Hadrons in Born approximation

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Aim: Address striking features of hadron data within QCD:

- $q\bar{q}$ and qqq quantum numbers, even for relativistic states (π, ρ, N,…)
- Freezing of gluon degrees of freedom at low scales (hybrids, glueballs)
- OZI rule: $\phi(1020) \to K\bar{K} \gg \phi(1020) \to \pi\pi\pi$
- Quark ↔ hadron duality (DIS, e^+e^-, hh, …)

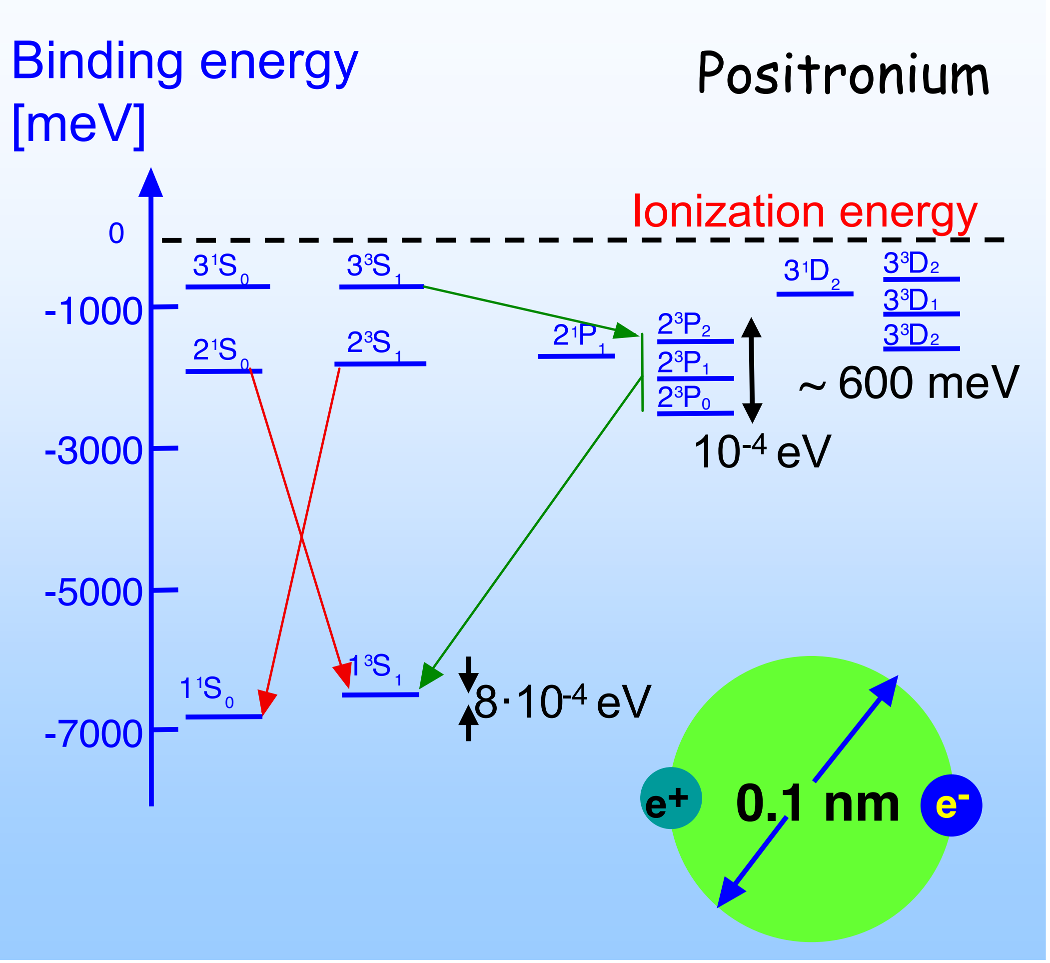
At face value: These phenomena indicate a weak coupling dynamics.

How is this consistent with relativistic binding and confinement?

How to proceed?

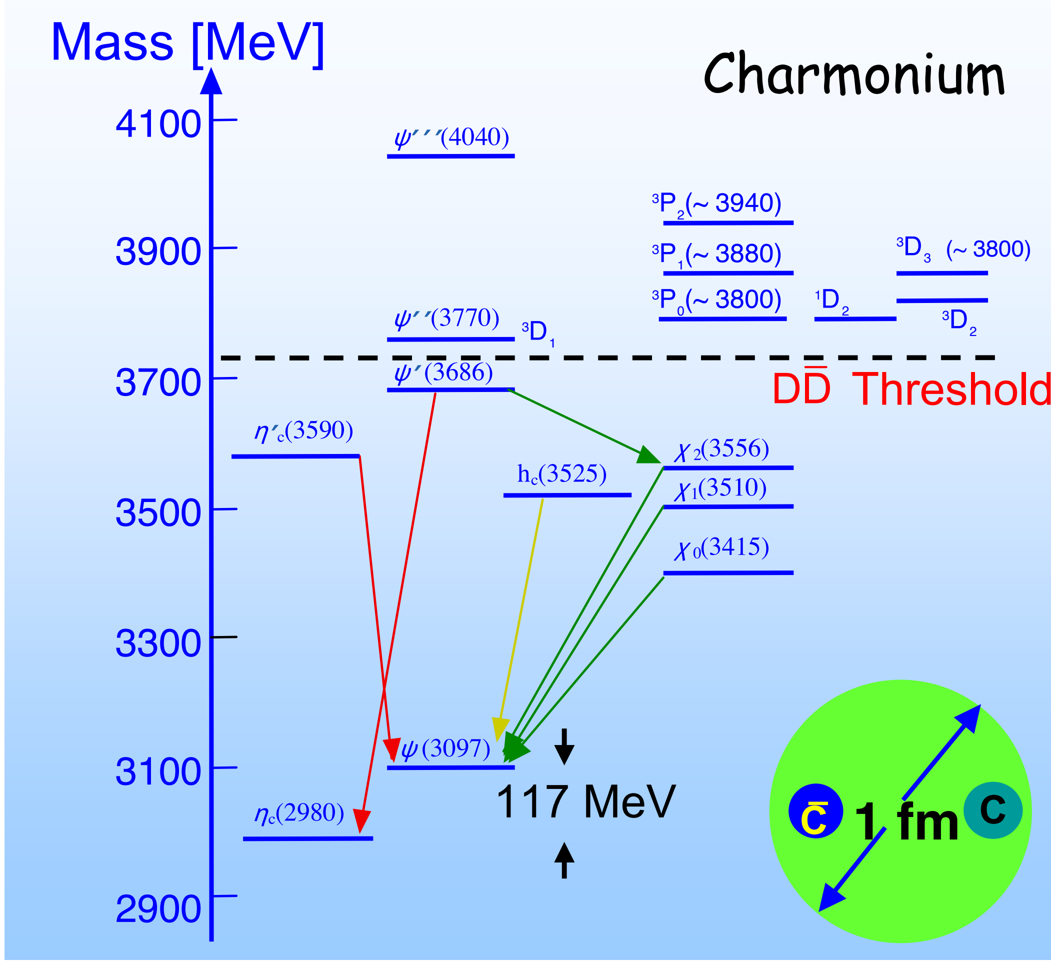
"The J/ψ is the Hydrogen atom of QCD"

QED



$$V(r) = -\frac{\alpha}{r}$$

QCD



$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

QED works for atoms

Example: Hyperfine splitting in Positronium

$$\Delta\nu_{QED} = m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right.$$

$$+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right]$$

$$\left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}$$

M. Baker et al, 1402.0876

A. Ishida et al, 1310.6923 : $\Delta\nu_{EXP} = 203.3941 \pm .003$ GHz

- Binding energy is perturbative in α and $\log(\alpha)$
- Wave function $\psi(r) \propto \exp(-m\alpha r)$ is of $\mathcal{O}(\alpha^\infty)$

How should one organize an expansion that starts with $\mathcal{O}(\alpha^\infty)$?

QM I: The Schrödinger equation

The Schrödinger equation with a classical potential is postulated:

$$\Big[-\frac{\boldsymbol{\nabla}^2}{2\mu}-\frac{\alpha}{|\boldsymbol{x}|}\Big]\varphi(\boldsymbol{x}) = E_b\,\varphi(\boldsymbol{x})$$

Classical potential: $eA^0(r) = -\dfrac{\alpha}{r}$ the obvious choice!

QFT: Adds $\mathcal{O}(\hbar^n)$ fluctuations around the classical field

$$\int [dA^\mu]\,\exp\big(iS[A^\mu]/\hbar\big)$$

Schrödinger atom is $\mathcal{O}(\hbar^0)$: Classical photon field, no loop contributions

Bound states *should* be expanded around the classical field

Perturbation theory expands around the zero field

Master formula for perturbative S-matrix

$$S_{fi} = {}_{\textbf{out}}\langle f|\left\{\mathrm{T}\exp\left[-i\int_{-\infty}^{\infty} dt\, H_I(t)\right]\right\}|i\rangle_{\textbf{in}}$$

Generates Feynman diagrams to arbitrary order for any scattering process

The free *in*- and *out*-states at $t = \pm\infty$ must overlap the physical i, f states.

Bound states have no overlap with free *in*- and *out*-states at $t = \pm\infty$

No finite order Feynman diagram for $e^+e^- \to e^+e^-$ has a positronium pole.

We need to expand around *in* and *out* states with their classical gauge field

A boundary condition
on the classical field equations may be the clue to confinement,
but cannot be imposed on free fields.

Quark loops: 10% effect in hadron spectrum

Lattice QCD:
Quenched approximation

Neglecting quark loops gives the light hadron spectrum at 10% accuracy

Light hadron spectrum in quenched approximation

	Expt.	m_K input Mass (GeV)	Deviation
K	0.4977	$\cdots$	$\cdots$
K^*	0.8961	0.858(09)	-4.2% 4.3σ
ϕ	1.0194	0.957(13)	-6.1% 4.8σ
N	0.9396	0.878(25)	-6.6% 2.5σ
Λ	1.1157	1.019(20)	-8.6% 4.7σ
Σ	1.1926	1.117(19)	-6.4% 4.1σ
Ξ	1.3149	1.201(17)	-8.7% 6.8σ
Δ	1.2320	1.257(35)	2.0% 0.7σ
Σ^*	1.3837	1.359(29)	-1.8% 0.9σ
Ξ^*	1.5318	1.459(26)	-4.7% 2.8σ
Ω	1.6725	1.561(24)	-6.7% 4.7σ

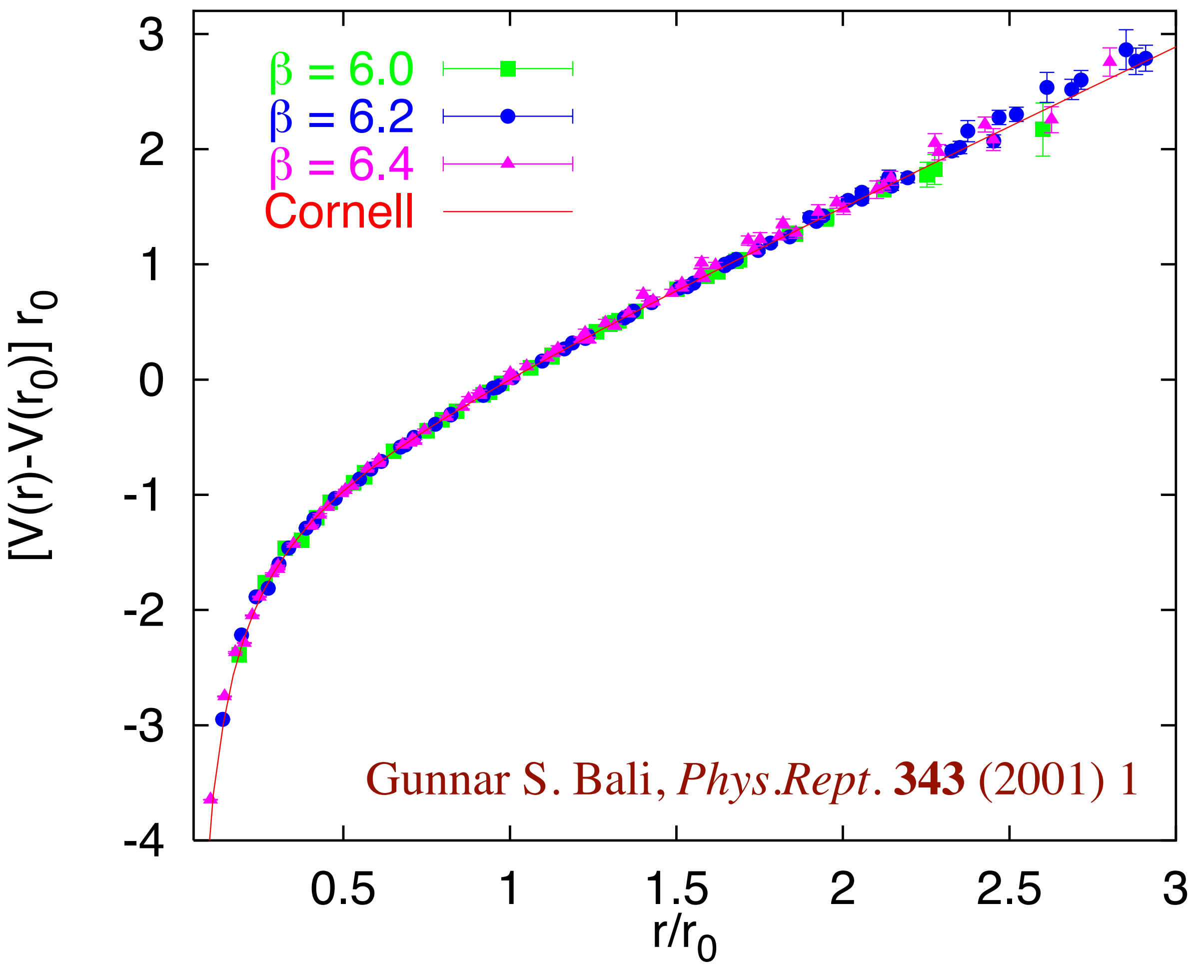
S. Aoki, et al., Phys. Rev. Lett. 84 (2000) 238

Heavy quark potential is classical

The static (heavy quark) potential of Lattice QCD agrees with the Cornell potential

Consistent with dominance of a *classical* gluon field

The quenched Wilson action SU(3) potential.



⇒ The Born approximation of QCD maintains confinement and chiral symmetry breaking.

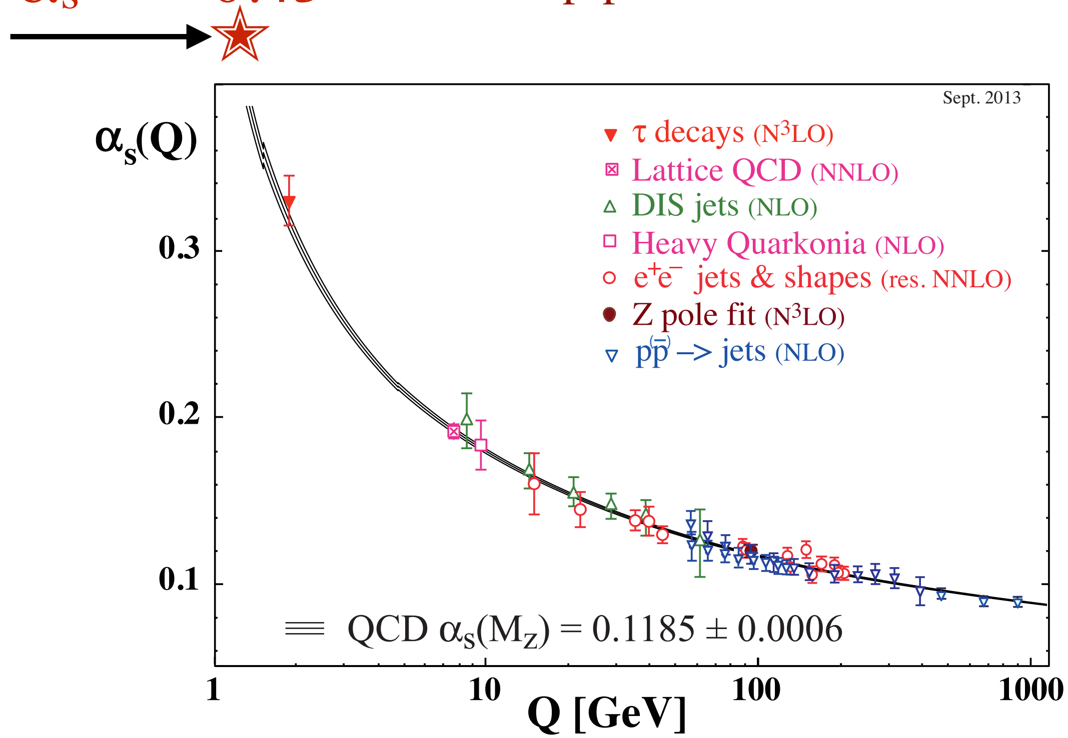
Two consequences of $\hbar \to 0$ in QCD

1. The suppression of loops,
   stops the running of α_s

   Gribov's prediction agrees with phenomenology:
   $\alpha_s(0)/\pi \approx 0.14$

$\Rightarrow$ PQCD corrections to $\mathcal{O}(\hbar^0)$ are relevant, as in QED.

$\alpha_s^{crit} \approx 0.43$ Gribov hep-ph/9902279



2. In the absence of loops, the QCD scale Λ_{QCD} cannot arise from renormalization.

$\Rightarrow$ Λ_{QCD} must arise from a boundary condition on the classical field equations.

Excluded in an expansion around free fields!

Positronium: Classical photon field

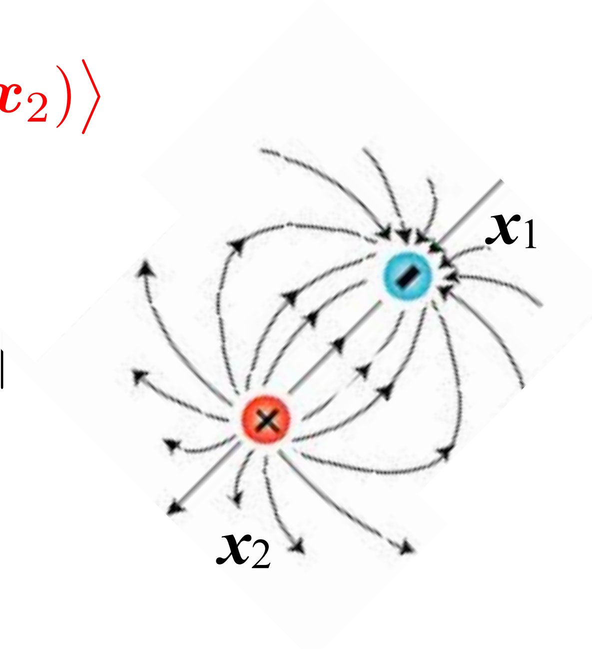
Non-relativistic dynamics: $A^j/A^0 = \mathcal{O}(\alpha)$: Transverse photons suppressed

$$\frac{\delta \mathcal{S}_{QED}}{\delta \hat{A}^0(t,\boldsymbol{x})} = 0 \qquad \Rightarrow \qquad -\boldsymbol{\nabla}^2 \hat{A}^0(t,\boldsymbol{x}) = e\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x})$$

The eigenvalue of the $\hat{A}^0$ field operator for $\left|e^-(\boldsymbol{x}_1)\, e^+(\boldsymbol{x}_2)\right\rangle$

is the classical field:

$$eA^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_2|}$$



Note: A^0 is determined instantaneously for all $\boldsymbol{x}$

It depends on $\boldsymbol{x}_1, \boldsymbol{x}_2$

$$\Longrightarrow \quad eA^0(\boldsymbol{x}_1) = -eA^0(\boldsymbol{x}_2) = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} \qquad \text{classical potential}$$

But: An external observer at $\boldsymbol{x}$ sees a dipole field

QCD Mesons

Color singlet
$q\bar{q}$ state at rest

$$|M\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t,\boldsymbol{x}_1)\, \Phi_{\alpha\beta}^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2)\, \psi_\beta^B(t,\boldsymbol{x}_2)\, |0\rangle$$

$$\Phi^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2) = \frac{1}{\sqrt{N_C}}\,\delta^{AB}\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

Each component $q^A(\boldsymbol{x}_1)\bar{q}^A(\boldsymbol{x}_2)$ has an $\mathcal{A}_a^0$ classical gluon field, which is a homogeneous solution of Gauss law:

$$\boldsymbol{\nabla}^2\mathcal{A}_a^0(t,\boldsymbol{x}) = 0$$

$$A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A) = \left[\boldsymbol{x}-\tfrac{1}{2}(\boldsymbol{x}_1+\boldsymbol{x}_2)\right]\cdot\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}\,T_a^{AA}\,6\Lambda^2$$

$$\sum_a\left[\boldsymbol{\nabla}_x A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A)\right]^2 = 12\Lambda^4 \qquad \mathcal{O}\left(\alpha_s^0\right)$$

Constant field energy density determines scale

$$\sum_A A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A) \propto \mathrm{Tr}\,T^{AA} = 0$$

External observer sees no field at any $\boldsymbol{x}$ (meson is a color singlet)

$\mathcal{A}_a^j$ is of $\mathcal{O}(g)$ Perturbative compared to $\mathcal{A}_a^0$

Meson spectra

$$\mathcal{H}_{QCD}\,|q\bar{q}\rangle = M\,|q\bar{q}\rangle$$

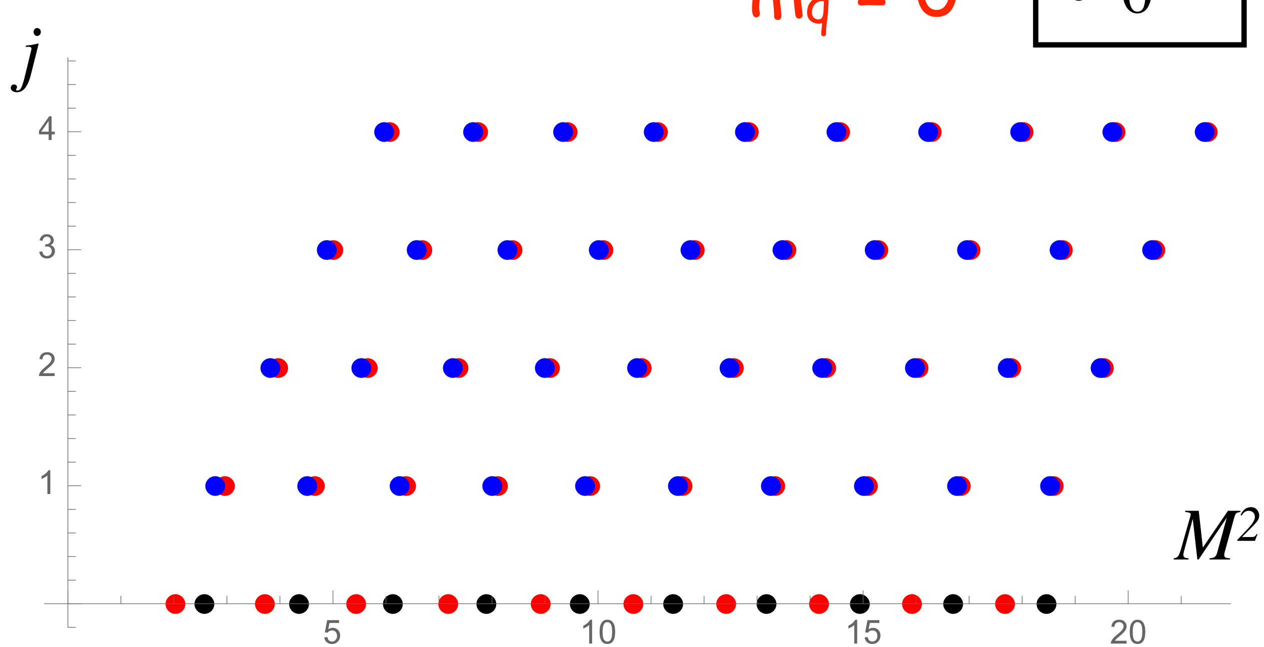
Bound state condition implies

$$i\boldsymbol{\nabla}\cdot\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\} + m\,[\gamma^0,\Phi(\boldsymbol{x})] = [M - V(\boldsymbol{x})]\,\Phi(\boldsymbol{x})$$

$$V(\boldsymbol{x}_1-\boldsymbol{x}_2) = \sum_a \tfrac{1}{2} g T_a^{AA}\left[A_a^0(\boldsymbol{x}_1) - A_a^0(\boldsymbol{x}_2)\right] = g\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

Three trajectories with different j^{PC} quantum numbers.
For $j = 0$: 0^{-+}, 0^{--} and 0^{++}

Spectrum similar to dual models



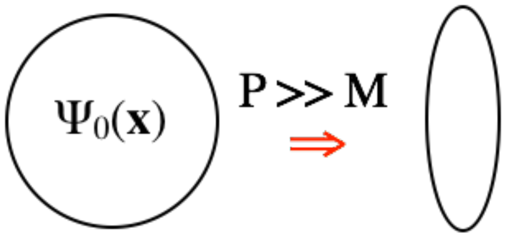
Promising prospects

The approach is guided by:

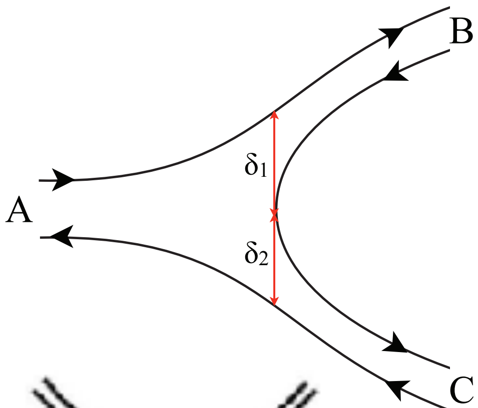
- Phenomenological observations
- QED/QCD framework: $\hbar$ expansion

Issues under study:

- Boost properties (IMF, spin)
- Phenomenology, e.g., DIS
- Chiral symmetry breaking



- String breaking (determined by $q\bar{q}$ states)
- Hadron loops, unitarity at $\hbar^0$



- Quark-hadron duality
- Hadron scattering amplitudes