BChPT x 1/Nc: Masses and Currents

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OUTLINE

- Why we need to combine ChPT and 1/Nc
- Baryons, spin-flavor symmetry and combined BChPT x 1/Nc
- Masses, sigma terms
- Vector charges in SU(3)
- Axial couplings in SU(3)
- Summary

Why we need to combine ChPT and 1/Nc

QCD expansion parameters: $m_q \quad (q = u, d, s); \qquad 1/N_c$

m_q and low energy/momenta $\rightarrow$ ChPT

$1/N_c \rightarrow$ N_c scalings of hadron masses and couplings

$1/N_c$ expansion
Pheno: OZI; VMD
LQCD @ varying N_c: string tension ; F_π; baryon masses

Need for combining ChPT and $1/N_c$ expansion

[Herrera-Siklody, Latorre, Pascual & Taron; Kaiser & Leutwyler]

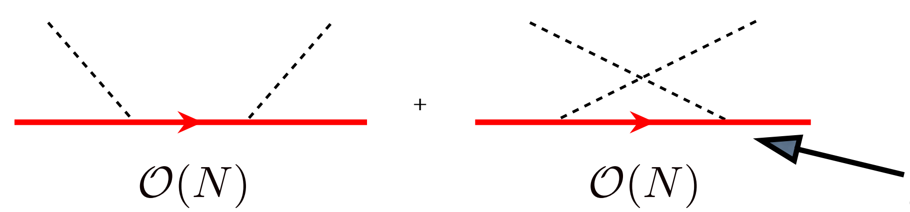
Effective theories need to agree with
chiral dynamics and $1/N_c$ power counting

Baryons

$$M_B = \mathcal{O}(N_c); \qquad g_{\pi B} = \mathcal{O}(\sqrt{N_c})$$

GBs couple strongly to baryons at large N_c
consistency of BChPT and $1/N_c$ expansion is crucial

Spin-flavor dynamical symmetry



$$g_A \frac{N_c}{F_\pi} \partial_i \pi^a X^{ia} = \mathcal{O}(\sqrt{N_c})$$

$$= i \frac{k^i k^j}{k^0} \frac{g_A^2 N^2}{F_\pi^2} [X^{ia}, X^{jb}] \quad \text{should be } \mathcal{O}(N^0)$$

$[X^{ia}, X^{jb}] = \mathcal{O}(1/N)$ key requirement at large Nc

$\{T^a, S^i, X^{ia}\}$ generate contracted $SU(2N_f)$ dynamical symmetry

classify baryons in multiplets of $SU(2N_f)$ with generators $\{T^a, S^i, G^{ia}\}$

$$G^{ia} = N_c X^{ia}$$

Large Nc baryons and chiral symmetry

$1/N_c$ $\times$ heavy baryon expansion is a natural combination [Jenkins]

LO chiral Lagrangian

$$\mathcal{L}_{\mathbf{B}}^{(1)} = \mathbf{B}^\dagger \left(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{HF}}{N_c} \hat{\vec{S}}^2 - \frac{c_1}{2\Lambda} \hat{\chi}_+ \right) \mathbf{B} \qquad g_A = \frac{5}{6} \mathring{g}_A$$

$\mathbf{B}$ is the baryon spin-flavor multiplet field

$N_f = 3$ states in $SU(2) \times SU(3)$: $[S, R] = [S, (2S, \frac{1}{2}(N - 2S))]$

LO all GB-baryon couplings given in terms of g_A

from Δ width: $g_A^{\Delta N} = 1.235 \pm 0.011$ vs $g_A^{NN} = 1.267 \pm 0.004$

Small scales: $p,\ M_{GB},\ m_\Delta - m_N = \mathcal{O}(1/N_c)$

Chiral and $1/N_c$ expansions do not commute!:
need to link power countings

ξ or small scale expansion:
$\mathcal{O}(p) = \mathcal{O}(1/N_c) = \mathcal{O}(\xi)$

NLO Lagrangians

$$\mathcal{L}_{\mathbf{B}}^{(2)} = \mathbf{B}^\dagger \Big(\big(\tfrac{z_1}{N_c} + \tfrac{z_2}{N_c}\hat{S}^2 + \tfrac{z_3}{\Lambda^2}\, N_c\, \chi_+^0\big)\, i\tilde{D}_0$$
$$+\big(-\frac{1}{2N_c m_0} + \tfrac{w_1}{\Lambda}\big)\vec{D}^2 + \big(\frac{1}{2N_c m_0} - \tfrac{w_2}{\Lambda}\big)\tilde{D}^2 + \tfrac{c_2}{\Lambda}\,\chi_+^0$$
$$+\tfrac{C_1^A}{N_c} u^{ia} S^i T^a + \tfrac{C_2^A}{N_c}\epsilon^{ijk} u^{ia}\{S^j, G^{ka}\}$$
$$+\kappa_0\, \epsilon^{ijk} F^0_{+ij} S^k + \kappa_1\, \epsilon^{ijk} F^a_{+ij} G^{ka} + \rho_0 F^0_{-0i} S^i + \rho_1 F^a_{-0i} G^{ia}$$
$$+\tfrac{\tau_1}{N_c} u_0^a G^{ia} D_i + \tfrac{\tau_2}{N_c^2} u_0^a S^i T^a D_i + \tfrac{\tau_3}{N_c}\nabla_i u_0^a S^i T^a + \tau_4 \nabla_i u_0^a G^{ia} + \cdots \Big)\mathbf{B}$$

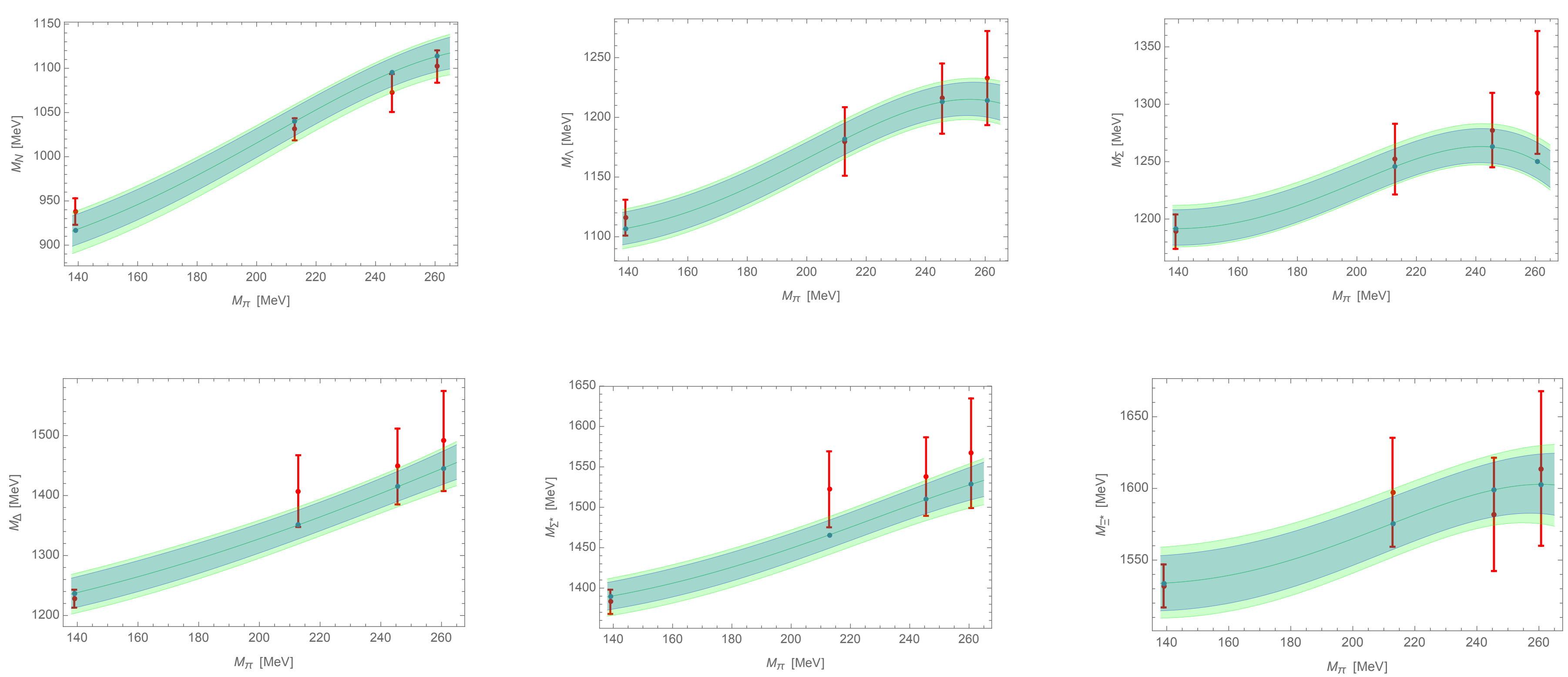
$$\mathcal{L}_{\mathbf{B}}^{(3)} = \mathbf{B}^\dagger \Big(\tfrac{z_4}{\Lambda^2}\, \tilde{\chi}_+\, i\tilde{D}_0 + \tfrac{z_5}{\Lambda^2}\, [i\tilde{D}_0,\, \tilde{\chi}_+] + \tfrac{c_3}{N_c\,\Lambda^3}\, \hat{\chi}_+^2$$
$$+\tfrac{h_1}{N_c^3}\hat{S}^4 + \tfrac{h_2}{N_c^2\Lambda}\hat{\chi}_+\hat{S}^2 + \tfrac{h_3}{N_c\Lambda}\chi_+^0\hat{S}^2 + \tfrac{h_4}{N_c\,\Lambda}\, \chi_+^a\{S^i, G^{ia}\}$$
$$+\tfrac{C_3^A}{N_c^2} u^{ia}\{\hat{S}^2, G^{ia}\} + \tfrac{C_4^A}{N_c^2} u^{ia} S^i S^j G^{ja}$$
$$+\tfrac{D_1^A}{\Lambda^2}\chi_+^0 u^{ia} G^{ia} + \tfrac{D_2^A}{\Lambda^2}\chi_+^a u^{ia} S^i + \tfrac{D_3^A(d)}{\Lambda^2} d^{abc}\chi_+^a u^{ib} G^{ic} + \tfrac{D_3^A(f)}{\Lambda^2} f^{abc}\chi_+^a u^{ib} G^{ic}$$
$$+g_E\, [D_i, F_{+i0}] + \alpha_1 \tfrac{i}{N_c}\epsilon^{ijk} F^a_{+0i} G^{ia} D_k + \beta_1 \tfrac{i}{N_c} F^a_{-ij} G^{ia} D_j + \cdots \Big)\mathbf{B}$$

Baryon Masses in SU(3) to one loop

WF renormalization factor is $\mathcal{O}(N_c)$!
plays key role in N_c power counting consistency in loops

mass corrections are $\mathcal{O}(N_c)$ (terms proportional to M_{GB}^3)

M_π dependency from LQCD ($M_K \sim 500$ MeV):
poor convergence above $M_\pi \sim 250$ MeV



[Alexandrou et al (2014), ETMC LQCD Coll.]

Mass relations

GMO

$$\Delta_{GMO} = \text{ Th: } 44 \pm 5 \text{ MeV vs Exp: } 30 \pm 10 \text{ MeV}$$

$$\Delta_{GMO} = -\left(\frac{\mathring{g}_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3}(M_K^3 - \frac{1}{4}M_\pi^3 - \frac{2}{\sqrt{3}}(M_K^2 - \frac{1}{4}M_\pi^2)^{\frac{3}{2}})\right.$$
$$\left. + \frac{2C_{HF}}{N_c}\left(- M_K^2 \log M_K^2 + \frac{1}{4}M_\pi^2 \log M_\pi^2 + (M_K^2 - \frac{1}{4}M_\pi^2)\log(\frac{4}{3}M_K^2 - \frac{1}{3}M_\pi^2)\right)\right) + \mathcal{O}(1/N_c^3)$$
$$= 37 \text{ MeV} + \mathcal{O}(1/N_c^3)$$

in large N_c, Δ_{GMO} is $\mathcal{O}(1/N_c)$

ES

$$\Delta_{ES} = m_{\Xi^*} - 2m_{\Sigma^*} + m_\Delta = \text{Th: } -6.5\text{MeV vs Exp: } -4 \pm 7\text{MeV}$$
$$= \mathcal{O}(1/N_c)$$

GR

$$\Delta_{GR} = m_{\Xi^*} - m_{\Sigma^*} - (m_\Xi - m_\Sigma) = 0, \quad \text{Exp: } 21 \pm 7 \text{ MeV},$$

$$\Delta_{GR} = \frac{h_2}{\Lambda}\frac{12}{N_c}(M_K^2 - M_\pi^2) + \underbrace{\text{O}(1/N_c) \text{ UV finite no-analytic terms}}_{\sim 68 \text{ MeV}}$$

πN σ-term

[J.M.Alarcon, I.Fernando & JLG]

$$\hat{\sigma} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \qquad \sigma_{\pi N} = \hat{\sigma} + \frac{2\hat{m}}{m_s}\sigma_s.$$

$$\hat{\sigma} = \underbrace{\frac{\hat{m}}{m_s - \hat{m}}\left(\frac{N_c+3}{6}m_\Xi + \frac{2N_c-3}{3}m_\Sigma - \frac{5N_c-3}{6}m_N\right)}_{@N_c=3:\sim 23\mathrm{MeV}} \leftarrow \mathcal{O}(N_c)$$

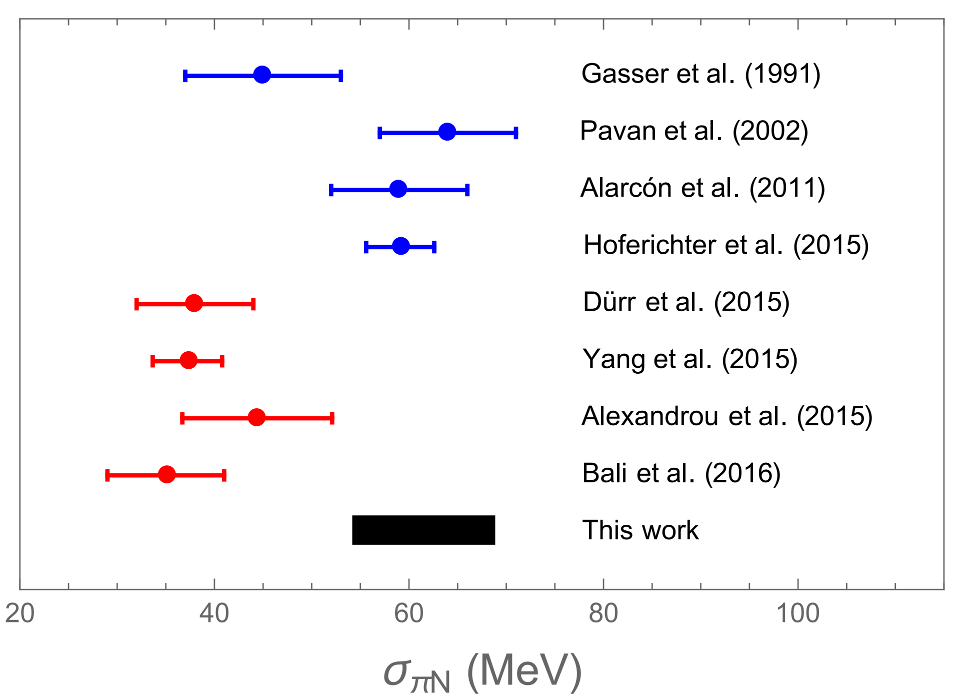
+UV finite non-analytic correction $\leftarrow \mathcal{O}(N_c)$

$$= 2.3\times 10^5\,\mathrm{MeV}^3 \times \frac{g_A^2}{F_\pi^2} \sim 30 - 50\mathrm{MeV}$$

40 % from 8 in loop and 60 % from 10

Chief uncertainty: values to be used for g_A, F_π in loop

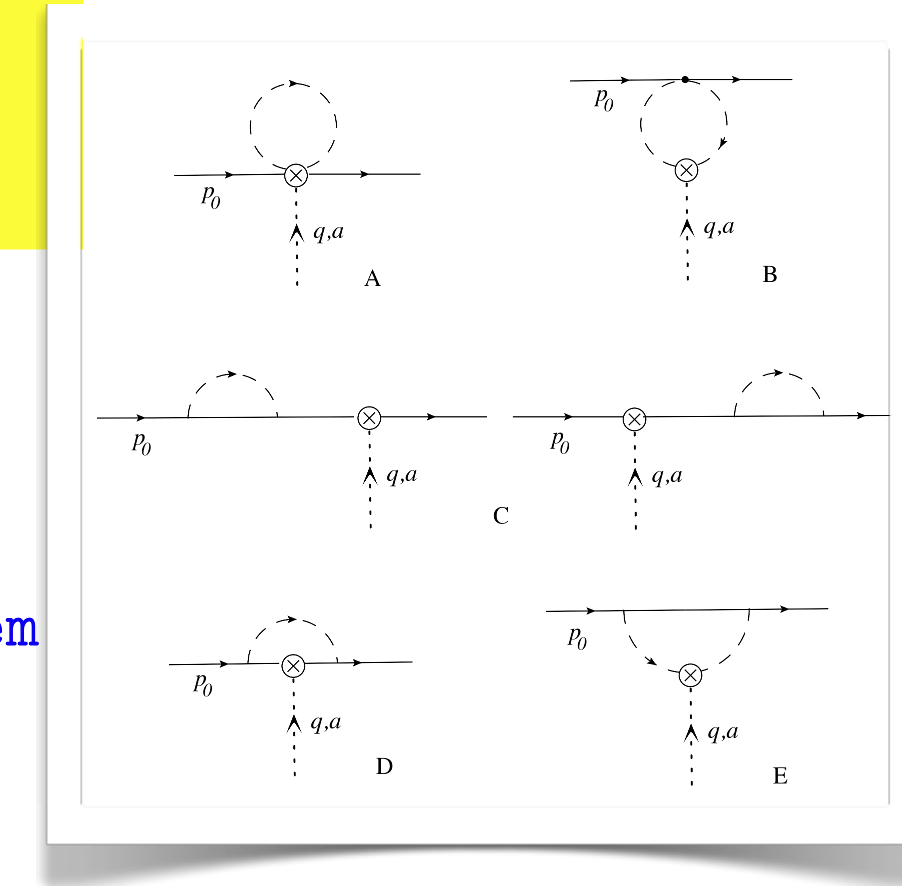
predicts $\sigma_{\pi N} > 50$ MeV



PRELIMINARY

Vector currents

[R.Flores-Mendieta & JLG; I.P.Fernando & JLG]



SU(3) breaking corrections to the vector currents:
$\mathcal{O}(\xi^2)$ corrections satisfying Ademollo-Gatto theorem
only non-analytic calculable corrections to AGTh

Vector charges

Charge	$\frac{f_1}{f_1^{SU(3)}}$	$\frac{f_1}{f_1^{SU(3)}} - 1$			
		[Flores-Mendieta & JLG:2014]	[Villadoro:2006]	[Lacour et al:2007]	[Geng et al:2009]
		HBChPT×$1/N_c$	HBChPT with 8 and 10	HBChPT only 8	RBChPT with 8 and 10
Λp	0.952	−0.048	−0.080	−0.097	−0.031
$\Sigma^- n$	0.966	−0.034	−0.024	0.008	−0.022
$\Xi^- \Lambda$	0.953	−0.047	−0.063	−0.063	−0.029
$\Xi^- \Sigma^0$	0.962	−0.038	−0.076	−0.094	−0.030

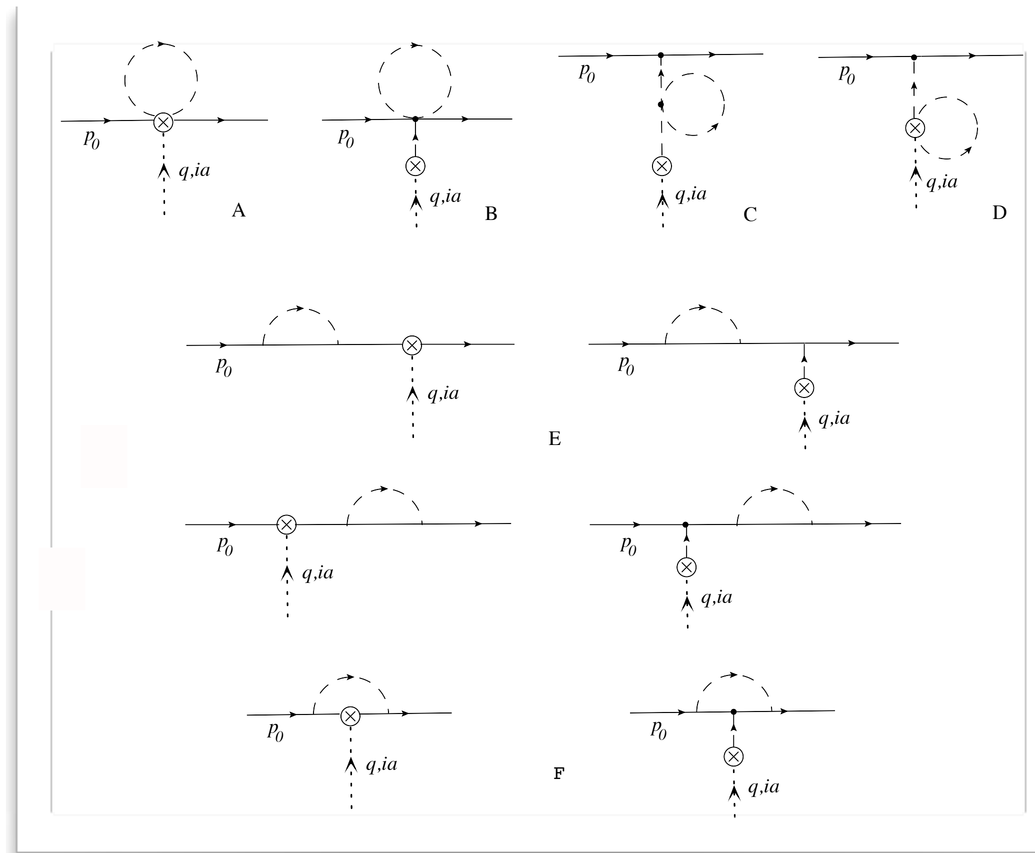
LQCD

$$f_1^{\Sigma\to N}(0) = -0.9662(43), \quad f_1^{\Xi\to\Sigma}(0) = +0.9742(28)$$

[S. Sasaki, (2017)]

Axial-vector currents

[Flores-Mendieta, Hernandez & Hofmann; Fernando & JLG] [SU(2): A. Calle-Cordon & JLG]



Definition of axial couplings

$$\langle B' \mid A^{ia} \mid B \rangle = \frac{6}{5} g_A^{aBB'} \langle B' \mid G^{ia} \mid B \rangle$$

countings of corrections to g_A's:

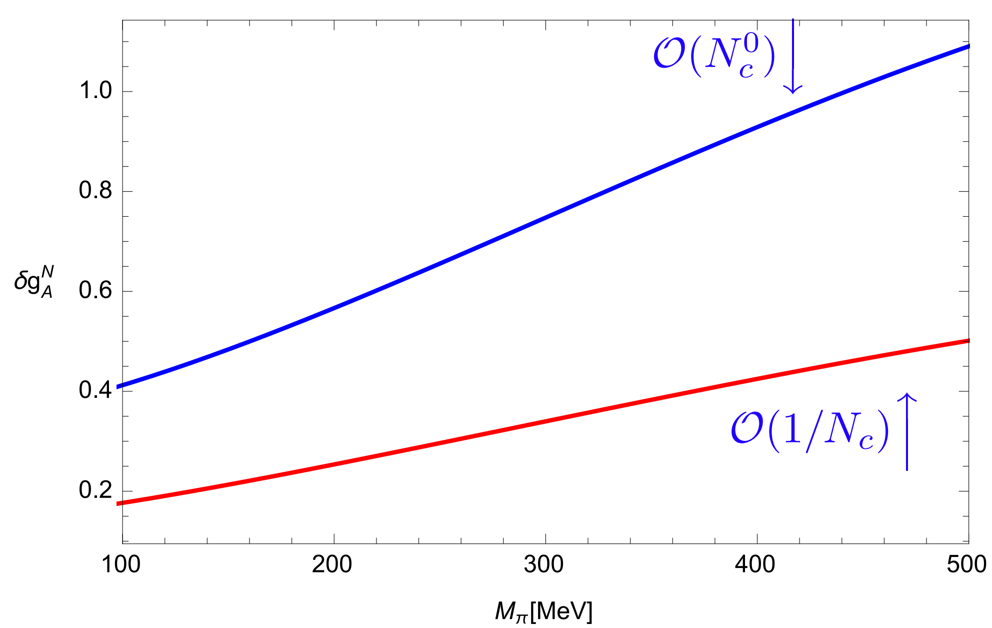
Diagram A: $\mathcal{O}(p^2/N_c)$

Diagrams E+F: $\mathcal{O}(1/N_c)$

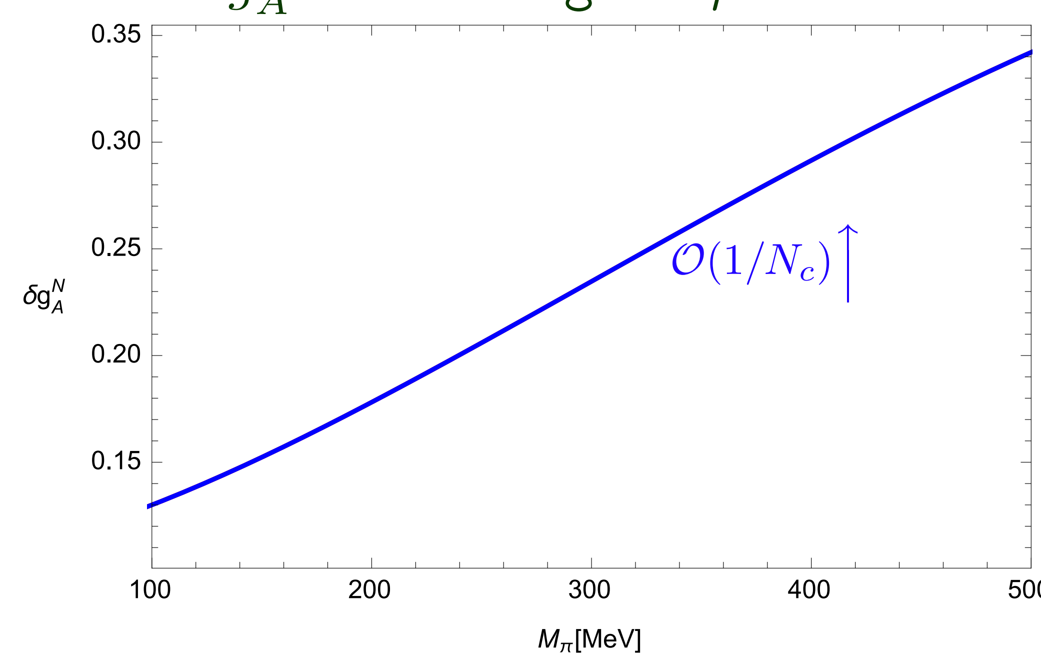
δg_A^N from Diags E+F: $\mu = 770\text{MeV}$

8 and Δ in loop

only **8** in loop



δg_A^N from Diag A: $\mu = 770\text{MeV}$



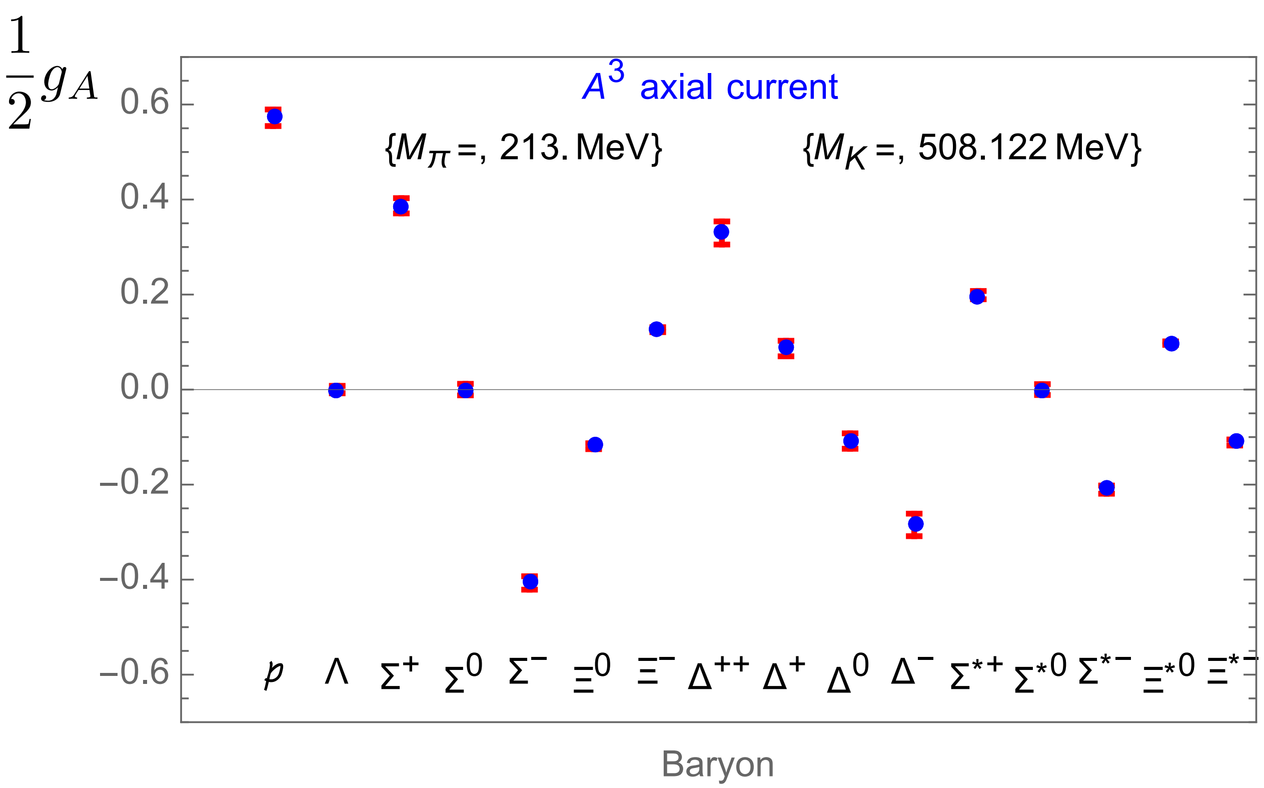
LQCD gA's

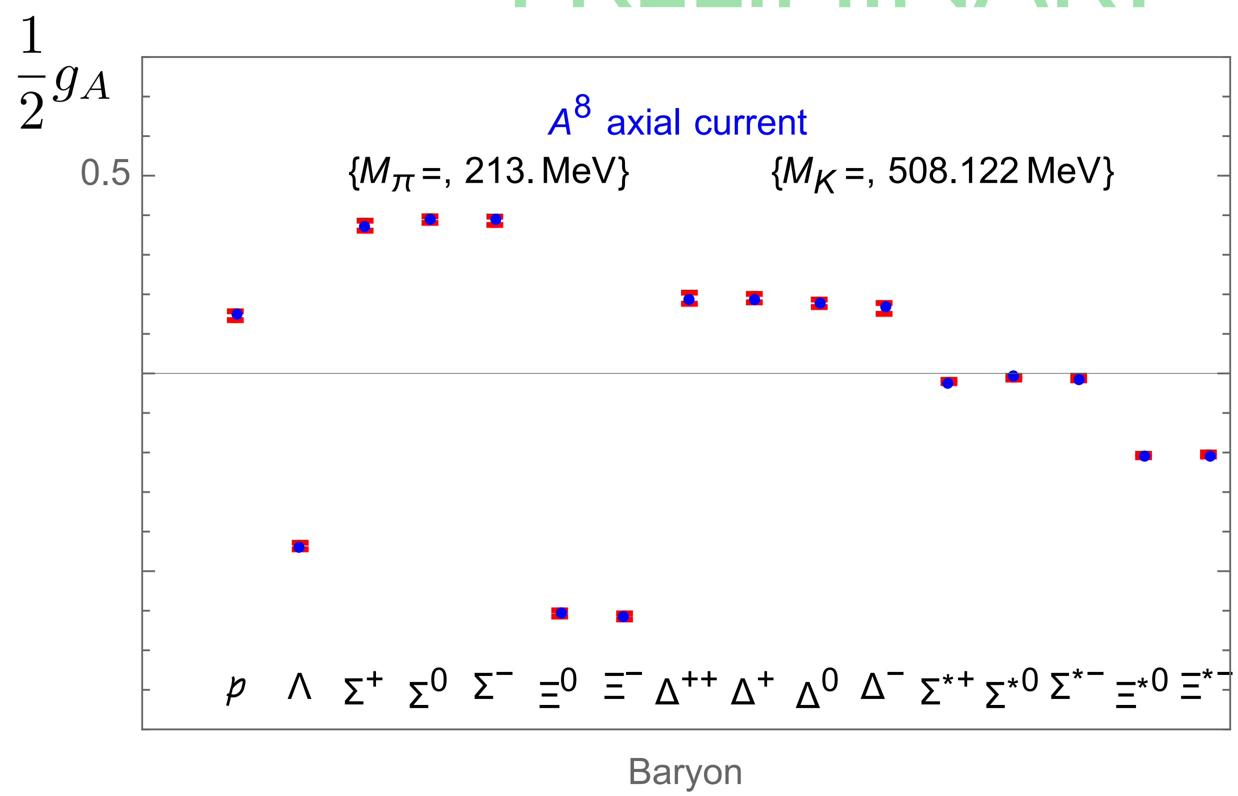
Key observed feature:@ fixed M_K, g_A's have little dependence on M_π

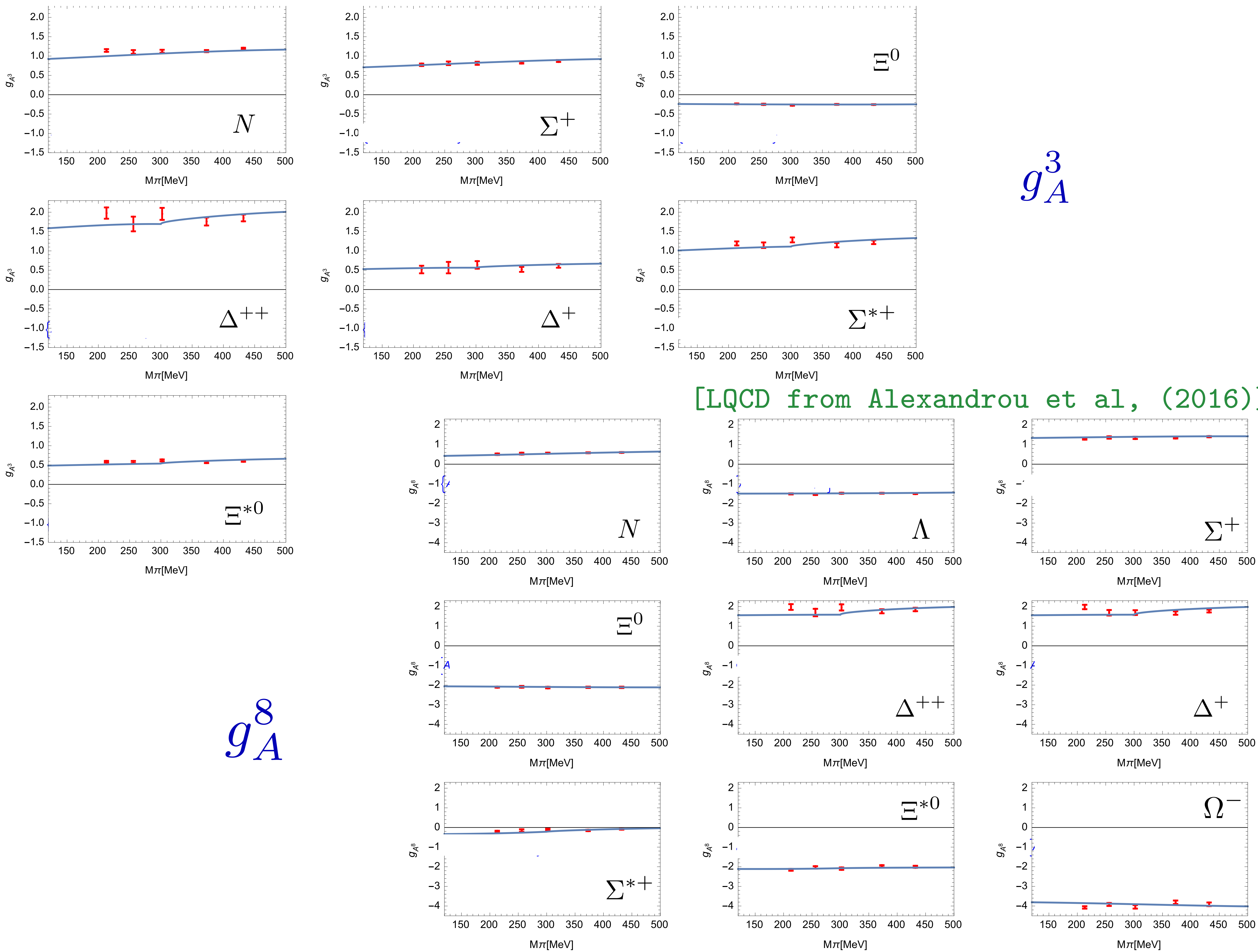
SU(3) calculation by Cyprus Group [Alexandrou et al, (2016)]
g_A^{3BB} and g_A^{8BB}

Fit	$\chi^2_{\rm dof}$	$\mathring{g}_A$	$\delta\mathring{g}_A$	C_1^A	C_2^A	C_3^A	C_4^A	D_1^A	D_2^A	D_3^A	D_4^A
LO	4	1.35	-	-	-	-	-	-	-	-	-
NLO Tree	0.6	1.31	-	-0.18	-	-	-	0.088	0.018	0.041	-
NLO Full 1	3.6	1.35	-.36	-2.7	-	-	6	-0.98	-0.08	-0.13	-
NLO Full 2	1.1	0.94	0	-1.03	-	-	2.1	-0.25	-0.02	-0.05	-

PRELIMINARY







g_A^3
N
Σ^+
Ξ^0
Δ^{++}
Δ^+
Σ^{*+}
Ξ^{*0}
Mπ[MeV]
[LQCD from Alexandrou et al, (2016)]
g_A^8
N
Λ
Σ^+
Ξ^0
Δ^{++}
Δ^+
Σ^{*+}
Ξ^{*0}
Ω^-

Summary and comments

- Consistency of BChPT with 1/Nc expansion improves convergence, especially important in SU(3) BChPT

- Axial couplings are a good testing ground thanks to inputs from LQCD

- Important predictions: calculable corrections to mass relations and to $\hat{\sigma}$ calculable corrections to SU(3) vector charges

- Significant correction to $g_A{}'s$ from LO to NNLO: -30% : need to be understood

- 1/Nc requirements impact broadly on BChPT