

Electric dipole transitions in potential nonrelativistic QCD

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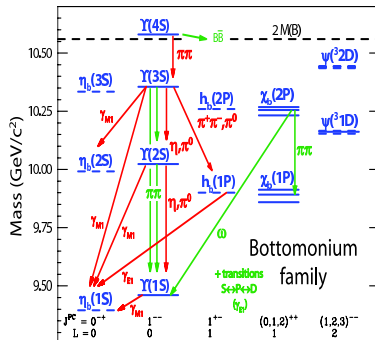
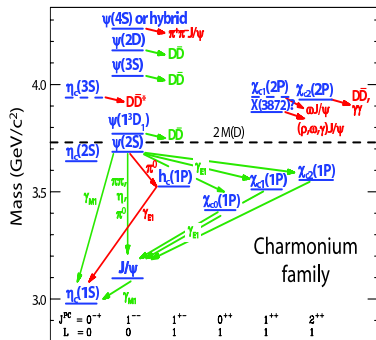
HADRON

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Spectroscopy and Structure



The charmonium and bottomonium systems were discovered in the 1970s
Experimentally clear spectrum of narrow states below the open-flavor threshold



E. Eichten *et al.*, Rev. Mod. Phys. 80 (2008) 1161.

- Heavy quarkonia are bound states made of a heavy quark and its antiquark ($c\bar{c}$ charmonium and $b\bar{b}$ bottomonium).
- They can be classified in terms of the quantum numbers of a nonrelativistic bound state → Reminds positronium [(e^+e^-) -bound state] in QED.
- Heavy quarkonium is a very well established multiscale system which can serve as an ideal laboratory for testing all regimes of QCD.

The nonrelativistic expansion

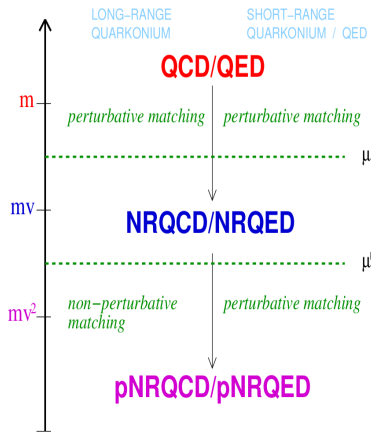
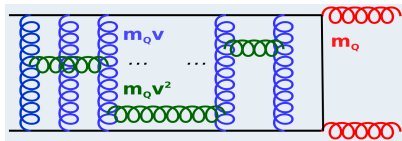
☞ Heavy quarkonium is a nonrelativistic system:

$$v_c \sim 0.55, \quad v_b \sim 0.32 \quad (v_{\text{light}} = 1.0)$$

☞ Heavy quarkonium is a multiscale system:

$$M \gg p \sim 1/r \sim Mv \gg E \sim Mv^2$$

☞ Scales are entangled in full QCD



☞ Systematic expansions in the small heavy-quark velocity v may be implemented at the Lagrangian level by constructing suitable effective field theories (EFTs):

- Expanding QCD in p/M , E/M leads to **NRQCD**.
→ G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D51 (1995) 1125.
- Expanding NRQCD in E/p leads to **pNRQCD**.
→ N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. B566 (2000) 275.

There is another scale in QCD: Λ_{QCD}

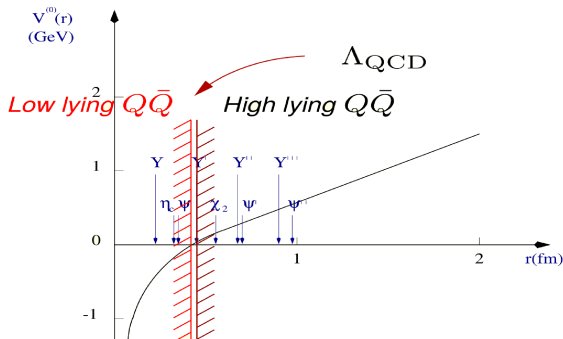
☞ The matching of QCD to NRQCD

$M \gg \Lambda_{\text{QCD}} \rightarrow$ Perturbative matching.

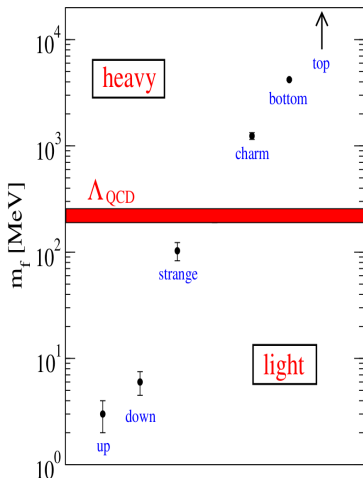
☞ The matching of NRQCD to pNRQCD

$p \sim 1/r \gg \Lambda_{\text{QCD}} \rightarrow$ Weak coupling regime.
Perturbative matching.

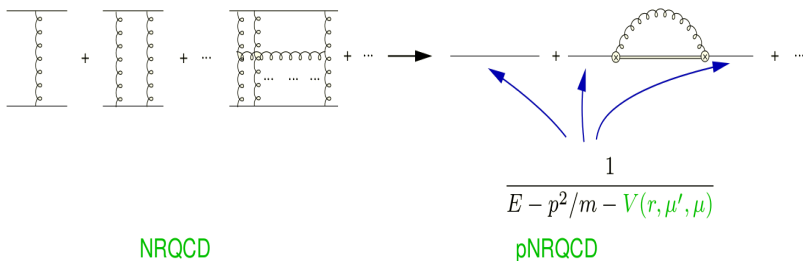
$p \sim 1/r \lesssim \Lambda_{\text{QCD}} \rightarrow$ Strong coupling regime.
Nonperturbative matching.



Quark masses (in $\overline{\text{MS}}$ at $\mu=2$ GeV)



- Physics at the scale Mv : Quarkonium formation.

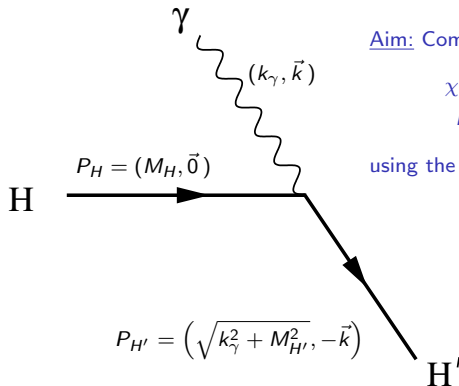


- The effective Lagrangian is organized as an expansion in $1/M$, $\alpha_s(M)$ and $1/p \sim r$:

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \sum_n \sum_k \frac{c_n(\alpha_s(M), \mu)}{M^n} \times V_{n,k}(r, \mu', \mu) r^k \times \mathcal{O}_k(\mu', Mv^2, \dots)$$

where a multipole expansion of the gluon field has been performed.

- The Wilson coefficients of pNRQCD depends on the distance r (and scales μ, μ'):
 - $V_{n,0}$ are the potentials in the Schrödinger equation.
 - $V_{n,k \neq 0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.



Aim: Compute the electric dipole (E1) transitions:

$$\chi_{bJ}(1P) \rightarrow \gamma \Upsilon(1S) \text{ with } J = 0, 1, 2$$

$$h_b(1P) \rightarrow \gamma \eta_b(1S)$$

using the EFT called potential nonrelativistic QCD.

- Electromagnetic transitions are often significant decay modes of heavy quarkonium states that are below the open-flavour threshold.
- They can be classified in a series of electric and magnetic multipoles. The electric dipole (E1) and the magnetic dipole (M1) are the most important ones.
- Large set of accurate experimental data taken by B-factories, τ -charm facilities and proton-proton colliders ask for a systematic and model-independent analysis.

- ☞ The LO decay width, which scales as $\sim k_\gamma^3/(mv)^2$, is

$$\Gamma_{E1}^{(0)} = \frac{4}{9} \alpha_{\text{em}} e_Q^2 k_\gamma^3 \left[I_3^{(0)}(n1 \rightarrow n'0) \right]^2$$

- ☞ A probe of the internal structure of hadrons:

$$I_N^{(k)}(n\ell \rightarrow n'\ell') = \int_0^\infty dr r^N R_{n'\ell'}^*(r) \left[\frac{d^k}{dr^k} R_{n\ell}(r) \right]$$

- ☞ Up to order k_γ^3/m^2 , the expressions we use for the decay rates under study are:

$$\Gamma(n^3P_J \rightarrow n'{}^3S_1 + \gamma) = \Gamma_{E1}^{(0)} \left\{ 1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60} \frac{I_5^{(0)}(n1 \rightarrow n'0)}{I_3^{(0)}(n1 \rightarrow n'0)} \right. \\ \left. + \left[\frac{J(J+1)}{2} - 2 \right] \left[- (1 + \kappa_Q^{\text{em}}) \frac{k_\gamma}{2m} + \frac{1}{m^2} (1 + 2\kappa_Q^{\text{em}}) \frac{I_2^{(1)}(n1 \rightarrow n'0) + 2I_1^{(0)}(n1 \rightarrow n'0)}{I_3^{(0)}(n1 \rightarrow n'0)} \right] \right\}$$

$$\Gamma(n^1P_1 \rightarrow n'{}^1S_0 + \gamma) = \Gamma_{E1}^{(0)} \left\{ 1 + R^{S=0} - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60} \frac{I_5^{(0)}(n1 \rightarrow n'0)}{I_3^{(0)}(n1 \rightarrow n'0)} \right\}$$

- $R^{S=1}(J)$ and $R^{S=0}$ include the initial and final state corrections due to higher order potentials and higher order Fock states.
- The remaining corrections within the brackets are the result of taking into account $\mathcal{O}(v^2)$ -suppressed electromagnetic interaction terms in the Lagrangian.

- ☞ The states are solutions of the Schrödinger equation:

$$H^{(0)}\psi_{n\ell m}^{(0)}(\vec{r}) = E_n^{(0)}\psi_{n\ell m}^{(0)}(\vec{r})$$

- ☞ Only the Coulomb-like term of the static potential is exactly included in the LO Hamiltonian ($C_F = 4/3$):

$$H^{(0)} = -\frac{\vec{\nabla}^2}{2m_r} + V_s^{(0)}(r) = -\frac{\vec{\nabla}^2}{2m_r} - \frac{C_F\alpha_s}{r}$$

- ☞ Therefore, $\psi_{n\ell m}^{(0)}(\vec{r})$ and $E_n^{(0)}$ can be written in the hydrogen-like form:

$$\psi_{n\ell m}^{(0)}(\vec{r}) = R_{n\ell}(r)Y_{\ell m}(\Omega_r) = N_{n\ell}\rho_n^\ell e^{-\frac{\rho_n}{2}}L_{n-\ell-1}^{2\ell+1}(\rho_n)Y_{\ell m}(\Omega_r)$$

$$E_n^{(0)} = -\frac{m_r C_F^2 \alpha_s^2}{2n^2}$$

$$\rho_n = 2r/na, \quad a = 1/m_r C_F \alpha_s, \quad N_{n\ell} = \sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]}}$$

These states are not eigenstates of the complete Hamiltonian due to higher order potentials and the presence of ultra-soft gluons that lead to singlet-to-octet transitions



One has to consider corrections to the wave function which can contribute to the decay rate at the required order of precision

Corrections due to higher order potentials (I)

☞ To account for $\mathcal{O}(v^2)$ -corrections to the decay width, we need to consider the complete Hamiltonian:

$$H = -\frac{\vec{\nabla}^2}{2m_r} + V_s(r) + \delta H$$

☞ The static potential is:

$$V_s(r) = V_s^{(0)}(r) \left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^n a_n(\nu, r) \right]$$

☞ The known $\mathcal{O}(\alpha_s^n)$ radiative corrections to the LO static potential are:

$$a_1(\nu, r) = a_1 + 2\beta_0 \ln(\nu e^{\gamma_E} r)$$

$$a_2(\nu, r) = a_2 + \frac{\pi^2}{3} \beta_0^2 + (4a_1\beta_0 + 2\beta_1) \ln(\nu e^{\gamma_E} r) + 4\beta_0^2 \ln^2(\nu e^{\gamma_E} r)$$

$$\begin{aligned} a_3(\nu, r) = & a_3 + a_1\beta_0^2\pi^2 + \frac{5\pi^2}{6}\beta_0\beta_1 + 16\zeta_3\beta_0^3 \\ & + \left(2\pi^2\beta_0^3 + 6a_2\beta_0 + 4a_1\beta_1 + 2\beta_2 + \frac{16}{3}C_A^3\pi^2 \right) \ln(\nu e^{\gamma_E} r) \\ & + \left(12a_1\beta_0^2 + 10\beta_0\beta_1 \right) \ln^2(\nu e^{\gamma_E} r) + 8\beta_0^3 \ln^3(\nu e^{\gamma_E} r) \\ & + \delta a_3^{US}(\nu, \nu_{US}). \end{aligned}$$

☞ The term δH encodes the relativistic corrections to the static potential and to the nonrelativistic kinetic operator:

$$\delta H = -\frac{\vec{\nabla}^4}{4m^3} + \frac{V^{(1)}}{m} + \frac{V_{\text{SI}}^{(2)}}{m^2} + \frac{V_{\text{SD}}^{(2)}}{m^2}$$

☞ At order $1/m^2$, we can split the contributions into spin-independent (SI) and spin-dependent (SD) terms:

$$V_{\text{SI}}^{(2)}(r) = V_r^{(2)}(r) + \frac{1}{2}\{V_{p^2}^{(2)}(r), -\vec{\nabla}^2\} + V_{L^2}^{(2)}(r) \vec{L}^2$$

$$V_{\text{SD}}^{(2)}(r) = V_{LS}^{(2)}(r) \vec{L} \cdot \vec{S} + V_{S^2}^{(2)}(r) \vec{S}^2 + V_{S_{12}}^{(2)}(r) S_{12}$$

☞ In the weak-coupling case, the above potentials read at leading (non-vanishing) order in perturbation theory:

$$\begin{aligned} V^{(1)}(r) &= -\frac{C_F C_A \alpha_s^2}{2r^2}, & V_r^{(2)}(r) &= \pi C_F \alpha_s \delta^{(3)}(\vec{r}), \\ V_{p^2}^{(2)}(r) &= -\frac{C_F \alpha_s}{r}, & V_{L^2}^{(2)}(r) &= \frac{C_F \alpha_s}{2r^3}, \\ V_{LS}^{(2)}(r) &= \frac{3C_F \alpha_s}{2r^3}, & V_{S^2}^{(2)}(r) &= \frac{4\pi C_F \alpha_s}{3} \delta^{(3)}(\vec{r}), \\ V_{S_{12}}^{(2)}(r) &= \frac{C_F \alpha_s}{4r^3}. \end{aligned}$$

- First order correction to the wave function ($|n\ell\rangle^{(0)} \equiv |n\ell\rangle$ and $E_n^{(0)} \equiv E_n$):

$$|n\ell\rangle^{(1)} = \sum_{n' \neq n} \frac{\langle n'\ell | V | n\ell \rangle}{E_n - E_{n'}} |n'\ell\rangle$$

- Second order correction to the wave function:

$$|n\ell\rangle^{(2)} = \sum_{k_1 \neq n} \left[\sum_{k_2 \neq n} \frac{\langle k_1\ell | V | k_2\ell \rangle \langle k_2\ell | V | n\ell \rangle}{(E_n - E_{k_1})(E_n - E_{k_2})} - \frac{\langle k_1\ell | V | n\ell \rangle \langle n\ell | V | n\ell \rangle}{(E_n - E_{k_1})^2} \right] |k_1\ell\rangle - \frac{1}{2} \sum_{k_2 \neq n} \frac{|\langle k_2\ell | V | n\ell \rangle|^2}{(E_n - E_{k_2})^2} |n\ell\rangle$$

- Identification with the Coulomb Green function:

$$\sum_{n'} \frac{|n'\ell\rangle \langle n'\ell|}{E_n - E_{n'}} - \sum_{n'=n} \frac{|n'\ell\rangle \langle n'\ell|}{E_n - E_{n'}} = (-1) \times \lim_{E \rightarrow E_n} \left(\frac{1}{E - H} - \frac{P_{n\ell}}{E - E_n} \right)$$

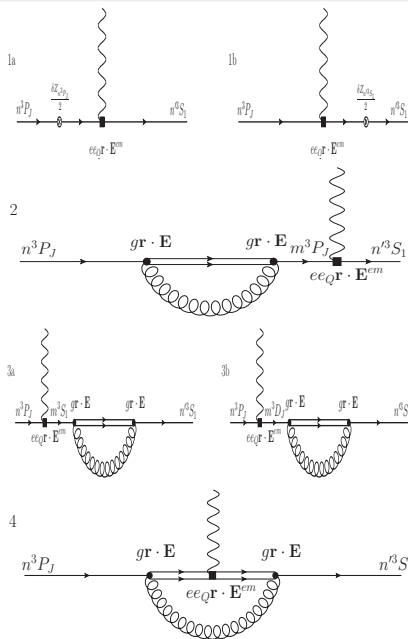
- Definition of the Coulomb Green function:

$$G(\vec{r}_1, \vec{r}_2; E) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_{\ell}(\hat{r}_1 \cdot \hat{r}_2) G_{\ell}(r_1, r_2; E)$$

$$G_{\ell}(r_1, r_2; E) = \sum_{\nu=\ell+1}^{\infty} m_r a^2 \left(\frac{\nu^4}{\lambda} \right) \frac{R_{\nu\ell}(\rho_{\lambda,1}) R_{\nu\ell}(\rho_{\lambda,2})}{\nu - \lambda}$$

Corrections due to higher order Fock states

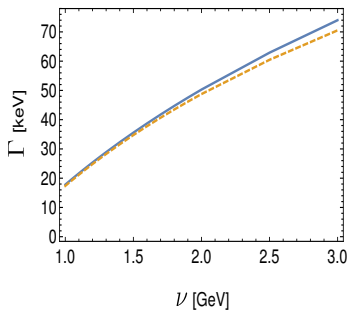
- Diagrams 1a and 1b correspond to the renormalization of the initial and final wave functions.
- Diagrams 2, 3a and 3b account for the correction of the initial and final wave functions due to the presence of octet states.
- Diagram 4 represents an electric dipole transition mediated by the intermediate octet state.
- The first two diagrams contribute to relative order $\Lambda_{\text{QCD}}^2/(mv)^2$ whereas the remaining ones scales as $\Lambda_{\text{QCD}}^3/(m^3v^4)$.
- We do not consider these contributions herein because in the (strict) weak-coupling regime, $E \sim mv^2 \gg \Lambda_{\text{QCD}}$, one can argue that they should be negligible.



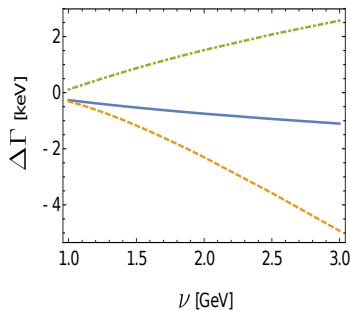
N. Brambilla *et al.*, Phys. Rev. D85 (2012) 094005.

$$\Gamma_{n^3P_J \rightarrow n'^3S_1} = \Gamma_{E_1}^{(0)} \left[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60} \frac{I_5^{(0)}(n1 \rightarrow n'0)}{I_3^{(0)}(n1 \rightarrow n'0)} \right. \\ \left. + \left(\frac{J(J+1)}{2} - 2 \right) \left(- (1 + \cancel{\frac{k_\gamma^2}{2m}}) \frac{k_\gamma}{2m} + \frac{1}{m^2} (1 + \cancel{\frac{k_\gamma^2}{2m}}) \frac{I_2^{(1)}(n1 \rightarrow n'0) + 2I_1^{(0)}(n1 \rightarrow n'0)}{I_3^{(0)}(n1 \rightarrow n'0)} \right) \right]$$

Combined effect



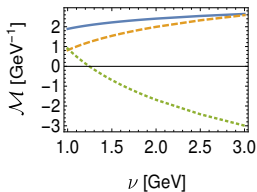
Individual contributions



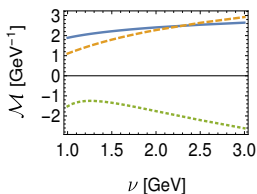
Observations:

- The leading order decay width depends strongly on the typical scale ν of the heavy quarkonia involved in the reactions.
- The relativistic effects from higher order electromagnetic terms are small, as expected. This can be understood analyzing each contribution separately.

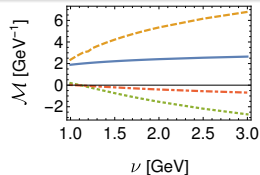
Wave function corrections [$\chi_{b1}(1P) \rightarrow \gamma \Upsilon(1S)$]



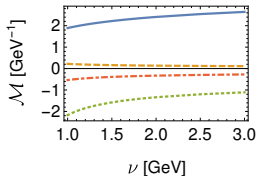
- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini,fin}}(V_0^{(1)})$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(V_0^{(1)})$



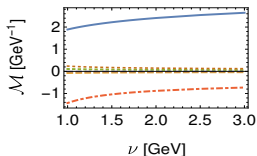
- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(V_0^{(2)})$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(V_0^{(2)})$



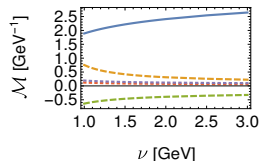
- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini,fin}}(V_0^{(1)})$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin,fin}}(V_0^{(1)})$
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini,fin}}(V_0^{(1)})$



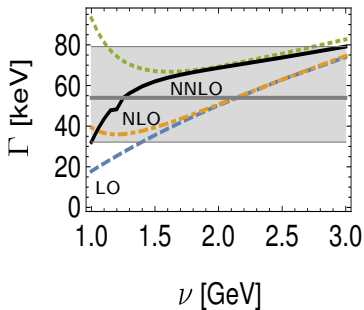
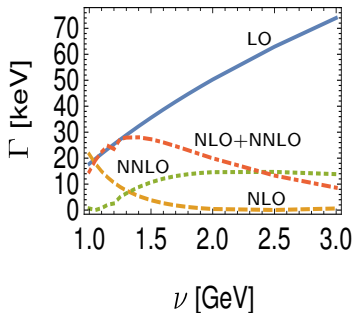
- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(V_r^{(1)})$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(V_r^{(1)})$
- - $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(V_r^{(2)})$



- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(L^2)$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(LS)$
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(S^2)$
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(S_{12})$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(S_{12})$



- \mathcal{M}_{LO}
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(p^2)$
- ⋯ $\mathcal{M}_{\text{NNLO}}^{\text{fin}}(p^2)$
- - $\mathcal{M}_{\text{NNLO}}^{\text{ini}}(p^4)$



Observations:

- LO, NLO and NNLO contributions depend strongly on the renormalization scale ν : running of $\alpha_s(\nu)$ and radiative terms ($\propto \log$ s) of the static potential.
- Subleading contributions are of the same order of magnitude than the leading order term.
- The renormalization scale dependence of the decay width is slightly reduced as the NLO and NNLO corrections are included.
- Setting the terms proportional to $a_1(\nu, r)$ and $a_2(\nu, r)$ to zero, the decay width exhibits a different ν -dependence (dotted green curve in the right panel).

Improving convergence: *The perturbative expansion in pNRQCD can be rearranged in such a way that the static potential is exactly included in the LO Hamiltonian*

☞ The new expansion scheme was applied in:

- Y. Kiyo *et al.*, Nucl. Phys. B841 (2010) 231, for studying inclusive EM decays of heavy quarkonium.
- A. Pineda *et al.*, Phys. Rev. D87 (2013) 074024 for computing magnetic dipole transitions between low-lying heavy quarkonia.



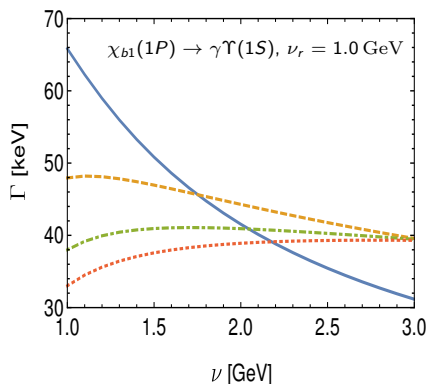
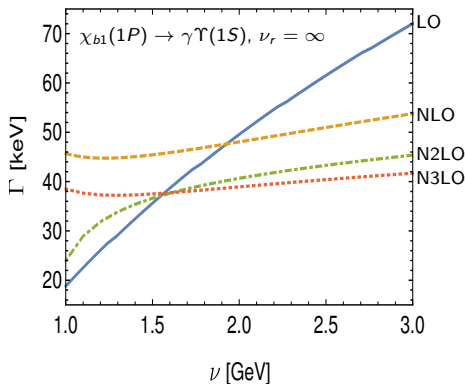
The exact treatment of the soft logarithms of the static potential made the factorization scale dependence much smaller

☞ We proceed herein to apply the same formalism to the electric dipole transitions under study:

$$H_{\text{new}}^{(0)} = -\frac{\vec{\nabla}^2}{2m_r} + V_s^{(N)}(r)$$

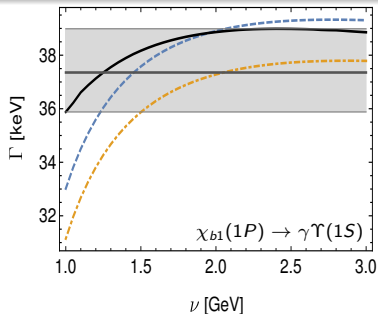
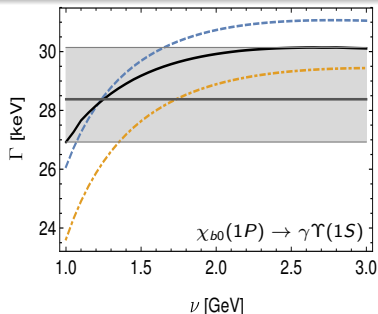
where the static potential is again approximated by a polynomial of order $N + 1$ in powers of α_s :

$$V_s^{(N)}(r) = -\frac{C_F \alpha_s(\nu)}{r} \left[1 + \sum_{k=1}^N \left(\frac{\alpha_s}{4\pi} \right)^k a_k(\nu, r) \right]$$



Observations:

- One gets a strong dependence on the factorization scale when solving the Schrödinger equation with only the Coulomb-like term of the static potential.
- The ν -scale dependence becomes mild as NLO, NNLO and NNNLO terms of the static potential are included in the Schrödinger equation.
- The convergence of the perturbative series has improved considerably with respect the purely analytical case. This feature is clearly seen at large values of ν .
- The convergence is seen at low values of ν once the correct log-arithmetically modulated short distance behavior of the static potential is included. ▶ ◀ ≡ ≡ ≡ ≡ ≡ ≡ ≡ ≡ ≡ ≡

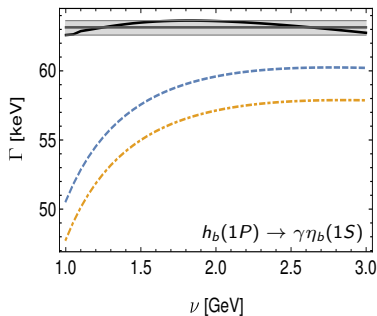
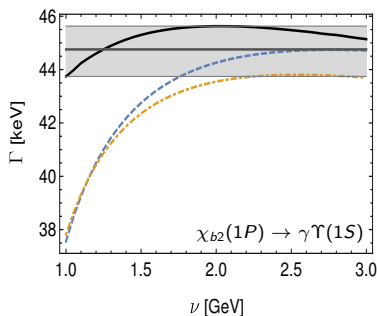


Legend:

- *Dashed blue curve*: leading order non-relativistic decay rate.
- *Dot-dashed orange curve*: + rel. contributions from higher order EM operators.
- *Solid black*: + rel. corrections to the initial and final wave functions.

Observations:

- The leading order decay width depends weakly on ν , this feature is translated to the cases in which relativistic corrections are included.
- Both relativistic contributions to the leading order decay rate are much more under control than in the purely analytical case.
- Higher order EM operators tend to diminish the LO decay rate whereas the opposite effect is found when corrections to the wave functions are incorporated.



Legend:

- *Dashed blue curve*: leading order non-relativistic decay rate.
- *Dot-dashed orange curve*: + rel. contributions from higher order EM operators.
- *Solid black*: + rel. corrections to the initial and final wave functions.

Observations:

- Similar conclusions to the ones already mentioned in the former slide apply here.
- The relativistic corrections stemming from higher order EM operators are clearly smaller whereas the effect due to corrected wave functions is 2-3 times bigger.
- The correction to the decay rate due to the modification of the initial and final state wave functions is much larger for the $h_b(1P) \rightarrow \gamma\Upsilon(1S)$ transition.

Final values for the decay widths:

$$\Gamma(\chi_{b0}(1P) \rightarrow \gamma\Upsilon(1S)) = 28_{-1}^{+2} \text{ keV},$$

$$\Gamma(\chi_{b1}(1P) \rightarrow \gamma\Upsilon(1S)) = 37_{-1}^{+2} \text{ keV},$$

$$\Gamma(\chi_{b2}(1P) \rightarrow \gamma\Upsilon(1S)) = 45_{-1}^{+1} \text{ keV},$$

$$\Gamma(h_b(1P) \rightarrow \gamma\eta_b(1S)) = 63_{-1}^{+1} \text{ keV}.$$

Comparison with several other theoretical approaches (in keV):

Mode	LO	NNLO	CQM	R	GI	BT	LFQM	SNR _{0/1}
$\chi_{b0}(1P) \rightarrow \gamma\Upsilon(1S)$	28.47	28.38	28.07	29.9	23.8	25.7	-	-
$\chi_{b1}(1P) \rightarrow \gamma\Upsilon(1S)$	36.04	37.35	35.66	36.6	29.5	29.8	-	-
$\chi_{b2}(1P) \rightarrow \gamma\Upsilon(1S)$	41.00	44.76	39.15	40.2	32.8	33.0	-	-
$h_b(1P) \rightarrow \gamma\eta_b(1S)$	55.22	63.15	43.7	52.6	35.7	-	37.5	55.8/36.3

Predicted total decay widths:

Mode	$\mathcal{B} = \Gamma_i/\Gamma$ (%)	Γ_i (keV)	Γ
$\chi_{b0}(1P) \rightarrow \gamma\Upsilon(1S)$	(1.76 ± 0.35)	28_{-1}^{+2}	$1.6_{-0.3}^{+0.3}$ MeV
$\chi_{b1}(1P) \rightarrow \gamma\Upsilon(1S)$	(33.9 ± 2.2)	37_{-1}^{+2}	110_{-8}^{+9} keV
$\chi_{b2}(1P) \rightarrow \gamma\Upsilon(1S)$	(19.1 ± 1.2)	45_{-1}^{+1}	234_{-16}^{+15} keV
$h_b(1P) \rightarrow \gamma\eta_b(1S)$	(52_{-5}^{+6})	63_{-1}^{+1}	121_{-12}^{+14} keV

We have computed the electric dipole transitions $\chi_{bJ}(1P) \rightarrow \gamma \Upsilon(1S)$ with $J = 0, 1, 2$ and $h_b(1P) \rightarrow \gamma \eta_b(1S)$ within the weak coupling version of a low-energy effective field theory called potential non-relativistic QCD

☞ Only the Coulomb term of the static potential is included in the LO Hamiltonian:

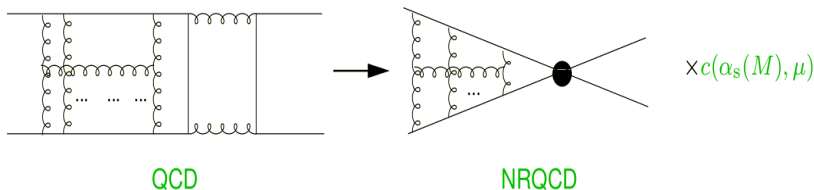
- The decay width reveals a severe dependence on the factorization scale ν : running of $\alpha_s(\nu)$ and radiative terms ($\propto \log s$) of the static potential.
- Most of the corrections to the decay rate induced by $1/m$ and $1/m^2$ potentials are relatively small and behave well with respect the renormalization scale ν .
- The general convergence of the perturbative series is not as good as expected for all the studied electric dipole transitions.

☞ The static potential is exactly included in the leading order Hamiltonian:

- The non-relativistic decay rate depends weakly on the factorization scale and this is translated to the result in which relativistic corrections are included.
- Relativistic corrections are much more under control: better convergence and small contributions.
- Our results are in fair agreement with the ones reported by other theoretical approaches [$\Gamma(h_b(1P) \rightarrow \gamma \eta_b(1S))$ is slightly higher].
- Total decay widths of the $\chi_{bJ}(1P)$ -family and the $h_b(1P)$ meson are predicted.

BACKUP SLIDES

- Physics at the scale M : Quarkonium annihilation and production.



- The effective Lagrangian is organized as an expansion in $1/M$ and $\alpha_s(M)$:

$$\mathcal{L}_{\text{NRQCD}} = \sum_n \frac{c_n(\alpha_s(M), \mu)}{M^n} \times \mathcal{O}_n(\mu, Mv, Mv^2, \dots)$$

- $\mathcal{L}_{\text{NRQCD}}$ is made of all low-energy operators \mathcal{O}_n that may be built from the effective degrees of freedom and are consistent with the symmetries of \mathcal{L}_{QCD} .
- The Wilson coefficients c_n encode the high energy physics. They are calculated by imposing that $\mathcal{L}_{\text{NRQCD}}$ and \mathcal{L}_{QCD} describe the same physics at $\mu = M$.

In summary...

- Provides a QM description from FT: the matching coefficients are the interaction potentials and the leading order dynamical equation is of the Schrödinger type.
- The degrees of freedom in pNRQCD (at weak coupling) are color singlet and octet fields and ultrasoft gluon fields.
- Account for non-potential terms as well. Singlet to Octet transitions via ultrasoft gluons provide loop corrections to the leading potential picture.
- The Quantum Mechanical divergences are canceled by the NRQCD matching coefficients.
- Poincaré invariance is realized via exact relations between different matching coefficients.

Potential nonrelativistic QCD is the state-of-the-art tool for addressing Quarkonium bound state properties

- Conventional meson spectrum: higher order perturbative corrections in v and α_s .
- Inclusive and semi-inclusive decays, E1 and M1 transitions, EM line-shapes.
- Doubly- and triply-heavy baryons.
- Precise extraction of Standard Model parameters: m_c , m_b , α_s , ...
- Exotic states such as gluelumps and hybrids.
- Properties of Quarkonium systems at finite temperature.

☞ One first makes the substitution

$$(m, V_s(r)) = (m_X + \delta m_X, V_{s,X}(r) - 2 \delta m_X)$$

where

$$\delta m_X^{(N)}(\nu_f) = \nu_f \sum_{k=0}^N \delta m_X^{(k)} \left(\frac{\nu_f}{\nu} \right) \alpha_s^{k+1}(\nu)$$

represents a residual mass that encodes the pole mass renormalon contribution, and X stands for the specific renormalon subtraction scheme.

☞ We mainly use the RS' scheme:

$$\delta m_{\text{RS}'}^{(0)} = 0$$

$$\delta m_{\text{RS}'}^{(1)} \left(\frac{\nu_f}{\nu} \right) = N_m \frac{\beta_0}{2\pi} S(1, b)$$

$$\delta m_{\text{RS}'}^{(2)} \left(\frac{\nu_f}{\nu} \right) = N_m \left(\frac{\beta_0}{2\pi} \right) \left[S(1, b) \frac{2d_0(\nu, \nu_f)}{\pi} + \left(\frac{\beta_0}{2\pi} \right) S(2, b) \right]$$

$$\delta m_{\text{RS}'}^{(3)} \left(\frac{\nu_f}{\nu} \right) = N_m \left(\frac{\beta_0}{2\pi} \right) \left[S(1, b) \frac{3d_0^2(\nu, \nu_f) + 2d_1(\nu, \nu_f)}{\pi^2} + \left(\frac{\beta_0}{2\pi} \right) S(2, b) \frac{3d_0(\nu, \nu_f)}{\pi} + \left(\frac{\beta_0}{2\pi} \right)^2 S(3, b) \right]$$

where

$$d_k(\nu, \nu_f) = \beta_k / 2^{1+2k} \ln \frac{\nu}{\nu_f}, \quad S(n, b) = \sum_{k=0}^n c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)},$$

b , c_0 , c_1 and c_2 are coefficients that only depend on the β 's and N_m is a constant.

The last improvement to the static potential that we consider herein is the absorption of the large logarithms into the running coupling constant.

$$V_{s,RS'}^{(N)}(\nu, \nu_f, \nu_r, r) = \begin{cases} V_s^{(N)} + 2\delta m_{RS'}^{(N)} \Big|_{\nu=\nu} \equiv \sum_{k=0}^N V_{s,RS'}^{(k)} \alpha_s^{k+1}(\nu) & \text{if } r > \nu_r^{-1}, \\ V_s^{(N)} + 2\delta m_{RS'}^{(N)} \Big|_{\nu=1/r} \equiv \sum_{k=0}^N V_{s,RS'}^{(k)} \alpha_s^{k+1}(1/r) & \text{if } r < \nu_r^{-1}, \end{cases}$$

- ☞ This has to be done carefully in order not to destroy the renormalon cancellation achieved order by order in α_s .
- ☞ Large values of ν_f imply a large infrared cutoff. This makes our scheme become closer to a $\overline{\text{MS}}$ -like scheme. Such schemes still achieve renormalon cancellation but jeopardize the power counting.
- ☞ Low values of ν_f are preferred with the constraint that one should still obtain the renormalon cancellation.
- ☞ By also taking a low value of $\nu_r = \nu_f$ we find that the convergence is accelerated and the scale dependence is significantly reduced.