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Angular Momentum And Early Time Gluon Fields

RAINER J FRIES

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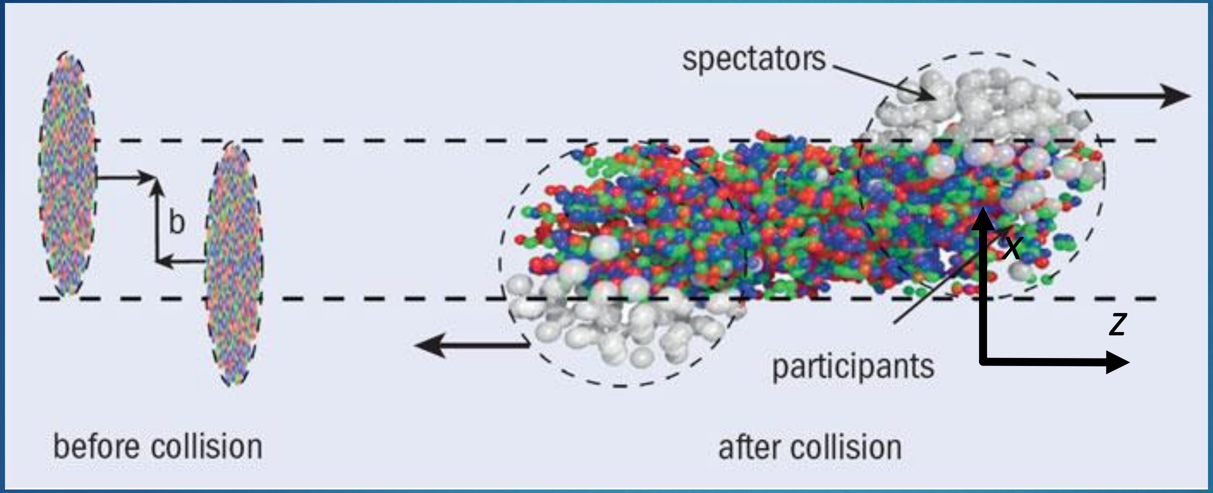
RJF, G. Chen, S. Somanathan, arXiv:1705:10779

S. Rose, RJF, in progress

Angular Momentum in Nuclear Collisions

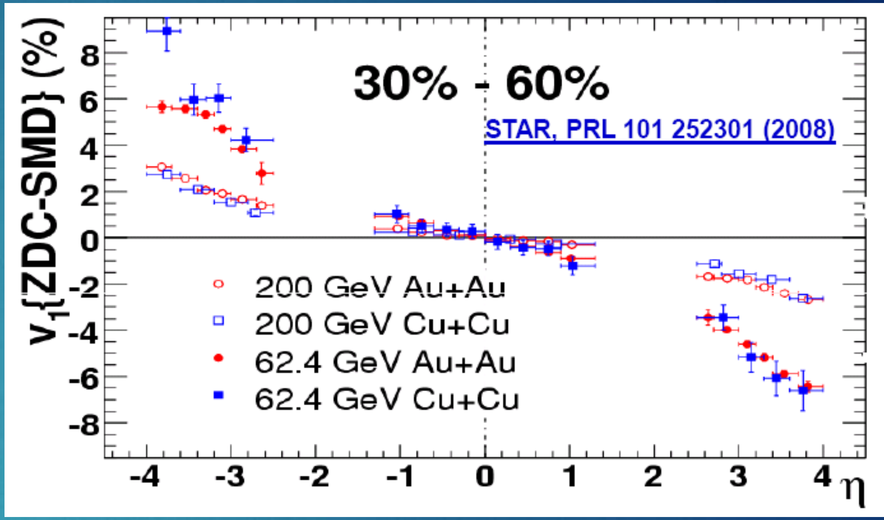
- Initial angular momentum present at finite impact parameter $L \sim \frac{b}{2} E_{CM}$.
- Carried by shear movement.



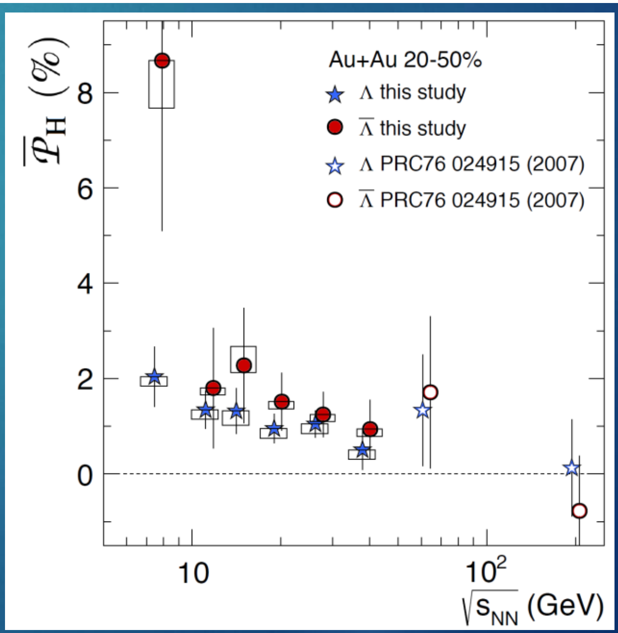
- Mostly neglected for decades. Justified?
- At LHC and top RHIC energies:
  - collision energy macroscopic, $E \sim 0.1$ mJ
  - $L \sim 10^6 \hbar \sim 10^{-25}$ mJs; far from macroscopic!

Angular Momentum in Nuclear Collisions

- Manifestations in the final state
- Rapidity-odd directed flow v_1



- Polarization of final state particles



- Effects decrease with collision energy.

[STAR, Nature 548 (2017)]

Angular Momentum in Nuclear Collisions

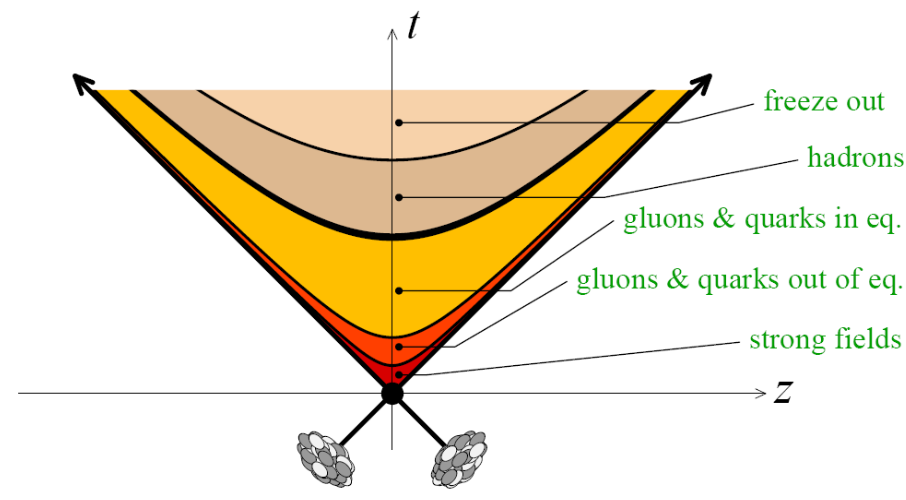
- Much recent theoretical work on vorticity, production of polarized particles, focus on lower energies.

  Liang and Wang; Csernai, Magas and Wang; Becattini, Csernai and Wang; Pang et al.; Jiang, Lin and Liao; ...

- Here: how is initial angular momentum transported to midrapidity in the limit of very high energy?
- Use the classical McLerran-Venugopalan model as an effective theory applicable at high energies and early times

Collisions at High Energy: Time Evolution

- Initial nuclear wave functions
- Strong classical gluon fields
- Local equilibration/hydrodynamization
- QGP/HG fluid close to local equilibrium
- Hadron gas and freeze-out



Expectations

- Caveat: we use boost symmetry in the calculation, good approximation at high energies.

- Infinite system: no global conservation laws.
- Boost symmetry apparently does not allow for rotation, only shear flow (as in the initial state).
- We are used to think of flow as a vector. The situation is more interesting when rank-2 tensors are involved.

Solving Yang-Mills Equations

- Color charges in nuclei → Nuclear fields A_1^i, A_2^i McLerran and Venugoplan (1994)
- Initial interaction of fields (LC gauge):

Kovner, McLerran, Weigert (1995)

$$A^i_{\perp(0)}(x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

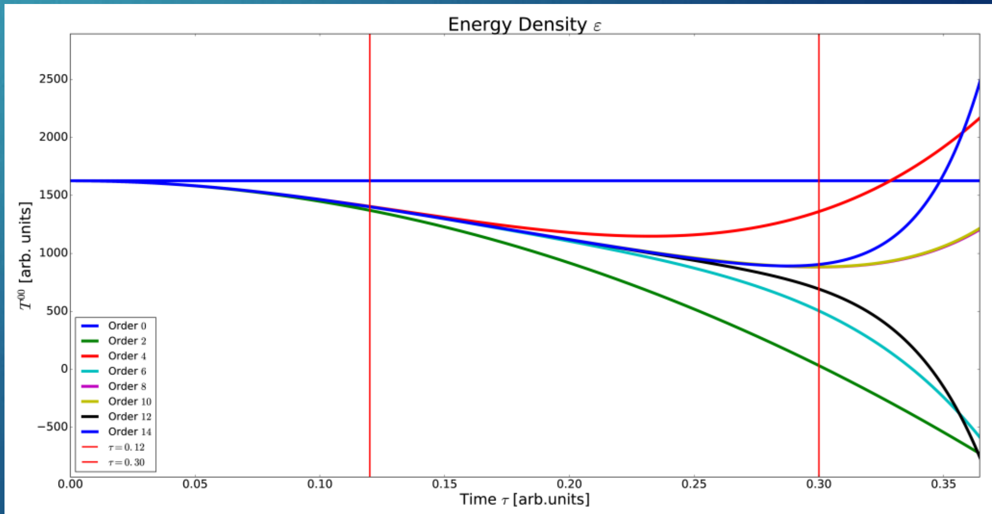
- Here: use recursive solution for the gauge field after the collision

$$A(\tau, x_\perp) = A_{(0)} + \frac{\tau^2}{8}[D^j,[D^j,A_{(0)}]] + \frac{\tau^4}{192}[D^k,[D^k,[D^j,[D^j,A_{(0)}]]]] + \frac{ig\tau^4}{48}\epsilon^{ij}[D^iA_{(0)}, D^jB_0] + \mathcal{O}(\tau^6)\,,$$

$$A^i_\perp(\tau, x_\perp) = A^i_{\perp(0)} + \frac{\tau^2}{4}\epsilon^{ij}[D^j, B_0] + \frac{\tau^4}{64}\epsilon^{ij}D^jD^kD^kB_0 - \frac{ig\tau^4}{64}[B_0, D^iB_0] + \frac{ig\tau^4}{16}[A_{(0)},[D^i,A_{(0)}]] + \mathcal{O}(\tau^6)$$

RJF, Kapusta, Li (2006)
Chen, RJF, Kapusta, Li (2015)

- Use high orders for realistic time evolution.



Energy Density and Energy Flow

- First: analytic estimates for event averages, lowest orders in time.
- Energy momentum tensor in Milne coordinates

$$T^{mn}_{\mathrm{YM}} = \begin{pmatrix} \varepsilon_0 - \frac{\tau^2}{8}(-2\triangle\epsilon_0 + \delta) & \frac{\tau}{2}\alpha^1 + \frac{\tau^3}{16}\xi^1 & \frac{\tau}{2}\alpha^2 + \frac{\tau^3}{16}\xi^2 & -\frac{\tau}{8}\nabla^i\beta^i \\ \frac{\tau}{2}\alpha^1 + \frac{\tau^3}{16}\xi^1 & \varepsilon_0 - \frac{\tau^2}{4}(-\triangle\varepsilon_0 + \delta - \omega) & \frac{\tau^2}{4}\gamma & \frac{1}{2}\beta^1 + \frac{\tau^2}{16}\zeta^1 \\ \frac{\tau}{2}\alpha^2 + \frac{\tau^3}{16}\xi^2 & \frac{\tau^2}{4}\gamma & \varepsilon_0 - \frac{\tau^2}{4}(-\triangle\varepsilon_0 + \delta + \omega) & \frac{1}{2}\beta^2 + \frac{\tau^2}{16}\zeta^2 \\ -\frac{\tau}{8}\nabla^i\beta^i & \frac{1}{2}\beta^1 + \frac{\tau^2}{16}\zeta^1 & \frac{1}{2}\beta^2 + \frac{\tau^2}{16}\zeta^2 & -\frac{\varepsilon_0}{\tau^2} + \frac{1}{8}(-2\triangle\varepsilon_0 + 3\delta) \end{pmatrix} + \mathcal{O}(\tau^4)$$

- Event averages:
  - Initial energy density

$$\varepsilon_0 = \frac{g^6 N_c\left(N_c^2 - 1\right)}{8\pi}\mu_1\mu_2 \ln^2\frac{Q^2}{m^2}$$

  - Initial rapidity-even and -odd transverse flow

$$\langle\alpha^i\rangle = -\varepsilon_0\frac{\nabla^i(\mu_1\mu_2)}{2\mu_1\mu_2},$$

$$\langle\beta^i\rangle = -\varepsilon_0\frac{\mu_2\nabla^i\mu_1 - \mu_1\nabla^i\mu_2}{\mu_1\mu_2}$$

RJF, Kapusta, Li (2006); Chen, RJF (2013); Chen, RJF, Kapusta, Li (2015)

Energy Density and Energy Flow

- First: analytic estimates for event averages, lowest orders in time.
- Energy momentum tensor in Milne coordinates:

$$T^{mn}_{\rm YM} = \begin{pmatrix} \varepsilon_0 - \frac{\tau^2}{8}(-2\triangle\epsilon_0 + \delta) & \frac{\tau}{2}\alpha^1 + \frac{\tau^3}{16}\xi^1 & \frac{\tau}{2}\alpha^2 + \frac{\tau^3}{16}\xi^2 & -\frac{\tau}{8}\nabla^i\beta^i \\ \frac{\tau}{2}\alpha^1 + \frac{\tau^3}{16}\xi^1 & \varepsilon_0 - \frac{\tau^2}{4}(-\triangle\varepsilon_0 + \delta - \omega) & \frac{\tau^2}{4}\gamma & \frac{1}{2}\beta^1 + \frac{\tau^2}{16}\zeta^1 \\ \frac{\tau}{2}\alpha^2 + \frac{\tau^3}{16}\xi^2 & \frac{\tau^2}{4}\gamma & \varepsilon_0 - \frac{\tau^2}{4}(-\triangle\varepsilon_0 + \delta + \omega) & \frac{1}{2}\beta^2 + \frac{\tau^2}{16}\zeta^2 \\ -\frac{\tau}{8}\nabla^i\beta^i & \frac{1}{2}\beta^1 + \frac{\tau^2}{16}\zeta^1 & \frac{1}{2}\beta^2 + \frac{\tau^2}{16}\zeta^2 & -\frac{\varepsilon_0}{\tau^2} + \frac{1}{8}(-2\triangle\varepsilon_0 + 3\delta) \end{pmatrix} + \mathcal{O}(\tau^4)$$

- Some components of the gluon Poynting vector carry angular momentum:
  - They all emerge from the rapidity-odd flow term β_i.

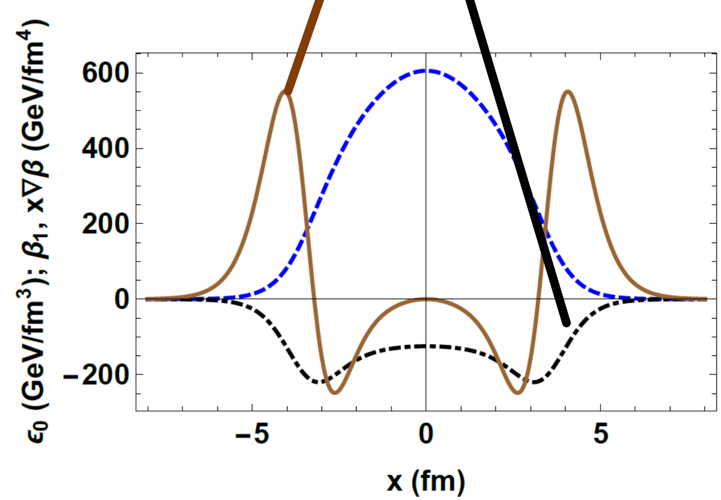
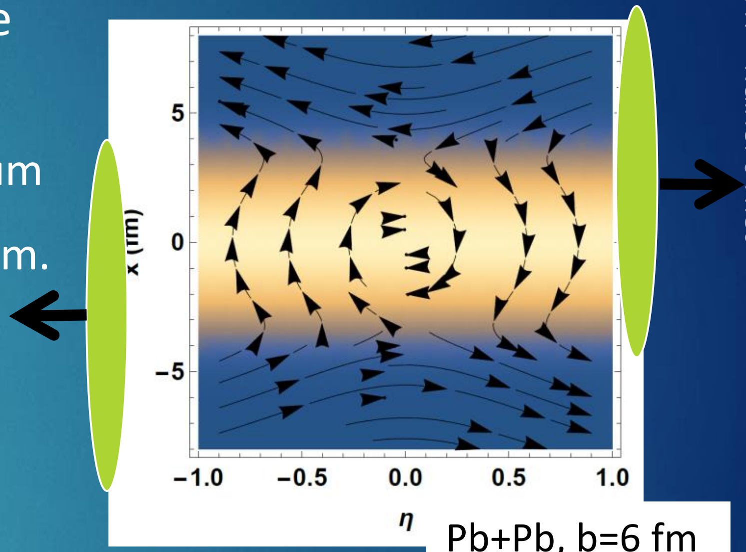


FIG. 1. The initial energy density ε_0 (dashed), directed flow in x-direction β^1 (dash-dotted), and the divergence of directed flow $x\nabla^i\beta^i$ weighted by x (solid line) as functions of coordinate x at $y = 0$ for Pb+Pb collisions at impact parameter $b = 6$ fm.

Angular Momentum

- Plot angular momentum carrying part of the Poynting vector (other terms omitted):
  - Vortex aligned with global angular momentum
  - Energy shear flow counter to global ang. Mom.



Pb+Pb, b=6 fm

- Angular momentum:

$$M^{\mu\nu\lambda} = r^\mu T^{\nu\lambda} - r^\nu T^{\mu\lambda}$$

$$L_i = \frac{1}{2}\epsilon^{ijk}\int_V d^3r M^{jk0}$$

- In a boost invariant system only $dL/d\eta$ makes sense.
- Lowest order in time:

$$\frac{\mathrm{d}L_2}{\mathrm{d}\eta} = \frac{\tau^3}{8}\left(\sinh^2\eta - 1\right)\int \mathrm{d}^2 r_\perp \beta^1 \,,$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}\eta} = \frac{\tau^3}{8}\sinh 2\eta \int \mathrm{d}^2 r_\perp \beta^1 \,.$$

Inbound longitudinal flow of L_2!

- Assuming a symmetric A+A collision.

RJF, G. Chen, S. Somanathan, arXiv:1705:10779

Angular Momentum

- Back of the envelope estimate ($\tau = 1/Q_s$)

$$\frac{dL_2}{d\eta} = \frac{1}{2} Q_s^{-3} R_A \bar{\varepsilon}_0$$

[R_A= nuclear radius, $\bar{\varepsilon}_0$=average energy density within radius R_A

- Scaling with impact parameter
  - Qualitatively similar to other models

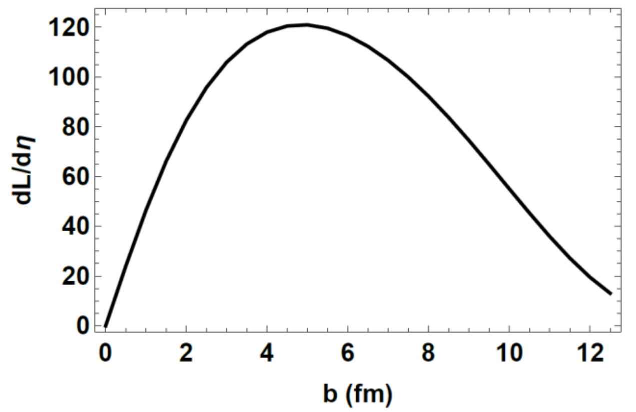


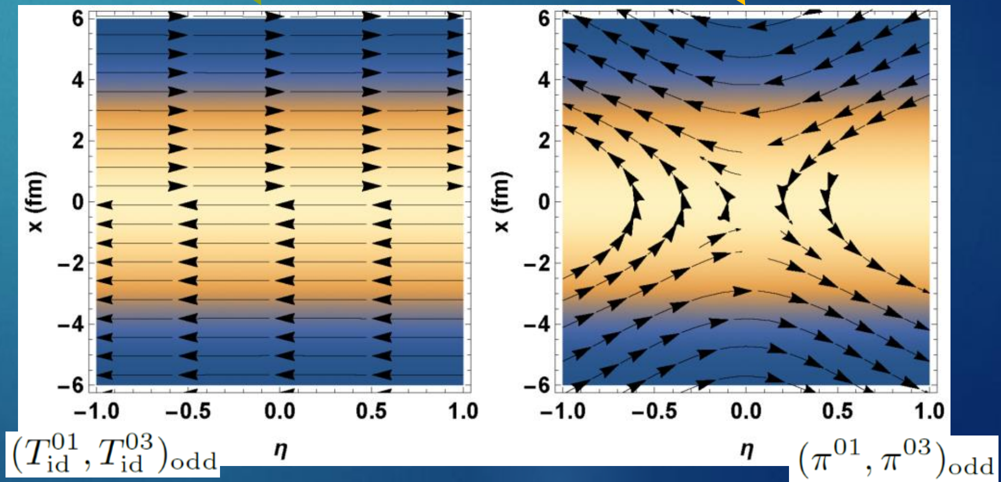
FIG. 3. The angular momentum per rapidity $dL_2/d\eta$ at $\eta = 0$ at a time $\tau = 0.25$ fm/c as a function of impact parameter b for Pb+Pb collisions as given by the leading terms in the power series in time.

Switching to Fluid Dynamics

- Rapid matching between classical YM and viscous fluid dynamics

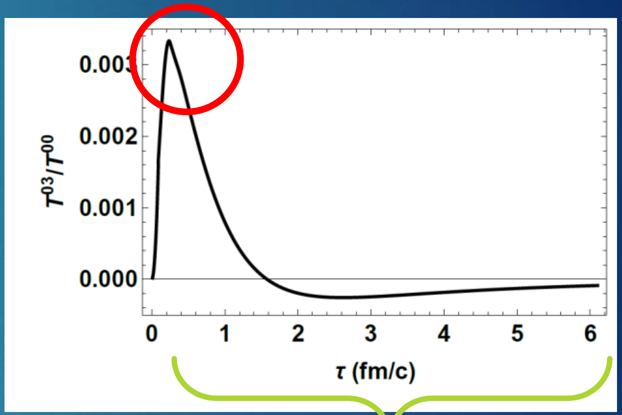
$$T_f^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

  at a time $\tau = \tau_0$, similar to IP-Glasma+MUSIC
- Enforce conservation laws, $\partial_\mu T_{total}^{\mu\nu} = 0$
- Keep viscous stress (or violate angular momentum conservation).
- Velocity field and dissipative flow contributing to angular momentum:
  - There are other flow contributions (radial flow, Bjorken flow, etc.) that do not contribute to angular momentum (not shown).
  - Longitudinal flow smaller than average radial flow.



Evolution in Fluid Dynamics

- Further time evolution in fluid dynamics:
  - Shear stress tensor decays
  - Shear flow dissipates
  - The naively expected picture holds at very late times in the collision.
  - Boost invariance suppresses rotation and Kelvin-Helmholtz instabilities.
- Matching guarantees continuity of the energy momentum tensor but not of the equations of motion.
  - Non-trivial dynamics in early times gluon fields from Gauss' Law, switching instantaneously to a dissipation law.



VIRAL fluid dynamics
$\eta/s = 1/4\pi$, $\tau_0 = 0.1$ fm/c

Full Simulations

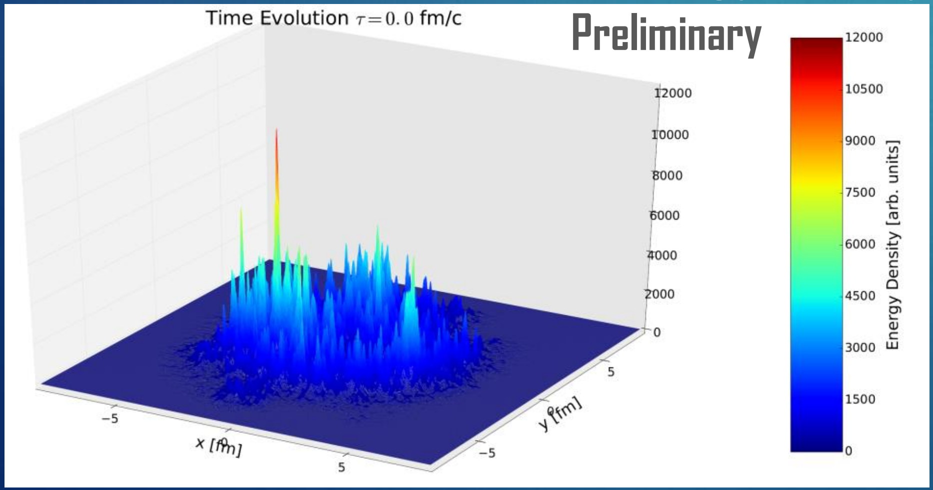
- Semi-analytic calculation
  - Numerical sampling of the nuclear charge distributions ρ_1, ρ_2.
  - Compute nuclear fields (LC gauge) A_1^i, A_2^i.
  - Use recursion relation for the fields after the collision

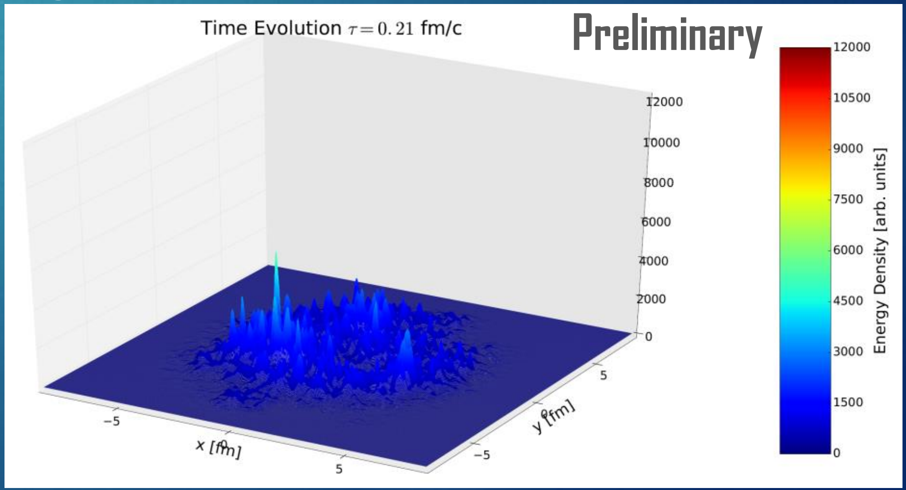
$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)} \right] \right]$$

$$A^i_{\perp(n)} = \frac{1}{n^2} \left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ji}_{(l)} \right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D^i_{(l)}, A_{(m)} \right] \right] \right)$$

Previous numerical solutions:
Krasnitz, Nara, Venugopalan (2003);
Lappi (2003); …
Fukushima, Gelis (2012); Schenke, Tribedy, Venugopalan (2012)

- Time evolution of the energy density, single event



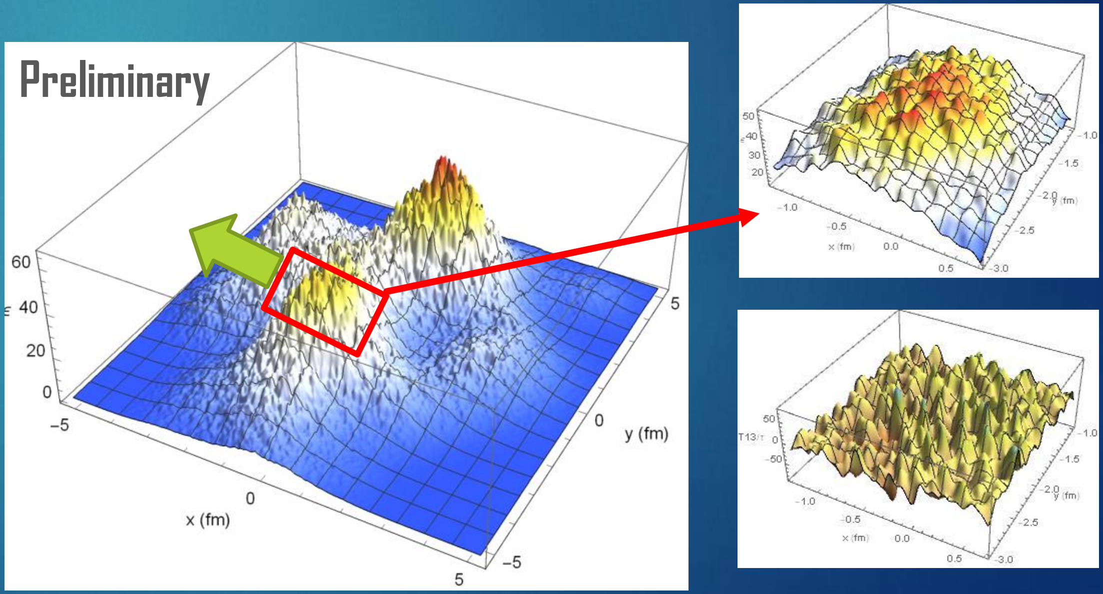


Angular Momentum with Fluctuations

- Dominated by fluctuations, dig out contributions to angular momentum
- Example: initial energy density, 200 events, same average charge distribution (sampled from Au+Au @ 6 fm)
- Directed flow can be identified, magnitude and sign consistent with the event-averaged calculation.

- Relation between integrated (per rapidity) longitudinal and rapidity-odd directed flow holds.

$$0 = \int \mathrm{d}^2 r_\perp \left(\beta^1 + x \nabla^i \beta^i \right)$$



Average $T^{13}/T^{00} \sim -2\%\ c$ at $\tau \sim 0.3$ fm

→ rapidity-odd directed flow of the local peak.

Conclusion and Outlook

- Boost invariant setup allows for angular momentum carried by gluon fields.
- Classical Yang-Mills phase: angular momentum built up through rapidity-odd (directed) flow of energy, driven by Gauss' Law.
- Matching to viscous fluid dynamics: shear flow at midrapdity aligned with total angular momentum, dissipates in boost-invariant viscous hydro.
  - Possible signals only from early times?
  - Sensitivity to switch off time of classical glue dynamics.
- Consistent with small signals at high energies. Going to lower energies: breaking of boost-invariance needed as a next step.
- First results from event-by-event simulations: large fluctuations in the event plane, event-averages match up.

Backup

Origin of Flow in YM

- Initial longitudinal fields E_0, B_0 → transverse fields through QCD versions of Ampere's, Faraday's and Gauss' Law.
  - Here *abelian* version for simplicity.
- Gauss' Law at fixed time *t*
  - Difference in long. flux ⟶ transverse flux
  - *rapidity-odd* and *radial*
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux ⟶ transverse field
  - *rapidity-even* and *curling* field
- Full classical QCD at $O(\tau^1)$:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

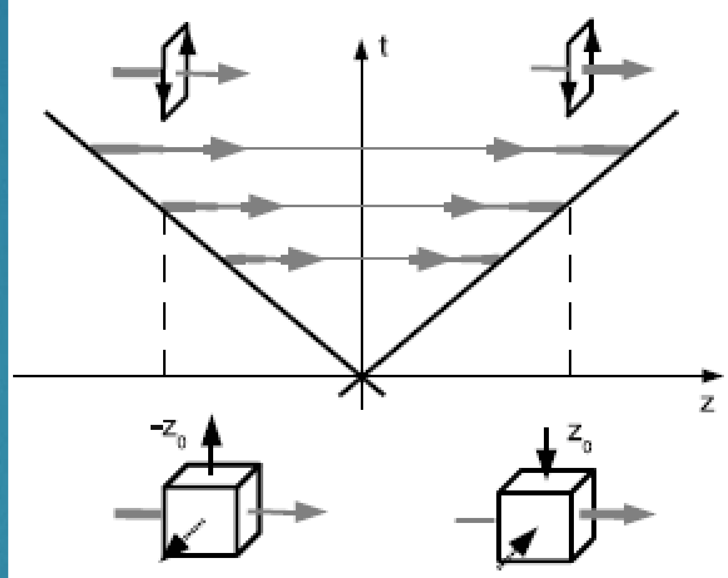


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Chen, RJF (2013)

Transverse Flow

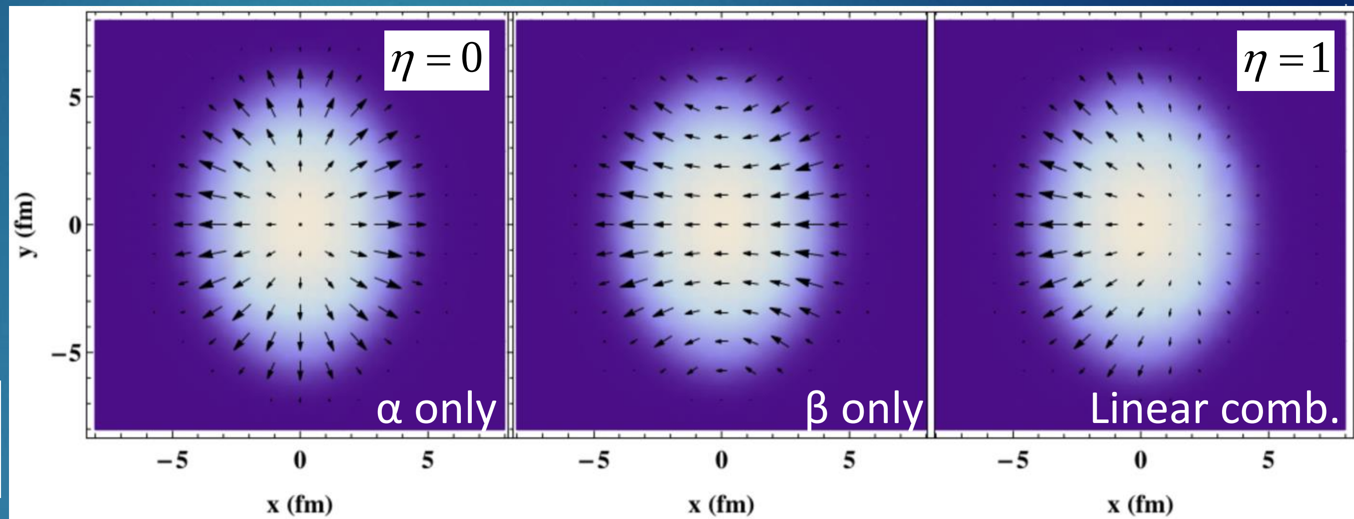
- Transverse Poynting vector: Transverse plane

Chen, RJF (2013)

Pb+Pb, b=6 fm

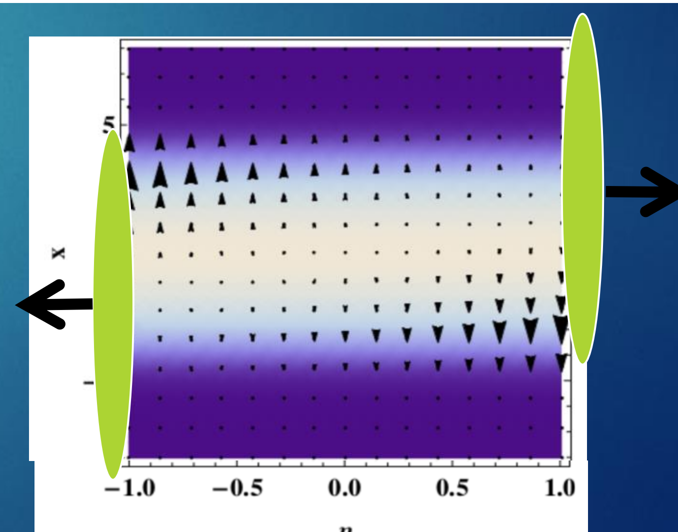
$$\alpha^i = -\varepsilon_0 \frac{\nabla^i (\mu_1 \mu_2)}{\mu_1 \mu_2}$$

$$\beta^i = -\varepsilon_0 \frac{\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2}{\mu_1 \mu_2}$$



- Transverse Poynting vector: Event Plane

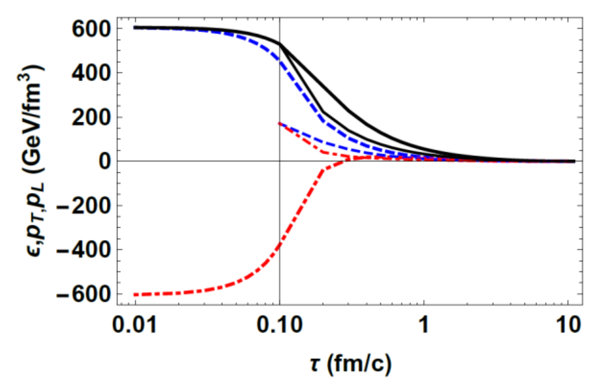
  (long. component suppressed)



- Radial and elliptic flow
- Rapidity-odd directed flow (from Gauss Law)
- Angular momentum

Initializing Fluid Dynamics

- Incomplete matching destroys energy and momentum conservation (thin lines = shear stress discarded at matching)



- Transverse flow dominates for matching at early times.

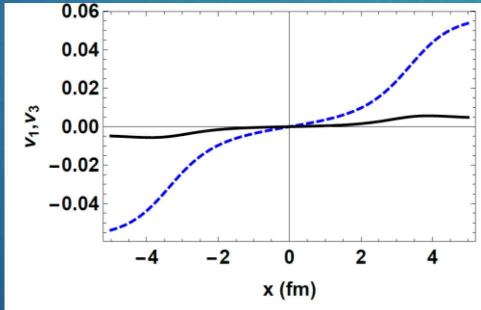


FIG. 5. Fluid flow vector components v_1 (dashed blue line) and v_3 (solid black line) as functions of x at $\eta = 0$, $y = 0$ obtained from the matching at $\tau = 0.1$ fm/c as described in the text for Pb+Pb collisions at impact parameter $b = 6$ fm. Because of the quadratic time-dependence of the longitudinal shear flow the build-up of v_3 is lagging behind v_1.

Resummation of the Time Evolution

- Generic arguments: convergence radius of the recursive solution ~$1/Q_s$.
- "Weak field" approximation to Yang Mills

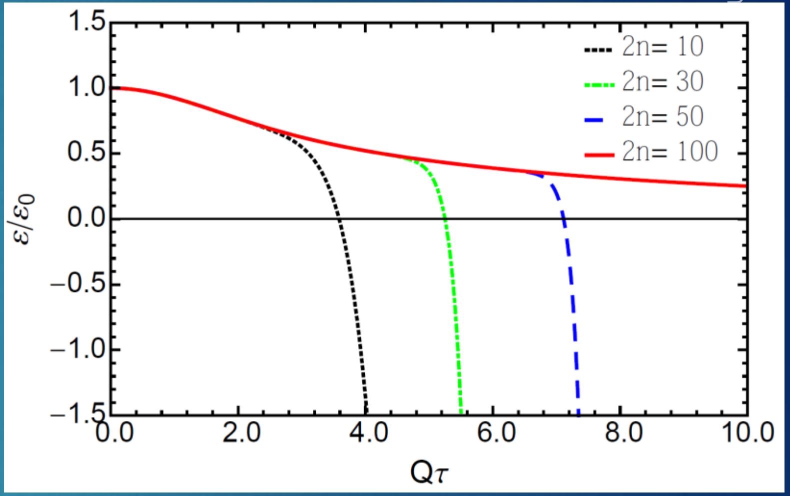
$$A^{\mathrm{LO}}(\tau, \mathbf{k}_\perp) = \frac{2A_{(0)}(\mathbf{k}_\perp)}{k_\perp \tau} J_1(k_\perp \tau)$$

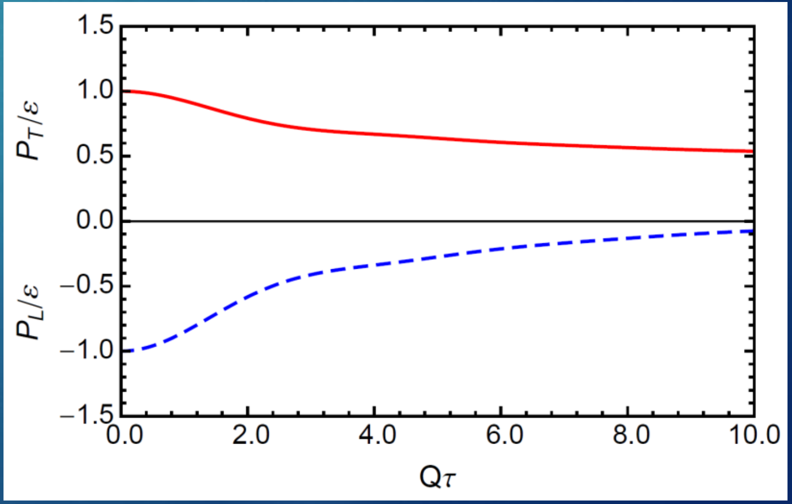
$$A_\perp^{i\,\mathrm{LO}}(\tau, \mathbf{k}_\perp) = A^i_{\perp(0)}(\mathbf{k}_\perp) J_0(k_\perp \tau)$$

  - Can be rederived from the recursive solution.
- Resumming $(Q\tau)^k$ terms: semi-closed form

$$\mathcal{A} = \varepsilon_0 + \frac{2\varepsilon_0}{\ln^2(Q^2/m^2)} \mathsf{G}_A(Q\tau) - \frac{\varepsilon_0}{\ln(Q^2/m^2)} (Q\tau)^2 \left[{}_3F_4\left(1, 1, \frac{3}{2}; 2, 2, 2, 2; -(Q\tau)^2\right) \right]$$

Li, Kapusta, 1602:09060





VIRAL 3+1 D Fluid Code

- KT for fluxes, 5th order WENO for spatial derivatives, 3rd order TVD Runge Kutta for time integration.
- Bulk and shear stress, vorticity
- Gubser and Sod-type tests: