

# Fitting and selecting scattering data

E. Ruiz Arriola

Universidad de Granada  
Atomic, Molecular and Nuclear Physics Department

XVII International Conference on Hadron Spectroscopy and Structure  
September 25th-29th, 2017  
Salamanca (Spain)

Rodrigo Navarro Pérez (Athens, Ohio)  
José Enrique Amaro Soriano (Granada),

# The question

- We have  $N$  DATA with UNCERTAINTIES

$$O_1 \pm \Delta O_1 \quad \dots \quad O_N \pm \Delta O_N$$

- We have a theory depending on  $M$ -PARAMETERS

$$O_1(p_1, \dots, p_M) \quad \dots \quad O_N(p_1, \dots, p_M)$$

- Does theory EXPLAIN data ?

YES (Validate)

NO (Falsify)

- Statistical Answer:

If uncertainties are a gaussian distribution

$$O_i^{\text{exp}} = O_i^{\text{th}} + \xi_i \Delta O_i$$

Define the LEAST SQUARES SUM

$$\chi_{\min}^2 \equiv \min_{p_1, \dots, p_M} \sum_{i=1}^N \left[ \frac{O_i(p_1, \dots, p_M) - O_i^{\text{exp}}}{\Delta O_i} \right]^2$$

The probability  $p$  that the theory explains the DATA is  $> 68\%$  if

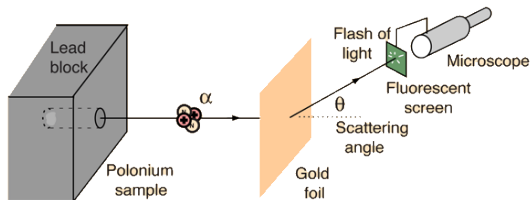
$$\frac{\chi_{\min}^2}{\nu} = 1 \pm \sqrt{\frac{2}{\nu}} \quad \nu = N - M \quad \text{d.o.f (degrees of freedom)}$$

# Theory versus experiment

- A good theory can tell us what experiments are wrong
- A good experiment can tell us what theories are wrong
  - ① Assume theory AND experiment to be correct
  - ② If we find no contradiction we validate theory and experiment
  - ③ Experiment , finite number of data, finite precision
  - ④ Theory , approximations
- Important questions
  - ① Does QCD describe hadronic interactions ?
  - ② Does ChPT describe low energy hadronic interactions ?
- Confidence level (statistics)  
Example: **AB scattering is described by scheme S with 68 percent confidence**

# SCATTERING

# Scattering experiment



- Scattering experiments measure FORCES
- Counting rates

$$\frac{R^2 N_{\text{out}}(\theta, \phi)}{N_{\text{in}}} \rightarrow \sigma(\theta, E) \equiv \frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$$

- Local Normalization

$$\sigma(\theta, E) \xrightarrow{\theta \ll 1} \sigma_{\text{Ruth}}(\theta, E)$$

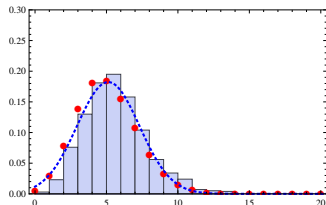
- Total Normalization (mean free path, no Coulomb)

$$l = \frac{1}{n\sigma_T} \quad , \quad \sigma_T = \int d\Omega \frac{d\sigma}{d\Omega}$$

# Counting statistics

- Binomial distribution ( $p$  scattering probability)

$$P_{N,k} = p^k (1-p)^{N-k} \binom{N}{k}, \quad \langle k \rangle = Np, \quad (\Delta k)^2 = \langle k^2 \rangle - \langle k \rangle^2 = Np(1-p)$$



- Binomial  $\rightarrow$  Poisson  $\rightarrow$  Gauss

$$P_{N,k} \xrightarrow[p \ll 1]{} \frac{e^{-Np} (Np)^k}{k!}$$
$$\xrightarrow[k \gg 1]{} \frac{e^{-(k-Np)^2/2}}{\sqrt{2\pi} \Delta k}$$

$p = 0.1 \quad N = 50$

- $N_{\text{out}} = \bar{N}_{\text{out}} \pm \Delta N_{\text{out}}, \quad \Delta N_{\text{out}} = \sqrt{\bar{N}_{\text{out}}}$

- $\sigma(\theta, E)$  is Gauss distributed

$$\sigma(\theta, E) = \bar{\sigma}(\theta, E) \pm \Delta(\theta, E)$$

# Partial wave expansion (No spin)

- Scattering amplitude

$$f(\theta, \phi) = \sum_{l=0}^{\infty} (2l+1) \frac{e^{2i\delta_l} - 1}{2ip} P_l(\cos \theta) \quad , \quad E = \frac{p^2}{2\mu}$$

- Schrödinger Equation

$$\left[ -\frac{\nabla^2}{2\mu} + V(\vec{x}) \right] \Psi(\vec{x}) = E\Psi(\vec{x})$$

- Spherical symmetry  $V(r)$

$$\Psi(\vec{x}) = \frac{u_l(r)}{r} Y_{lm}(\hat{x})$$

- Reduced radial equation

$$-u_l''(r) + \left[ \frac{l(l+1)}{r^2} + 2\mu V(r) \right] u_l(r) = p^2 u_l(r)$$

- Asymptotic conditions

$$u_l(r) \underset{r \rightarrow 0}{\rightarrow} r^{l+1} \quad , \quad u_l(r) \underset{r \rightarrow \infty}{\rightarrow} \sin \left( pr - \frac{l\pi}{2} + \delta_l \right)$$

- GOAL : Determine  $V(r) \pm \Delta V(r)$

# Finite range forces

- Meson exchange picture  $\rightarrow$  Longest range  $\equiv$  Lightest particle

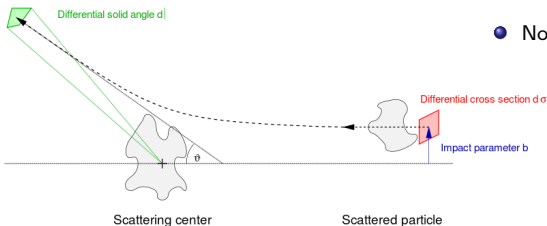
$$r_c \sim \frac{\hbar}{m_\pi c} \sim 1.4\text{fm}$$

- Impact parameter

$$|\vec{L}| = |\vec{x} \wedge \vec{p}| \rightarrow bp \quad L^2 = l(l+1)^2 \sim (l+1/2)^2 \rightarrow l + \frac{1}{2} = bp$$

- No scattering condition

$$\begin{aligned} V(r) &\sim 0 & r \gtrsim r_c \\ \delta_l(p) &\sim 0 & b \gtrsim r_c \\ \rightarrow l_{\max} + \frac{1}{2} &\sim pr_c \sim p/m_\pi \end{aligned}$$



- Truncation in the partial wave expansion

$$f(\theta, \phi) = \sum_{l=0}^{l_{\max}} (2l+1) \frac{e^{2i\delta_l} - 1}{2ip} P_l(\cos \theta) = \frac{e^{2i\delta_0} - 1}{2ip} + 3 \frac{e^{2i\delta_1} - 1}{2ip} \cos \theta + \dots$$



# FITTING

# Single energy fits

- Complete data in a GIVEN energy  $E$

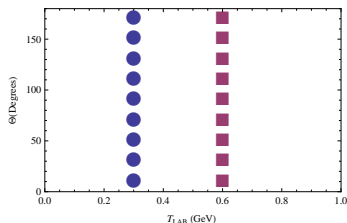
$$\sigma(\theta, E) \rightarrow \sigma(\theta_1, E), \dots, \sigma(\theta_N, E)$$

- Fitting function  $\rightarrow$  Fitting parameters  $\delta_1(E), \dots, \delta_l(E)$

$$\chi^2(\delta_1(E), \dots, \delta_l(E), \nu) = \sum_{i=1}^N \left[ \frac{\sigma^{\text{exp}}(\theta_i, E) - \nu \sigma^{\text{th}}(\theta_i, \delta_1(E), \dots, \delta_l(E))}{\Delta \sigma(\theta_i, E)} \right]^2 + \left( \frac{1 - \nu}{\Delta \nu} \right)^2$$

- Normalization is COMMON for ONE energy
- Phase-shifts are “experimental” and MODEL INDEPENDENT

$$\delta_l^{\text{exp}}(E) \pm \Delta \delta_l^{\text{exp}}(E) \quad , \quad l = 0, \dots, l_{\text{max}}$$



# Multienergy analysis

- Incomplete Data in several energies and angles

$$\sigma(\theta, E) \rightarrow \sigma(\theta_1, E_1), \dots, \sigma(\theta_N, E_N)$$

- The need for interpolation (Smoothness in  $(\theta, E)$ )
- Fitting function *to* Fitting MODEL DEPENDENT parameters  $\mathbf{p} = (p_1, \dots, p_M)$

$$\chi^2(\mathbf{p}, \nu) = \sum_{i=1}^N \left[ \frac{\sigma(\theta_i, E_i)^{\text{exp}} - \nu \sigma^{\text{th}}(\theta_i, E_i, \mathbf{p})}{\Delta\sigma(\theta_i, E_i)} \right]^2 + \left( \frac{1 - \nu}{\Delta\nu} \right)^2$$

- The statement

$$\sigma(\theta_i, E_i)^{\text{exp}} = \nu_0 \sigma^{\text{th}}(\theta_i, E_i, \mathbf{p}_0) \pm \Delta\sigma(\theta_i, E_i)$$

- Too large  $\chi^2/\nu$ 
  - Bad model  $\rightarrow$  SELECT MODEL
  - Bad data  $\rightarrow$  SELECT DATA
  - Bad model and data

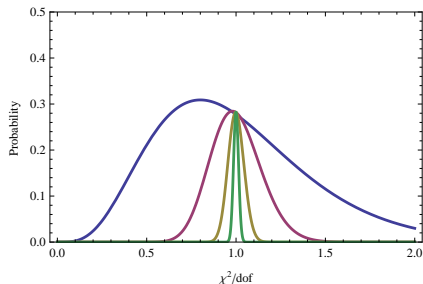
# The $\chi^2$ -test

- If  $\xi_n \in N(0, 1)$

$$P_\nu(\chi^2) = \prod_{n=1}^N \left( \int_{-\infty}^{\infty} d\xi_i \frac{e^{-\xi_i^2/2}}{\sqrt{2\pi}} \right) \delta(\chi^2 - \sum_{n=1}^N \xi_n^2) = \frac{e^{-\chi^2} \chi^{\nu-2}}{2^{\nu/2} \Gamma(\frac{\nu}{2})}$$

- Mean and Variance

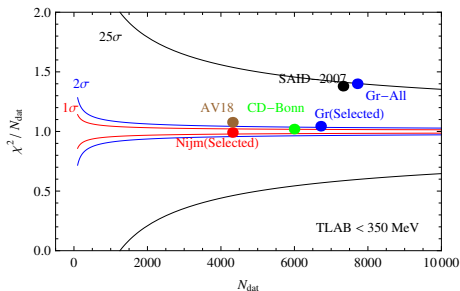
$$\langle \chi^2 \rangle = \nu, \quad \langle (\chi^2 - \langle \chi^2 \rangle)^2 \rangle = 2\nu^2, \quad \rightarrow \chi^2 = \nu \pm \sqrt{2\nu}$$



$\nu$	$\chi^2/\nu$ (68%)
10	$1 \pm 0.447$
100	$1 \pm 0.141$
1000	$1 \pm 0.044$
10000	$1 \pm 0.014$

# To believe or not to believe

$N_{\text{data}}$	HJ62	Reid68	TR575	Paris80	Urb81	Arg84	BonnR	Bonn89	Nijm93
1787	13.5	2.9	3.4	4.5	6.0	7615	1090	25.5	1.8

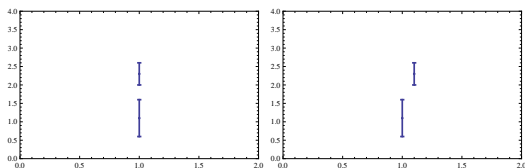


$$\chi_{\min}^2 / \nu = 1 \pm \sqrt{2/\nu}$$

- Charge dependence in OPE
- Magnetic-Moments, Vacuum polarization, ...

# The need for selection

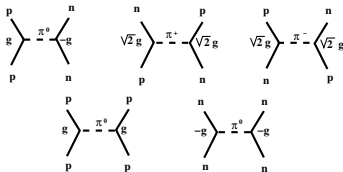
- Example: THE SAID DATABASE
  - PP Data No=25188 Chi2= 48225.043
  - NP Data No=12962 Chi2= 26079.973
  - $\pi$ N 41926 Chi2= 166585.05
  - $\pi$ N 2599 Chi2= 4586.26 (TLAB  $i$  300)
- Which experiments are INCOMPATIBLE ?



- Contribution the  $\chi^2$  will be large (discard BOTH ?)
- If errorbar includes BOTH no contribution to the  $\chi^2$
- Incompatibility is DESTRUCTIVE
- Real physical effect ?

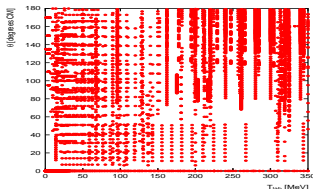
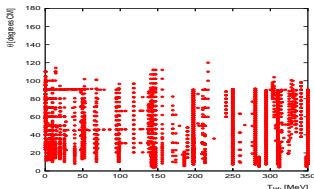
# Long distances

- Nucleons exchange JUST one pion



- Low energies (about pion production) 8000 pp + np scattering data (polarizations etc.)
- Granada coarse grained analysis (2016) (isospin breaking !!)

$$g_p^2/(4\pi) = 13.72(7) \neq g_n^2/(4\pi) = 14.91(39) \neq g_c^2/(4\pi) = 13.81(11) \quad \chi^2/\nu = 1.02$$



# COARSE GRAINING NN (LOW ENERGY)



# Nucleon-Nucleon Scattering

- Scattering amplitude

$$\begin{aligned} M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g - h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\ &+ (g + h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \\ \mathbf{l} &= \frac{\mathbf{k}_f + \mathbf{k}_i}{|\mathbf{k}_f + \mathbf{k}_i|} \quad \mathbf{m} = \frac{\mathbf{k}_f - \mathbf{k}_i}{|\mathbf{k}_f - \mathbf{k}_i|} \quad \mathbf{n} = \frac{\mathbf{k}_f \wedge \mathbf{k}_i}{|\mathbf{k}_f \wedge \mathbf{k}_i|} \end{aligned}$$

- 5 complex amplitudes  $\rightarrow$  24 measurable cross-sections and polarization asymmetries
- Partial Wave Expansion

$$\begin{aligned} M_{m'_s, m_s}^s(\theta) &= \frac{1}{2ik} \sum_{J, l', l} \sqrt{4\pi(2l+1)} Y_{m'_s - m_s}^{l'}(\theta, 0) \\ &\times C_{m_s - m'_s, m'_s, m_s}^{l', s, J} (S_{l, l'}^{J, s} - \delta_{l', l}) C_{0, m_s, m_s}^{l, s, J}, \end{aligned} \quad (1)$$

- S-matrix

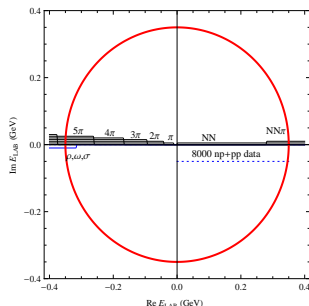
$$S^J = \begin{pmatrix} e^{2i\delta_{J-1}^{J,1}} \cos 2\epsilon_J & ie^{i(\delta_{J-1}^{J,1} + \delta_{J+1}^{J,1})} \sin 2\epsilon_J \\ ie^{i(\delta_{J-1}^{J,1} + \delta_{J+1}^{J,1})} \sin 2\epsilon_J & e^{2i\delta_{J+1}^{J,1}} \cos 2\epsilon_J \end{pmatrix}, \quad (2)$$

# Analytical Structure

- $s = 4(M_N^2 + p^2) \rightarrow E_{\text{LAB}} = 2p^2/M_N$
- Partial Wave Scattering Amplitude analytical for  $|p| \leq m_\pi/2$

$$T_{ll'}^J(p) \equiv S_{ll'}^J(p) - \delta_{l,l'} = p^{l+l'} \sum_n C_{n,l,l'} p^{2n}$$

- Nucleons behave as elementary (AT WHAT SCALE ?)



- Nucleons are heavy  $\rightarrow$  Local Potentials

$$V_{n\pi}(r) \sim \frac{g^{2n}}{r} e^{-nm_\pi r}$$

- Crucial long range electromagnetic effects are local

# The number of parameters (for $E_{\text{LAB}} \leq 350 \text{ MeV}$ )

- At what distance look nucleons point-like ?

$$r > 2\text{fm}$$

- When is OPE the **ONLY** contribution ?

$$r_c > 3\text{fm}$$

- What is the minimal resolution where interaction is elastic ?

$$p_{\text{max}} \sim \sqrt{M_N m_\pi} \rightarrow \Delta r = 1/p_{\text{max}} = 0.6\text{fm}$$

- How many partial waves must be fitted ?

$$l_{\text{max}} = p_{\text{max}} r_c = r_c / \Delta r = 5$$

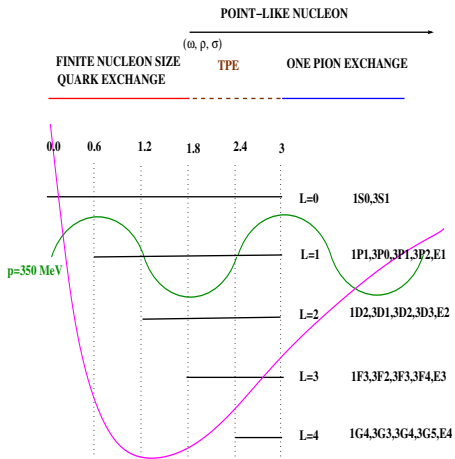
- Minimal distance where centrifugal barrier dominates

$$\frac{l(l+1)}{r_{\text{min}}^2} \leq p^2$$

- How many parameters ?

$$({}^1S_0, {}^3S_1), ({}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2), ({}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3), ({}^1F_3, {}^3F_2, {}^3F_3, {}^3F_4)$$

$$2 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 50$$



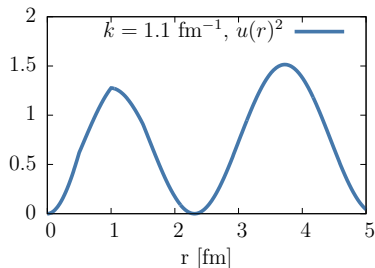
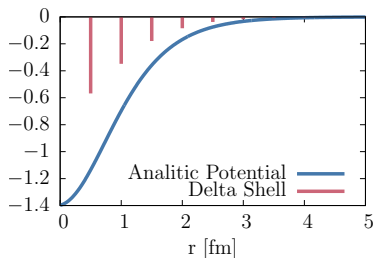
# Delta Shell Potential

- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu} \delta(r - r_i)$$

[Aviles, Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold  $\Delta k \sim 2 \text{ fm}^{-1}$
- Optimal sampling,  $\Delta r \sim 0.5 \text{ fm}$



# Delta Shell Potential

- 3 well defined regions
- Innermost region  $r \leq 0.5$  fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region  $0.5 \leq r \leq 3.0$  fm
  - Unknown interaction
  - $\lambda_i$  parameters fitted to scattering data
- Outermost region  $r \geq 3.0$  fm
  - Long range interaction
  - Described by OPE and **EM effects**
    - Coulomb interaction  $V_{C1}$  and relativistic correction  $V_{C2}$  (pp)
    - Vacuum polarization  $V_{VP}$  (pp)
    - Magnetic moment  $V_{MM}$  (pp and np)

# Fitting NN observables

- Delta shell potential in every partial wave

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{\alpha\beta}} \sum_{n=1}^N (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \quad r \leq r_c = 3.0\text{fm}$$

- Strength coefficients  $\lambda_n$  as fit parameters
- Fixed and equidistant concentration radii  $\Delta r = 0.6$  fm
- EM interaction is crucial for pp scattering amplitude

$$V_{C1}(r) = \frac{\alpha'}{r},$$

$$V_{C2}(r) \approx -\frac{\alpha\alpha'}{M_p r^2},$$

$$V_{VP}(r) = \frac{2\alpha\alpha'}{3\pi r} \int_1^\infty dx e^{-2m_e r x} \left[ 1 + \frac{1}{2x^2} \right] \frac{(x^2 - 1)^{1/2}}{x^2},$$

$$V_{MM}(r) = -\frac{\alpha}{4M_p^2 r^3} [\mu_p^2 S_{ij} + 2(4\mu_p - 1)\mathbf{L}\cdot\mathbf{S}]$$

# STATISTICS



# Self-consistent fits

- We test the assumption

$$O_i^{\text{exp}} = O_i^{\text{th}} + \xi_i \Delta O_i \quad i = 1, \dots, N_{\text{Data}} \quad \xi_i \in N[0, 1]$$

- Least squares minimization  $\mathbf{p} = (p_1, \dots, p_M)$

$$\chi^2(\mathbf{p}) = \sum_{i=1}^N \left( \frac{O_i^{\text{exp}} - F_i(\mathbf{p})}{\Delta O_i^{\text{exp}}} \right)^2 \rightarrow \min_{\mathbf{p}} \chi^2(\mathbf{p}) \equiv \chi^2(\mathbf{p}_0) \quad (3)$$

- Are residuals Gaussian ?

$$R_i = \frac{O_i^{\text{exp}} - O_i^{\text{th}}}{\Delta O_i} \quad O_i^{\text{th}} = F_i(\mathbf{p}_0) \quad i = 1, \dots, N \quad (4)$$

If  $R_i \in N[0, 1]$  self-consistent fit.

- Normality test for a finite sample with N elements  $\rightarrow$  Probability (Confidence level) p-value

$$\chi_{\min}^2 = 1 \pm \sigma \sqrt{\frac{2}{\nu}} \quad \nu = N_{\text{Dat}} - N_{\text{Par}} \quad p = 1 - \int_0^{\sigma} dt \frac{e^{-t^2}}{\sqrt{2\pi}}$$

Histograms, Moments, Kolmogorov-Smirnov, Tail Sensitive QQ-plots

# Normality tests

- Does the sequence

$$x_1^{\text{exp}} \leq x_2^{\text{exp}} \leq \dots \leq x_N^{\text{exp}} \in N[0, 1]$$

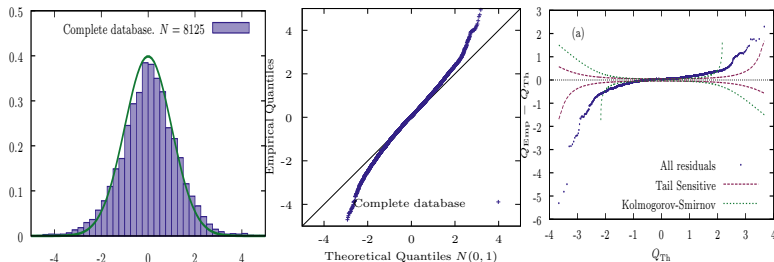
- We compute the theoretical points

$$\frac{n}{N+1} = \int_{-\infty}^{x_n^{\text{th}}} dt \frac{e^{-t^2/2}}{\sqrt{2\pi}}$$

- The Q-Q plot is  $x_n^{\text{th}}$  vs  $x_n^{\text{exp}}$

- For large  $N$

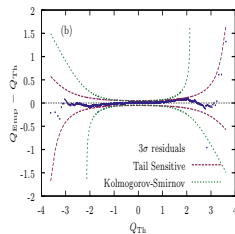
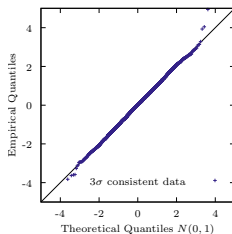
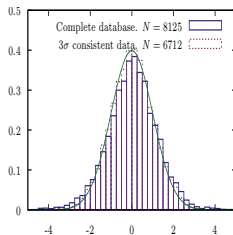
$$x_n^{\text{th}} - x_n^{\text{exp}} = \mathcal{O}(1/\sqrt{N})$$



# Granada-2013 np+pp database

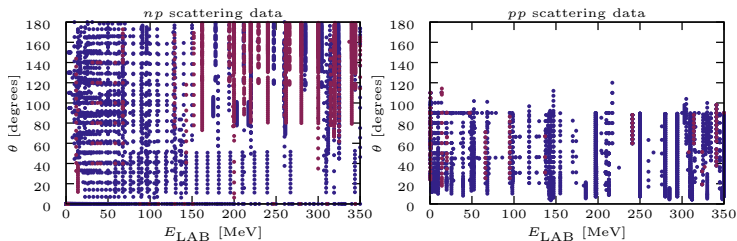
## Selection criterium

- Mutually incompatible data. Which experiment is correct? Is any of the two correct?
- Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  - 1 Fit to all data ( $\chi^2/\nu > 1$ )
  - 2 Remove data sets with improbably high or low  $\chi^2$  ( $3\sigma$  criterion)
  - 3 Refit parameters
  - 4 Re-apply  $3\sigma$  criterion to all data
  - 5 Repeat until no more data is excluded or recovered

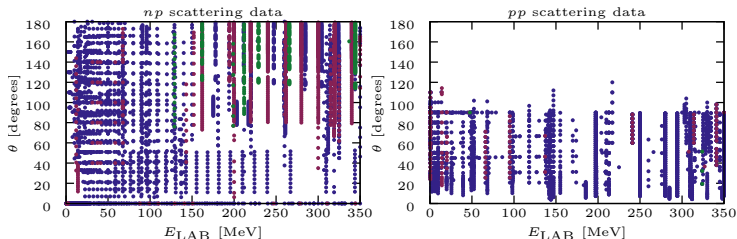


# Selection of data

Usual Nijmegen  $3\sigma$  criterion (1677 rejected data)

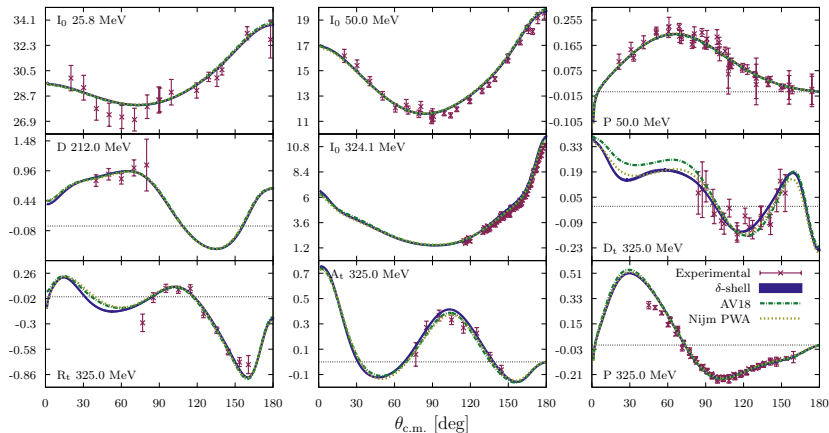


300 recovered data with Granada procedure (consistent database)



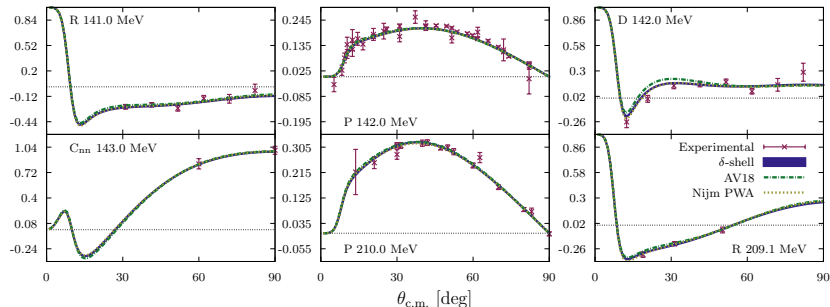
# Scattering Observables

- Comparing with Potentials and Experimental data
- np data



# Scattering Observables

- Comparing with Potentials and Experimental data
- pp data



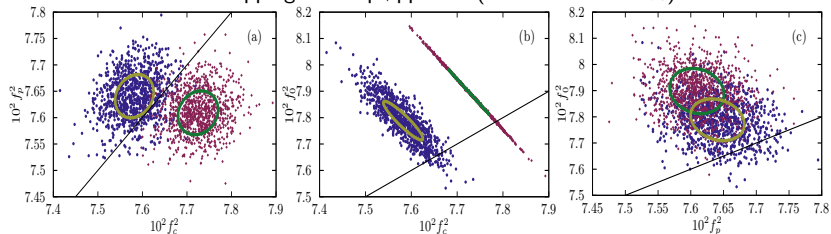
- $\chi^2/d.o.f. = 1.06$  with  $N = 2747|_{pp} + 3691|_{np}$

[RNP, Amaro & Ruiz-Arriola. Phys.Rev.C88 (2013) 024002]

# STATISTICAL CONSEQUENCES

# Coupling constants

Bootstrapping 6713 np+pp data (benchmark  $\sim 0.5\%$ )



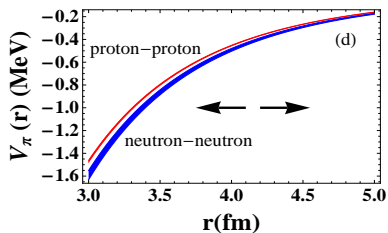
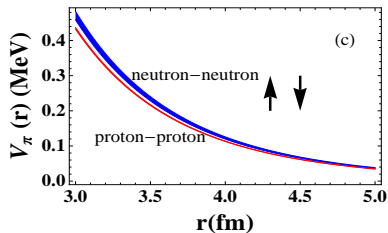
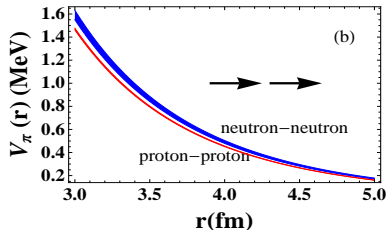
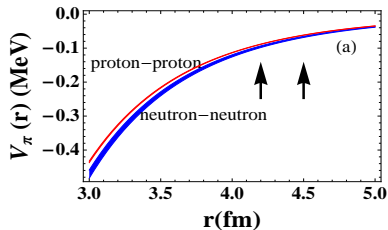
Fits to the Granada-2013 database.

$f^2$	$f_0^2$	$f_c^2$	CD-waves	$\chi_{pp}^2$	$\chi_{np}^2$	$N_{\text{Dat}}$	$N_{\text{Par}}$	$\chi^2/\nu$
0.075	idem	idem	$^1S_0$	3051	3951	6713	46	1.051
0.0761(3)	idem	idem	$^1S_0$	3051	3951	6713	46+1	1.051
-	-	-	$^1S_0, P$	2999	3951.40	6713	46+3	1.043
0.0759(4)	0.079(1)	0.0763(6)	$^1S_0, P$	3045	3870	6713	46+3+9	1.039



# Neutron-Neutron vs Proton-Proton (Polarized)

nn interaction is more intense than pp interaction



# Arqueological Flashback

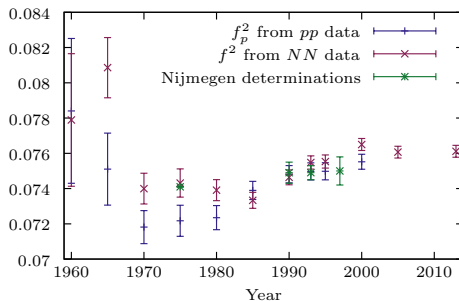
- Can we witness isospin breaking in the couplings ?

$$\left. \frac{dg}{g} \right|_{\text{QCD}} = \mathcal{O}\left(\alpha, \frac{m_u - m_d}{\Lambda_{\text{QCD}}}\right) = \mathcal{O}\left(\alpha, \frac{M_n - M_p}{\Lambda_{\text{QCD}}}\right) \sim 0.01 - 0.02$$

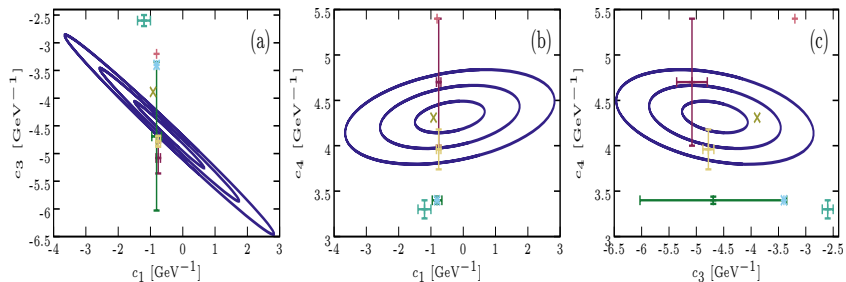
- Statistically yes ! Granada NN  $N = 6713$

$$\left. \frac{dg}{g} \right|_{\text{stat}} = \mathcal{O}\left(\frac{\Delta N_{\text{Dat}}}{N_{\text{Dat}}}\right) = \mathcal{O}\left(\frac{1}{\sqrt{N_{\text{Dat}}}}\right) \rightarrow N \sim 7000 - 10000$$

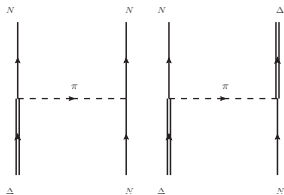
- Chronological recreation of pion-nucleon coupling constants



# Chiral Two Pion Exchange from Granada-2013 np+pp database



# $f_{\pi N\Delta}$ from Granada-2013 np+pp database



- NN potential in the Born-Oppenheimer approximation

$$\bar{V}_{NN,NN}^{1\pi+2\pi+\dots}(\mathbf{r}) = V_{NN,NN}^{1\pi}(\mathbf{r}) + 2 \frac{|V_{NN,N\Delta}^{1\pi}(\mathbf{r})|^2}{M_N - M_\Delta} + \frac{1}{2} \frac{|V_{NN,\Delta\Delta}^{1\pi}(\mathbf{r})|^2}{M_N - M_\Delta} + \mathcal{O}(V^3),$$

- Bulk of TWO-Pion Exchange Chiral forces reproduced
- Fit with  $r_e = 1.8\text{fm}$  to  $N = 6713pp + np$  scattering data

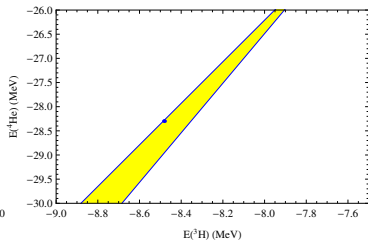
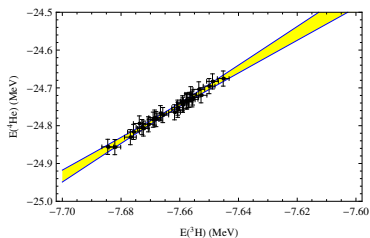
$$f_{\pi N\Delta}/f_{\pi NN} = 2.178(14) \quad \chi^2/\nu = 1.12 \rightarrow h_A = 1.397(9)$$

# Tjon-Lines: numerical accuracy of $A = 2, 3, 4$ Nuclei

(with A. Nogga)

$$\Delta E_{\text{triton}}^{\text{stat}} = 15\text{KeV}$$

$$\Delta E_{\alpha}^{\text{stat}} = 50\text{KeV}$$



- 4-Body forces are masked by numerical noise in the 3 and 4 body calculation if

$$\Delta_t^{\text{num}} > 1\text{KeV}$$

$$\Delta_t^{\text{num}} > 20\text{KeV}$$

# To count or not to count: The Falsification of Chiral Forces

- We can fit CHIRAL forces to ANY energy and look if counterterms are compatible with zero within errors
- We find that if  $E_{\text{LAB}} \leq 125\text{MeV}$  Weinberg counting is INCOMPATIBLE with data.
- You have to promote D-wave counterterms.  
N2LO-Chiral TPE + N3LO-Counterterms  $\rightarrow$  Residuals are normal  
[Piarulli,Girlanda,Schiavilla,Navarro Pérez,Amaro,RA, PRC](#)
- We find that if  $E_{\text{LAB}} \leq 40\text{MeV}$  TPE is INVISIBLE
- We find that peripheral waves predicted by 6th-order chiral perturbation theory ARE NOT consistent with data within uncertainties

$$|\delta^{\text{Ch,N5LO}} - \delta^{\text{PWA}}| > \Delta\delta^{\text{PWA,stat}}$$

5  $\sigma$  incompatible

# CONCLUSIONS

# Conclusions

- Fitting and selecting scattering databases allows to pose and ANSWER important questions
- Coarse graining is a simple method to analyze and select data using a statistical framework
- Self-consistent databases allow to determine coupling constants
- Validate power countings in NN (Weinberg is not)
- It could be a possible way to do Nuclear Physics AND Hadronic physics



# ACKNOWLEDGEMENTS

# The Botijo Collaboration

- Quique Amaro (Granada) Rodrigo Navarro Pérez (Livermore), E.R.A. (Granada)



- I thank Eduardo Garrido, Andreas Nogga, James Vary, Pieter Maris, Ignacio Ruiz Simó, Pedro Fernández-Soler, Jacobo Ruiz de Elvira for collaborations.

# References

- [1] **Coarse graining Nuclear Interactions**  
Prog. Part. Nucl. Phys. **67** (2012) 359
- [2] **Error estimates on Nuclear Binding Energies from Nucleon-Nucleon uncertainties**  
arXiv:1202.6624 [nucl-th].
- [3] **Phenomenological High Precision Neutron-Proton Delta-Shell Potential**  
Phys.Lett. B724 (2013) 138-143.
- [4] **Nuclear Binding Energies and NN uncertainties**  
PoS QNP 2012 (2012) 145
- [5] **Effective interactions in the delta-shells potential**  
Few Body Syst. 54 (2013) 1487.
- [6] **Nucleon-Nucleon Chiral Two Pion Exchange potential vs Coarse grained interactions**  
PoS CD12 (2013) 104.
- [7] **Partial Wave Analysis of Nucleon-Nucleon Scattering below pion production**  
Phys.Rev. C88 (2013) 024002, Phys.Rev. C88 (2013) 6, 069902.
- [8] **Coarse-grained potential analysis of neutron-proton and proton-proton scattering below the pion production threshold**  
Phys.Rev. C88 (2013) 6, 064002, Phys.Rev. C91 (2015) 2, 029901.
- [9] **Coarse grained NN potential with Chiral Two Pion Exchange**  
Phys.Rev. C89 (2014) 2, 024004.
- [10] **Error Analysis of Nuclear Matrix Elements** Few Body Syst. 55 (2014) 977-981.
- [11] **Partial Wave Analysis of Chiral NN Interactions** Few Body Syst. 55 (2014) 983-987.

- [12] **Statistical error analysis for phenomenological nucleon-nucleon potentials**  
Phys.Rev. C89 (2014) 6, 064006.
- [13] **Error analysis of nuclear forces and effective interactions**J.P.G42(2015)3,034013.
- [14] **Bootstrapping the statistical uncertainties of NN scattering data** Phys.Lett. B738 (2014) 155-159.
- [15] **Triton binding energy with realistic statistical uncertainties** (with E. Garrido)  
Phys.Rev. C90 (2014) 4, 047001.
- [16] **The Low energy structure of the Nucleon-Nucleon interaction: Statistical vs Systematic Uncertainties** J. Phys. G **43**, no. 11, 114001 (2016)
- [17] **Low energy chiral two pion exchange potential with statistical uncertainties**  
Phys.Rev. C91 (2015) 5, 054002.
- [18] **Minimally nonlocal nucleon-nucleon potentials with chiral two-pion exchange including  $\Delta$  resonances** (with M. Piarulli, L. Girlanda, R. Schiavilla).  
Phys.Rev. C91 (2015) 2, 024003.
- [19] **The Falsification of Nuclear Forces** EPJ Web Conf. 113 (2016) 04021
- [20] **Statistical error propagation in ab initio no-core full configuration calculations of light nuclei** (with P. Maris, J.P. Vary) Phys.Rev. C92 (2015) no.6, 064003
- [21] **Uncertainty quantification of effective nuclear interactions**  
Int.J.Mod.Phys. E25 (2016) no.05, 1641009
- [22] **Binding in light nuclei: Statistical NN uncertainties vs Computational accuracy** (with A. Nogga) J. Phys. Conf. Ser. **742**, no. 1, 012001 (2016)
- [23] **Precise Determination of Charge Dependent Pion-Nucleon-Nucleon Coupling Constants** arXiv:1606.00592 (PRC in press)
- [24] **Three pion nucleon coupling constants**  
Mod.Phys.Lett.A, Vol.31, No.28(2016)1630027

- [25] **Self-consistent statistical error analysis of  $\pi\pi$  scattering** R. Navarro Pérez, E. Ruiz Arriola and J. Ruiz de Elvira. Phys. Rev. D **91**, 074014 (2015)
- [26] **The falsification of Chiral Nuclear Forces** EPJ Web Conf. **137**, 09006 (2017)
- [27] **Coarse grained short-range correlations** I. Ruiz Simo, R. Navarro Perez, J. E. Amaro and E. Ruiz Arriola. Phys. Rev. C **95**, no. 5, 054003 (2017)
- [28] **Low energy peripheral scaling in Nucleon-Nucleon Scattering** I. Ruiz Simo, J. E. Amaro, E. Ruiz Arriola and R. Navarro Perez. arXiv:1705.06522 [nucl-th]
- [29] **Coarse graining of NN inelastic interactions up to 3 GeV: Repulsive vs Structural core** P. Fernandez-Soler and E. Ruiz Arriola. arXiv:1705.06093 [nucl-th]
- [30] **Coarse graining  $\pi\pi$  scattering** J. Ruiz de Elvira, and E. Ruiz Arriola. (2016, unpublished)
- [31] **Statistical limits to the precision of nuclear physics ab initio calculations for triton and helium** (with A. Nogga), (2015,unpublished)