Collectivity from Interference

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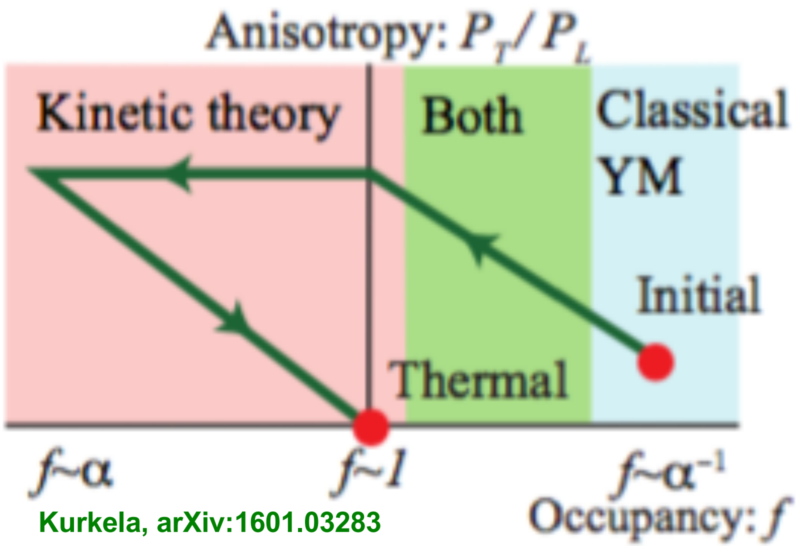
based on: B. Blok, C. Jäkel, M. Strikman and UAW, arXiv:1708.08241

IS2017, Krakow, 21 Sept 2017

Fluid dynamics ⇔ final state interactions ⇔ Jet quenching

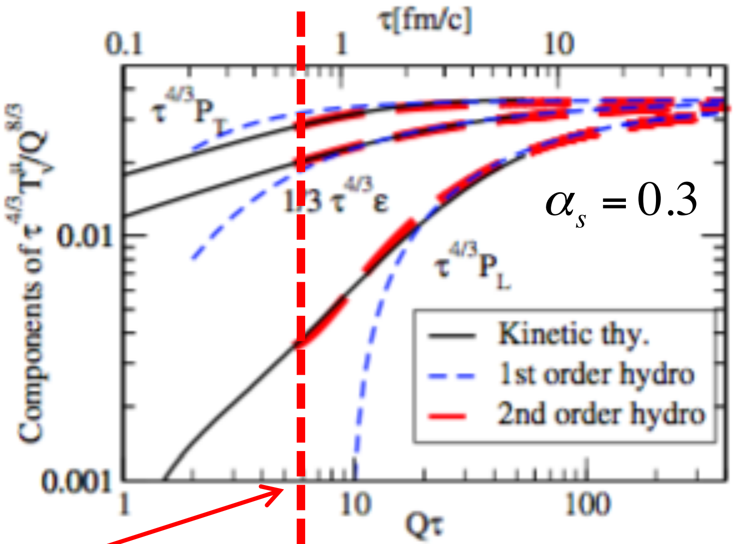
- Bottom-up thermalization formalizes relation between fluid dynamics and jet quenching

R.Baier, A.H. Mueller, D. Schiff, D.T. Son, 2001



Kurkela, arXiv:1601.03283

Kurkela&Zhu, PRL115 (2015)182301



Partonic distributions f(p) governed by **Boltzmann equation.**
We know:

- f(p) hydrodynamizes on sub-fermi time scale
- "Hydro" & "jet quenching" arise from the same collision kernels

$$\partial_t f(p,t) = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

Berges, Eppelbaum, Kurkela, Moore, Schlichting, Venugopalan, …

2->2 collision kernel

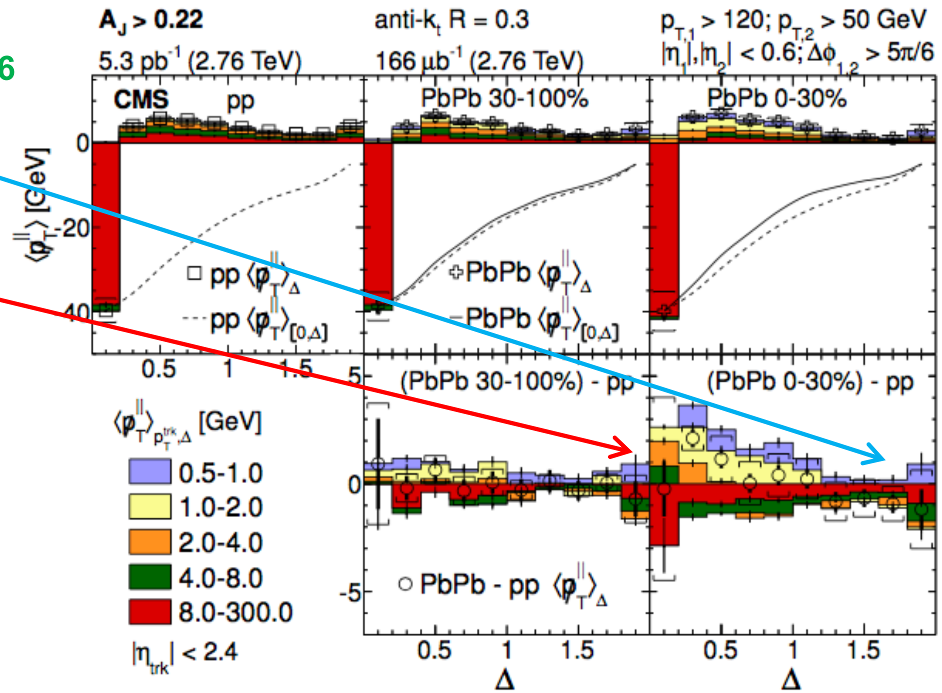
LPM splitting kernel

Jet quenching and fluid dynamics = two manifestations of the same physics

Jet thermalization (a.k.a. quenching) in AA

CMS Coll., JHEP 01 (2016) 006

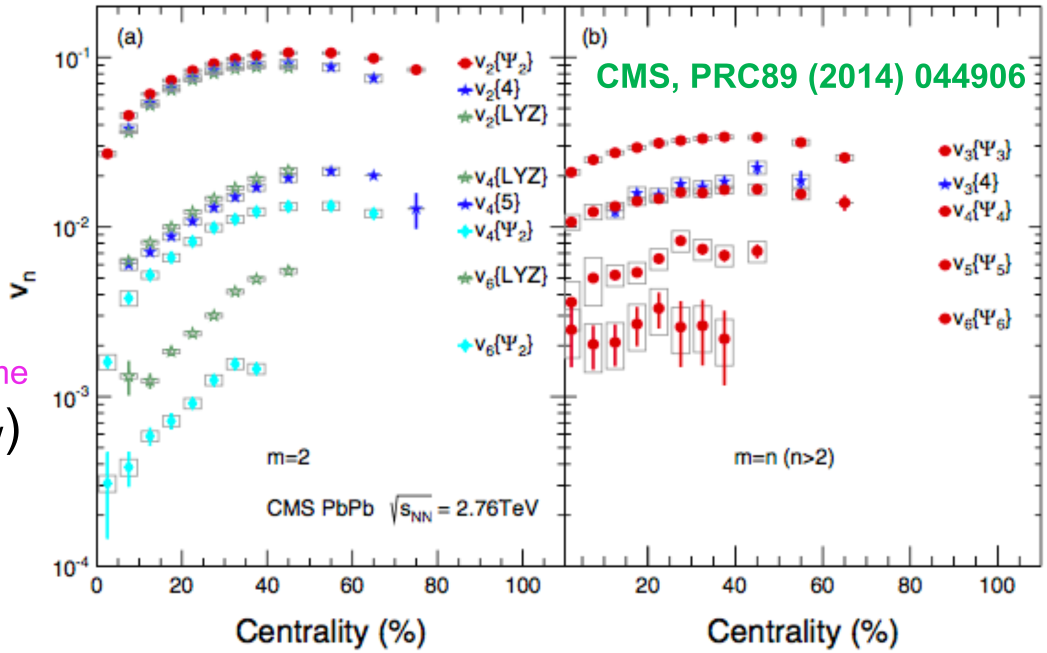
Softening and **isotropization** of jet structure



⇔ hallmarks of local equilibration

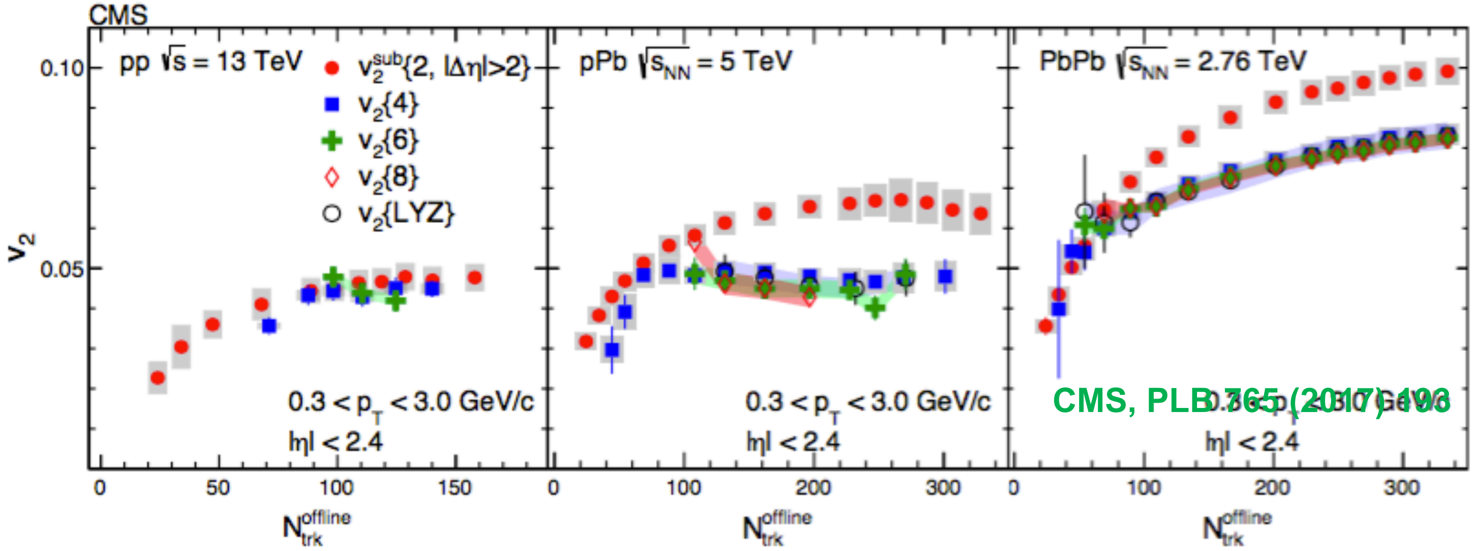
⇔ Dynamically <u>consistent with fluid dynamic interpretation</u> of momentum asymmetries v_n

(i.e. it is conceivable that the same physics / same collision kernels underlie jet quenching and flow)



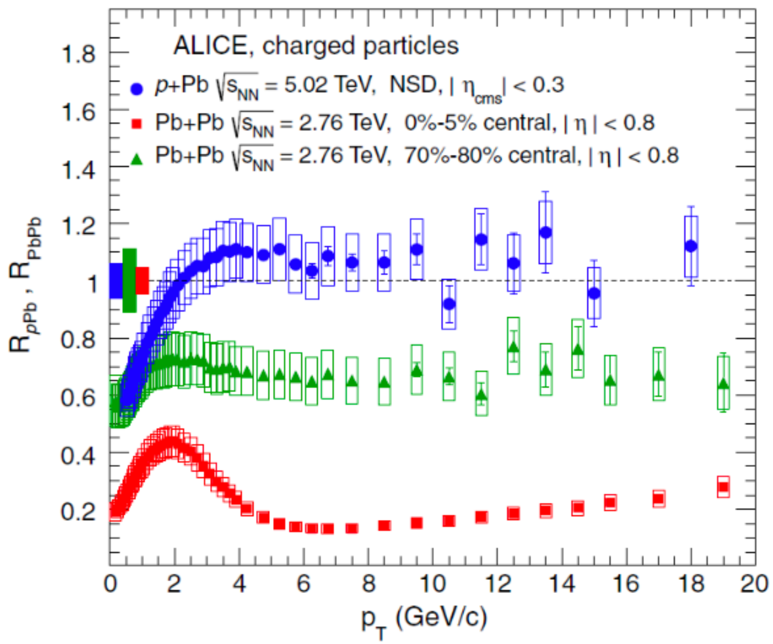
Fluid dynamic interpretation of v_n in pp and pA

is problematic (from this perspective)



- ➢ since v_n not accompanied by jet quenching. This indicates a lack of final interactions (that are needed to build up flow)

ALICE Collaboration, PRL 110, 082302 (2013)



Solutions to the "Flow w/o quenching" puzzle in pp / pA

- 1. Quantitative Explanation: maintain that v_n result from final state interactions

  - **small jet quenching effects must be seen in pp/pA**
    for techniques to detect them, see e.g. **Mangano, Nachman arXiv:1708.08369**
  - Theory improvements needed to relate jet quenching and v_n signals.

- 2. High-density Scenario: azimuthal correlations from a **saturated initial state ("CGC")**

  Altinoluk, Armesto, Beuf, Dumitru, Gotsman, Jalilian-Marian, Iancu, Kovner, Lappi, Levin, Lublinsky, McLerran, Skokov, Schlichting, Venugopalan, ….

  - UE (underlying event) physics in pp multi-purpose MC event generators based on dilute system of up to O(10) MPIs (multi parton interactions)
  - If saturated initial state needed to describe pp UE, then dramatic implications: **Torbjorn go home.**
  - **One needs to understand whether initial density effects are necessary for azimuthal correlations.**

… solutions to the "Flow w/o quenching" puzzle, cont'd...

- 3. High-density Scenario: **strongly coupled fluid paradigm** (à la AdS/CFT) for pp/pA

  ⇒
  - **small jet quenching effects must be seen in pp/pA**
  - **UE model radically different from that in MC generators**

- 4. Low-density Scenario: fluid dynamics negligible,
  azimuthal correlations from escape mechanism

  Liang He et al., Phys. Lett. B753 (2016) 506-510; AMTP

  - mechanism to be understood quantitatively outside a MC code

  ⇒
  - **small jet quenching effects must be seen in pp/pA**
  - mild extension of UE model of multi-purpose MC generators

- 5. Low-density Scenario: **Collectivity from interference**

  B.Blok, C. Jäkel, M. Strikman, UAW, arXiv:1708.08241

  - No initial density and no initial asymmetry, no final state interactions
  - Contribution to v_n from QM interference & color correlations?

  ⇒
  - **does not imply jet quenching in pp/pA**
  - natural extension of UE model of multi-purpose MC generators

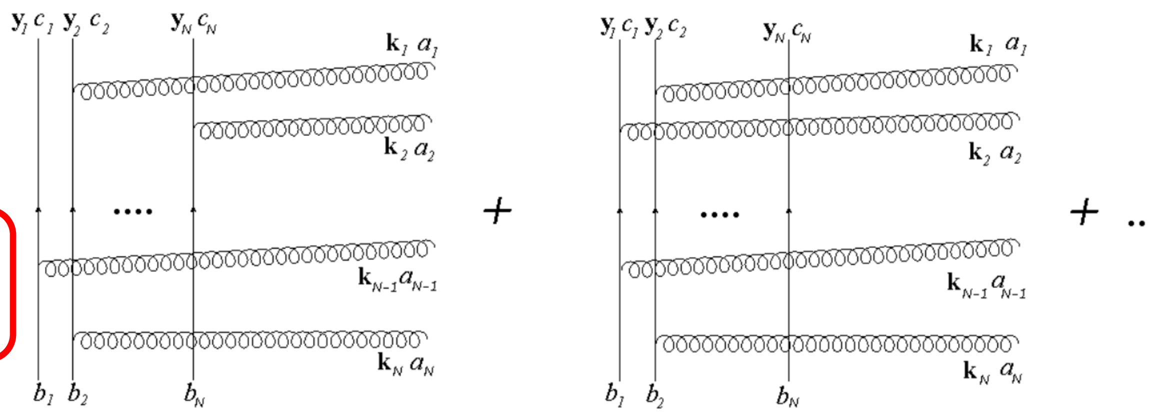
This Talk

QCD-based picture of multi-parton emission in pp

- We want to calculate m-parton emission from the square of production amplitudes off N sources

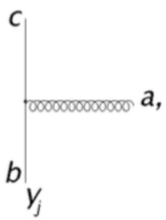
$$\boxed{\frac{dN}{d\mathbf{k}_1 \ldots d\mathbf{k}_m} = \int \left(\prod_{i=1}^{N} d\mathbf{y}_i \right) \rho(\{y_i\})\, \hat{\sigma}\left(\{\mathbf{k}_j\}, \{y_i\}\right)}$$

without final state interactions



How does this spectrum depend on: 1. number of sources (MPIs) N
2. number of emitted gluons m ?

- Diagrammatic rules



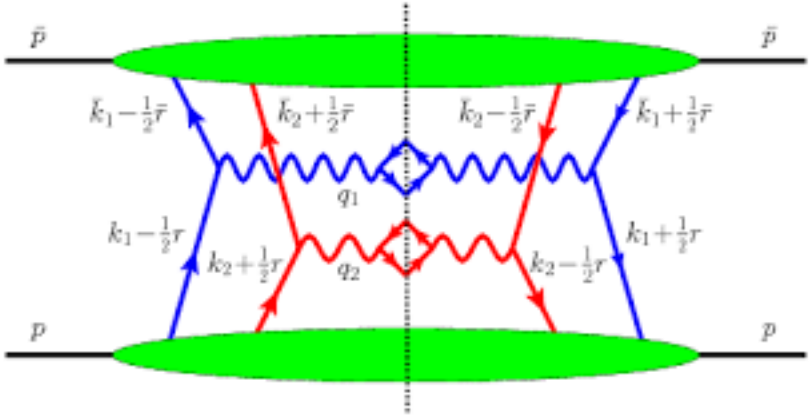
$$= \boxed{T^a_{b_i c_i}} \int d\mathbf{x}\, \vec{f}(\mathbf{x}-\mathbf{y}) \boxed{e^{i\,\mathbf{k}.\mathbf{x}}} \equiv T^a_{b_i c_i} \boxed{\vec{f}(\mathbf{k})} \exp\left[i\,\mathbf{y}.\mathbf{k}\right]$$

- Keep track of color and phases exactly.
  (basis for understanding QCD interference effects)

- Simplifications (to make calculation of m-particle emission possible)
  - Don't specify **kinematics**.
  - Flat rapidity dependence of f(k).
  - Gluons do not cross.

Geometrical information from MPIs in pp

- Double hard interactions set **scale σ_{eff}**



$$\sigma_{2\ MPIs} = \frac{\sigma_1\ \sigma_2}{\sigma_{\text{eff}}}$$

$$\sigma_{N\ MPIs} = \frac{\sigma_1\ \dots\ \sigma_N}{K_N}$$

- Generalized parton distribution functions (GPDs) carry geometrical information

$$\frac{1}{K_N} = \int \left(\prod_{i=1}^{N} \frac{d\mathbf{\Delta}_i}{(2\pi)^2} \right) \frac{G_N(\{x_i\}, \{Q_i^2\}, \{\mathbf{\Delta}_i\})\, G_N(\{x_i'\}, \{Q_i^2\}, \{\mathbf{\Delta}_i\})}{\prod_{i=1}^{N}(f(x_i, Q_i^2)\, f(x_i', Q_i^2))} \delta^{(2)} \left(\sum_{i=1}^{N} \mathbf{\Delta}_i \right)$$

- Probabilistic interpretation of GPDs in a mean-field approximation

Blok, Dokshitzer, Frankfurt, Strikman, PRD 83, 071501 (2011)

$$G_N(\{x_i\}, \{Q_i^2\}, \{\mathbf{\Delta}_i\}) = \prod_{i=1}^{N} G_1(x_i, Q_i, \mathbf{\Delta}_i) = \prod_{i=1}^{N} f(x_i, Q_i) F_{2g}(\mathbf{\Delta}_i)$$

- Only purpose for the following: $F_{2g}^2(\Delta) = \exp(-B\Delta_i^2)$

$$\boxed{B = 2\,\text{GeV}^{-2} \qquad \longleftrightarrow \qquad \sigma_{\text{eff}} \approx 20\,\text{mb}}$$

LHC data set scale of parameter B

- Density distribution of colliding partons in pp

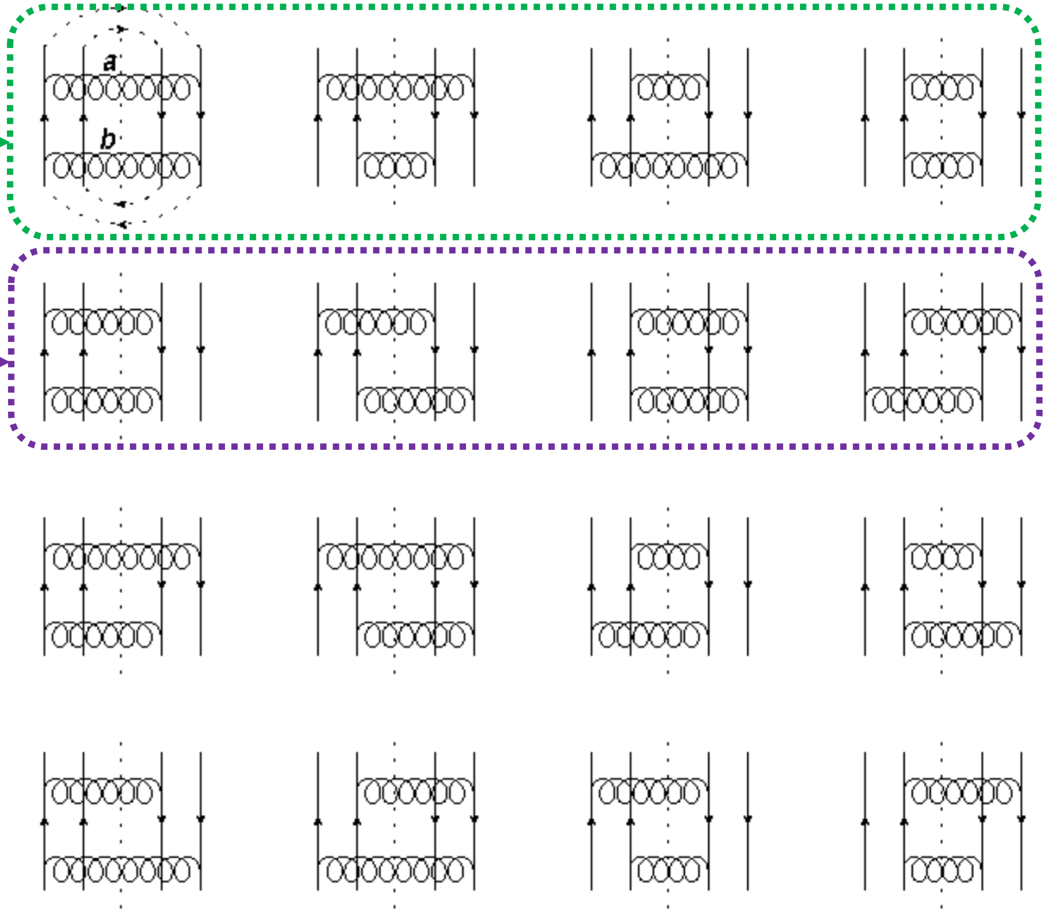
$$\rho(\{\mathbf{y}_i\}, \mathbf{b}) = \prod_j \frac{1}{(4\pi B)^2} \exp\left[-\frac{\mathbf{y}_j^2}{4B}\right] \exp\left[-\frac{(\mathbf{y}_j - \mathbf{b})^2}{4B}\right]$$

Simplest case: emitting m=2 gluons from N=2 sources

- Color can be read easily from diagrams

$$\boxed{\mathrm{Tr}\left[T^aT^bT^bT^a\right]\,\mathrm{Tr}\left[\mathbb{1}\right] = N_c^2\,\left(N_c^2-1\right)^2}$$

$$\boxed{\mathrm{Tr}\left[T^aT^b\right]\,\mathrm{Tr}\left[T^bT^a\right] = N_c^2\,\left(N_c^2-1\right)}$$



$$\text{phases} \propto e^{i\mathbf{k}.\mathbf{y}_i}\ \left(\propto e^{-i\mathbf{k}.\mathbf{y}_i}\right)$$

in amplitude (complex conj. amplitude)

$$\boxed{\frac{dN}{d\mathbf{k}_1 d\mathbf{k}_2} \propto \left|\vec{f}(\mathbf{k}_1)\right|^2 \left|\vec{f}(\mathbf{k}_2)\right|^2 \left[1 + \frac{\left(e^{-B(\mathbf{k}_1+\mathbf{k}_2)^2} + e^{-B(\mathbf{k}_1-\mathbf{k}_2)^2}\right)}{(N_c^2-1)}\right]}$$

- For $B = 1/Q_s^2$, this QCD dipole radiation agrees with CGC calculations.

Altinoluk et al, PLB 751 (2015) 448; PLB 752 (2016) 113
Lappi, Schenke, Schlichting, Venugopalan JHEP 1601 (2016) 061

- Saturated initial state not assumed here.

Dipole radiation for m gluons from N sources

- General structure of result to leading order in N_c and N

$$\hat{\sigma} \propto (N_c^2-1)^N N_c^m \left(\prod_{i=1}^{m} \left| \vec{f}(\mathbf{k}_i) \right|^2 \right)$$
$$\times \left\{ N^m + F^{(2)}_{\mathrm{corr}}(N,m) \frac{N^{m-2}}{(N_c^2-1)} \sum_{(ab)} \sum_{(ij)} 4\cos(\mathbf{k}_a.\Delta\mathbf{y}_{ij})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{ij}) \right.$$
$$\left. + O\left(\frac{1}{N} \frac{1}{(N_c^2-1)} \right) + O\left(\frac{1}{(N_c^2-1)^2} \right) \right\} .$$

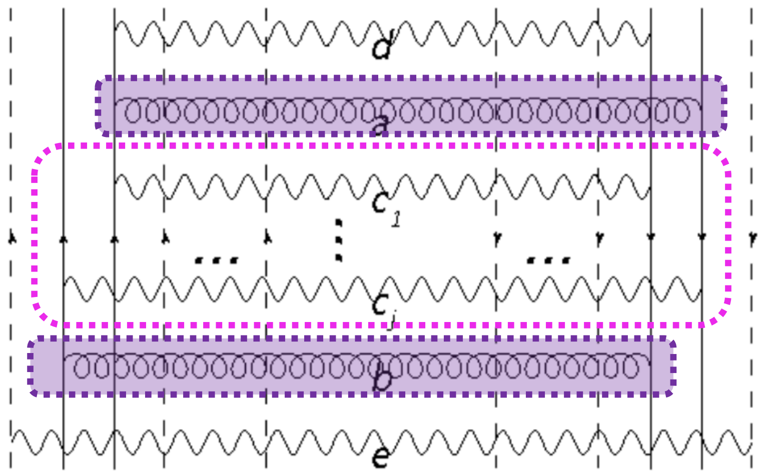
sum over $\frac{m(m-1)}{2}$ emitted gluon pairs

sum over $\frac{N(N-1)}{2}$ emitting pairs of sources

- Origin of **color correction factor**: unlike QED, **diagonal gluon emissions** do not act incoherently

Diagonal gluons sandwiched btw **off-diagonal ones** contribute with color trace reduced by factors 2:

$$\boxed{T^{c_j} T^a T^{c_j} = \frac{1}{2} N_c T^a}$$ instead of $$T^d T^d = N_c \mathbb{1} .$$



- This effect can be resummed exactly

$$F^{(2)}_{\mathrm{corr}}(N,m) = \frac{1}{N_{\mathrm{incoh}}} \sum_{j=0}^{m-2} N^{m-2-j}(m-1-j) \left(\sum_{l=0}^{j} \binom{j}{l} 2^l (N-2)^{j-l} \frac{1}{2^l} \right)$$
$$= \frac{2}{m(m-1)} N^{1-m} \left(N(N-1)^m + mN^m - N^{1+m} \right) .$$

From the spectra to v_n's and higher order cumulants

- Once spectrum is known, azimuithal phase space averages can be formed

$$T(k_1,k_2) = \binom{m}{2} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2 \, \exp\left[in(\phi_1-\phi_2)\right] \left(\int \prod_{b=3}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

- Suitably normalized, these define v_n's (2^{nd} order cumulants)

$$\overline{T}(k_1,k_2) = \binom{m}{2} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2 \left(\int \prod_{b=3}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

$$\boxed{v_n^2\{2\}(k_1,k_2) \equiv \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle(k_1,k_2) \equiv \frac{T(k_1,k_2)}{\overline{T}(k_1,k_2)}}$$

- Higher order cumulants obtained in close similarity

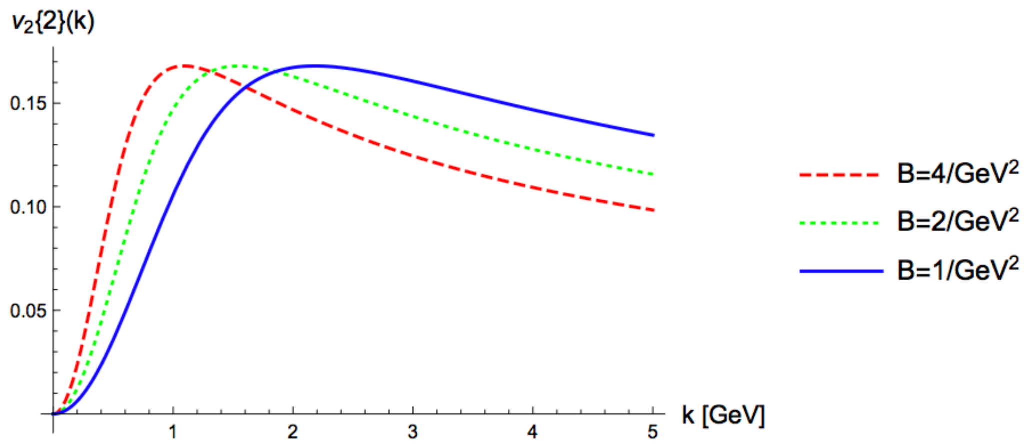
$$S(k_1,k_2,k_3,k_4) = \binom{m}{4} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2\, d\phi_3\, d\phi_4\, e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \left(\int \prod_{b=5}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

$$\langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c = \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle - \langle\langle e^{in(\phi_1-\phi_3)}\rangle\rangle\langle\langle e^{in(\phi_2-\phi_4)}\rangle\rangle - \langle\langle e^{in(\phi_1-\phi_4)}\rangle\rangle\langle\langle e^{in(\phi_2-\phi_3)}\rangle\rangle$$

2nd order cumulant: v_2

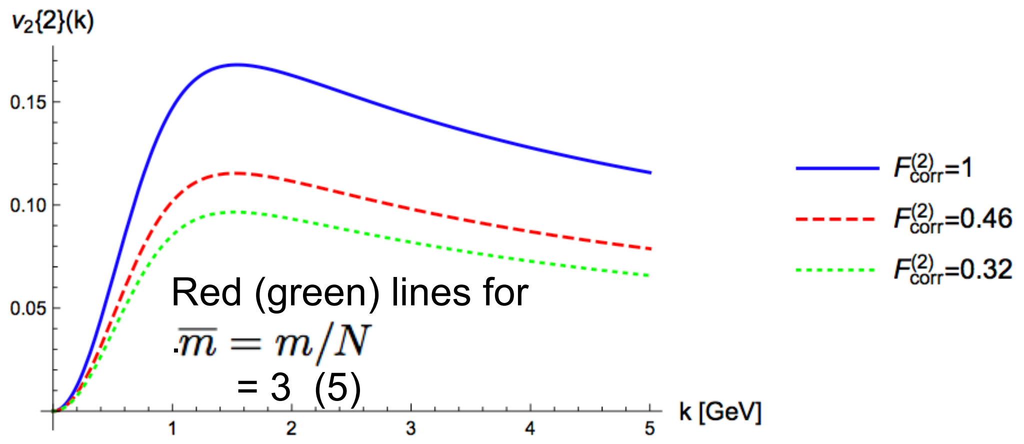
$$v_2^2\{2\}(k_1,k_2) \equiv \langle\langle e^{i2(\phi_1-\phi_2)}\rangle\rangle(k_1,k_2)$$
$$\equiv \frac{F_{\rm corr}^{(2)}(N,m)\int_\rho \frac{1}{N^2}\sum_{(ij)} 2^2 J_2(k_1\Delta y_{ij})\; J_2(k_2\Delta y_{ij})}{(N_c^2-1)+F_{\rm corr}^{(2)}(N,m)\int_\rho \frac{1}{N^2}\sum_{(ij)} 2^2 J_0(k_1\Delta y_{ij})\; J_0(k_2\Delta y_{ij})} + O\left(\frac{1}{(N_c^2-1)^2}\right)$$

- Partonic v_2, may be modified by hadronization
- Signal persists to multi-GeV region



- For fixed average multiplicity per source, $\overline{m} = m/N$

$$\lim_{m\to\infty} F_{\rm corr}^{(2)}(m/\overline{m},m) = \frac{2\overline{m}+2e^{-\overline{m}}-2}{\overline{m}^2}\,.$$



Higher order cumulants require higher-order spectrum

- Complete result in large N limit: (contains sums over source doublets, triplets, quadruplets, pairs of doublets, …)

B. Blok, C.Jäkel, M. Strikman, UAW, arXiv:1708.08241

$$
\begin{aligned}
\hat{\sigma} \propto\; & N_c^m \left(N_c^2-1\right)^N \left(\prod_{i=1}^{m} \left|\vec{f}(\mathbf{k}_i)\right|^2\right) N^{m-4} \\
\times \Big\{ & N^4 + F_{\mathrm{corr}}^{(2)}(N,m) \frac{N^2}{(N_c^2-1)} \sum_{(ab)} \sum_{(lm)} 2^2 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{lm}\right) \\
& + F_{\mathrm{corr}}^{(3i)}(N,m) \frac{N}{(N_c^2-1)^2} \sum_{(abc)} \sum_{(lm)(mn)(nl)} 2^3 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \\
& \qquad \times \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{mn}\right) \cos\left(\mathbf{k}_c.\Delta\mathbf{y}_{nl}\right) \\
& + F_{\mathrm{corr}}^{(4i)}(N,m) \frac{1}{(N_c^2-1)^2} \sum_{(lm),(no)} \sum_{(ab)(cd)} 2^4 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{lm}\right) \\
& \qquad \times \cos\left(\mathbf{k}_c.\Delta\mathbf{y}_{no}\right) \cos\left(\mathbf{k}_d.\Delta\mathbf{y}_{no}\right) \\
& + F_{\mathrm{corr}}^{(4ii)}(N,m) \frac{1}{(N_c^2-1)^3} \sum_{(lm)(mn)(no)(ol)} \sum_{(abcd)} 2^4 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{mn}\right) \\
& \qquad \times \cos\left(\mathbf{k}_c.\Delta\mathbf{y}_{no}\right) \cos\left(\mathbf{k}_d.\Delta\mathbf{y}_{ol}\right) \\
& + F_{\mathrm{corr}}^{(5)}(N,m) \frac{N^{-1}}{(N_c^2-1)^3} \sum_{[(lm)(mn)(nl)](op)} \sum_{(abc)(de)} 2^2 \cos\left(\mathbf{k}_d.\Delta\mathbf{y}_{op}\right) \cos\left(\mathbf{k}_e.\Delta\mathbf{y}_{op}\right) \\
& \qquad \times 2^3 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{mn}\right) \cos\left(\mathbf{k}_c.\Delta\mathbf{y}_{nl}\right) \\
& + F_{\mathrm{corr}}^{(6)}(N,m) \frac{N^{-2}}{(N_c^2-1)^3} \sum_{(lm)(no)(pq)} \sum_{(ab)(cd)(ef)} 2^2 \cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{lm}\right) \cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{lm}\right) \\
& \qquad \times 2^2 \cos\left(\mathbf{k}_c.\Delta\mathbf{y}_{no}\right) \cos\left(\mathbf{k}_d.\Delta\mathbf{y}_{no}\right) 2^2 \cos\left(\mathbf{k}_e.\Delta\mathbf{y}_{pq}\right) \cos\left(\mathbf{k}_f.\Delta\mathbf{y}_{pq}\right) \\
& + O\left(\frac{1}{N}\right) + O\left(\frac{1}{(N_c^2-1)^4}\right) \Big\},
\end{aligned}
$$

4^{th} order cumulant up to $1/(N_c^2-1)^2$

- To simplify higher-order cumulant calculations, assume now $B\,k_i^2 \ll 1$.

$$
\begin{aligned}
\langle\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c = &\,(Bk_1^2)\,(Bk_2^2)\,(Bk_3^2)\,(Bk_4^2) \\
&\Big\{\frac{1}{(N_c^2-1)^2}\left(2\,F_{\rm corr}^{(4i)} - 2F_{\rm corr}^{(2)}F_{\rm corr}^{(2)} + O\left(N^{-1}\right)\right) \\
&\quad + \frac{1}{(N_c^2-1)^3}\Big(2\,F_{\rm corr}^{(6)}(m-4)(m-5) - 4\,F_{\rm corr}^{(2)}\,F_{\rm corr}^{(4i)}(m-2)(m-3) \\
&\qquad +4\,F_{\rm corr}^{(5)}(m-4) - 4\,F_{\rm corr}^{(2)}F_{\rm corr}^{(3)}(m-2) \\
&\qquad +2\,F_{\rm corr}^{(4ii)} + 8\left(F_{\rm corr}^{(2)}\right)^3 - 4\,F_{\rm corr}^{(4i)}F_{\rm corr}^{(2)}\Big) + O\left(N^{-1}\right)\Big\} \\
&+O\left(1/(N_c^2-1)^4\right) .
\end{aligned}
$$

- Limiting cases:

$$
\lim_{N\to\infty} F_{\rm corr}^{(*)}(N,m)\Big|_{m=\text{const.}} = 1
$$

$$
\lim_{N\to\infty} F_{\rm corr}^{(2)}(N,N\overline{m})^2 = \lim_{N\to\infty} F_{\rm corr}^{(4i)}(N,N\overline{m}) = \left(\frac{2\overline{m}+2e^{-\overline{m}}-2}{\overline{m}^2}\right)^2
$$

$$
\Rightarrow \quad \boxed{v_2^4\{4\}(k_1,k_2,k_3,k_4) = 0 + O\left(1/N\right) + O\left(1/(N_c^2-1)^3\right)}
$$

4^{th} order cumulant up to $1/(N_c^2-1)^3$ and at large N

- **The limit $N \to \infty$ for constant multiplicity m to order $O(1/(N_c^2-1)^3)$**

$$\boxed{\begin{aligned}\langle\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c = (Bk_1^2)\,(Bk_2^2)\,(Bk_3^2)\,(Bk_4^2)\,\frac{2}{(N_c^2-1)^3}\,(7+m-m^2)\\ +O\left(1/(N_c^2-1)^4\right)\,.\end{aligned}}$$

- **QCD interference results in a negative 4-th order cumulant**
  that can be interpreted as a collective phenomenon (for large m)

$$v_n\{4\} \equiv \sqrt[4]{-\langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c}\,.$$

$$\boxed{v_2\{4\}(k) \simeq \frac{1}{(N_c^2-1)^{3/4}}2^{1/4}\sqrt{m}\,B\,k^2}$$

- The density of this system is $\rho = \dfrac{N}{B}$.

  Result does not depend on parton density in the initial state.
  CGC result for $v_2\{4\}$ has same N_c but different m- (N?)-dependencies, see e.g. Skokov 1412.5191,
  Skokov & McLerran, 1510.08072.pdf

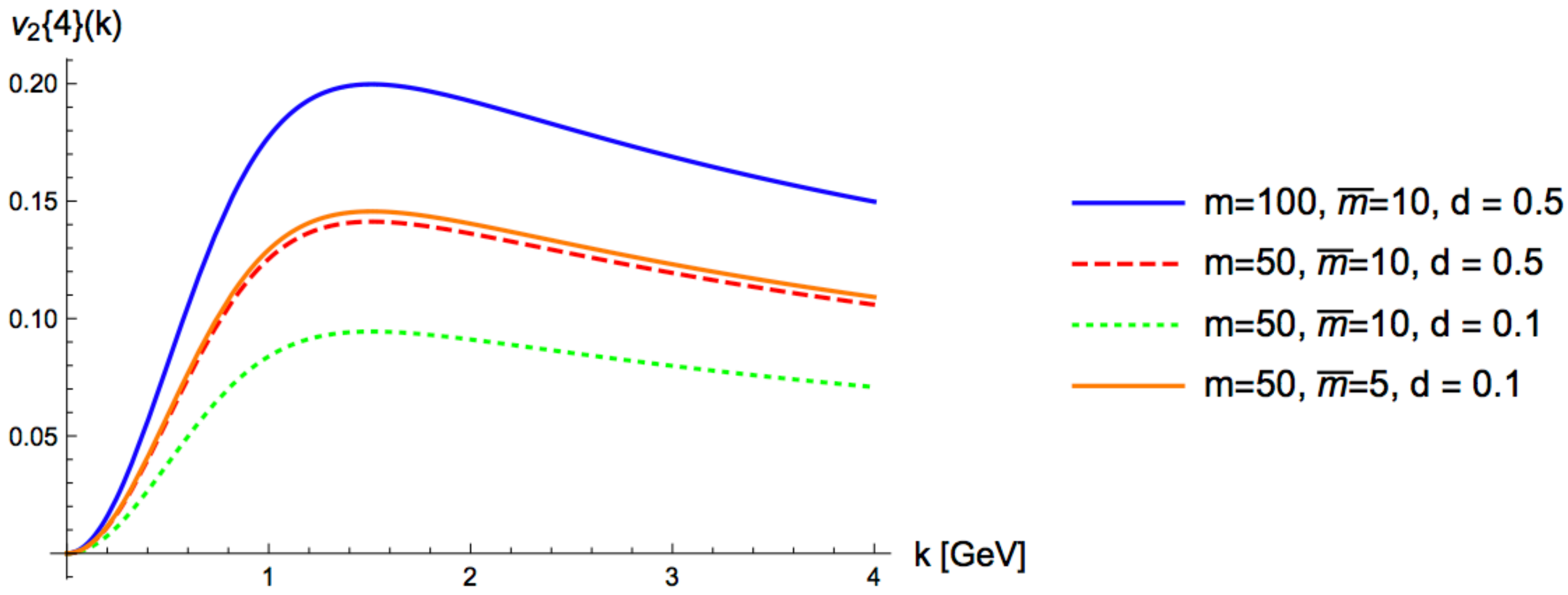
- Limited range of validity: expand in powers of $\frac{1}{(N_c^2-1)}$,
  find m-dependence from integrating out gluons.

  valid for m up to

$$m_{\text{max}} \sim \sqrt{N_c^2-1}/a$$

$$a \equiv \frac{\int d\mathbf{q}\int d\mathbf{z}\,J_0(qz)\,\rho(\mathbf{z})\,f(\mathbf{q})^2}{\int d\mathbf{q}\,|\vec{f}(\mathbf{q})|^2}$$

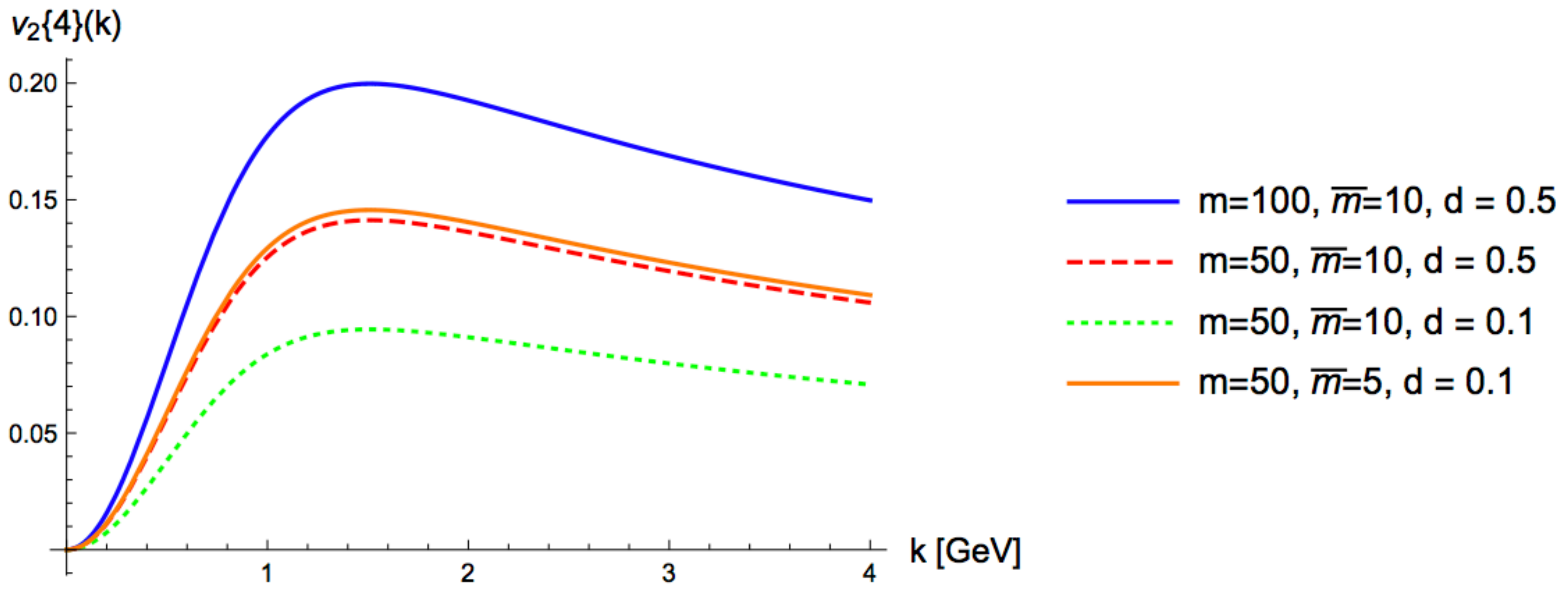
…4th order cumulant, cont'd …



parton-parton interactions ("sources"). In an expansion in powers of $1/(N_c^2 - 1)$ and to leading order in the number of sources N, we calculate interference effects in the m-particle spectra and we determine from them the second and fourth order cumulant momentum anisotropies $v_n\{2\}$ and $v_n\{4\}$. Without invoking any azimuthal asymmetry and any density dependent non-linear dynamics in the incoming state, and without invoking any interaction in the final state, we find that QCD interference alone can give rise to values for $v_n\{2\}$ and $v_n\{4\}$, n even, that persist unattenuated for increasing number of sources, that may increase with increasing multiplicity and that agree with measurements in proton-proton (pp) collisions in terms of the order of magnitude of the signal and the approximate shape of the transverse momentum dependence. We further find that the non-abelian features

B. Blok, C.Jäkel, M. Strikman, UAW, arXiv:1708.08241

…4^{th} order cumulant, cont'd …



Input: QCD final state interference
without initial state asymmetry
without density effects
without final state interactions

Output: $v_n\{2\}$, $v_n\{4\}$ in proton-proton, n even
a no-interaction baseline calculation
persisting unattenuated for many sources N
increasing possibly with multiplicity m
numerically sizeable effect

B. Blok, C.Jäkel, M. Strikman, UAW,
arXiv:1708.08241

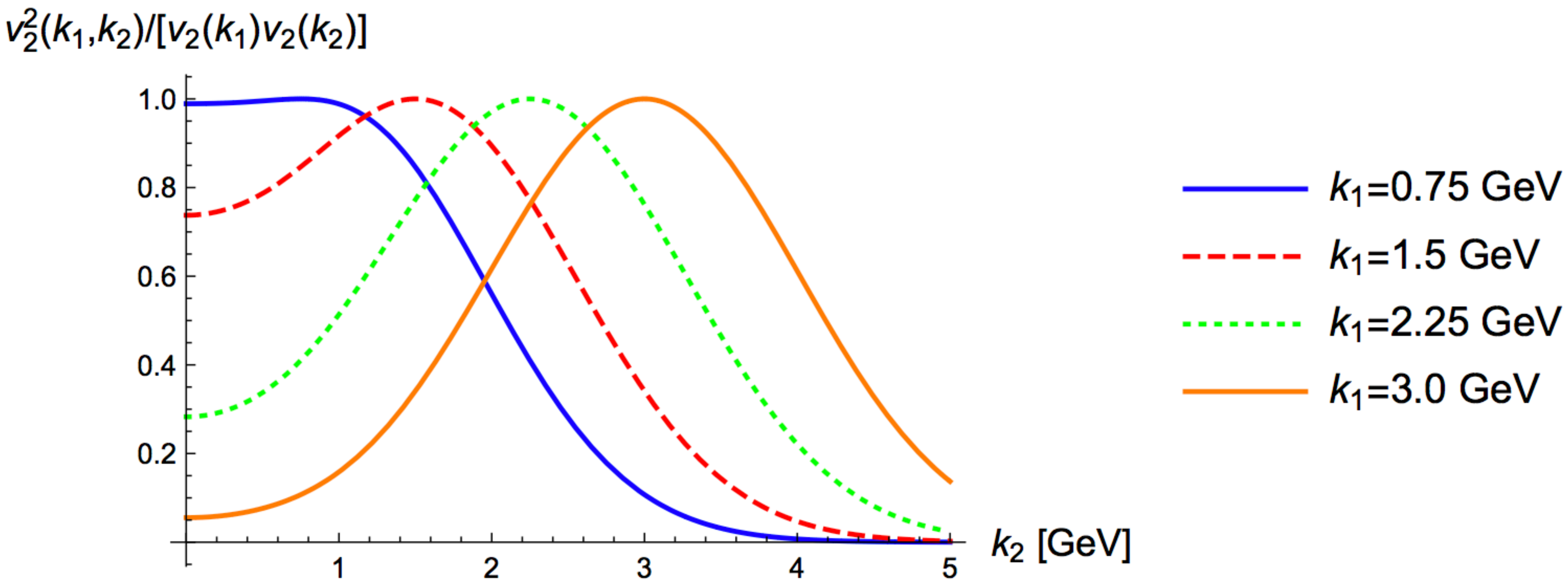
Further properties of v_n's

- Relation between different azimuthal coefficients shares commonalities with fluid dynamics. For instance, at small k,

$$v_4\{2\}(k) = \text{const.}\; v_2\{2\}(k)\, v_2\{2\}(k)$$

- There is a factorization breaking as in other dynamical models:

$$v_2^2\{2\}(k_1, k_2) \neq v_2\{2\}(k_1)\, v_2\{2\}(k_2)$$



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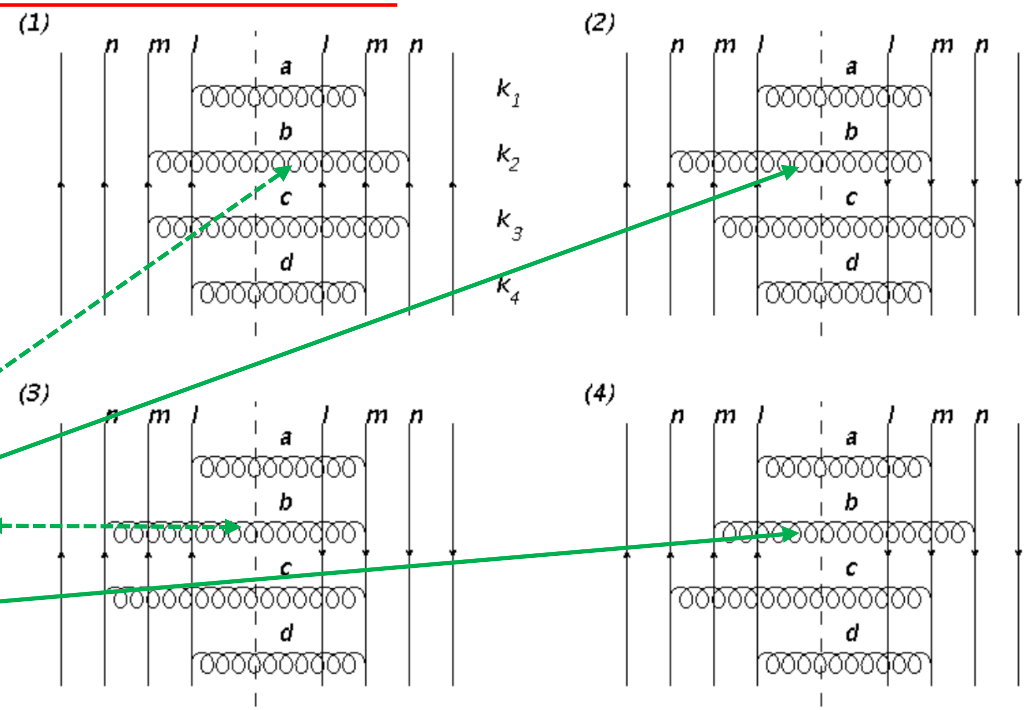
… odd harmonics …

- Arise in the present set-up from a purely non-abelian mechanism
- Interchanging sources btw amplitude and complex conjugate amplitude affects **phase** and **color factor**



$$\mathrm{Tr}\left[\mathbb{1}\right]\ \mathrm{Tr}\left[T^cT^b\right]\ \mathrm{Tr}\left[T^bT^cT^dT^a\right]\ \mathrm{Tr}\left[T^aT^d\right] = N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\left[\mathbb{1}\right]\ \mathrm{Tr}\left[T^bT^c\right]\ \mathrm{Tr}\left[T^cT^dT^bT^a\right]\ \mathrm{Tr}\left[T^aT^d\right] = \boxed{\frac{1}{2}}N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\left[\mathbb{1}\right]\ \mathrm{Tr}\left[T^bT^c\right]\ \mathrm{Tr}\left[T^dT^cT^bT^a\right]\ \mathrm{Tr}\left[T^aT^d\right] = N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\left[\mathbb{1}\right]\ \mathrm{Tr}\left[T^cT^b\right]\ \mathrm{Tr}\left[T^bT^dT^cT^a\right]\ \mathrm{Tr}\left[T^aT^d\right] = \boxed{\frac{1}{2}}N_c^4\ (N_c^2-1)^2$$

- The different color factor lead to odd harmonics (in an abelian theory, there would be none). Amplitudes sketched above are proportional to phase and color factors

$$e^{i\,\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}}\left(e^{i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}} + \boxed{\frac{1}{2}}e^{-i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}\right) + e^{-i\,\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}}\left(\boxed{\frac{1}{2}}e^{i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}} + e^{-i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}\right)$$
$$= 3\cos\left(\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)\ \cos\left(\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}\right) - \sin\left(\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)\ \sin\left(\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)\ . \qquad ($$

- Contributions suppressed by ~1/N.

Conclusions

This model is an attempt to disentangle in the calculation of v_n's QCD interference effects from effects that depend on parton density in the initial state. We find

… the no-interaction baseline including QCD interference effects can make a sizeable if not dominant contribution to the measured v_n coefficients in pp collisions.

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- to get closer to phenomenology:
  Contributions to O(1/N) remain to be quantified fully.
  Effects of rapidity dependence, hadronization,
  … to be understood
  .
- Relations to other approaches like escape mechanism and CGC?
  … to be clarified
  .
- What about AA?
  Rescattering destroys interference.
  => interference effects cannot be added incoherently
  to flow contribution.

END