Radiative decays of light-quark mesons to a pion revisited in the covariant oscillator quark model

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OUTLINE

- Introduction
- The covariant oscillator quark model
- Electromagnetic currents for quark-antiquark meson systems
- Radiative decay widths of light-quark mesons to a pion
- Supplementary discussions
- Conclusions

INTRODUCTION (1)

- Difficulty in dealing with pions within the constituent quark model
  - The pion has an exceptionally light mass compared with other ground-state hadrons.
  - Decay form factors, which come from overlaps between initial- and final-state wave functions, have anomalous behaviors due to too small mass of the pion.
  - In actual applications to decay processes the "physical or symmetric mass" ambiguity exists on the treatment of the pion and other hadron masses.

INTRODUCTION (2)

- The cause of these difficulties in view of QCD
  - The pion is a pseudo-Nambu-Goldstone boson associated with the spontaneous breaking of chiral symmetry in QCD.
  - The NG boson nature of the pion is not incorporated into the constituent quark model, including the present covariant oscillator quark model (COQM).

INTRODUCTION (3)

- A possible approach to the solution of the difficulty with pions in the COQM
  - The pion mass is treated as an additional parameter which should be determined by comparing the model predictions with the corresponding experiments.
  - For the other hadrons their masses are taken to be the corresponding experimental values.

INTRODUCTION (4)

- In the COQM hadrons themselves and their interactions are formulated in a manifestly covariant way. It should, therefore, be noted that there is no ambiguity associated with a choice of the relativistic or nonrelativistic phase space, as in the nonrelativistic quark model.

INTRODUCTION (5)

- Consideration on the dimension of bilocal meson fields in the COQM
  - Bilocal meson fields for quark-antiquark meson systems have been so far treated as the bosonic fields independent of constituent quark flavors in our previous works.
  - It is argued that the bilocal meson fields should be bosonic for light-quark meson systems, while fermionic for heavy-light and heavy-heavy meson systems.

THE COVARIANT OSCILLATOR QUARK MODEL (1)

- Basic framework of the COQM

In the COQM quark-antiquark meson systems are described by the bilocal field

$$\Psi(x_1, x_2)_\alpha{}^\beta = \Psi(X, x)_\alpha{}^\beta$$

where x_1^μ (x_2^μ) is the space-time coordinate, α (β) the Dirac spinor index of the constituent quark (antiquark), and the center-of-mass and relative coordinates are defined, respectively, by

$$X^\mu = \frac{m_1 x_1^\mu + m_2 x_2^\mu}{m_1 + m_2}, \ x^\mu = x_1^\mu - x_2^\mu$$

THE COVARIANT OSCILLATOR QUARK MODEL (2)

with the constituent quark (antiquark) mass m_1 (m_2).

The bilocal meson field is required to satisfy the Klein-Gordon-type equation

$$\left(-\frac{\partial^2}{\partial X_\mu \partial X^\mu} - \mathcal{M}^2(x)\right)\Psi(X,x)_\alpha{}^\beta = 0$$

with the squared-mass operator, in the pure confining force limit,

$$\mathcal{M}^2(x) = 2(m_1+m_2)\left(\frac{1}{2\mu}\frac{\partial^2}{\partial x_\mu x^\mu} + U(x)\right),\ U(x) = -\frac{1}{2}K x_\mu x^\mu + \text{const.}$$

THE COVARIANT OSCILLATOR QUARK MODEL (3)

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass and K is the spring constant.

A solution of the above Klein-Gordon-type equation can be written as

$$\Psi(X, x)_\alpha{}^\beta = N e^{\mp i P_\mu X^\mu} \Phi(v, x)_\alpha{}^\beta, \quad v^\mu = P^\mu / M$$

where N is the normalization constant for the plane wave, P^μ and M are the center-of-mass momentum and mass, respectively, of the whole meson system, and $\Phi(v, x)_\alpha{}^\beta$ is the internal wave function which is given by

THE COVARIANT OSCILLATOR QUARK MODEL (4)

the direct product of eigenfunctions of the squared-mass operator and the Bargmann-Wigner spinor functions, defined by the direct tensor product of respective Dirac spinors, with the meson four velocity v^{μ}, for the constituent quark and antiquark.

THE COVARIANT OSCILLATOR QUARK MODEL (5)

- Key features of the COQM

In order to freeze the redundant freedom of relative time for the four-dimensional harmonic oscillator, which gives here the squared-mass operator, the definite-metric-type subsidiary condition is adopted. The space-time wave functions satisfying this condition is normalizable and leads to the desirable asymptotic behavior of electromagnetic form factors of hadrons.

THE COVARIANT OSCILLATOR QUARK MODEL (6)

The eigenvalues of the squared-mass operator are given by

$$M_N^2 = M_0^2 + N\Omega, \quad \Omega = 2(m_1 + m_2)\sqrt{\frac{K}{\mu}}$$

where $N = 2N_r + L$, N_r (L) being the radial (orbital angular momentum) quantum number. This squared-mass formula gives linear Regge trajectories with the slope Ω^{-1}, in accord with the well-known experimental fact.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (1)

- Lagrangians for the free bilocal meson fields

The above Klein-Gordon-type equation is rewritten in terms of the quark and antiquark coordinates as

$$2(m_1+m_2)\left(\sum_{i=1}^{2}\frac{-1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}\partial x_i^{\mu}}-U(x_1,x_2)\right)\Psi(x_1,x_2)_\alpha{}^\beta=0$$

or, equivalently,

$$\left(\sum_{i=1}^{2}\frac{-1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}\partial x_i^{\mu}}-U(x_1,x_2)\right)\Psi(x_1,x_2)_\alpha{}^\beta=0$$

Electromagnetic currents for quark-antiquark meson systems (2)

This equation is derived from either of the actions

$$S_{\text{free}}^{(\text{KG,S})} = \int d^4x_1 \int d^4x_2 \; \mathcal{L}_{\text{free}}^{(\text{KG,S})}(\Psi, \partial_{1\mu}\Psi, \partial_{2\mu}\Psi)$$

with the Klein-Gordon-like Lagrangian

$$\mathcal{L}_{\text{free}}^{(\text{KG})} = \overline{\Psi}(x_1, x_2) 2(m_1 + m_2) \left(\sum_{i=1}^{2} \frac{-1}{2m_i} \frac{\overleftarrow{\partial}}{\partial x_{i\mu}} \frac{\overrightarrow{\partial}}{\partial x_i^{\mu}} - U(x_1, x_2) \right) \Psi(x_1, x_2)$$

and the Schrödinger-like Lagrangian

$$\mathcal{L}_{\text{free}}^{(\text{S})} = \overline{\Psi}(x_1, x_2) \left(\sum_{i=1}^{2} \frac{-1}{2m_i} \frac{\overleftarrow{\partial}}{\partial x_{i\mu}} \frac{\overrightarrow{\partial}}{\partial x_i^{\mu}} - U(x_1, x_2) \right) \Psi(x_1, x_2)$$

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (3)

where the bilocal field $\Psi(x_1, x_2)$ has the dimension of bosons [M^1] and fermions [$M^{3/2}$] for $\mathcal{L}_{\rm free}^{\rm (KG)}$ and $\mathcal{L}_{\rm free}^{\rm (S)}$, respectively.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (4)

- Electromagnetic currents of quark-antiquark meson systems

The interaction of quark-antiquark meson systems with an electromagnetic field can be obtained by the minimal substitutions

$$\frac{\partial}{\partial x_i^\mu} \to \frac{\partial}{\partial x_i^\mu} + ieQ_iA_\mu(x_i)$$

in the free Lagrangians $\mathcal{L}_{\mathrm{free}}^{(\mathrm{KG,S})}$, in which the heuristic prescription by Feynman, Kislinger, and Ravndal (1971) is adopted as

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (5)

the following replacements

$$\frac{\overleftarrow{\partial}}{\partial x_{i\mu}}\frac{\overrightarrow{\partial}}{\partial x_i^\mu} \to \frac{\overleftarrow{\partial}}{\partial x_i^\mu}\gamma^\mu\gamma^\nu\frac{\overrightarrow{\partial}}{\partial x_i^\nu}$$

Here $Q_i \ (i = 1, 2)$ are the quark and antiquark charges in units of e. Then the action for the electromagnetic interaction of meson systems is obtained, up to the first order of e, as

$$S_{\rm int}^{\rm (KG,S)} = \int d^4x_1 \int d^4x_2 \sum_{i=1}^{2} j_i^{{\rm (KG,S)}\mu}(x_1, x_2) A_\mu(x_i) \equiv \int d^4X J^{{\rm (KG,S)}\mu}(X) A_\mu(X)$$

with the conserved currents

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (6)

$$j_i^{(\mathrm{KG})\mu}(x_1,x_2) = 2(m_1+m_2)\left\langle \overline{\Psi}(x_1,x_2)\frac{-ieQ_i}{2m_i}\left(\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}} - ig_M^{(i)}\sigma^{\mu\nu}\left(\frac{\overrightarrow{\partial}}{\partial x_i^\nu}+\frac{\overleftarrow{\partial}}{\partial x_i^\nu}\right)\right)\Psi(x_1,x_2)\right\rangle$$

and

$$j_i^{(\mathrm{S})\mu}(x_1,x_2) = \frac{j_i^{(\mathrm{KG})\mu}(x_1,x_2)}{2(m_1+m_2)}$$

where $\langle\cdots\rangle$ means taking trace concerning the Dirac indices and $g_M^{(i)}$ are the parameters related to the anomalous magnetic moments of constituent quarks. The electric charges of meson systems are given by the diagonal elements

$$Q_{\mathrm{meson}}^{(\mathrm{KG,S})} = \int d^3X\ \langle i|J^{(\mathrm{KG,S})0}(X)|i\rangle$$

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (7)

- Features of the Klein-Gordon-like current
  - The meson charge is given by

$$Q^{(\mathrm{KG})}_{\mathrm{meson}} = (Q_1 + Q_2)e$$

    which reproduces the physical one correctly.
  - The currents $j_i^{(\mathrm{KG})\mu}(x_1, x_2)$ do not have the absolute mass scales of quarks. This would seem to be natural for light-quark meson systems from the viewpoint of QCD in the chiral limit.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (8)

- Features of the Schrödinger-like current
  - In this case the meson charge becomes

$$Q^{(\mathrm{S})}_{\mathrm{meson}} = \frac{M}{m_1 + m_2}(Q_1 + Q_2)e$$

where M is the meson mass. This expression does not generally coincide with the physical meson charge. However, in the heavy quark limit, it gives the correct charge. This means that the Schrödinger-like current is applicable to heavy-light and heavy-heavy meson systems.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (9)

- If the meson masses for nonrelativistic quark systems are written as

$$M_n = (m_1 + m_2) + \mathcal{E}_n$$

with the n-th excitation energy $\mathcal{E}_n$, then the meson charge can be expressed as

$$Q^{(\mathrm{S})}_{\mathrm{meson}} = \left(1 + \frac{\mathcal{E}_n}{M_0}\right)(Q_1 + Q_2)e$$

where M_0 is a mass of the ground-state meson.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (10)

From this expression it is found that the applicability of the schrödinger-like current is estimated by the ratio of the excitation energy to the ground-state mass.

- The currents $j_i^{(\mathrm{S})\mu}(x_1, x_2)$, unlike $j_i^{(\mathrm{KG})\mu}(x_1, x_2)$, have the absolute mass scales of quarks. This is a desirable feature in describing the nonrelativistic quark systems.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (11)

- The dimension of bilocal meson fields

  From the above considerations, it is concluded that the dimension of bilocal meson fields is bosonic for light-quark meson systems, while fermionic for heavy-light and heavy-heavy meson systems.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (1)

- Radiative decay widths of light-quark mesons

The above effective electromagnetic interactions $S_{\rm int}^{\rm (KG,S)}$ describe systematically all the electromagnetic interactions of quark-antiquark meson systems.

For the radiative transitions of light-quark mesons the decay width is obtained, following the usual procedure with the Klein-Gordon-like interaction $S_{\rm int}^{\rm (KG)}$, as

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (2)

$$\Gamma = \frac{1}{2J_i + 1} \frac{|\mathbf{q}|}{8\pi M_i^2} \sum_{\text{spin}} |\mathcal{M}_{fi}|^2$$

where M_i (J_i) are the mass (spin) of the initial-state meson and $|\mathrm{q}|$ is the photon three-momentum.

In the actual applications of the above formula to radiative decay widths of light-quark mesons the physical masses are used for initial- and final-state mesons, except for the pion.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (3)

- Parameter determination

In this talk we restrict ourselves to a discussion on the radiative decays of I=1 mesons with nonstrange quarks. Then the present radiative decay model has two parameters, Ω and g_M, the inverse of the Regge slope and the parameter related to the anomalous magnetic moment of u and d quarks.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (4)

Here we take the value of $\Omega = 1.14\ \mathrm{GeV}^2$ and determine a value of g_M so as to fit the experimental width for $\rho \to \pi\gamma$.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (5)

- Choices for the effective pion mass
  - Case A
    Assuming that the pion lies on the spin-singlet, b_1-π_2, trajectory, the effective pion mass is 0.476 GeV. Calculating the numerical width for $\rho \to \pi\gamma$ with this pion mass, $g_M = 1.51$ is obtained.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (6)

- Case B

  Assuming that the pion and $\pi_2(1670)$ lie on the same trajectory with $\Omega = 1.14\ \mathrm{GeV}^2$, the effective pion mass is 0.719 GeV. In this case the width for $\rho \to \pi\gamma$ is calculated with this pion mass only for the exponential factor of the formula, while the physical pion mass for the other part. This treatment gives $g_M = 0.743$.

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (7)

- Numerical results of the radiative decay widths in keV

Decay	Case A	Case B	Experiment
$\rho \to \pi\gamma$	68 (fit)	68 (fit)	68 ± 7
$a_1(1260) \to \pi\gamma$	464	394	640 ± 246
$a_2(1320) \to \pi\gamma$	340	250	311 ± 25
$b_1(1235) \to \pi\gamma$	40	52	230 ± 60
$\pi_2(1670) \to \pi\gamma$	88	150	181 ± 29

RADIATIVE DECAY WIDTHS OF LIGHT-QUARK MESONS TO A PION (8)

- ✓ Experimental widths are taken from PDG 2017.
- ➢ The agreement with experiment is satisfactory, though that for $\pi_2(1670) \to \pi\gamma$ less so in Case A, except for $b_1(1235) \to \pi\gamma$.
- ➢ It should be noted that only the electric (convection) current, independent of g_M, contributes to the decay processes $b_1(1235) \to \pi\gamma$ and $\pi_2(1670) \to \pi\gamma$.

SUPPLEMENTARY DISCUSSIONS (1)

- A comment on the decay $f_1(1285) \to \rho^0\gamma$

The present model predicts, using the parameter values fixed above, the decay width for $f_1(1285) \to \rho^0\gamma$ to be

$$\begin{aligned}\Gamma(f_1(1285) \to \rho^0\gamma) &= 752 \text{ keV} \quad (\text{Case A}) \\ &= 369 \text{ keV} \quad (\text{Case B})\end{aligned}$$

where $f_1(1285)$ is taken to be the pure $u\bar{u} + d\bar{d}$ state.

SUPPLEMENTARY DISCUSSIONS (2)

The experimental width for the decay $f_1(1285) \to \rho^0\gamma$ is as follows:

$$\begin{aligned}\Gamma(f_1(1285) \to \rho^0\gamma) &= 1203 \pm 331 \text{ keV} \quad (\text{PDG 2017}) \\ &= 545 \pm 253 \text{ keV} \quad (\text{CLAS 2016}) \\ &= 636 \pm 240 \text{ keV} \quad (\text{VES 1995})\end{aligned}$$

where the PDG 2017 values are used for the total width and $\eta\pi\pi$ branching ratio of $f_1(1285)$.

The present model calculation strongly supports the experimental results of VES 1995 and CLASS 2016.

CONCLUSIONS

- The dimension of bilocal meson fields is bosonic for light-quark meson systems, while fermionic for heavy-light and heavy-heavy meson systems.
- The calculated results for the radiative decays of $a_1,\ a_2,\ b_1,\ \pi_2$ mesons to a pion are in fair agreement with experiment, except for $b_1(1235) \to \pi\gamma$.
- For the decay $f_1(1285) \to \rho^0\gamma$ the present model strongly supports the experimental results of VES 1995 and CLASS 2016.