Exact Solutions for a Rehadronizing, Expanding Fireball

- with lattice QCD Equation of State -

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Talk at Low-X 2016 meeting: arXiv:1610.02197 [nucl-th]

I. Introduction

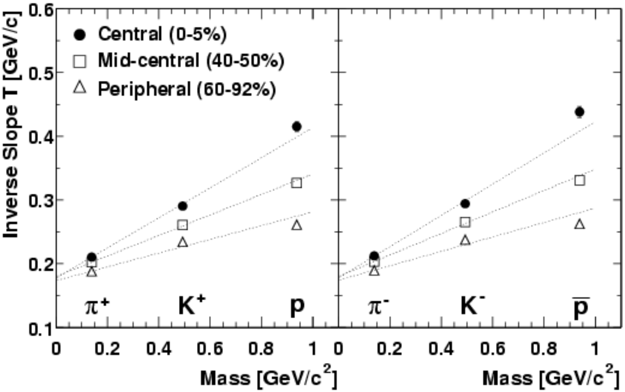
Motivation

- Deeper understanding of rehadronization
- More precise description of the fireball evolution
- Mass dependence of inverse slope

New solution

- Non-relativistic, expanding fireball
- Hadro-chemical and kinetic freeze-out stage
- Multi-component hadronic matter
- Equation of state is from lattice QCD

I. Introduction



$$T = T_f + m\langle u_t \rangle^2 \Longrightarrow T_i = T_f + m_i \langle u_t \rangle^2$$

PHENIX Collaboration: arXiv:nucl-ex/0307022

II. Hydro equations

Non-relativistic, perfect fluid hydrodynamics

- Strongly coupled quark matter - QM ($T > T_c$)

$$\frac{\partial \sigma}{\partial t} + \nabla(\sigma \vec{v}) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla(\varepsilon \vec{v}) = -p\nabla\vec{v}$$

$$T\sigma(\partial_t + \vec{v}\nabla)\vec{v} = -\nabla p$$

- Chemically frozen, mc. hadronic matter - HM ($T < T_c$)

$$\frac{\partial n_i}{\partial t} + \nabla(n_i \vec{v}) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla(\varepsilon \vec{v}) = -p\nabla\vec{v}$$

$$\sum_i m_i n_i \left(\frac{\partial}{\partial t} + \vec{v}\nabla\right)\vec{v} = -\nabla p$$

II. Hydro equations

Temperature equations

- Strongly coupled quark matter - QM ($T > T_c$)

$$\varepsilon = \kappa_{QM}(T)p$$

$$p = \frac{\sigma T}{1+\kappa}$$

$$(1+\kappa_{QM})\left[\frac{d}{dT}\frac{\kappa_{QM}T}{1+\kappa_{QM}}\right](\partial_t + \vec{v}\nabla)T + T\nabla\vec{v} = 0$$

- Chemically frozen hadronic matter - HM ($T < T_c$)

$$\varepsilon = \kappa_{HM}(T)p$$

$$p = \sum_i p_i = T\sum_i n_i$$

$$\left[\frac{d}{dT}\kappa_{HM}T\right](\partial_t + \vec{v}\nabla)T + T\nabla\vec{v} = 0$$

T. Csörgő, M.I. Nagy: arXiv:1309.4390

III. Multi-component solution

Triaxial (X≠Y≠Z) solution

- Velocity field ($\omega_0 = 0$)

$$v_x = \frac{\dot{X}(t)}{X(t)} r_x, \qquad v_y = \frac{\dot{Y}(t)}{Y(t)} r_y, \qquad v_z = \frac{\dot{Z}(t)}{Z(t)} r_z$$

- Entropy and particle density

$$\sigma(\vec{r},t) = \sigma_0 \frac{V_0}{V} exp\left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2}\right)$$

$$n_i(\vec{r},t) = n_{i,c} \frac{V_c}{V} exp\left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2}\right)$$

Landau's idea

$$\frac{\sigma(\vec{r},t_c)}{\sigma(\vec{r}=0,t_c)} = \frac{n_i(\vec{r},t_c)}{n_i(\vec{r}=0,t_c)}$$

S.V. Akkelin, T. Csörgő and others: arXiv:hep-ph/0012127 (for single component)

III. Multi-component solution

Spheroidal (X=Y≠Z), rotating solution

- Velocity field ($\omega_0 \neq 0$)

$$v_x = \frac{\dot{R}(t)}{R(t)} r_x - \omega r_y, \qquad v_y = \frac{\dot{R}(t)}{R(t)} r_y + \omega r_x, \qquad v_z = \frac{\dot{Z}(t)}{Z(t)} r_z$$

$$\omega(t) = \omega_0 \frac{R_0^2}{R^2(t)}$$

- Entropy and particle density

$$\sigma(\vec{r}, t) = \sigma_0 \frac{V_0}{V} exp\left(-\frac{r_x^2}{2R^2} - \frac{r_y^2}{2R^2} - \frac{r_z^2}{2Z^2}\right)$$

$$n_i(\vec{r}, t) = n_{i,c} \frac{V_c}{V} exp\left(-\frac{r_x^2}{2R^2} - \frac{r_y^2}{2R^2} - \frac{r_z^2}{2Z^2}\right)$$

T. Csörgő, M.I. Nagy: arXiv:1309.4390 (for single component)

III. Multi-component solution

- Boundary conditions (B=before, A=after)

  t_c: the "moment" of the chemical freeze-out

$$T_B(t_c, \not{\vec{r}}) = T_A(t_c, \not{\vec{r}})$$
$$\vec{v}_B(t_c) = \vec{v}_A(t_c)$$
$$\kappa_{QM}(T_B(t_c)) = \kappa_{HM}(T_A(t_c))$$
$$\{X_B(t_c), Y_B(t_c), Z_B(t_c)\} = \{X_A(t_c), Y_A(t_c), Z_A(t_c)\}$$

Ansatz

Even for the hadronic matter phase, the scales are independent of the particle species:

$$\{X_i, Y_i, Z_i\} = \{X, Y, Z\}, \quad \forall i.$$

III. Multi-component solution

Dynamical equations

- Strongly coupled quark matter ($T > T_c$)

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{1}{1+\kappa(T)} \qquad (\omega_0 = 0)$$

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{1}{1+\kappa(T)} \qquad (\omega_0 \neq 0)$$

$$(1+\kappa_{QM})\left[\frac{d}{dT}\frac{\kappa_{QM}T}{1+\kappa_{QM}}\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$$

- Chemically frozen, mc. hadronic matter ($T < T_c$)

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{\langle m \rangle} \qquad (\omega_0 = 0)$$

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{T}{\langle m \rangle} \qquad (\omega_0 \neq 0)$$

$$\frac{d\left(\kappa_{HM}T\right)}{dT}\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$$

III. Multi-component solution

Compare to the single component hadronic matter

- Ellipsoidal symmetry ($\omega_0 = 0$)

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{m}$$

- Spheroidal symmetry ($\omega_0 \neq 0$)

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{T}{m}$$

- Difference: $m \Longleftrightarrow \langle m \rangle$

$$\langle m \rangle = \frac{\sum_i m_i n_{i,c}}{\sum_i n_{i,c}} \approx 280\ MeV$$

Conclusion

The X, Y and Z scales are independent of the particle species.

M. Kaneta, N. Xu: arXiv:nucl-th/0405068

IV. Observables

Inverse slope parameter

- Single particle spectrum of the MC scenario

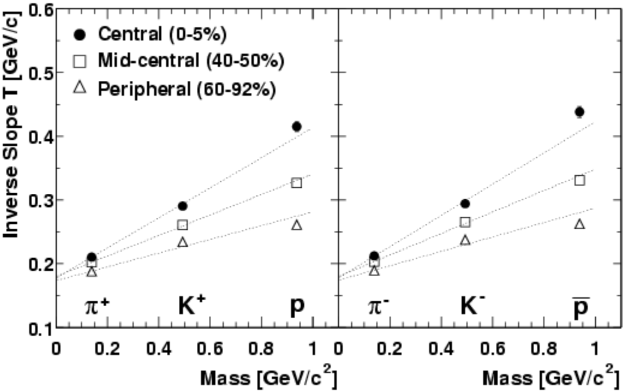
$$N_{1,i}(p_i) \propto exp\left(-\frac{p_{x,i}^2}{2m_i T_{x,i}} - \frac{p_{y,i}^2}{2m_i T_{y,i}} - \frac{p_{z,i}^2}{2m_i T_{z,i}}\right)$$

Inverse slope	**Single-component**	**Multi-component**
	$T_x = T_f + m\dot{X}_f^2$	$T_{x,i} = T_f + m_i\dot{X}_f^2$
$\omega_0 = 0$	$T_y = T_f + m\dot{Y}_f^2$	$T_{y,i} = T_f + m_i\dot{Y}_f^2$
(*ellipsoidal*)	$T_z = T_f + m\dot{Z}_f^2$	$T_{z,i} = T_f + m_i\dot{Z}_f^2$
	$T_x = T_f + m\left(\dot{R}_f^2 + \omega_f^2 R_f^2\right)$	$T_{x,i} = T_f + m_i\left(\dot{R}_f^2 + \omega_f^2 R_f^2\right)$
$\omega_0 \neq 0$	$T_y = T_f + m\left(\dot{R}_f^2 + \omega_f^2 R_f^2\right)$	$T_{y,i} = T_f + m_i\left(\dot{R}_f^2 + \omega_f^2 R_f^2\right)$
(*spheroidal*)	$T_z = T_f + m\dot{Z}_f^2$	$T_{z,i} = T_f + m_i\dot{Z}_f^2$

T. Csörgő, S.V. Akkelin and others: arXiv:hep-ph/0108067v4

T. Csörgő, M.I. Nagy, I.F. Barna: arXiv:1511.02593v1

IV. Observables



$$T_i = k_1 \cdot m_i + k_2$$

PHENIX Collaboration: arXiv:nucl-ex/0307022

IV. Observables

HBT-radii

- Two particle correlation function of the MC scenario

$$C_{2,i}(\vec{q}) \propto exp\left(-q_x^2 R_{x,i}^2 - q_y^2 R_{y,i}^2 - q_z^2 R_{z,i}^2\right)$$

	Single-component	Multi-component
	$R_x^{-2} = X_f^{-2}\left[1 + \frac{m}{T_f}\dot{X}_f^2\right]$	$R_{x,i}^{-2} = X_f^{-2}\left[1 + \frac{m_i}{T_f}\dot{X}_f^2\right]$
$\omega_0 = 0$	$R_y^{-2} = Y_f^{-2}\left[1 + \frac{m}{T_f}\dot{Y}_f^2\right]$	$R_{y,i}^{-2} = Y_f^{-2}\left[1 + \frac{m_i}{T_f}\dot{Y}_f^2\right]$
(*ellipsoidal*)	$R_z^{-2} = Z_f^{-2}\left[1 + \frac{m}{T_f}\dot{Z}_f^2\right]$	$R_{z,i}^{-2} = Z_f^{-2}\left[1 + \frac{m_i}{T_f}\dot{Z}_f^2\right]$
	$R_x^{-2} = R_f^{-2}\left[1 + \frac{m}{T_f}\left(\dot{R}_f^2 + R_f^2\omega_f^2\right)\right]$	$R_{x,i}^{-2} = R_f^{-2}\left[1 + \frac{m_i}{T_f}\left(\dot{R}_f^2 + R_f^2\omega_f^2\right)\right]$
$\omega_0 \neq 0$	$R_y^{-2} = R_f^{-2}\left[1 + \frac{m}{T_f}\left(\dot{R}_f^2 + R_f^2\omega_f^2\right)\right]$	$R_{y,i}^{-2} = R_f^{-2}\left[1 + \frac{m_i}{T_f}\left(\dot{R}_f^2 + R_f^2\omega_f^2\right)\right]$
(*spheroidal*)	$R_z^{-2} = Z_f^{-2}\left[1 + \frac{m}{T_f}\dot{Z}_f^2\right]$	$R_{z,i}^{-2} = Z_f^{-2}\left[1 + \frac{m_i}{T_f}\dot{Z}_f^2\right]$

T. Csörgő, S.V. Akkelin and others: arXiv:hep-ph/0108067v4

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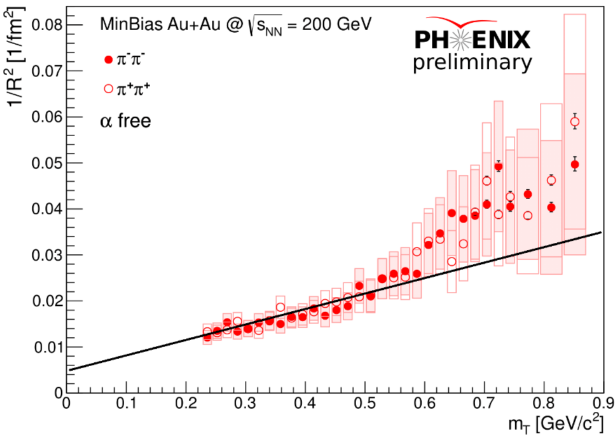
IV. Observables



$$R_i^{-2} = c_1 \cdot m_i + c_2$$

D. Kincses: CPOD 2016

V. Equation of state

Three classes of solution

- $T = T(\not{\vec{r}}, t)$, $\kappa = \kappa(T)$
- $T = T(\vec{r}, t)$, $\kappa = constant$
- $T = T(\vec{r}, t)$, $\kappa = \kappa(T)$ (New!)

Equations of the $\kappa(T)$ function

- Strongly coupled quark matter - QM ($T > T_c$)

$$\frac{d}{dT}\left[\frac{T\kappa_{QM}(T)}{1+\kappa_{QM}(T)}\right] = \frac{\kappa_Q}{1+\kappa_{QM}(T)}$$

- Chemically frozen hadronic matter - HM ($T < T_c$)

$$\frac{d}{dT}\left[T\kappa_{HM}(T)\right] = \frac{\kappa_c T_c - \kappa_f T_f}{T_c - T_f}$$

T. Csörgő, M.I. Nagy: arXiv:1309.4390

V. Equation of state

Solutions for the $\kappa(T)$ function

- Strongly coupled quark matter - QM ($T > T_c$)

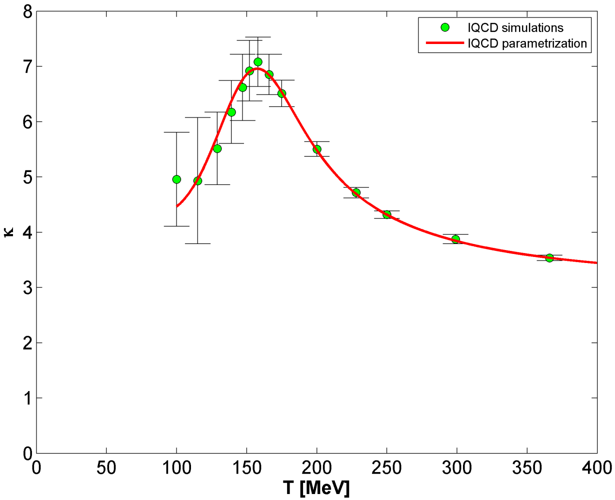
$$\kappa_{QM}(T) = \frac{\kappa_Q \left(\frac{T}{T_c}\right)^{1+\kappa_Q} + \frac{\kappa_c - \kappa_Q}{\kappa_c + 1}}{\left(\frac{T}{T_c}\right)^{1+\kappa_Q} - \frac{\kappa_c - \kappa_Q}{\kappa_c + 1}},$$

- Chemically frozen hadronic matter - HM ($T < T_c$)

$$\kappa_{HM}(T) = \frac{\kappa_c T_c - \kappa_f T_f}{T_c - T_f} - \frac{\kappa_c - \kappa_f}{T_c - T_f} \frac{T_c T_f}{T},$$

- κ in the crossover range?
- Fitting to the lattice simulations is in progress

V. Equation of state



Sz. Borsányi, G. Endrődi and others: arXiv:1007.2580

VI. Time evolution

Initial and final state conditions

- $R_0 = Z_0 = 5\ fm$
- $\dot{R}_0 = \dot{Z}_0 = 0$
- $\theta_0 = 0,\ \ \omega_0 = 0.05\ c/fm$
- $T_f = 140\ MeV,\ \ \langle m \rangle = 280\ MeV$

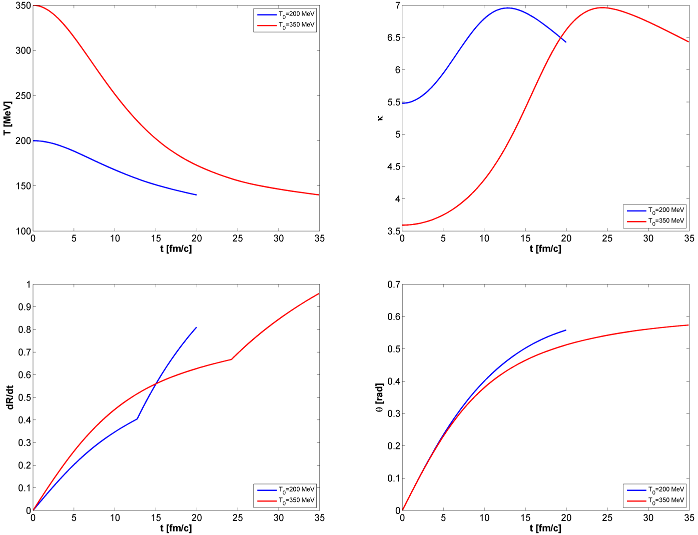
A new effect in the hydro description

- The medium has a second explosion
- Starts just after the conversion to the hadron gas

Condition of the 2nd explosion

$$\frac{1}{1+\kappa_c} < \frac{T_c}{\langle m \rangle}$$

VI. Time evolution



VII. Summary

- Previous solutions describe a single component transition
- New solution for multi-component hadronic matter
- Same scales characterize the dynamics for all particle types
- The multi-c. scenario doesn't complicate the dynamics
- Experimental results are in agreement with our theory
- New hydro parametrization of lattice QCD EoS (in progress)
- Hadrochemical freeze-out leads to a second explosion
- The searching of the relativistic generalization has started