Analytical and numerical performance models of a Heisenberg Vortex Tube

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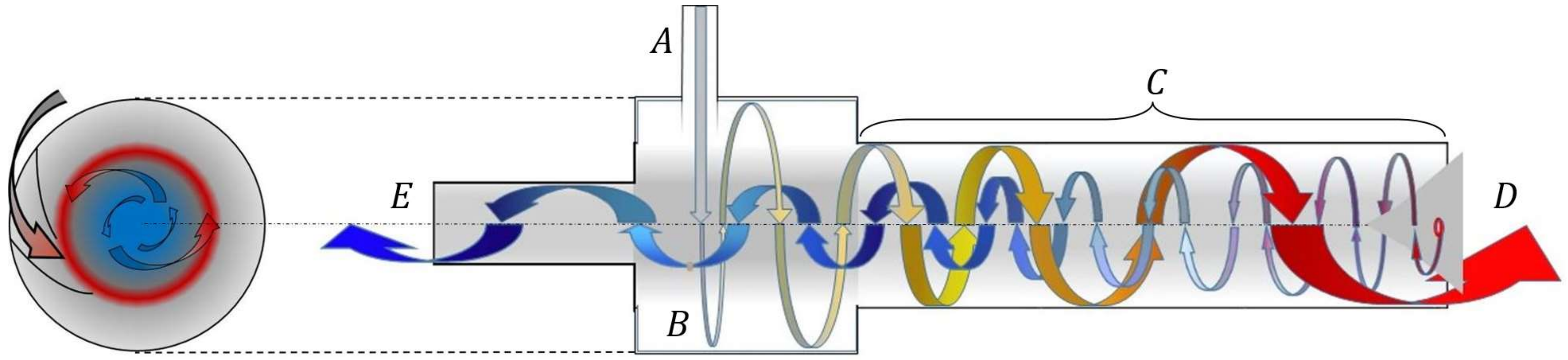
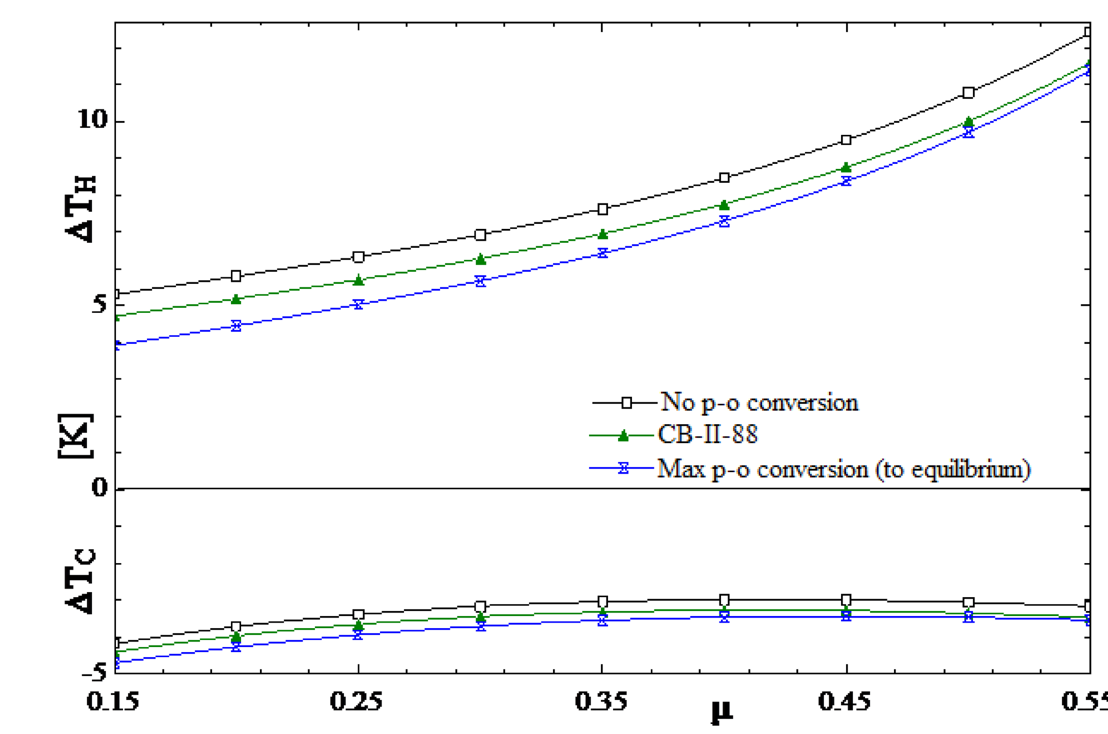
Abstract

Analytical and numerical investigations of a Heisenberg Vortex Tube (HVT) are performed to estimate the cooling potential with cryogenic hydrogen. The Ranque-Hilsch Vortex Tube (RHVT) is a device that tangentially injects a compressed fluid stream into a cylindrical geometry to promote enthalpy streaming and temperature separation between inner and outer flows. The HVT is the result of lining the inside of a RHVT with a hydrogen catalyst.

This is the first concept to utilize the endothermic heat of para-orthohydrogen conversion to aid primary cooling. A review of 1st order vortex tube models available in the literature is presented and adapted to accommodate cryogenic hydrogen properties. These first order model predictions are compared with 2-D axisymmetric Computational Fluid Dynamics (CFD) simulations.



Conceptual schematic of a counter-flow Heisenberg Vortex Tube (HVT): ***A.*** Inlet injection plenum, ***B.*** Vortex Generator, ***C.*** Centrifuge with catalytic liner, ***D.*** Hot annular plug outlet, ***E.*** Cold centerline diffuser.



Plot of temperature drop versus cold flow fraction at various conversion rates at a $PR = 2$. Equilibrium conditions are associated with the p-o heat of conversion potential at the hot outlet. μ is the cold flow fraction. ΔT_C is the differential in cold outlet to reservoir temperature K. ΔT_H is the differential in total temperature from hot outlet to reservoir K.

- EHE model predicts an increase in temperature separation and efficiency at higher cold flow fractions due to the reduced boundary layer thickness.
- CB-II-88 catalyst converts approximately 50% of conversion potential.

Literature-sourced 1st Order Models

	Model	Year	Description	Assumptions[a]	Predictive performance Equations[b]
1	Polihronov and Straatman	2012	Rotating duct	A, I, R	$T_H - T_R = \frac{c^2}{2c_p}$ $\quad T_R - T_C = \frac{c^2}{2c_p}$
2	Eiamsa-ard and Promvonge	2008	2nd Law Estimate	A, I, R	$COP = \frac{\mu c_p (T_R - T_C)}{\left(\frac{\gamma}{\gamma-1}\right) RT_R \left[\left(\frac{p_R}{p_C}\right)^{\gamma - \frac{1}{\gamma}} - 1\right]}$ $COP_{rev} = \frac{1}{\frac{T_R}{T_C} - 1}$ $\quad T_H = \frac{T_R - T_C \mu}{1-\mu}$
3	Ahlborn and Gordon	2000	Secondary flow	A, I, R, N	$T_R - T_C = T_R \left[1 - \frac{1 + \frac{2Bx\mu}{1+Bx}}{1 + \frac{(\gamma-1)x(1+\mu)}{\gamma}}\right]$ $T_H - T_R = T_R \left[\frac{2Bx\mu}{1+Bx}\right]$
4	Liew et al.	2012	Maxwell's demon	A, I, R, N, T	$\frac{T_H}{T_C} = \left[\frac{p_H}{p_C}\right]^{\frac{\gamma-1}{\gamma}} \left[1 + \frac{\gamma-1}{2} Ma_\theta^2\right]$ $\frac{p_R}{p_C} = exp\left[\frac{\gamma}{2} Ma_\theta^2\right] \left[\frac{R_{vc}}{R_{vt}}\right]^{\gamma Ma_\theta^2} \left[1 + \frac{\gamma-1}{2} Ma_\theta^2\right]^{\frac{\gamma}{\gamma-1}}$

[a]Where *A* represents the adiabatic assumption (no losses in the form of heat transfer to surrounding system). *I* represents ideal gas assumption, *R* represents the integration of REFPROP fluid properties into the model. *N* represents the nozzle effects are present in the model, and *T* represents total temperature measurement.

[b]Where T_R is reservoir temperature, T_C is the cold outlet temperature T_H is the hot outlet temperature, c is the speed of sound, c_p is the specific heat at constant pressure, COP is Coefficient of Performance, COP_{rev} is the isentropic Coefficient of Performance, $\mu = \frac{\dot{m}_C}{\dot{m}_{in}}$ is the cold mass fraction, γ is the ratio of specific heats, R is the individual gas constant for H_2, p_R is the reservoir pressure, p_C is the pressure at the cold outlet, p_H is the pressure at the hot outlet. $B = 3(\gamma-1)/(4\gamma)$, $x = (p_R - p_C)/(p_R + 2p_C) < \gamma/2$, Ma_θ is the swirl Mach number, R_{vc} is the radius of the vortex chamber, R_{vt} is the radius of the vortex tube, $\dot{m}$, $\dot{m}_o$, $\dot{m}_i$, $\dot{m}_C$, and $\dot{m}_H$ represent the total mass flow into the tube, mass flow in the outer flow layer, mass flow of the inner flow layer, mass flow exiting the cold outlet, and mass flow exiting the hot outlet, respectively, $\dot{m}_c$ and $\dot{m}_{in}$ are the cold-stream and total (inlet) mass flow rates, respectively. T_o and T_i represent the outer and inner cell temperatures. v_o is the tangential velocity in the outer flow. $\dot{q}_{io}$ represents the heat flux between inner and outer flows.

Review of H_2 vortex tube studies

		Analysis type		Results[a]					
Report	Year	Fluid	Method	PR	T_R	ΔT_C	ΔT_H	ΔT_{Total}	μ
A.F. Johnson	1947	normal-H_2	Experimental	6.6	294	-15.9	-	-	-
Elser and Hoch	1951	normal-H_2	Experimental	6	285	-	-	74	0.5
T. Dutta et al.	2013	normal-H_2	FLUENT® w/REFPROP	3	115	-10	25	35	0.22
				3	115	-7	9	16	0.54
Bunge et al.	2017	normal-H_2	FLUENT® w/REFPROP	1.73	77	-2.81	1.15	3.96	0.36
				2	75	-6.41	0.46	6.87	0.70
Shoemake et al.	2017	para-H_2	Experimental w/o catalyst	1.79	73	-1.08	2.16	3.24	0.37
			w/Ruthenium	1.96	74	-1.70	1.13	2.83	0.42

[a]Where T_R is temperature of the reservoir (inlet fluid) before centrifugal acceleration (K), ΔT_{Total} is the total differential in total temperature from hot outlet to cold outlet. Each row corresponds to the respective method.

Extended Heat Exchanger (EHE) Model

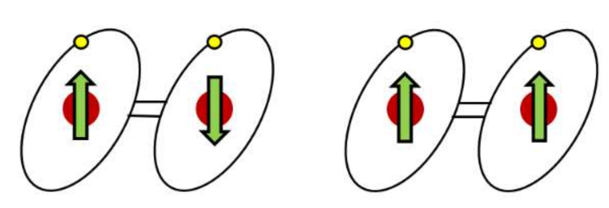
The EHE model accounts for turbulent heat pumping from the inner to outer flow under a pressure gradient. The inner flow cross-section is approximated as a forced turbulent vortex and the radial heat pumping is driven by turbulent fluctuations of fluid particles in the presence of a pressure gradient.

Energy balance, inner flow: $\dot{m}_i c_p \frac{dT_i}{dx} = -\dot{q}_{io}$

Where the term of the left is the temperature differential, the first term on the right represents the turbulent heat transfer as in equation (3) below. The enthalpy streaming is a result of particle transport from a higher or lower pressure region as it contracts or expands. The energy balance is similar to the outer flow with the addition of the p-o conversion term.

Energy balance, outer flow: $\dot{m}_o c_p \frac{dT_o}{dx} = \dot{q}_{io} - \dot{m}_{po} A_{vt} \Delta h_{po}$ Turbulence model: $\dot{q}_{io} = -c_p \rho l^2 \frac{dV}{dr}\left[\frac{dT}{dr} - \left(\frac{dT}{dP}\right)_s \frac{dP}{dr}\right]$

The last term represents the p-o conversion which extracts energy from an endothermic reaction in the outer flow where current conversion units measure per area of catalyst on vortex tube wall. Methods for calculating the enthalpy of formation for p-o conversion are described elsewhere. Investigation of the p-o transition catalysis mechanisms on a molecular level reveals why the near supersonic azimuthal vortex tube flow boosts catalyst performance. The two main factors which dictate the probability for a para- molecule to transition into the ortho-hydrogen state are the kinetic energy which the molecule impinges on the catalyst and the spatial interaction with the catalyst perturbation mechanism. Molecular hydrogen has the potential to stick or become physisorbed to a catalyst surface. However, the proton-perturbation distance has the most influence on the conversion rate which changes at the distance raised to the eighth and sixth powers in two separate models. There is also an optimum surface residence time for a molecule to conduct the p-o conversion energy. This time lengthens at low temperatures beyond typically increases with decreasing temperature. Therefore, the conversion mechanisms at work in the high shear and turbulent flow of the hydrogen in the vortex tube have potential to enable fast transitions through intense interaction and mixing. From the system level of the vortex tube flow, the conversion can be used for extracting enthalpy adjacent to the catalyst. As the catalyst interacts with the high shear boundary layer the equilibrium o-p fraction is shifted to favor a higher ortho fraction due to the increased temperature.



para-hydrogen (lower energy state) *ortho*-hydrogen (higher energy state)

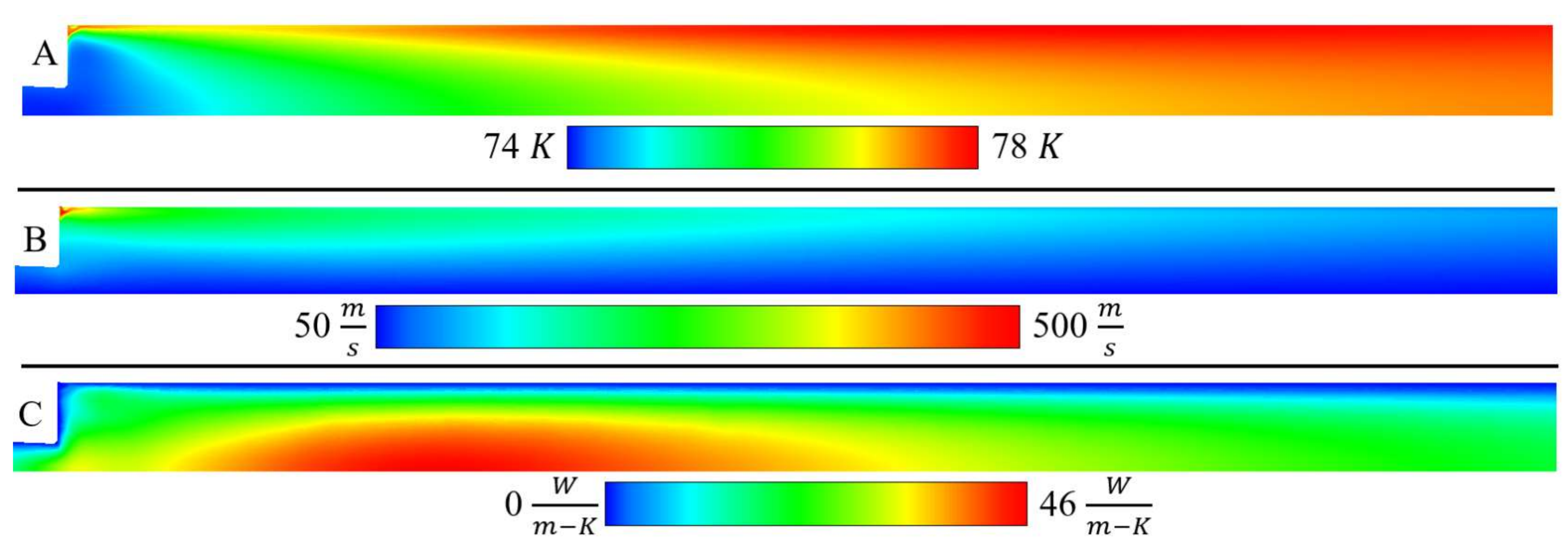
This is modeled by the percentage of conversion is modelled by the following conversion potential:

$$\beta^{cp} = \frac{Maximum\ p-o\ conversion\ percentage}{Estimated\ p-o\ conversion\ percentage}$$

A mixture of equilibrium hydrogen will lack a conversion potential without a temperature potential. The equilibrium orthohydrogen fraction at an inlet of 77K is 49.4%. This equilibrium ortho-hydrogen fraction increases to approximately 55.5% at the outer wall from the predictive theoretical EHE model outer wall temperature. Therefore, the conversion potential percentage potential is 6.1%. We can estimate the power extraction possible given the endothermic para-orthohydrogen enthalpy of conversion of 683 kJ/kg at an average of 81 K and total mass flow rate of 0.15 g/s. This power extraction equals 6.25 W in addition to the fluid mechanical cooling power.

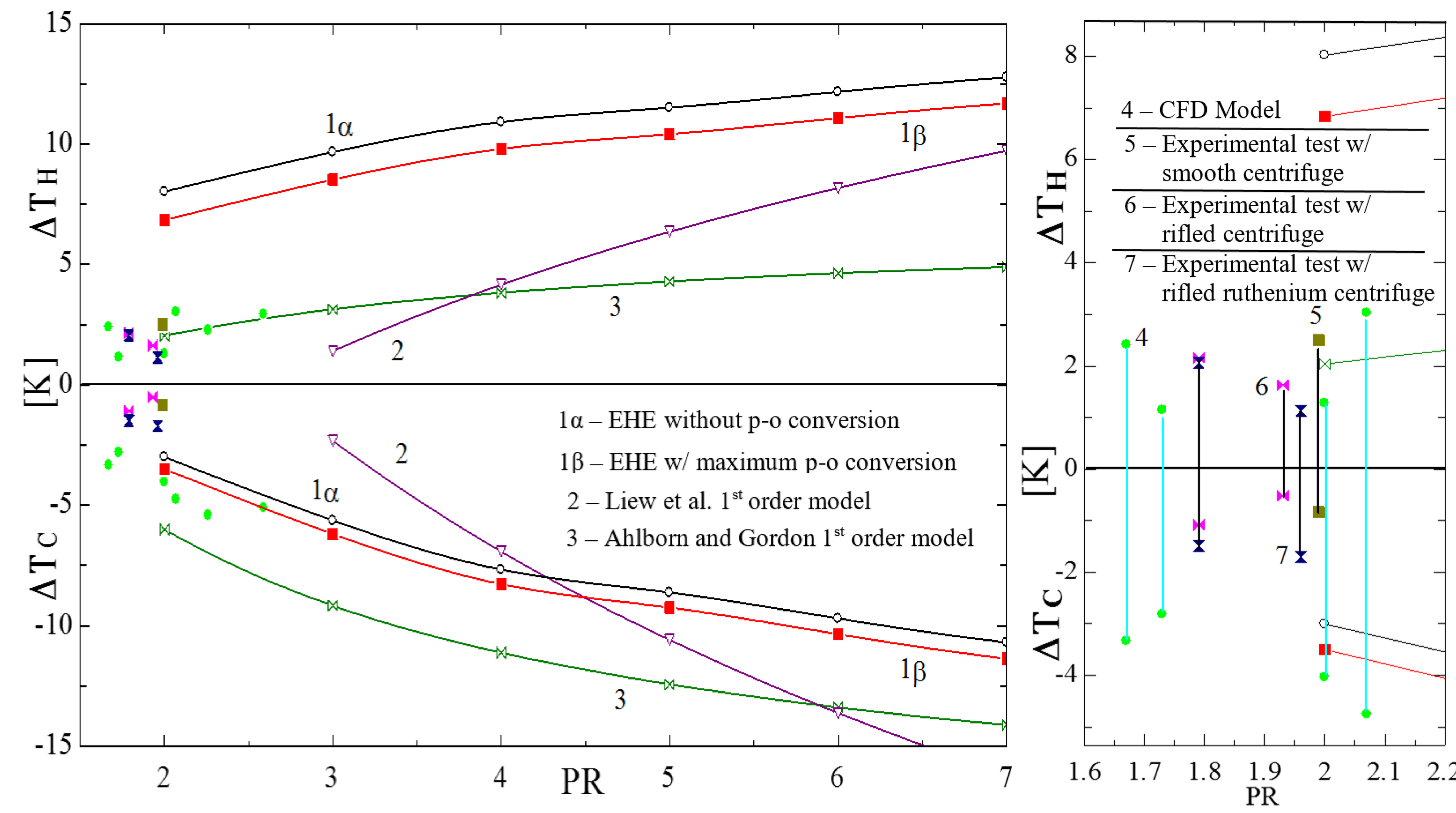
A particle absorbs or rejects heat when its temperature due to expansion/compression in the 2nd term becomes different than the local temperature in the 1st term when turbulence causes random shifts in the radial direction. A forward-time Crank-Nicolson finite difference scheme is used to solve for the temperatures of both inner and outer flows.

2D Axisymmetric Swirl CFD Model



Performance plots of 2D axisymmetric CFD model with REFPROP integrated. Inlet and cold end on left and hot end further downstream on right. $PR = 1.73$, $\mu = 0.37$. A. Total temperature contour. B. Tangential swirl velocity. C. Effective thermal conductivity.

Model Comparison



Temperature separation performance comparison of 1st order models, 2D axisymmetric CFD model, and experimental measurements with and without zoom perspectives. Where 1α represents the EHE model without p-o conversion, 1β - EHE model with maximum p-o conversion potential, 2 – Liew et al. model, 3 – Ahlborn and Gordon [6] model, 4 - 2D CFD model ($\mu = 0.37 \pm 0.035$), 5 – HVT experimental test w/ smooth centrifuge, 6 – HVT experimental test w/ rifled centrifuge, 7 – HVT experimental test w/ rifled ruthenium centrifuge. PR is Pressure Ratio of the total stagnation pressure of the upstream reservoir to total stagnation of the cold end outlet.

Highlights

- The length to diameter ratio needed for complete enthalpy streaming is 5 based on CFD study.
- This is confirmed through experimental test where indium extruded into the entrance of the centrifuge and still created a temperature separation.
- Operating at higher cold flow fractions has the potential to increase temperature separation and efficiency.
- Para-orthohydrogen conversion is confirmed to have significant impact of vortex tube performance.
- Predicted performance boost of para-ortho transition with 1st order EHE: 10%
- Measured performance boost of experimental HVT with ruthenium catalyst: 57%

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