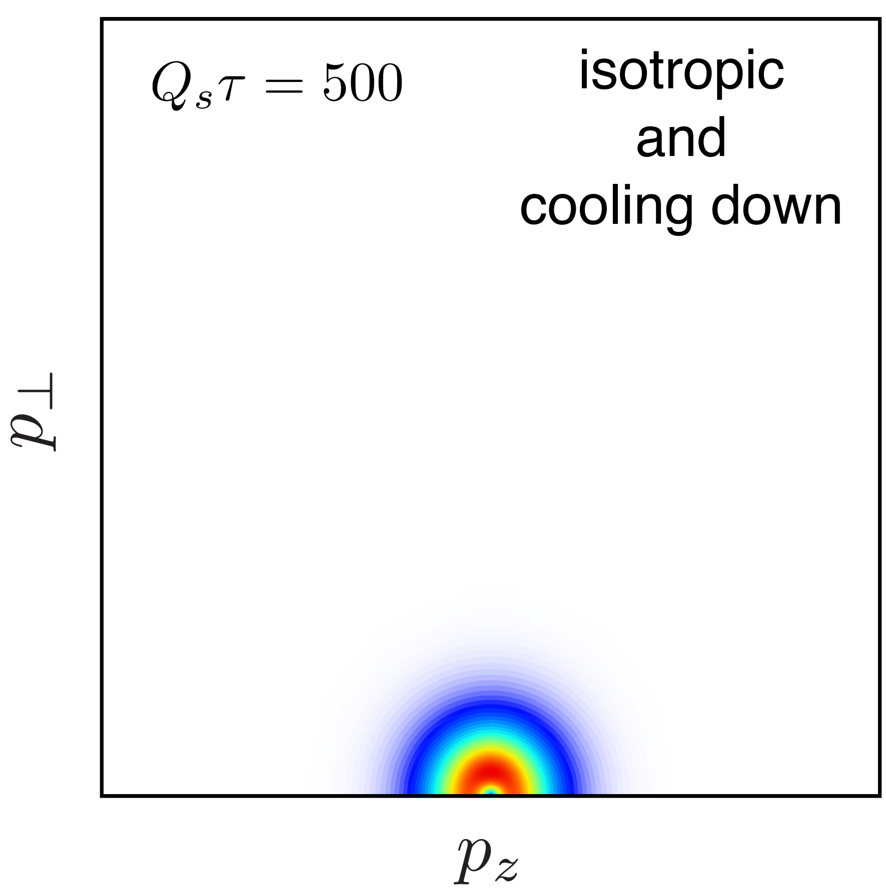

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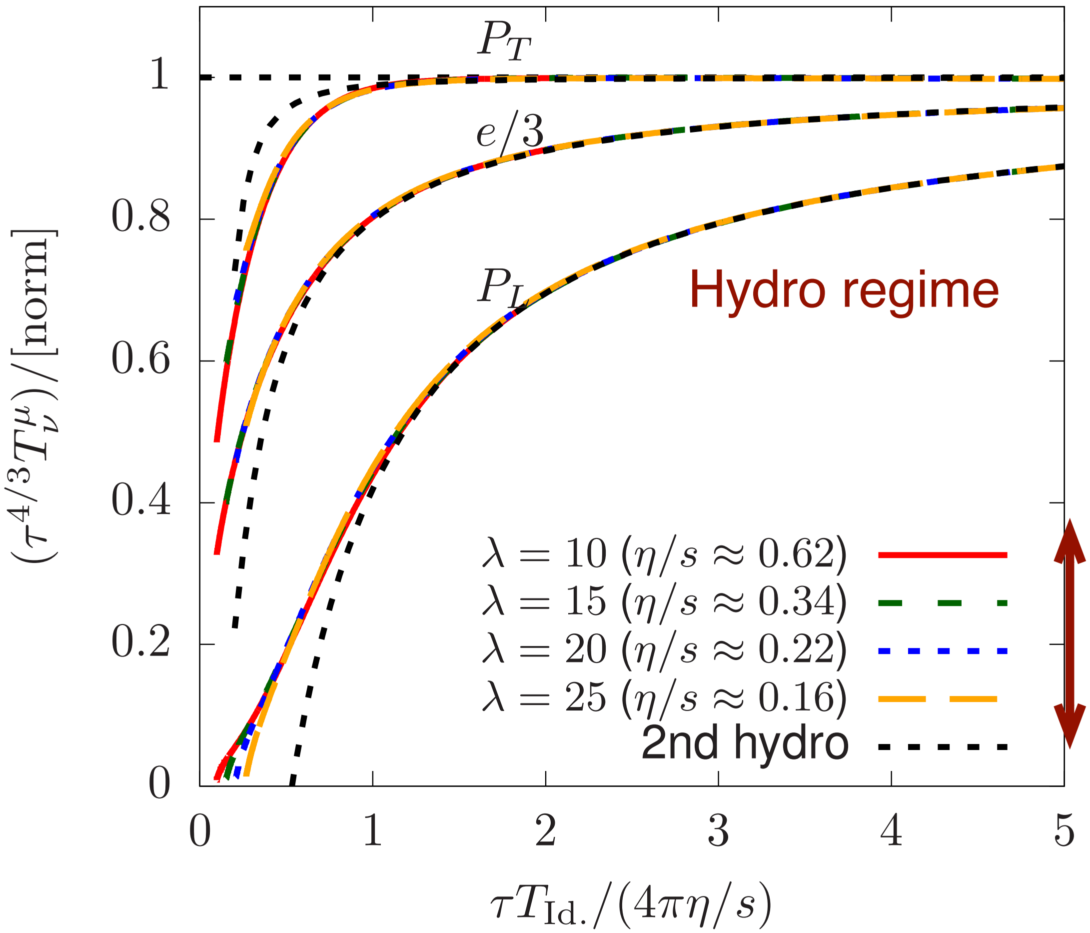
When does the background stress tensor approach second order hydrodynamics?



Different values of coupling give different η/s

In terms of eta/s, all couplings thermalize at same scaled time

Keegan, Kurkela, Romatschke, Schee, Zhu

Gives a basis for interpolating from weak coupling results to stronger coupling

Measure time in a physical relaxation time given by $\tau_R \equiv \eta/sT$ instead of α_s:

$$\frac{\tau}{\tau_R} \equiv \frac{\tau T_{\mathrm{eff}}(\tau)}{\eta/s} \qquad \text{with} \qquad \tau_R \equiv \frac{\eta}{s T_{\mathrm{eff}}}$$

Can start hydro when $\tau T_{\mathrm{Id}}/4\pi\eta/s \sim 1$

Translating earliest hydro starting time into physical units:

1. At late times the dynamics is ideal hydro: $T_{\rm Id}(\tau) = \Lambda_T/(\Lambda_T\tau)^{1/3}$

$$\lim_{\tau\to\infty} \tau T^3(\tau) = \Lambda_T^2$$

This integration constant determines dN/dy at the end of hydro

2. Hydro fits to multiplicity give:

$$\left\langle \tau e^{3/4} \right\rangle\Big|_{\tau=1.2\,\mathrm{fm}} = \underbrace{1.6\,\mathrm{GeV}^2 \propto \Lambda_T^2}_{\text{highly constrained!}}$$

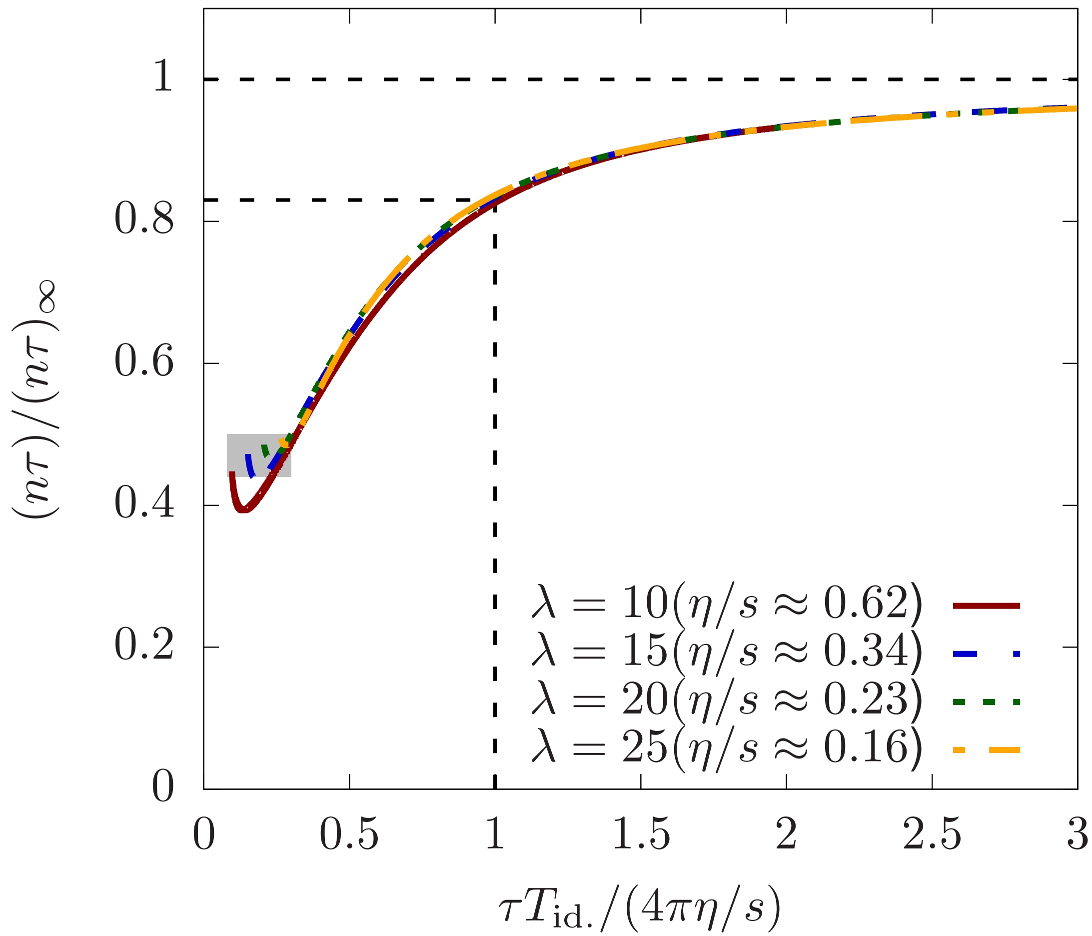
3. The estimate for $\tau_{\rm hydro}$:

$$\frac{\tau_{\rm hydro} T_{\rm Id}(\tau_{\rm hydro})}{4\pi(\eta/s)} = 1 \quad \text{with} \quad T_{\rm Id}(\tau) = \frac{\Lambda_T}{(\Lambda_T\tau)^{1/3}}$$

Find that hydrodynamics is applicable for times later than:

$$\tau_{\rm hydro} \approx 0.85\,\mathrm{fm} \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{1.6\,\mathrm{GeV}}{\langle \tau e^{3/4}\rangle}\right)^{1/2} \left(\frac{\nu_{\rm eff}}{16}\right)^{3/8}$$

How much to gluons multiply during the equilibration process?



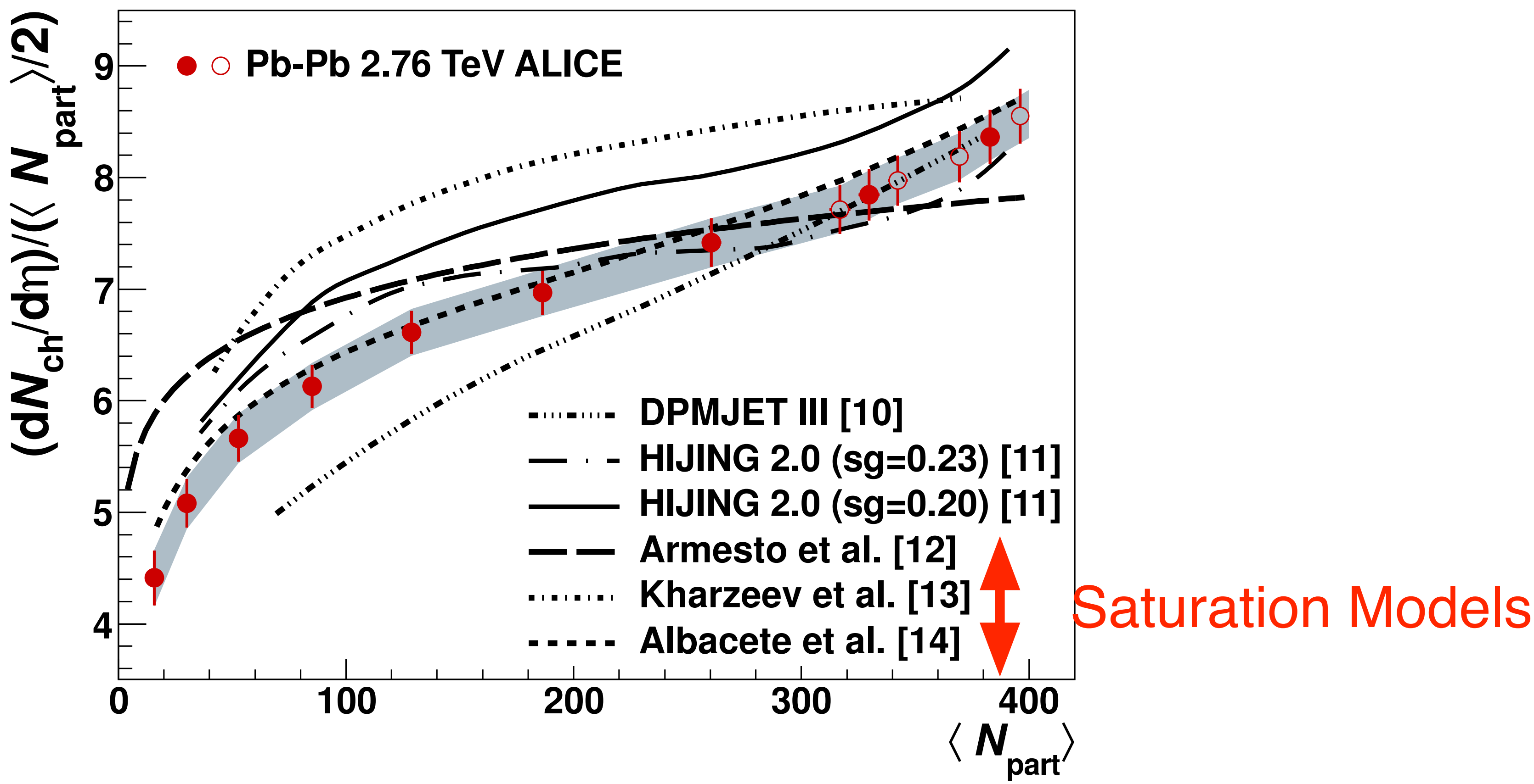
$$n\tau = \frac{1}{A}\frac{dN}{dy}$$

The final gluon multiplicity is 2.5 times the initial gluon multiplicity

independent of the coupling or η/s!

Does parton saturation explain hadron multiplicities at RHIC?

3. **Does parton saturation at high density explain hadron multiplicities at RHIC?**
R. Baier (Bielefeld U.), Alfred H. Mueller (Columbia U.), D. Schiff (Orsay, LPT), D.T. Son (Washington U., Seattle).
Apr 2002. 8 pp.
Published in **Phys.Lett. B539 (2002) 46-52**



We should compute the gluon multiplication factors versus centrality . . .

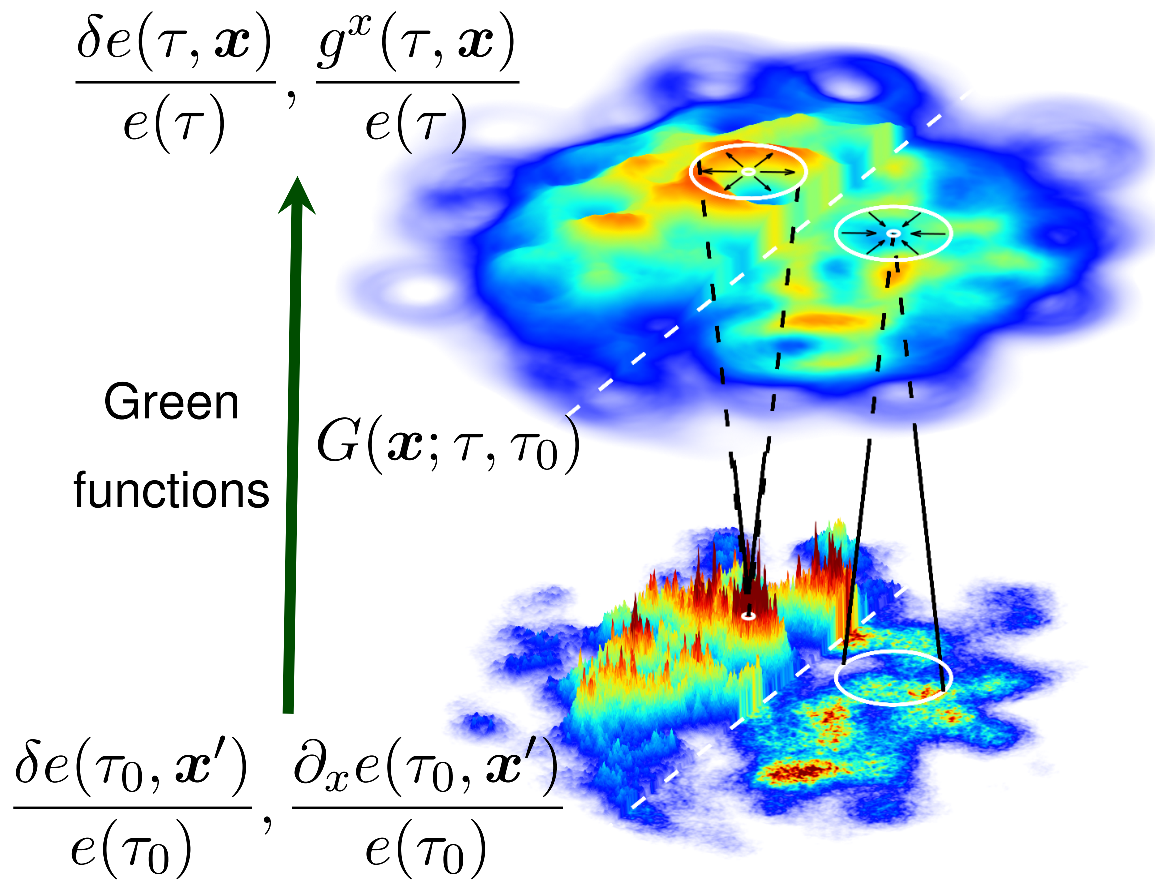
Outline

✓ Evolution of the background: “bottom-up” thermalization

II. Evolution of the perturbations

The Green functions fourier mode by fourier mode:

- Compute the response in “bottom-up” to an initial perturbation, $\delta f_{\boldsymbol{k}} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$
  - ⋆ Then sum them up



Properties of Green Functions

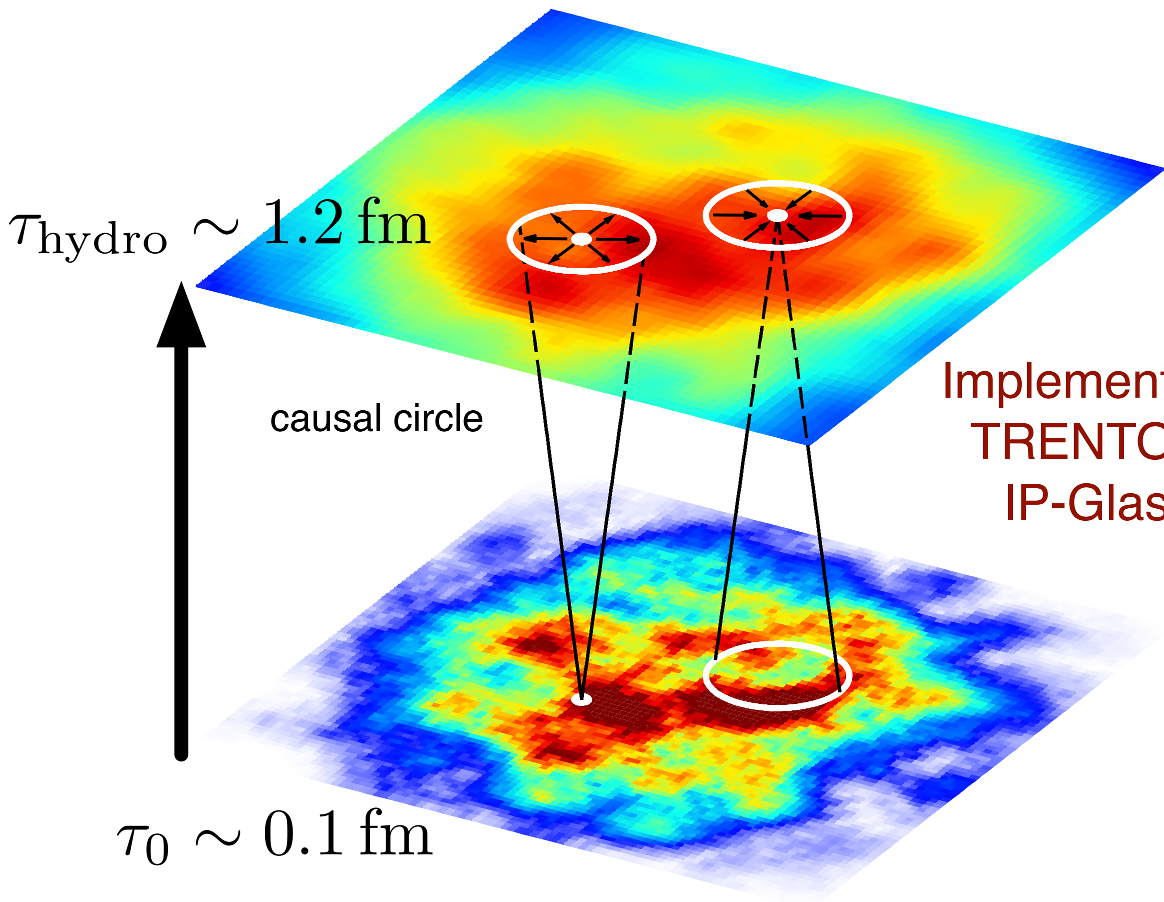
1. Has free streaming for $k\to\infty$
2. Has hydro for $k\to 0$
3. Depends on η/s and time through

$$\frac{\tau T_{\rm Id}(\tau)}{4\pi(\eta/s)}$$

For hydro need:

$$\frac{\tau T_{\rm Id}(\tau)}{4\pi(\eta/s)} > 1$$

A practical algorithm for implementation:



Implemented for TRENTO and IP-Glasma

(i) For each point, average the energy in causal circle

- ⋆ Find the scaling time corresponding to τ_0 and $\tau_{\rm hydro}$ for given η/s and energy

(ii) Propagate background and perturbations in scaled time

- ⋆ Sometimes need to regulate the response

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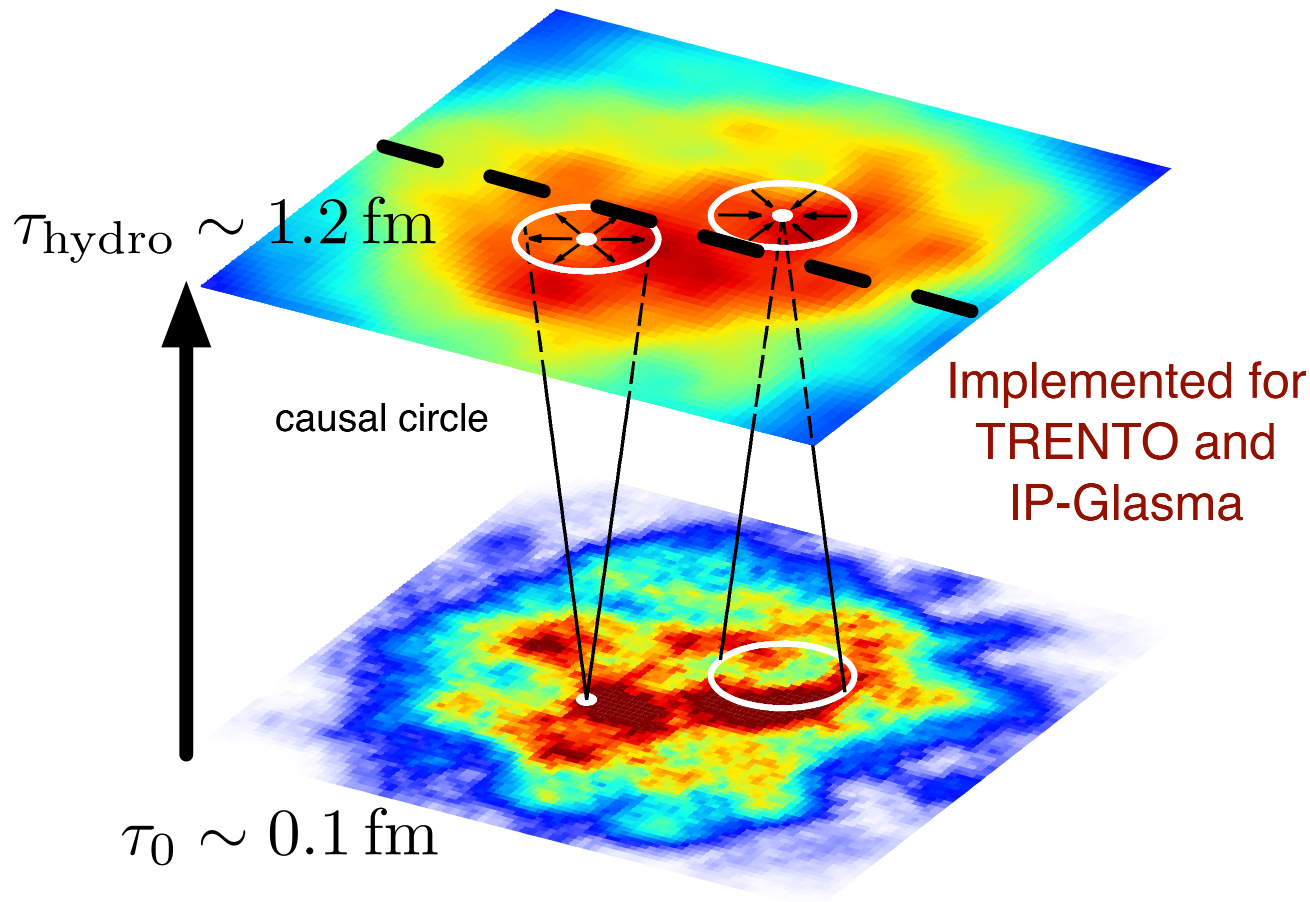

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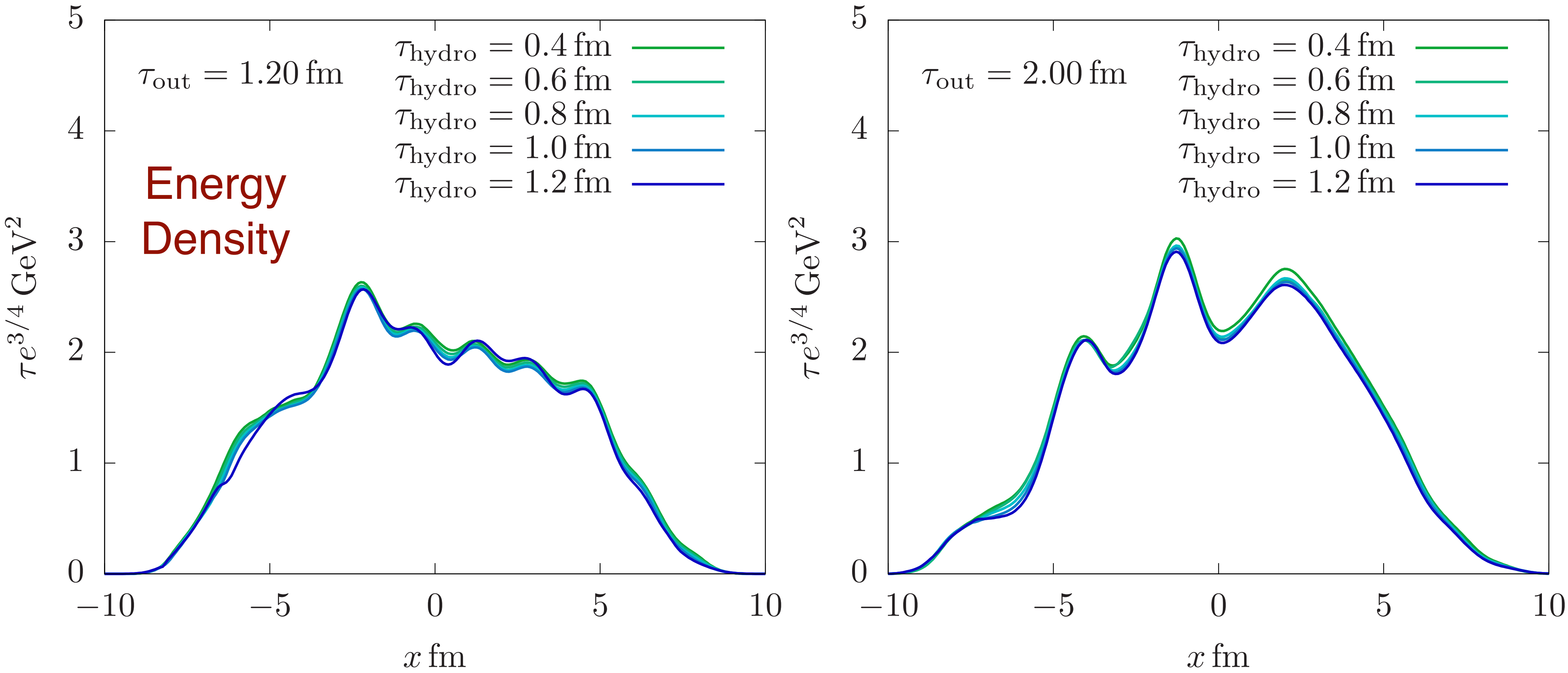
Do hydro results depend on $\tau_{\rm hydro}$?

1. Implementation in TRENTO. $\eta/s = 2/4\pi$. Central LHC.

2. Kinetics runs from $\tau_0 = 0.1$ up to $\tau_{\rm hydro}$



Remarkably insensitive to $\tau_{\rm hydro}$ as we want !

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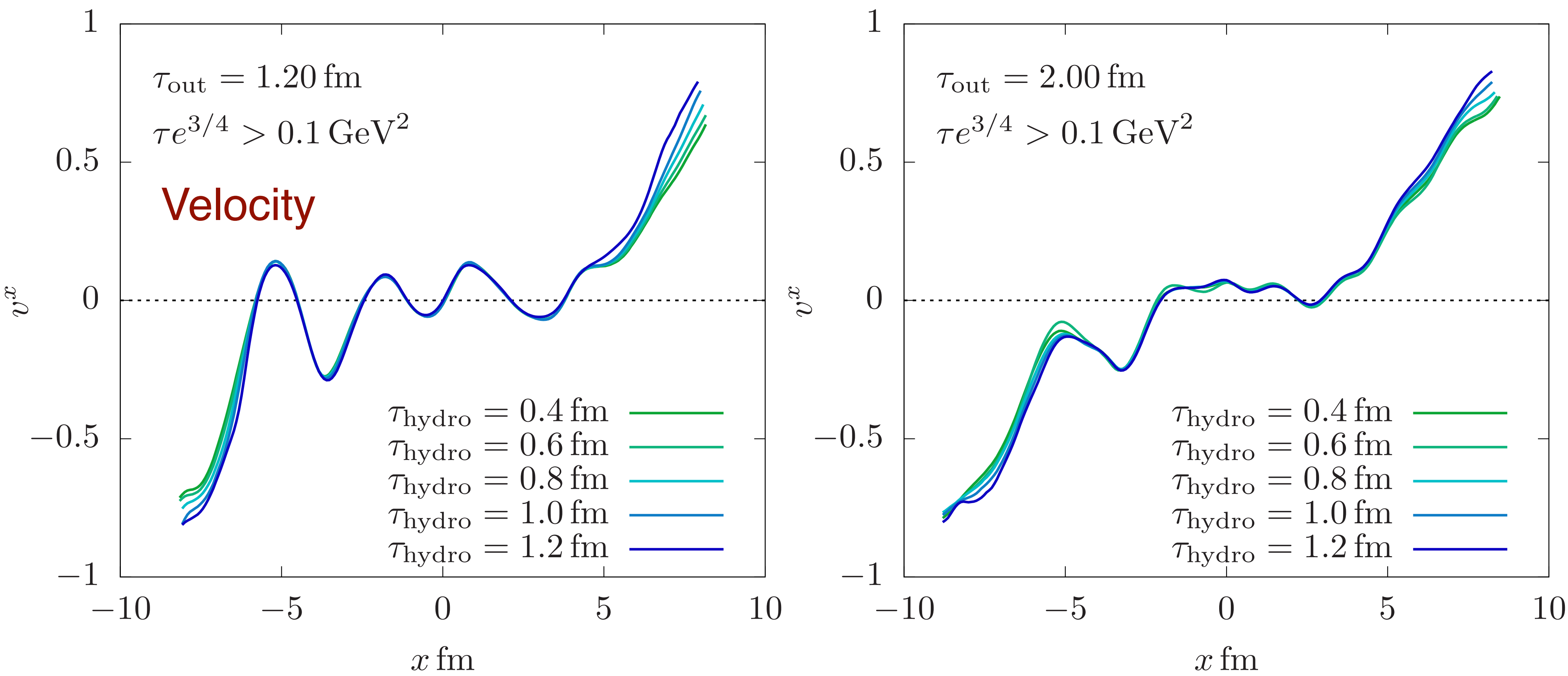
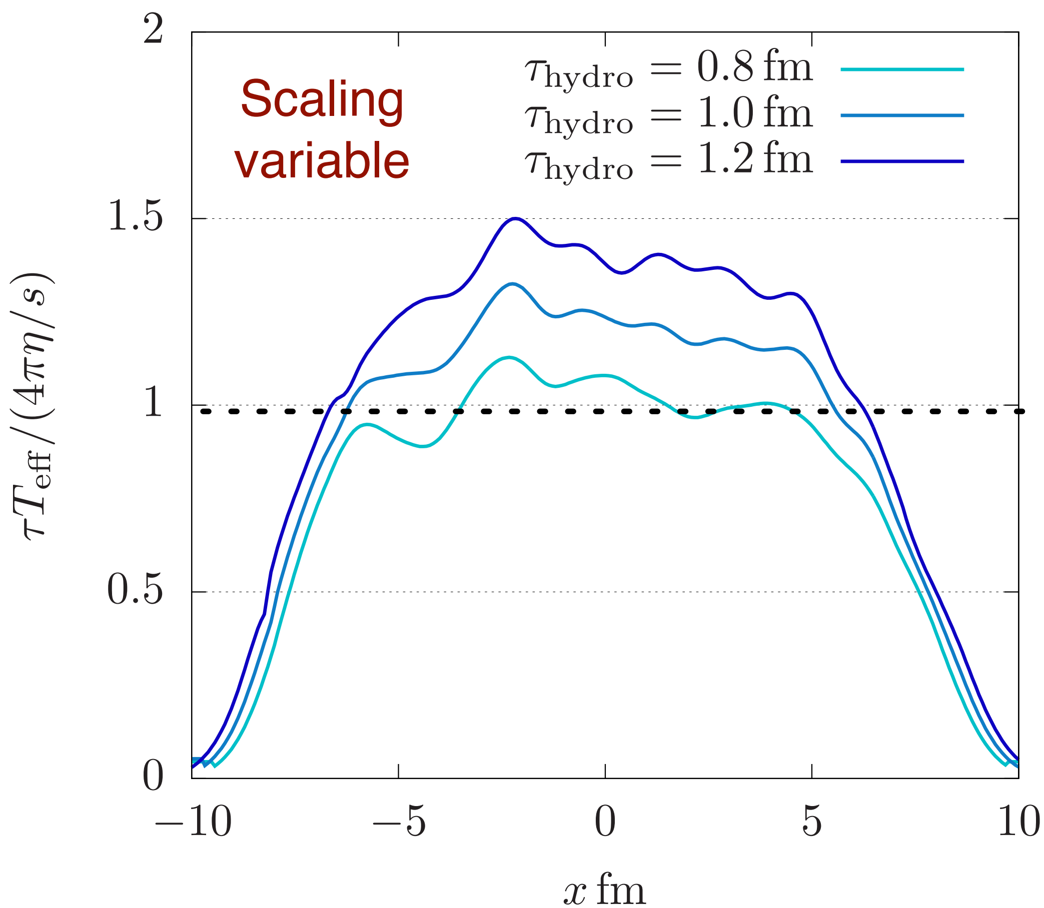


Remarkably insensitive to $\tau_{\rm hydro}$ as we want !

Are the constitutive relations are satisfied at late times?

- For times sufficiently late times Navier-Stokes should be valid:

$$\pi^{\mu\nu} = \underbrace{-\eta\sigma^{\mu\nu}}_{\text{navier stokes}} \qquad \text{for} \qquad \frac{\tau T_{\rm eff}}{4\pi\eta/s} > 1$$



Expect the cells near the line to be equilibrated and obey constitutive equations

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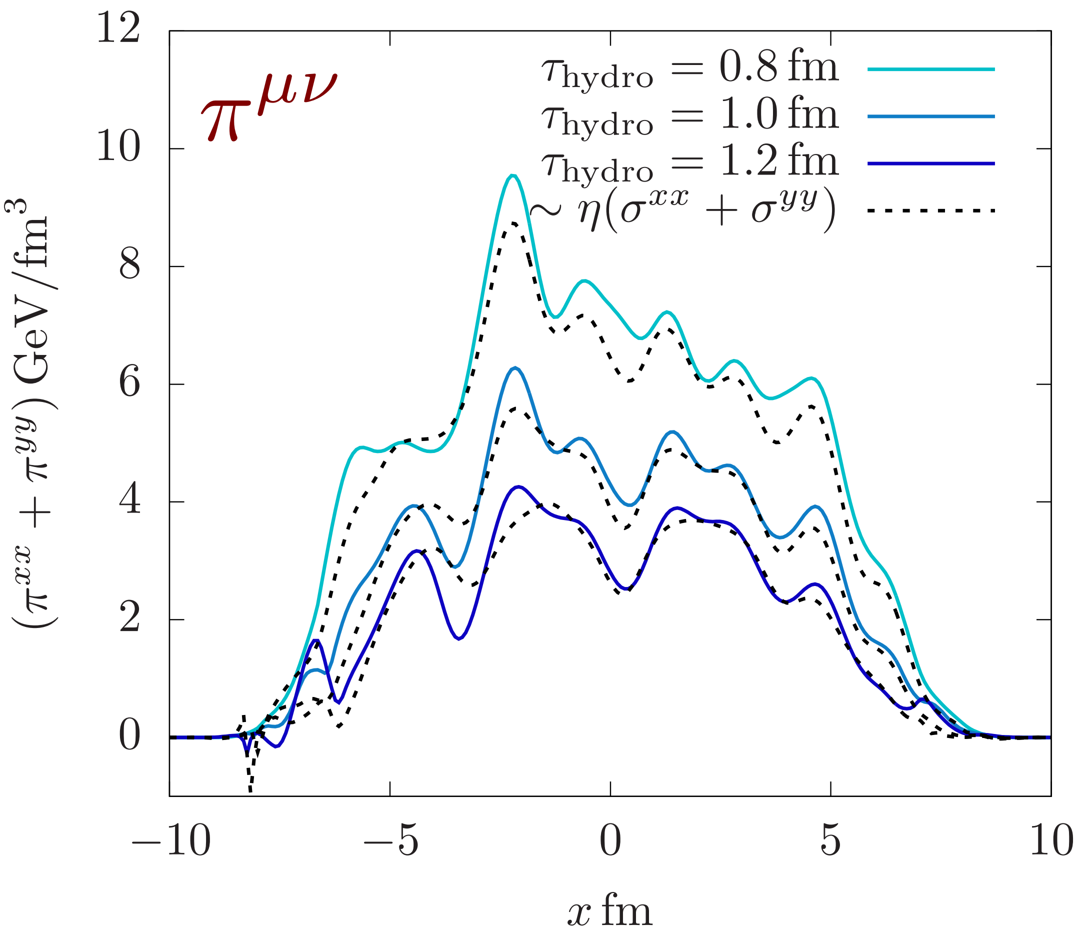

Black lines are navier stokes.

Color lines are kinetics

Constitutive relations are satisfied!

Summary

1. Still under the spell of "bottom-up" after all these years.
   - ⋆ A big next step are non-linear corrections – especially for small systems!
2. The tool is easy to use and fast. Use it!
   - ⋆ It gracefully connects any initial state to fully developed hydro

Things left out – see Mazeliauskas:

1. Hadronic observables – no surprises!
2. Comparison with other approaches:
   - ⋆ Free streaming:
   - ⋆ The Pratt pre-flow is a low k limit of our results

Code has a tentative name – `KøMPøST`.

Thank You!