

Analytic rotational solutions of the non-relativistic hydrodynamical equations

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Introduction

- Exact hydrodynamical models:
 - Hydrodynamics relies only on local thermal equilibrium and conservation laws
 - Exact solutions: analytic insight into time evolution
 - Simple formulas for observables
- Rotation in heavy-ion collisions:
 - Non-central heavy-ion collisions: non-zero angular momentum
 - Time evolution of rotation: unveils expansion dynamics, stiffness of Equation of State (EoS)
 - Important to define observables which are sensitive to rotation

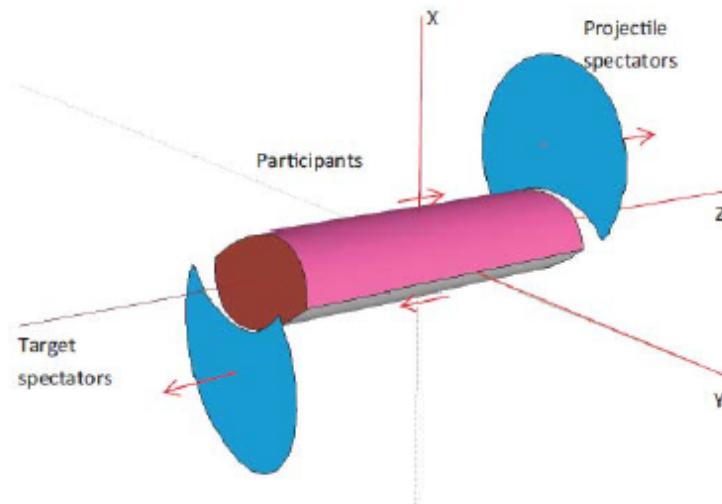


Figure taken from arXiv:1610.08678

- Numerical models of rotation:
 - Many interesting results
 - Predict differential HBT as sensitive but entangled method
 - Λ^0 polarization: measurable effect
 - Need for exact models

Basic ideas

- This work follows the footsteps of:
 Csizmadia, Csörgő, Lukács, PLB443 (1998) 21
 Csörgő, Acta Phys. Polon. B37 (2006) 483
 Csörgő, et al., PRC67 (2003) 034094
- Recent work: rotating but spheroidal exact solutions:
 Csörgő, Nagy, PRC89 (2014) 044901
 Csörgő, Nagy, Barna, PRC93 (2016) 024916
- Rotation (spheroidal models):
 - Vorticity: Λ^0 polarization, etc.
 - Effective radial flow
- Aim #1: three-axis (non-spheroidal) expanding rotating exact solutions
- Aim #2: effect of rotation (with simple formulas for observables)

- To be solved: NR hydrodynamical equations
 - EoS: $\varepsilon = \kappa(T)p$, $p = nT$.
 - Equations in massive case:

$$\begin{aligned}\partial_t n + \nabla(n\mathbf{v}) &= 0, \\ \partial_t \varepsilon + \nabla(\varepsilon\mathbf{v}) &= -p(\nabla\mathbf{v}), \\ \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} &= -\nabla p/(m_0 n), \\ \Rightarrow \partial_t \sigma + \nabla(\sigma\mathbf{v}) &= 0.\end{aligned}$$

- Rewritten for n , T , \mathbf{v} :

$$\begin{aligned}(\partial_t + \mathbf{v}\nabla)n &= -n\nabla\mathbf{v}, \\ \frac{dT\kappa(T)}{dT}(\partial_t + \mathbf{v}\nabla)T &= -T\nabla\mathbf{v}, \\ Tnm_0(\partial_t + \mathbf{v}\nabla)\mathbf{v} &= -\nabla(nT).\end{aligned}$$

Equations in rotating coordinate system...

- Solution easier to find in co-rotating coordinate system K' (rotation in x - z plane, around y axis by $\vartheta(t)$ angle)

$$r'_x = r_x \cos \vartheta - r_z \sin \vartheta,$$

$$r'_z = r_x \sin \vartheta + r_z \cos \vartheta,$$

$$v'_x = v_x \cos \vartheta - v_z \sin \vartheta - \dot{\vartheta} r'_z,$$

$$v'_z = v_x \sin \vartheta + v_z \cos \vartheta + \dot{\vartheta} r'_x.$$

Equations in lab frame K	in rotating frame K'
$\partial_t n + \nabla(n\mathbf{v}) = 0,$ $\frac{d(\kappa T)}{dT} (\partial_t + \mathbf{v}\nabla)T + T\nabla\mathbf{v} = 0,$ $m_0 n (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla(nT).$	$\partial'_t n + \nabla'(n\mathbf{v}') = 0,$ $\frac{d(\kappa T)}{dT} (\partial'_t + \mathbf{v}'\nabla')T + T\nabla'\mathbf{v}' = 0,$ $(\partial'_t + \mathbf{v}'\nabla')\mathbf{v}' = -\frac{\nabla'(nT)}{m_0 n} + \mathbf{f}',$ $\mathbf{f}' = 2\mathbf{v}' \times \boldsymbol{\Omega} + \boldsymbol{\Omega} \times (\mathbf{r}' \times \boldsymbol{\Omega}) + \mathbf{r}' \times \dot{\boldsymbol{\Omega}}.$

where $\boldsymbol{\Omega} = (0, \dot{\vartheta}, 0)$.

- Inertial forces \mathbf{f}' linear in \mathbf{v}' : linear velocity profile can be generalized

... and their new solutions

In both frames:

$$\begin{aligned} H_x &= \frac{\dot{X}}{X}, \quad H_y = \frac{\dot{Y}}{Y}, \quad H_z = \frac{\dot{Z}}{Z}, \\ V &= (2\pi)^{3/2} XYZ, \quad n = n_0 \frac{V_0}{V} \exp(-s/2), \\ \dot{\vartheta} &\equiv \frac{\omega}{2}, \quad \omega = \omega_0 \frac{R_0^2}{R^2}, \quad R = \frac{X+Z}{2}, \end{aligned}$$

$$\begin{aligned} \frac{d[T\kappa(T)]}{dT} \frac{\dot{T}}{T} + \frac{\dot{V}}{V} &= 0 \quad \text{if } \kappa(T) \neq \text{const}, \\ T &= T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \quad \text{if } \kappa(T) = \text{const} \\ X(\ddot{X} - \omega^2 R) &= Y\ddot{Y} = Z(\ddot{Z} - \omega^2 R) = \frac{T}{m_0}, \end{aligned}$$

in laboratory frame K :

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} + \left(\frac{1}{Z^2} - \frac{1}{X^2} \right) [\sin^2 \vartheta (r_x^2 - r_z^2) + \sin(2\vartheta) r_x r_z]$$

$$\mathbf{v}(\mathbf{r}, t) = \mathbf{v}_H(\mathbf{r}, t) + \mathbf{v}_R(\mathbf{r}, t)$$

$$\mathbf{v}_H(\mathbf{r}, t) = \begin{pmatrix} (H_x \cos^2 \vartheta + H_z \sin^2 \vartheta) r_x \\ H_y r_y \\ (H_x \sin^2 \vartheta + H_z \cos^2 \vartheta) r_z \end{pmatrix} + (H_z - H_x) \frac{\sin(2\vartheta)}{2} \begin{pmatrix} r_z \\ 0 \\ r_x \end{pmatrix}$$

$$\mathbf{v}_R(\mathbf{r}, t) = \dot{\vartheta} \begin{pmatrix} r_z \\ 0 \\ -r_x \end{pmatrix} + \dot{\vartheta} \begin{pmatrix} \left(\frac{X}{Z} \cos^2 \vartheta + \frac{Z}{X} \sin^2 \vartheta \right) r_z \\ 0 \\ -\left(\frac{X}{Z} \sin^2 \vartheta + \frac{Z}{X} \cos^2 \vartheta \right) r_x \end{pmatrix} + \dot{\vartheta} \left(\frac{X}{Z} - \frac{Z}{X} \right) \frac{\sin(2\vartheta)}{2} \begin{pmatrix} r_x \\ 0 \\ -r_z \end{pmatrix}$$

in the co-rotating frame K' :

$$s = \frac{r'_x{}^2}{X^2} + \frac{r'_y{}^2}{Y^2} + \frac{r'_z{}^2}{Z^2}$$

$$\mathbf{v}'(\mathbf{r}', t) = \mathbf{v}'_H(\mathbf{r}', t) + \mathbf{v}'_R(\mathbf{r}', t)$$

$$\mathbf{v}'_H(\mathbf{r}', t) = \begin{pmatrix} H_x r'_x \\ H_y r'_y \\ H_z r'_z \end{pmatrix}$$

$$\mathbf{v}'_R(\mathbf{r}', t) = \dot{\vartheta} \begin{pmatrix} \frac{X}{Z} r'_z \\ 0 \\ -\frac{Z}{X} r'_x \end{pmatrix}$$

- Many possibilities, this one (Gaussian n , homogeneous T) is a simple case

Observables

- Solution: simplest (Gaussian) one: analytic formulas possible
- Freeze-out condition: simplest one: $T(t_f) = T_f = 140 \text{ MeV}$.
- Source function: final thermal distribution

$$S(\mathbf{r}, \mathbf{p}) \propto \frac{n(t_f, \mathbf{r})}{T_f^{3/2}} \exp \left\{ -\frac{(\mathbf{p} - m\mathbf{v}(t_f, \mathbf{r}))^2}{2mT_f} \right\}$$

- Observables:
 - Single-particle spectrum
 - Flow coefficients ($v_1, v_2, v_3 \dots$)
 - Two-particle HBT correlations

- Single-particle spectrum:

$$N_1(\mathbf{p}) \equiv E \frac{dn}{d^3\mathbf{p}} \propto E \int d^3\mathbf{r} S(\mathbf{p}, \mathbf{r})$$

- Flow coefficients:

$$\begin{aligned} \frac{dn}{d^3\mathbf{p}} &= \frac{E}{2\pi p_t} \frac{dn}{dp_t dy} \times \\ &\times \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos [n(\varphi - \Psi_n)] \right] \end{aligned}$$

- Two-particle HBT correlations:

$$C(\mathbf{K}, \mathbf{q}) \approx 1 + \lambda \frac{|\tilde{S}_{\mathbf{K}}(\mathbf{q})|^2}{|\tilde{S}_{\mathbf{K}}(0)|^2}$$

$$S_{\mathbf{K}}(\mathbf{q}) \equiv \int d^3\mathbf{r} e^{i\mathbf{q}\mathbf{r}} S(\mathbf{r}, \mathbf{K})$$

Illustration: time evolution of ellipsoid axes

Now skipping some formulas...

- For suitable initial conditions
- Expansion is sensitive to EoS: illustrated here by varying κ

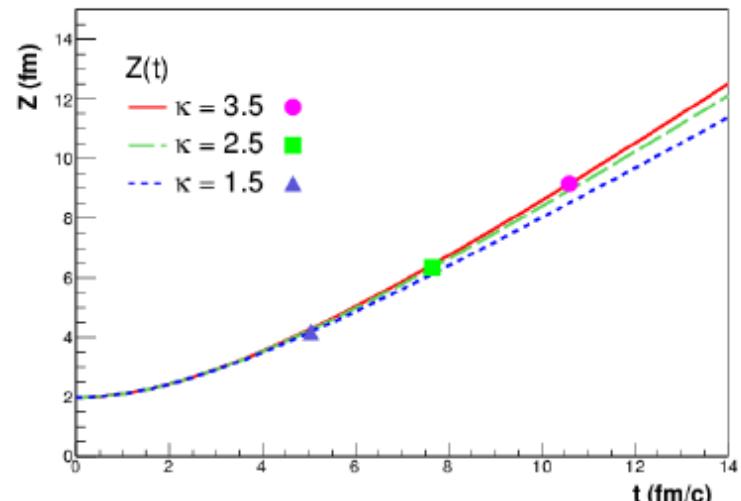
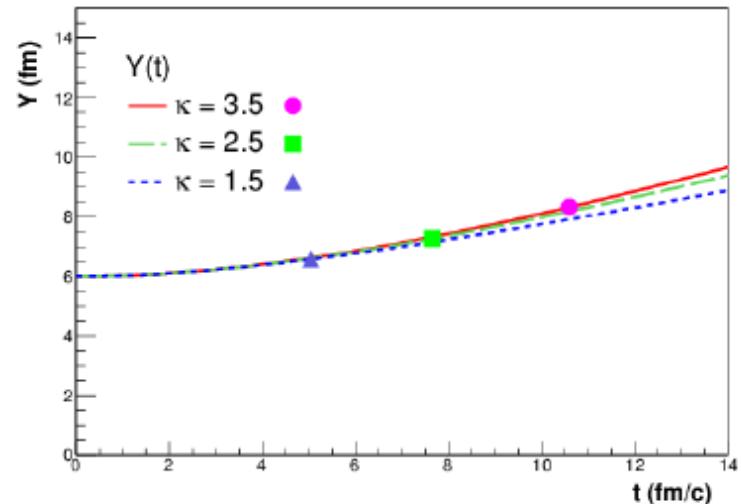
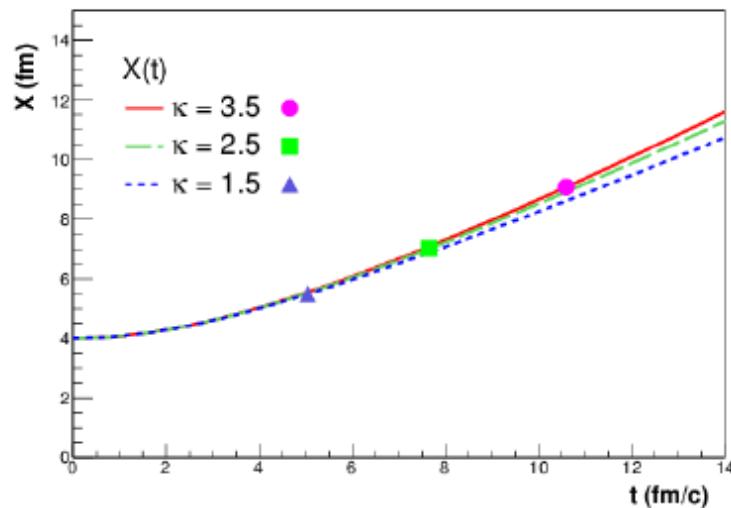


Illustration: time evolution of T, ω

- Initial conditions: same as before
- Effect of EoS is two-fold:
 - adiabatic expansion with stiffer EoS leads to slower temperature change: more time before freeze-out
 - stiffer EoS leads to faster expansion \rightarrow faster decrease of $\omega(t)$.
- Interplay of these effects is sensitive to precise initial conditions

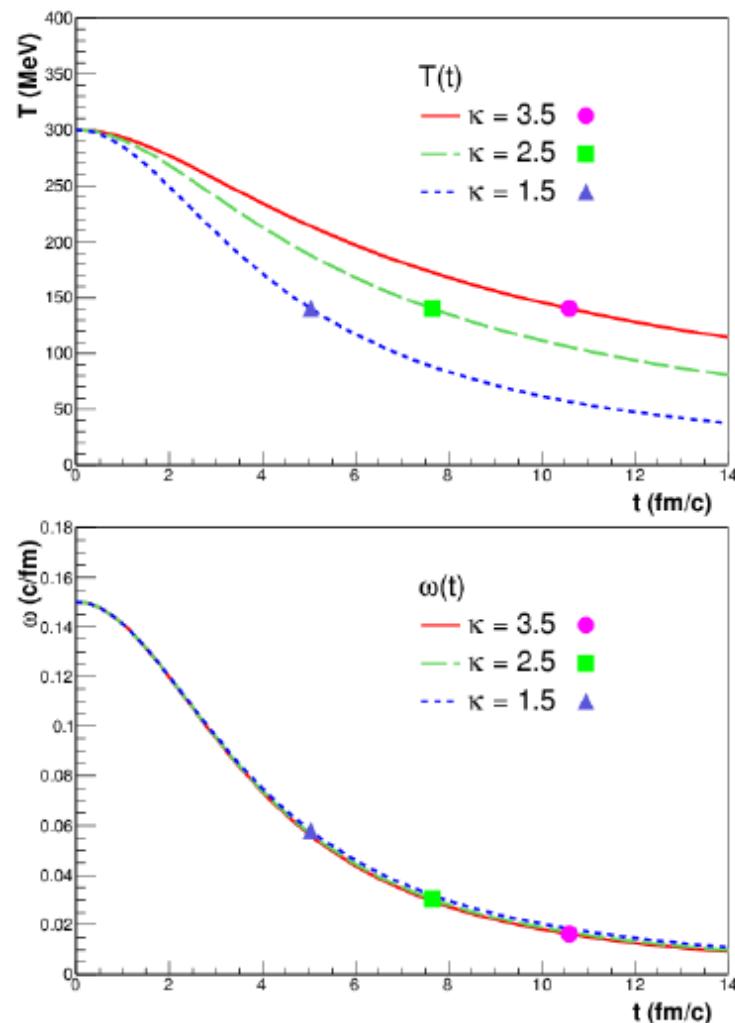
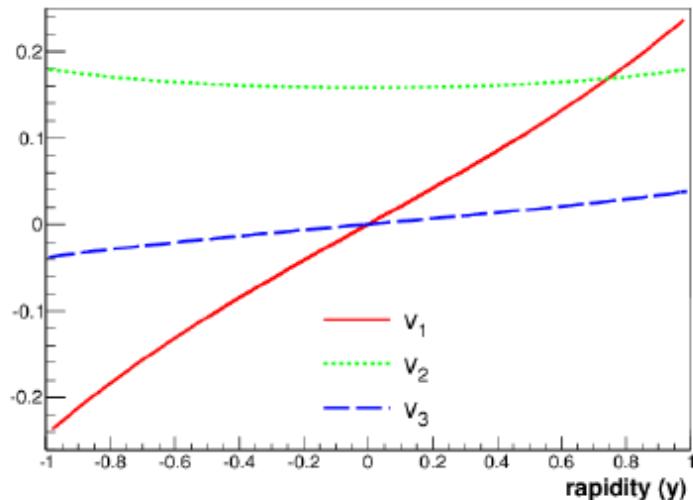
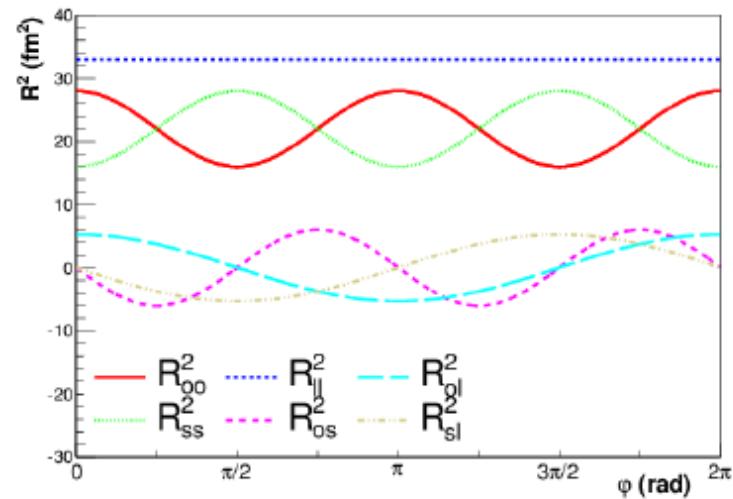


Illustration: final observables (flow coefficients, HBT radii)



Single-particle spectrum: Gaussian, tilted in K' !

$$\frac{dn}{d^3\mathbf{p}'} \propto \exp \left(-\frac{1}{2m} p'_k (\mathbf{T}')_{kl}^{-1} p'_l \right)$$



HBT correlations: Gaussian, tilted in K' with different angles!

$$C(\mathbf{K}, \mathbf{q}) = 1 + \lambda \exp \left(- \sum_{k,l=x,y,z} q_k R_{kl}^2 q_l \right)$$

Signatures of rotation

Coordinate-space ellipsoid at the beginning of time evolution

Final coordinate-space ellipsoid at freeze-out

“Momentum-space ellipsoid” (eigenframe of single-particle spectrum)

“HBT-space ellipsoid” (eigenframe of HBT correlations)

- Tilt angle of momentum space ellipsoid:

$$\tan(2\vartheta'_p) = \frac{2T'_{xz}}{T'_{xx} - T'_{zz}} = \frac{2\omega R}{\dot{X} + \dot{Z}}$$

- Tilt angle of HBT ellipsoid:

$$\tan(2\vartheta'_{\text{HBT}}) = \frac{2XZT'_{xz}}{X^2 T'_{zz} - Z^2 T'_{xx}}$$

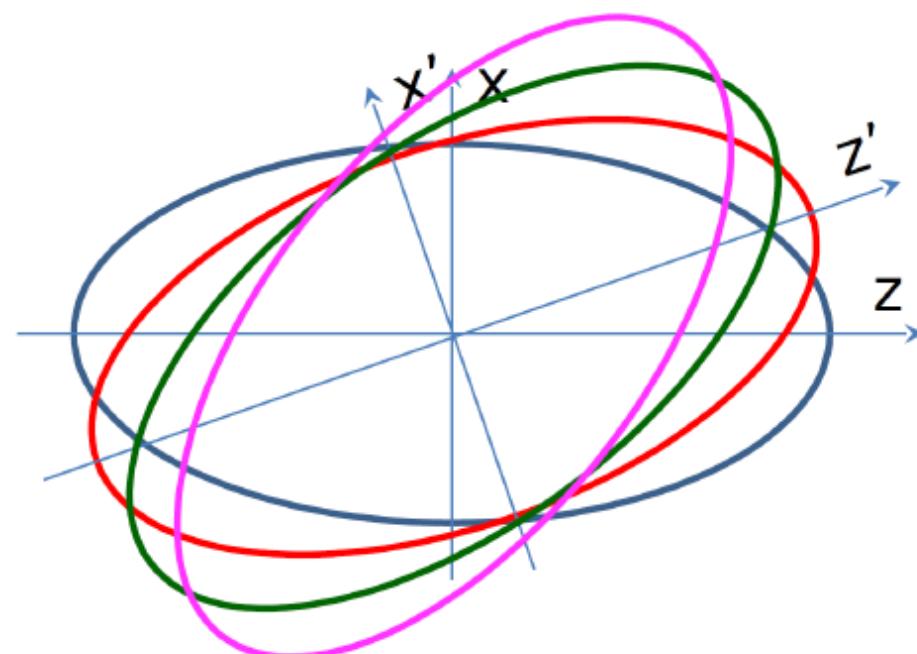
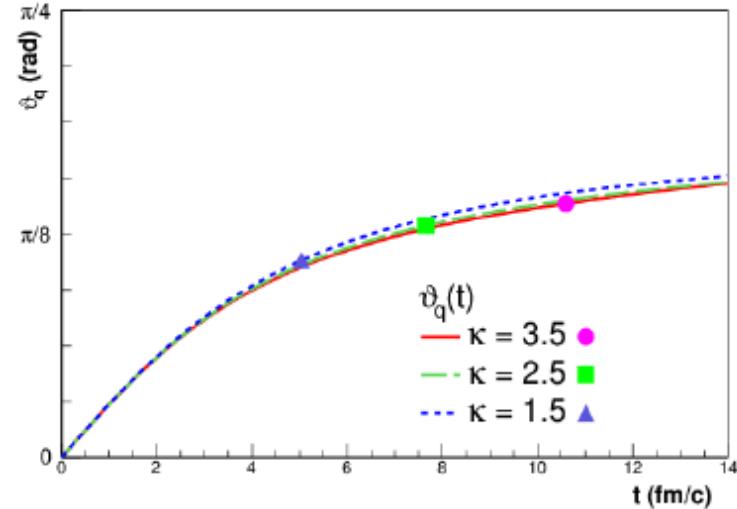
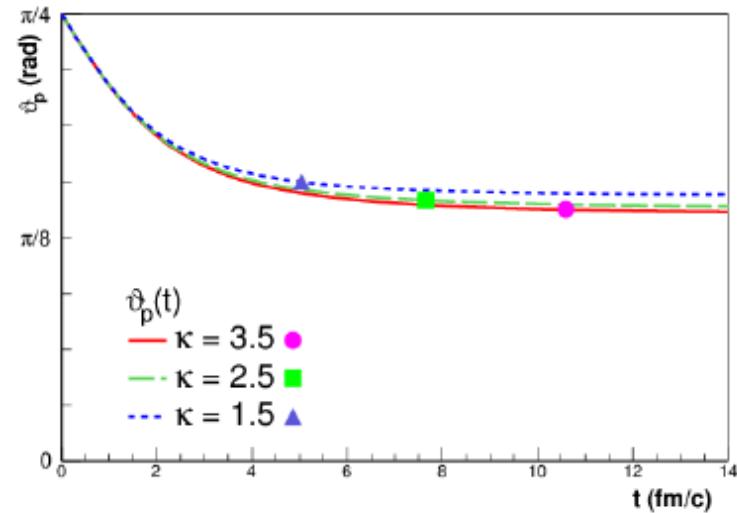
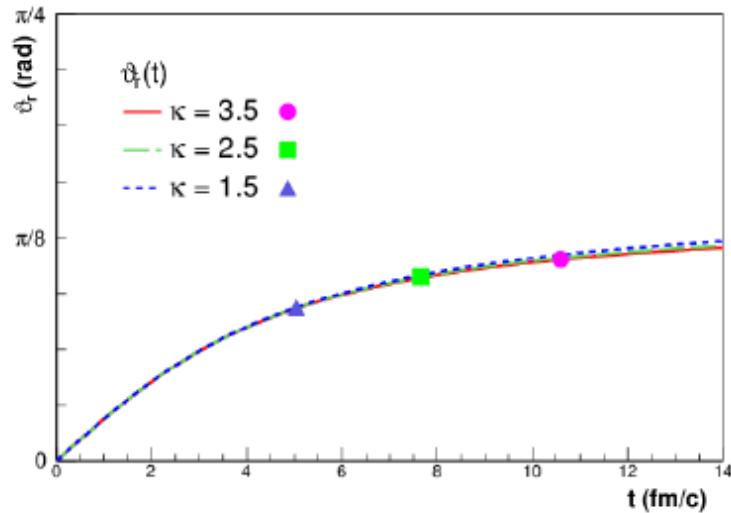


Illustration: time evolution of tilt angles



Summary and outlook

- Hydrodynamical models: time evolution vs. observables
- New solutions (triaxial, rotating, expanding)
- Generalization of previous solutions (rotating but spheroidal & triaxial but non-rotating); relativistic generalization needed!
- Signals of rotation:
 - Directed & third flow vs. rapidity ($v_1(y)$, $v_3(y)$)
 - 1st order order oscillations of R_{ol} , R_{sl} HBT radii vs. pair azimuth angle
 - tilt angle of HBT \neq tilt angle of spectrum \neq tilt angle of coordinate ellipsoid
- Tilt angle measurement: complicated task...
- Also: promising! Mapping of QCD phase diagram by investigating EoS stiffness
- Further steps
 - Relativistic generalization (first steps already taken)
 - *Experimental measurement...*

Thank you for your attention!

