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POSSIBLE EFFECT OF MIXED PHASE AND DECONFINEMENT UPON SPIN CORRELATIONS IN THE $\Lambda\bar{\Lambda}$ PAIRS GENERATED IN RELATIVISTIC HEAVY-ION COLLISIONS

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Spin correlations for the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, generated in relativistic heavy-ion collisions, and related angular correlations at the joint registration of space-parity nonconserving hadronic decays of two hyperons are theoretically analyzed. The correlation tensor components can be derived by the method of "moments" – as a result of averaging the combinations of trigonometric functions of proton (antiproton) flight angles over the double angular distribution of flight directions for products of two decays. The properties of the "trace" of the correlation tensor (a sum of three diagonal components), determining the angular correlations as well as the relative fractions of the triplet states and singlet state of respective pairs, are discussed. In this report, spin correlations for two identical particles ($\Lambda\Lambda$) and two non-identical particles ($\Lambda\bar{\Lambda}$) are generally considered from the viewpoint of the conventional model of one-particle sources. Within this model, correlations vanish at enough large relative momenta. However, under these conditions (especially at ultrarelativistic energies), in the case of two non-identical particles ($\Lambda\bar{\Lambda}$) the two-particle annihilation sources – quark-antiquark and two-gluon ones – start playing a noticeable role and lead to the difference of the correlation tensor from zero. In particular, such a situation may arise, when the system passes through the "mixed phase" and – due to the multiple production of free quarks and gluons in the process of deconfinement of hadronic matter – the number of two-particle sources strongly increases.

1 Introduction

Spin correlations for the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, generated in heavy ion collisions, and respective angular correlations at the joint registration of hadronic decays of two hyperons with space parity nonconservation give important information about the character and mechanism of multiple processes. The advantage of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ systems over other ones is due to the fact that the P-odd decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ serve as effective analyzers of spin state of the Λ and $\bar{\Lambda}$ particles. In connection with this, spin correlations in the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ systems can be rather easily distinguished and studied experimentally by the method of "moments" over the background of a large amount of produced secondary particles. This fact is especially meaningful for the investigations of multiple generation at modern and future ion colliders like RHIC, LHC, NICA, since the polarization parameters – especially for the $\Lambda\bar{\Lambda}$ pair – are very sensitive to the scenario of process after the act of collision of relativistic heavy ions .

2 General structure of the spin density matrix of the pairs $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$

The spin density matrix of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, just as the spin density matrix of two spin-1/2 particles in general, can be presented in the following form [1,2,3]:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{P}_2) + \\ + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}]; \tag{1}$$

in doing so, $tr_{(1,2)}\hat{\rho}^{(1,2)} = 1$.

Here $\hat{I}$ is the two-row unit matrix, $\hat{\boldsymbol{\sigma}} = (\ \hat{\sigma}_x,\ \hat{\sigma}_y,\ \hat{\sigma}_z\ )$ is the vector Pauli operator $(x, y, z \to 1, 2, 3\ )$, $\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarization vectors of first and second particle ($\mathbf{P}_1 = \langle \hat{\boldsymbol{\sigma}}^{(1)} \rangle$, $\mathbf{P}_2 = \langle \hat{\boldsymbol{\sigma}}^{(2)} \rangle$), $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$ are the correlation tensor components . In the general case $T_{ik} \neq P_{1i}P_{2k}$. The tensor with components $C_{ik} = T_{ik} - P_{1i}P_{2k}$ describes the spin correlations of two particles .

The respective one-particle density matrices are as follows:

$$\hat{\rho}^{(1)} = \frac{1}{2}(\hat{I}^{(1)} + \mathbf{P}_1\hat{\boldsymbol{\sigma}}^{(1)}), \qquad \hat{\rho}^{(2)} = \frac{1}{2}(\hat{I}^{(2)} + \mathbf{P}_2\hat{\boldsymbol{\sigma}}^{(2)}). \tag{2}$$

The "trace" of the correlation tensor is $T = T_{xx} + T_{yy} + T_{zz} =$ $= \langle \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)} \rangle$. The eigenvalues of the operator $\hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)}$ equal $\lambda_t = 1$ for three triplet states (total spin $S = 1$) and $\lambda_s = -3$ for the singlet state (total spin $S = 0$) .

3 Spin correlations and angular correlations at joint registration of decays of two Λ particles into the channel $\Lambda \to p + \pi^-$

Any decay with the space parity nonconservation may serve as an analyzer of spin state of the unstable particle [3].

The normalized angular distribution at the decay $\Lambda \to p + \pi^-$ takes the form:

$$\frac{d\, w(\mathbf{n})}{d\,\Omega_{\mathbf{n}}} = \frac{1}{4\,\pi}(1 + \alpha_\Lambda \mathbf{P}_\Lambda\, \mathbf{n})\,. \tag{3}$$

Here $\mathbf{P}_\Lambda$ is the polarization vector of the Λ particle, $\mathbf{n}$ is the unit vector along the direction of proton momentum in the rest frame of the Λ particle, α_Λ is the coefficient of P-odd angular asymmetry ($\alpha_\Lambda = 0.642$). The decay $\Lambda \to p + \pi^-$ selects the projections of spin of the Λ particle onto the direction of proton momentum; the analyzing power equals $\boldsymbol{\xi} = \alpha_\Lambda \mathbf{n}$.

Now let us consider the double angular distribution of flight directions for protons formed in the decays of two Λ particles into the channel $\Lambda \to p + \pi^-$, normalized by unity (the analyzing powers are $\boldsymbol{\xi}_1 = \alpha_\Lambda \mathbf{n}_1$, $\boldsymbol{\xi}_2 = \alpha_\Lambda \mathbf{n}_2$). It is described by the following formula [2,3]:

$$\frac{d^2\, w(\mathbf{n}_1, \mathbf{n}_2)}{d\,\Omega_{\mathbf{n}_1}\, d\,\Omega_{\mathbf{n}_2}} = \frac{1}{16\,\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_1\, \mathbf{n}_1 + \alpha_\Lambda \mathbf{P}_2\, \mathbf{n}_2 + \alpha_\Lambda^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik} n_{1i} n_{2k}\right], \tag{4}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are polarization vectors of the first and second Λ particle, T_{ik} are the correlation tensor components, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the respective rest frames of the first and second Λ particle, defined in the common (unified) coordinate axes of the c.m. frame of the pair ($i, k = \{1, 2, 3\} = \{x, y, z\}$) .

By using the method of moments, the components of polarization vectors and correlation tensor may be determined as a result of averaging combinations of trigonometric functions of angles of proton flight over the double angular distribution [2,3]:

$$P_{1i} = \frac{3}{\alpha_\Lambda}\langle n_{1i}\rangle, \quad P_{2k} = \frac{3}{\alpha_\Lambda}\langle n_{2k}\rangle, \quad T_{ik} = \frac{9}{\alpha_\Lambda^2}\langle n_{1i} n_{2k}\rangle. \tag{5}$$

Here

$$\langle ....\rangle \equiv \int (....) \left(\frac{d^2\, w(\mathbf{n}_1, \mathbf{n}_2)}{d\,\Omega_{\mathbf{n}_1}\, d\,\Omega_{\mathbf{n}_2}} \right) d\,\Omega_{\mathbf{n}_1}\, d\,\Omega_{\mathbf{n}_2}; \tag{6}$$

$$n_{1x} = \sin\theta_1 \cos\phi_1; \quad n_{1y} = \sin\theta_1 \sin\phi_1; \quad n_{1z} = \cos\theta_1;$$
$$n_{2x} = \sin\theta_2 \cos\phi_2; \quad n_{2y} = \sin\theta_2 \sin\phi_2; \quad n_{2z} = \cos\theta_2, \tag{7}$$

where θ_1 and ϕ_1, θ_2 and ϕ_2 are the polar and azimuthal angles of emission of protons in the rest frames of the first and second Λ particle, respectively – with respect to the unified system of coordinate axes ;

$d\,\Omega_{\mathbf{n}_1} = \sin\theta_1 d\theta_1 d\phi_1$ and $d\,\Omega_{\mathbf{n}_2} = \sin\theta_2 d\theta_2 d\phi_2$ are the elements of solid angles of proton emission .

The double angular distribution may be integrated over all angles except the angle θ between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$:

$$\cos\theta = \mathbf{n}_1 \mathbf{n}_2 = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2). \tag{8}$$

At this integration, the solid angle element $d\,\Omega_{\mathbf{n}_2}$ can be defined, without losing generality, in the coordinate frame with the axis z being parallel to the vector $\mathbf{n}_1$, and the solid angle element $d\,\Omega_{\mathbf{n}_1}$ is defined in the coordinate frame where the polarization parameters are specified:

$$d\,\Omega_{\mathbf{n}_2} = \sin\theta\, d\theta\, d\phi, \qquad d\,\Omega_{\mathbf{n}_1} = \sin\theta_1\, d\theta_1\, d\phi_1;$$

here ϕ is the azimuthal angle of rotation of the vector $\mathbf{n}_2$ around the vector $\mathbf{n}_1$.

So, the angular correlation between the proton momenta at the decays of two Λ particles is expressed as follows:

$$d\,w(\cos\theta) = \left(\int \frac{d^2\, w(\mathbf{n}_1, \mathbf{n}_2)}{d\,\Omega_{\mathbf{n}_1}\, d\,\Omega_{\mathbf{n}_2}}\, d\phi d\,\Omega_{\mathbf{n}_1} \right) \sin\theta d\theta. \tag{9}$$

The angular correlation, being described by the formula [2,3,4,5]

$$d\,w(\cos\theta) = \frac{1}{2}\,(\,1 + \frac{1}{3}\,\alpha_\Lambda^2 T\, \cos\theta)\sin\theta d\,\theta, \tag{10}$$

is determined only by the “trace” of the correlation tensor $T = W_t - 3W_s$, and it does not depend on the polarization vectors (single-particle states may be unpolarized).

So, finally we have:

$$d\,w(\cos\theta) = \frac{1}{2}\,[\,1 - \alpha_\Lambda^2\,(W_s - \frac{W_t}{3})\cos\theta]\sin\theta d\,\theta, \tag{11}$$

W_s and W_t are relative fractions of the singlet state and triplet states, respectively .

4 Correlations at the joint registration of the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$

Due to CP invariance, the coefficients of P-odd angular asymmetry for the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ have equal absolute values and opposite signs: $\alpha_{\bar{\Lambda}} = -\alpha_\Lambda = -0.642$. The double angular distribution for this case is as follows [2,3]:

$$\frac{d^2\,w(\mathbf{n}_1,\mathbf{n}_2)}{d\,\Omega_{\mathbf{n}_1}\,d\,\Omega_{\mathbf{n}_2}} = \frac{1}{16\,\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_\Lambda\,\mathbf{n}_1 - \alpha_\Lambda \mathbf{P}_{\bar{\Lambda}}\,\mathbf{n}_2 - \alpha_\Lambda^2\,\sum_{i=1}^{3}\sum_{k=1}^{3}\,T_{ik} n_{1i} n_{2k}\right], \tag{12}$$

(here $-\alpha_\Lambda = +\alpha_{\bar{\Lambda}}$ and $-\alpha_\Lambda^2 = +\alpha_\Lambda \alpha_{\bar{\Lambda}}$) .

Thus, the angular correlation between the proton and antiproton momenta in the rest frames of the Λ and $\bar{\Lambda}$ particles is described by the expression:

$$d\,w(\cos\theta) = \frac{1}{2}\,(\,1 - \frac{1}{3}\,\alpha_\Lambda^2 T\,\cos\theta)\sin\theta d\,\theta = \frac{1}{2}\,[\,1 + \alpha_\Lambda^2\,(W_s - \frac{W_t}{3})\cos\theta]\sin\theta d\,\theta, \tag{13}$$

where θ is the angle between the proton and antiproton momenta .

5 Spin correlations at the generation of $\Lambda\Lambda$ pairs in multiple processes

Now let us consider the production of $\Lambda\Lambda$ pairs in multiple processes from the viewpoint of the model of one-particle sources ("consitituents") [6], which is widely applied now for describing the momentum–energy correlations and related spin correlations of both the identical and non-identical particles .

It should be noted that the method of correlation femtoscopy, based on the source model, has been used successfully enough for studying the correlations in processes of collision of elementary particles . But, by its essence, this approach is the most adequate one namely for investigating the processes of multiple generation of hadrons, leptons and photons in collisions of relativistic heavy ions .

a) The Fermi-statistics effect leads not only to the momentum-energy $\Lambda\,\Lambda$-correlations at small relative momenta (correlation femtoscopy), but to the spin correlations as well .

The following relation holds, in consequence of the symmetrization or antisymmetrization of the total wave function of any identical particles with nonzero spin (bosons as well as fermions) [8] :

$$(-1)^{S+L} = 1.$$

Here S is the total spin and L is the orbital momentum in the c.m. frame of the pair. At the momentum difference $q = p_1 - p_2 \to 0$ the states with nonzero orbital momenta "die out", and only states with $L = 0$ and even total spin S survive .

Since the Λ-particle spin is equal to 1/2, at $q \to 0$ the $\Lambda\,\Lambda$ pair is generated only in the singlet state with $S = 0$.

Meantime, at the 4-momentum difference $q \neq 0$ there are also triplet states generated together with the singlet state .

Within the conventional model of one-particle sources emitting unpolarized particles, the triplet states with spin projections +1, 0 and −1 are produced with equal probabilities . If correlations are neglected, the singlet state is generated with the same probability, – the relative "weights" are $\widetilde{W}_t = 3/4$, $\widetilde{W}_s = 1/4$.

When taking into account the Fermi statistics and s-wave final-state interaction, which is essential at close momenta (at orbital momenta $L \neq 0$ the contribution of final-state interaction is suppressed), the fractions of triplet states and the singlet state become proportional to the quantities [7,9]:

$$W_t(q) = \frac{3}{4}\,(1 - \langle\cos qx\rangle), \qquad W_s(q) = \frac{1}{4}(1 + \langle\cos qx\rangle + 2\,B_{int}(q)); \quad (14)$$

here $q = p_1 - p_2$ is the difference of 4-momenta, $x = x_1 - x_2$ is the difference of 4-coordinates of two sources .

In the above formula ,

$$\langle \cos qx \rangle = \int W(x) \cos qx \, d^4x$$

is the Fermi-statistics contribution; here $W(x)$ is the distribution of difference of 4-coordinates of two sources; $B_{int}(q)$ is the contribution of s-wave final-state interaction of two Λ particles. In doing so,

$$R(q) = W_t(q) + W_s(q) = 1 - \frac{1}{2} \langle \cos qx \rangle + \frac{1}{2} B_{int}(q) \tag{15}$$

is the correlation function describing the momentum–energy correlations of two Λ particles with close momenta .

The correlation function $R(q)$ represents the ratio of the two-particle spectrum to the non-correlated background, which is constructed usually as a product of one-particle spectra from different events at the same values of momenta. In terms of inclusive cross sections we have [9]:

$$\frac{d^6\sigma}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} = \frac{R(q)}{\sigma_{\text{tot}}} \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} \left(\frac{d^3\sigma}{d^3\mathbf{p}_1} \right) \left(\frac{d^3\sigma}{d^3\mathbf{p}_2} \right), \tag{16}$$

where n is the multiplicity and σ_{tot} is the total interaction cross-section (for the Poisson distribution of multiplicity we have $\langle n(n-1) \rangle / \langle n \rangle^2 = 1$) .

b) The spin density matrix of two Λ particles with close momenta at the emission of unpolarized Λ particles has the following structure:

$$\hat{\rho}^{(1,2)} = \frac{W_s(q)\, \hat{\rho}_s + W_t(q)\, \hat{\rho}_t}{W_s(q) + W_t(q)} =$$
$$= \frac{1}{R(q)} \left[\left(\frac{1}{4} (1 + \langle \cos qx \rangle + 2\, B_{int}(q) \right) \hat{\rho}_s + \frac{3}{4} (1 - \langle \cos qx \rangle) \hat{\rho}_t \right]. \tag{17}$$

Here

$$\hat{\rho}_s = \frac{1}{4} (\hat{I}^{(1)} \otimes \hat{I}^{(2)} - \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)})$$

is the density matrix of the singlet state, and

$$\hat{\rho}_t = \frac{1}{4} (\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{1}{3} \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)})$$

is the density matrix of the unpolarized triplet state, averaged over the spin projections $\lambda = +1, 0, -1$:

$$\hat{\rho}_t = \frac{1}{3}(\hat{\rho}_{t\,+1} + \hat{\rho}_{t\,0} + \hat{\rho}_{t\,-1}); \qquad \hat{\rho}_s + 3\,\hat{\rho}_t = \hat{I}^{(1)} \otimes \hat{I}^{(2)}.$$

It is easy to see that Eq. (17) for $\hat{\rho}^{(1,2)}$ can be rewritten in the form:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\,(\hat{I}^{(1)} \otimes \hat{I}^{(2)} - \frac{\langle \cos qx \rangle + B_{int}(q)}{2\,R(q)}\,\hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)}). \tag{18}$$

The correlation tensor components [2]

$$T_{ik} = C_{ik} = -\frac{\langle \cos qx \rangle + B_{int}(q)}{2 - \langle \cos qx \rangle + B_{int}(q)}\,\delta_{ik} \tag{19}$$

depend upon the momentum difference as well as upon the space–time parameters of the generation region; the "trace" of the correlation tensor amounts to

$$T = \sum_i T_{ii} = -3\,\frac{\langle \cos qx \rangle + B_{int}(q)}{2 - \langle \cos qx \rangle + B_{int}(q)}. \tag{20}$$

Thus, on account of the effects of quantum statistics and final-state interaction, at small relative momenta two identical particles, initially unpolarized ($\mathbf{P}_1 = \mathbf{P}_2 = 0$) and non-correlated by spins, remain unpolarized as well but their spins become correlated .

At $q \to 0$ we obtain: $\langle \cos qx \rangle \to 1$, $T_{ik} \to -\,\delta_{ik}$ (singlet state).

On the other hand, in the limit of large q: $\langle \cos qx \rangle \to 0$, $B_{int}(q) \to 0$, $R(q) \to 1$, $T_{ik} \to 0$, i.e. both the momentum–energy and spin correlations vanish .

c) Now let us consider the emission of Λ particles with equal polarization vectors $\widetilde{\mathbf{P}}_1 = \widetilde{\mathbf{P}}_2 = \widetilde{\mathbf{P}}$ [2].

It should be noted that, at the stage of emission by sources, correlations are absent.

The fraction of the triplet state with the total spin projection $\lambda = +1$ onto the direction of $\widetilde{\mathbf{P}}$ and the respective constituent of the spin density matrix are as follows :

$$\widetilde{W}_{t\,1} = \frac{(1 + \widetilde{P})^2}{4}; \qquad \hat{\rho}_{t\,1} = \frac{1}{4}\,(\hat{I}^{(1)} + \hat{\boldsymbol{\sigma}}^{(1)}\,\mathbf{l}) \otimes (\hat{I}^{(2)} + \hat{\boldsymbol{\sigma}}^{(2)}\,\mathbf{l}); \tag{21}$$

here $\widetilde{P} = |\widetilde{\mathbf{P}}|$ and $\mathbf{l}$ is the unit vector directed along $\widetilde{\mathbf{P}}$.

Analogously, we have the following fractions and spin density matrix constituents:

for the triplet state with total spin projection $\lambda = -1$:

$$\widetilde{W}_{t\,-1} = \frac{(1-\widetilde{P})^2}{4}; \qquad \hat{\rho}_{t\,1} = \frac{1}{4}\,(\hat{I}^{(1)} - \hat{\boldsymbol{\sigma}}^{(1)}\,\mathbf{l}) \otimes (\hat{I}^{(2)} - \hat{\boldsymbol{\sigma}}^{(2)}\,\mathbf{l}); \qquad (22)$$

for the triplet state with total spin projection $\lambda = 0$:

$$\widetilde{W}_{t\,0} = \frac{1-\widetilde{P}^2}{4}; \qquad \hat{\rho}_{t\,0} = \frac{1}{4}\,[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)} - 2\,(\hat{\boldsymbol{\sigma}}^{(1)}\,\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\,\mathbf{l})], \quad (23)$$

and for the singlet state:

$$\widetilde{W}_{s} = \frac{1-\widetilde{P}^2}{4}; \qquad \hat{\rho}_{s} = \frac{1}{4}\,(\hat{I}^{(1)} \otimes \hat{I}^{(2)} - \hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)}). \qquad (24)$$

In doing so, the fractions of spin states $\widetilde{W}_{t\,1}$, $\widetilde{W}_{t\,-1}$ $\widetilde{W}_{t\,0}$, $\widetilde{W}_{s}$ obey the normalization condition

$$\widetilde{W}_{t\,1} + \widetilde{W}_{t\,-1} + \widetilde{W}_{t\,0} + \widetilde{W}_{s} = 1,$$

and the primary spin density matrix is described by the expression :

$$\hat{\rho} = \widetilde{W}_{t\,1}\,\hat{\rho}_{t\,1} + \widetilde{W}_{t\,-1}\,\hat{\rho}_{t\,-1} + \widetilde{W}_{t\,0}\,\hat{\rho}_{t\,0} + \widetilde{W}_{s}\,\hat{\rho}_{s} = \frac{1}{4}\,(\hat{I}^{(1)} + \widetilde{P}\,\hat{\boldsymbol{\sigma}}^{(1)}) \otimes (\hat{I}^{(2)} + \widetilde{P}\,\hat{\boldsymbol{\sigma}}^{(2)}). \qquad (25)$$

At low relative momenta, on account of Fermi statistics and final-state interaction, the fractions of triplet states and singlet state change and become proportional to the following quantities:

$$W_{t\,1}(q) = \frac{(1+\widetilde{P})^2}{4}\,(1 - \langle \cos qx \rangle), \qquad (26)$$

$$W_{t\,-1}(q) = \frac{(1-\widetilde{P})^2}{4}\,(1 - \langle \cos qx \rangle), \qquad (27)$$

$$W_{t\,0}(q) = \frac{1-\widetilde{P}^2}{4}\,(1 - \langle \cos qx \rangle), \qquad (28)$$

$$W_{s}(q) = \frac{1-\widetilde{P}^2}{4}\,(1 + \langle \cos qx \rangle + 2\,B_{int}(q)). \qquad (29)$$

The inclusive cross-section of generation of the $\Lambda\Lambda$ pair with close momenta is proportional to the correlation function describing the momentum–energy correlations:

$$R(q) = W_{t\,1}(q) + W_{t\,-1}(q) + W_{t\,0}(q) + W_s(q) =$$
$$= 1 - \frac{1+\widetilde{P}^2}{2}\langle\cos qx\rangle + \frac{1-\widetilde{P}^2}{2}B_{int}(q). \tag{30}$$

In doing so, the "renormalized" density matrix is determined by the relation:

$$\hat{\rho} = \frac{1}{R(q)}\left(W_{t\,1}(q)\hat{\rho}_{t\,1} + W_{t\,-1}(q)\hat{\rho}_{t\,-1} + W_{t\,0}(q)\hat{\rho}_{t\,0} + W_s(q)\hat{\rho}_s\right). \tag{31}$$

In accordance with this, the polarization parameters of the Λ particles, renormalized due to the effects of Fermi statistics and s-wave final-state interaction, take the form:

$$\mathbf{P}_1 = \mathbf{P}_2 = \frac{1}{R(q)}(1 - \langle\cos qx\rangle)\,\widetilde{\mathbf{P}}; \tag{32}$$

$$T_{ik} = \frac{1}{R(q)}\left[(1 - \langle\cos qx\rangle)\widetilde{P}_i\widetilde{P}_k - \frac{1-\widetilde{P}^2}{2}\left(\langle\cos qx\rangle + B_{int}(q)\right)\delta_{ik}\right]. \tag{33}$$

Irrespective of the primary polarization $\widetilde{P}$, at the momentum difference $q \to 0$ only the singlet state of the $\Lambda\Lambda$ pair is realized, and the renormalized polarization vectors $\mathbf{P}_1 = \mathbf{P}_2$ tend to zero . The s-wave final-state interaction amplifies the predominant role of the singlet state. If $\widetilde{P} = 1$, then in the limit $q \to 0$ the generation of $\Lambda\Lambda$ pairs is forbidden – in full accordance with the Pauli principle .

d) In the c.m. frame of the $\Lambda\Lambda$ pair we have: $q = \{0, 2\mathbf{k}\}$, where $\mathbf{k}$ is the momentum of one of the particles. In doing so, the momentum $\mathbf{k}$ is connected with the relative momentum $\mathbf{q}$ in the laboratory frame by the Lorentz transformation [9] (we use the unit system with $\hbar = c = 1$) :

$$\mathbf{k} = \frac{1}{2}\left[\mathbf{q} + (\gamma - 1)\frac{(\mathbf{q}\mathbf{v})\mathbf{v}}{|\mathbf{v}|^2} - \gamma\mathbf{v}\,q_0\right]; \tag{34}$$

here $\mathbf{v} = (\mathbf{p}_1 + \mathbf{p}_2)/(\varepsilon_1 + \varepsilon_2)$ is the velocity of the $\Lambda\Lambda$ pair in the laboratory frame, $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor, $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ and $q_0 = \varepsilon_1 - \varepsilon_2$.

The Lorentz transformations of 4-coordinates are given by the expressions :

$$\mathbf{r}^* = \mathbf{r} + (\gamma - 1)\,\frac{(\mathbf{r}\mathbf{v})\mathbf{v}}{|\mathbf{v}|^2} - \gamma \mathbf{v}\,t, \qquad t^* = \gamma(t - \mathbf{v}\mathbf{r}), \tag{35}$$

where $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ and $t = t_1 - t_2$.

The interference term connected with identity (quantum statistics) is determined by the expression:

$$\langle \cos qx \rangle = \langle \cos 2\mathbf{k}\mathbf{r}^* \rangle = \int W_{\mathbf{v}}(\mathbf{r}^*) \cos(2\mathbf{k}\mathbf{r}^*) d^3\mathbf{r}^*, \tag{36}$$

where

$$W_{\mathbf{v}}(\mathbf{r}^*) = \int W(x)\,d\,t^* = \int W(\mathbf{r}^*, t^*)\,d\,t^*$$

is the distribution of coordinate difference between two sources in the c.m. frame of the $\Lambda\Lambda$ pair .

Meantime, the contribution of s-wave final-state interaction is expressed as follows (at the sizes of the generation region in the c.m. frame, exceeding the effective radius of interaction of two Λ particles):

$$B_{int}(q) = B^{(\Lambda\Lambda)}(\mathbf{k}, \mathbf{v}) = \int W_{\mathbf{v}}(\mathbf{r}^*)\, b(\mathbf{k}, \mathbf{r}^*) d^3\mathbf{r}^*, \tag{37}$$

where the function $b(\mathbf{k}, \mathbf{r}^*)$ has the structure [2,7,9]:

$$b(\mathbf{k}, \mathbf{r}^*) = |f^{(\Lambda\Lambda)}(k)|^2\,\frac{1}{(r^*)^2} + 2\,\mathrm{Re}\left(f^{(\Lambda\Lambda)}(k)\,\frac{e^{ikr^*}\cos \mathbf{k}\mathbf{r}^*}{r^*}\right) -$$

$$-2\,\pi\,|f^{(\Lambda\Lambda)}(k)|^2\, d_0^{(\Lambda\Lambda)} \delta^3(\mathbf{r}^*). \tag{38}$$

Here $k = |\mathbf{k}|$, $r^* = |\mathbf{r}^*|$, $f^{(\Lambda\Lambda)}(k)$ is the amplitude of low-energy $\Lambda\Lambda$ scattering. In the framework of the effective radius theory [8,10]:

$$f^{(\Lambda\Lambda)}(k) = a_0^{(\Lambda\Lambda)}(1 + \frac{1}{2}\, d_0^{(\Lambda\Lambda)}\, a_0^{(\Lambda\Lambda)}\, k^2 - i\,k\, a_0^{(\Lambda\Lambda)})^{-1}, \tag{39}$$

where, by definition, $(-a_0^{(\Lambda\Lambda)})$ is the length of s-wave scattering and

$$d_0^{(\Lambda\Lambda)} = \frac{1}{k}\,\frac{d}{d\,k}\left(\mathrm{Re}\,\frac{1}{f^{(\Lambda\Lambda)}(k)}\right)$$

is the effective radius .

The integral (37), with expression (38) inside, approximately takes into account the difference of the true wave function of two interacting Λ particles with the momenta $\mathbf{k}$ and $(-\mathbf{k})$ at small distances from the asymptotic wave function of continuous spectrum [7,11] .

In the case of Gauss distribution of 4-coordinates of two independent sources in the laboratory frame, with the mean-square radius $\sqrt{\langle \mathbf{r}_1^2 \rangle} = \sqrt{\langle \mathbf{r}_2^2 \rangle} = \sqrt{3} r_0$ and the mean-square emission time $\sqrt{\langle t_1^2 \rangle} = \sqrt{\langle t_2^2 \rangle} = \tau_0$, we obtain for the function $W_{\mathbf{v}}(\mathbf{r}^*)$ [9]:

$$W_{\mathbf{v}}(\mathbf{r}^*) = \frac{1}{8\pi^{3/2}\,\gamma\, r_0^2 \sqrt{r_0^2 + v^2\tau_0^2}} \exp\left[-\frac{\mathbf{r}^{*2} - (\mathbf{r}^*\mathbf{n})^2}{4r_0^2} - \frac{(\mathbf{r}^*\mathbf{n})^2}{4\gamma^2(r_0^2 + v^2\tau_0^2)} \right]. \tag{40}$$

In doing so,

$$\langle \cos(2\mathbf{k}\mathbf{r}^*) \rangle = \exp\left[-4k^2 r_0^2 - 4\gamma^2 v^2 (\mathbf{k}\mathbf{n})^2 (r_0^2 + \tau_0^2) \right], \tag{41}$$

and the contribution of s-wave final-state $\Lambda\Lambda$ interaction at the momentum $k = 0$ (maximum value) is as follows [7]:

$$B^{(\Lambda\Lambda)}(0, \mathbf{v}) = \frac{1}{\gamma\rho} \left[\frac{1}{2} \frac{(a_0^{(\Lambda\Lambda)})^2}{r_0} \left(A - \frac{d_0^{(\Lambda\Lambda)}}{2\sqrt{\pi} r_0} \right) + \frac{2}{\sqrt{\pi}} C\, a_0^{(\Lambda\Lambda)} \right], \tag{42}$$

where

$$\rho = \sqrt{r_0^2 + v^2\tau_0^2}, \quad A = \frac{1}{u} \arcsin u, \quad C = \frac{1}{2u} \ln \frac{1+u}{1-u}; \quad u = \frac{v\sqrt{r_0^2 + \tau_0^2}}{\rho}.$$

6 Spin correlations at the generation of $\Lambda\bar{\Lambda}$ pairs in multiple processes

In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\bar{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state . At small relative momenta, the s-wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$), in the case of non-identical particles ($\Lambda\bar{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum–energy correlations has the following structure (in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair):

$$R(\mathbf{k},\mathbf{v}) = 1 + \frac{3}{4}\,B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + \frac{1}{4}\,B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}). \tag{43}$$

The spin density matrix of the $\Lambda\bar{\Lambda}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\bar{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4\,R(\mathbf{k},\mathbf{v})}\,\hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)}, \tag{44}$$

and the components of the correlation tensor are as follows:

$$T_{ik} = \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4 + 3\,B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}\,\delta_{ik}; \tag{45}$$

here the contributions of final-state triplet and singlet $\Lambda\bar{\Lambda}$ interaction are determined by the expression (analogously to Eqs. (37,38) for the $\Lambda\Lambda$ interaction [2,7], with the replacement $\cos \mathbf{kr}^* \to e^{i\,\mathbf{kr}^*}$ in Eq.(38) owing to the non-identity of the particles Λ and $\bar{\Lambda}$ [9]) :

$$B_{s(t)}^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = |f_{s(t)}^{(\Lambda\bar{\Lambda})}(k)|^2 \,\langle \frac{1}{(r^*)^2}\rangle + 2\,\mathrm{Re}\left(f_{s(t)}^{(\Lambda\bar{\Lambda})}(k)\,\langle \frac{e^{ikr^*}\,e^{i\,\mathbf{kr}^*}}{r^*}\rangle \right) -$$

$$-\frac{2\pi}{k}\,|f_{s(t)}^{(\Lambda\bar{\Lambda})}(k)|^2\,\frac{d}{d\,k}\left(\mathrm{Re}\,\frac{1}{f_{s(t)}^{(\Lambda\bar{\Lambda})}(k)}\right)\,W_{\mathbf{v}}(0), \tag{46}$$

where $f_{s(t)}^{(\Lambda\bar{\Lambda})}(k)$ is the amplitude of the s-wave low-energy singlet (triplet) $\Lambda\bar{\Lambda}$ scattering .

Let us remark that some information on the low-energy $\Lambda\bar{\Lambda}$ interaction may be obtained, e.g., by investigating the annihilation process $p\bar{p} \to \Lambda\bar{\Lambda}$.

At sufficiently large values of k, one should expect that [7]:

$$B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0, \qquad B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0.$$

In this case the angular correlations in the decays $\Lambda \to p+\pi^-$, $\bar{\Lambda} \to \bar{p}+\pi^+$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \qquad T = 0.$$

7 Angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ and the "mixed phase"

Thus, at sufficiently large relative momenta (for $k \gg m_\pi$) one should expect that the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$, connected with the interaction of the Λ and $\bar{\Lambda}$ hyperons in the final state (i.e. with one-particle sources) are absent . However, if at the considered energy the dynamical trajectory of the system passes through the so-called "mixed phase", then the two-particle sources, consisting of the free quark and antiquark , start playing a noticeable role . For example, the process $s\,\bar{s} \to \Lambda\bar{\Lambda}$ may be discussed .

In this process, the charge parity of the pairs $s\bar{s}$ and $\Lambda\bar{\Lambda}$ is equal to $C = (-1)^{L+S}$, where L is the orbital momentum and S is the total spin of the fermion and antifermion . Meantime, the CP parity of the fermion–antifermion pair is $CP = (-1)^{S+1}$.

In the case of one-gluon exchange, $CP = 1$, and then $S = 1$, i.e. the $\Lambda\bar{\Lambda}$ pair is generated in the triplet state; in doing so, the "trace" of the correlation tensor $T = 1$.

Even if the frames of one-gluon exchange are overstepped, the quarks s and $\bar{s}$, being ultrarelativistic, interact in the triplet state ($S = 1$) . In so doing, the primary CP parity is $CP = 1$, and, due to the CP parity conservation, the $\Lambda\bar{\Lambda}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x . Then at large relative momenta

$$T = x > 0.$$

Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the two-photon process $\gamma\gamma \to e^+e^-$ [12], in this case the "trace" of the correlation tensor is described by the formula (here the process $g\,g \to \Lambda\bar{\Lambda}$ is implied) :

$$T = 1 - \frac{4\,(1-\beta^2)}{1 + 2\,\beta^2\,\sin^2\theta - \beta^4 - \beta^4\,\sin^4\theta}, \tag{47}$$

where β is the velocity of Λ (and $\bar{\Lambda}$) in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame (see [12]). At small β ($\beta \ll 1$) the $\Lambda\bar{\Lambda}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$), whereas at $\beta \approx 1$ – in the triplet state ($S = 1$, $T = 1$). Let us remark that at ultrarelativistic velocities β

(i.e. at extremely large relative momenta of Λ and $\bar{\Lambda}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\Lambda\bar{\Lambda}$ pair ($T = 1$).

8 Summary

So, it is surely advisable to investigate the spin correlations of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs produced in relativistic heavy ion collisions (see also our respective papers, e.g. [13 -19]) .

The spin correlations, as well as the momentum–energy ones, make it possible to determine the space–time characteristics of the multiple particle generation region . In doing so, the best way of studying the spin correlations is the method of angular correlations – method of moments.

Finally, it should be emphasized that, in the general case, the appearance of angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ with the nonzero values of the "trace" of the correlation tensor T at large relative momenta of the Λ and $\bar{\Lambda}$ particles may testify to the passage of the system through the "mixed phase" [13 -19] .

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