Aim: Address striking features of hadron data within QCD:

• q$q\bar{$q$} and qqq quantum numbers, even for relativistic states ($\pi, q, N, \ldots$)
• Freezing of gluon degrees of freedom at low scales (hybrids, glueballs)
• OZI rule: $\phi(1020) \rightarrow K\bar{K} \gg \phi(1020) \rightarrow \pi \pi \pi$
• Quark ↔ hadron duality (DIS, $e^+e^-, hh, \ldots$)

At face value: These phenomena indicate a weak coupling dynamics.

How is this consistent with relativistic binding and confinement?

How to proceed?
“The J/ψ is the Hydrogen atom of QCD”

**QED**

\[ V(r) = -\frac{\alpha}{r} \]

**QCD**

\[ V(r) = c r - \frac{4}{3} \frac{\alpha_s}{r} \]

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**Positronium**

- 1S0
- 2S1
- 3S1
- 3P1
- 2P0
- 2P1
- 2P2

**Ionization energy**

\[ \sim 600 \text{ meV} \]

\[ 8 \cdot 10^{-4} \text{ eV} \]

**Binding energy [meV]**

- 0
- 1000
- 3000
- 5000
- 7000

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**Charmonium**

- \( \psi'(4040) \)
- \( \psi(3770) \)
- \( \psi(3686) \)
- \( \eta_c(3590) \)
- \( \eta_c(2980) \)
- \( \eta_c(2980) \)
- \( \eta_c(2980) \)
- \( \psi(3097) \)
- \( \chi_c(3510) \)
- \( \chi_c(3556) \)
- \( \chi_c(3515) \)

**DD Threshold**

- 1 fm

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Paul Hoyer Hadron 2017
Example: Hyperfine splitting in Positronium

\[
\Delta \nu_{QED} = m_e \alpha^4 \left\{ \frac{7}{12} - \frac{\alpha}{\pi} \left( \frac{8}{9} + \frac{\ln 2}{2} \right) \right. \\
+ \frac{\alpha^2}{\pi^2} \left[ - \frac{5}{24} \pi^2 \ln \alpha + \frac{1367}{648} - \frac{5197}{3456} \pi^2 + \left( \frac{221}{144} \pi^2 + \frac{1}{2} \right) \ln 2 - \frac{53}{32} \zeta(3) \right] \right. \\
- \frac{7\alpha^3}{8\pi} \ln^2 \alpha + \frac{\alpha^3}{\pi} \ln \alpha \left( \frac{17}{3} \ln 2 - \frac{217}{90} \right) + O(\alpha^3) \right\} = 203.39169(41) \text{ GHz}
\]

M. Baker et al, 1402.0876

A. Ishida et al, 1310.6923 : \( \Delta \nu_{\text{Exp}} = 203.3941 \pm 0.003 \text{ GHz} \)

- **Binding energy** is perturbative in \( \alpha \) and \( \log(\alpha) \)
- **Wave function** \( \psi(r) \propto \exp(-m \alpha r) \) is of \( O(\alpha^\infty) \)

How should one organize an expansion that starts with \( O(\alpha^\infty) \) ?
The Schrödinger equation with a classical potential is postulated:

\[
\left[ -\frac{\nabla^2}{2\mu} - \frac{\alpha}{|\mathbf{x}|} \right] \varphi(\mathbf{x}) = E_b \varphi(\mathbf{x})
\]

**Classical potential:** \( eA^0(r) = -\frac{\alpha}{r} \) the obvious choice!

**QFT:** Adds \( \mathcal{O}(\hbar^n) \) fluctuations around the classical field

\[
\int [dA^\mu] \exp \left( iS[A^\mu]/\hbar \right)
\]

Schrödinger atom is \( \mathcal{O}(\hbar^0) \) : Classical photon field, no loop contributions

Bound states *should* be expanded around the classical field

Perturbation theory expands around the zero field
Master formula for perturbative S-matrix

\[ S_{fi} = \text{out}\left< f\right| \left\{ T\exp \left[ -i \int_{-\infty}^{\infty} dt \, H_I(t) \right] \right\} \left< i\right| \text{in} \]

Generates Feynman diagrams to arbitrary order for any scattering process.

The free \( \text{in} \)- and \( \text{out} \)-states at \( t = \pm \infty \) must overlap the physical \( i, f \) states.

Bound states have no overlap with free \( \text{in} \)- and \( \text{out} \)-states at \( t = \pm \infty \)

No finite order Feynman diagram for \( e^+e^- \rightarrow e^+e^- \) has a positronium pole.

We need to expand around \( \text{in} \) and \( \text{out} \) states with their classical gauge field.

A boundary condition

on the classical field equations may be the clue to confinement,
but cannot be imposed on free fields.
Quark loops: 10% effect in hadron spectrum

Light hadron spectrum in quenched approximation

<table>
<thead>
<tr>
<th>mass</th>
<th>input</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>0.4977</td>
<td>( m_K )</td>
</tr>
<tr>
<td>( K^* )</td>
<td>0.8961</td>
<td>0.858(09)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>1.0194</td>
<td>0.957(13)</td>
</tr>
<tr>
<td>( N )</td>
<td>0.9396</td>
<td>0.878(25)</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>1.1157</td>
<td>1.019(20)</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>1.1926</td>
<td>1.117(19)</td>
</tr>
<tr>
<td>( \Xi )</td>
<td>1.3149</td>
<td>1.201(17)</td>
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<tr>
<td>( \Delta )</td>
<td>1.2320</td>
<td>1.257(35)</td>
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<tr>
<td>( \Sigma^* )</td>
<td>1.3837</td>
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</tr>
<tr>
<td>( \Xi^* )</td>
<td>1.5318</td>
<td>1.459(26)</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>1.6725</td>
<td>1.561(24)</td>
</tr>
</tbody>
</table>

Lattice QCD: Quenched approximation

Neglecting quark loops gives the light hadron spectrum at 10% accuracy
Heavy quark potential is classical

The static (heavy quark) potential of Lattice QCD agrees with the Cornell potential

Consistent with dominance of a \textit{classical} gluon field

\[\Rightarrow\]

The Born approximation of QCD maintains confinement and chiral symmetry breaking.

Two consequences of $\hbar \to 0$ in QCD

1. The suppression of loops, stops the running of $\alpha_s$

Gribov's prediction agrees with phenomenology:

$$\alpha_s(0)/\pi \approx 0.14$$

$\Rightarrow$ PQCD corrections to $\mathcal{O}(\hbar^0)$ are relevant, as in QED.

2. In the absence of loops, the QCD scale $\Lambda_{QCD}$ cannot arise from renormalization.

$\Rightarrow$ $\Lambda_{QCD}$ must arise from a boundary condition on the classical field equations.

Excluded in an expansion around free fields!
Positronium: Classical photon field

Non-relativistic dynamics: \( A^0 / A^0 = \mathcal{O}(\alpha) \): Transverse photons suppressed

\[
\frac{\delta S_{QED}}{\delta \hat{A}^0(t, \mathbf{x})} = 0 \quad \Rightarrow \quad -\nabla^2 \hat{A}^0(t, \mathbf{x}) = e\psi^\dagger(t, \mathbf{x})\psi(t, \mathbf{x})
\]

The eigenvalue of the \( \hat{A}^0 \) field operator for \( |e^-(\mathbf{x}_1) e^+(\mathbf{x}_2)\rangle \)

is the classical field:

\[
eA^0(\mathbf{x}; \mathbf{x}_1, \mathbf{x}_2) = \frac{\alpha}{|\mathbf{x} - \mathbf{x}_1|} - \frac{\alpha}{|\mathbf{x} - \mathbf{x}_2|}
\]

Note: \( A^0 \) is determined instantaneously for all \( \mathbf{x} \)

It depends on \( \mathbf{x}_1, \mathbf{x}_2 \)

\[
\Rightarrow \quad eA^0(\mathbf{x}_1) = -eA^0(\mathbf{x}_2) = -\frac{\alpha}{|\mathbf{x}_1 - \mathbf{x}_2|}
\]

classical potential

But: An external observer at \( \mathbf{x} \) sees a dipole field
QCD Mesons

Color singlet

q\bar{q} state at rest

Each component q^A(x_1)\bar{q}^A(x_2) has an \( A^0_a \) classical gluon field, which is a homogeneous solution of Gauss law:

\[
A^0_a(x; x_1, x_2, A) = \left[ x - \frac{1}{2} (x_1 + x_2) \right] \cdot \frac{x_1 - x_2}{|x_1 - x_2|} T^{AA}_a 6\Lambda^2
\]

\[
\sum_a \left[ \nabla_x A^0_a(x; x_1, x_2, A) \right]^2 = 12\Lambda^4 \quad \mathcal{O}(\alpha_s^0)
\]

Constant field energy density determines scale

\[
\sum_A A^0_a(x; x_1, x_2, A) \propto \text{Tr} T^{AA} = 0
\]

External observer sees no field at any \( x \) (meson is a color singlet)

\( A^0_a \) is of \( \mathcal{O}(g) \) Perturbative compared to \( A^0_a \)
Meson spectra

\[ \mathcal{H}_{QCD} |q\bar{q}\rangle = M |q\bar{q}\rangle \]

Bound state condition implies

\[ i \nabla \cdot \{ \gamma^0 \gamma, \Phi(x) \} + m \left[ \gamma^0, \Phi(x) \right] = \left[ M - V(x) \right] \Phi(x) \]

\[ V(x_1 - x_2) = \sum_a \frac{1}{2} g T_{AA}^a [A_a^0(x_1) - A_a^0(x_2)] = g \Lambda^2 |x_1 - x_2| \]

Three trajectories with different \( j^{PC} \) quantum numbers.

For \( j = 0 \): \( 0^{--}, 0^{--} \) and \( 0^{++} \)

\[ m_q = 0 \]

Spectrum similar to dual models
Promising prospects

The approach is guided by:

- Phenomenological observations
- QED/QCD framework: $\hbar$ expansion

Issues under study:

- Boost properties (IMF, spin)
- Phenomenology, e.g., DIS
- Chiral symmetry breaking

- String breaking (determined by $q\bar{q}$ states)
- Hadron loops, unitarity at $\hbar^0$

- Quark-hadron duality
- Hadron scattering amplitudes